Coursework 1 for MATH0070

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May 21, 2001

Aim of the coursework

Produce approximations to $a = 17^{1/m}$ for $m = 2, 3, \ldots$ using partial quotients and the continued fractions method.

General Description

My coursework consists of two procedures written in Maple. They are

- `cw1`: This is the main procedure which should be called in order to get the results that the coursework requires.
- `quot`: This is a procedure which computes partial quotient of an input number, and returns the best approximation to that number using continued fractions.

Detailed Description of `cw1`

The main procedure `cw1` must be called with 2 passing parameters `start_m` and `end_m`. These parameters specify the limits for `m` (ie, `start_m` $\leq$ `m` $\leq$ `end_m`) in the equation $a = 17^{1/m}$. The procedure acts as follows

- Sets up a matrix which will be the return value (**line 13**). The choice of a matrix was purely due to the fact that is it more readable than a table or a list.
- For every `row` of the matrix, the following happens:

- Calculate $a = 17^{1/m}$ for the current value of m. Put the result in column 1 (**line 21**).
- Convert the above value to float, and puts it in column 2 (**line 22**).
- Finds an approximation to the above float value, by calling procedure `quot`, and by specifying a limit of 51 partial quotients. The returned fraction is stored in column 3 (**line 23**).
- Calculates the difference between the values of column 2 and column 3 (**line 24**).
- Increment `m`, and repeat the whole process for the next `row`.

The procedure returns the whole matrix as a result (**line 28**).

Detailed Description of `quot`

Procedure `quot` accepts two parameters, `x` and `n`. Its purpose is to calculate the first `n` partial quotients for `x`, keeping track of the largest quotient. Then, it calculates an approximation p/q for `x`, according to the largest quotient.

After setting up the initial values (**lines 10-15**), the procedure enters the main loop (**line 18**). This loop goes on until either `n` partial quotients have been computed, or a quotient with 0 error margin was computed.

The partial quotients are computed using the formula $\mathtt{q_i} = floor(1/\mathtt{e_{i-1}})$ (**line 20**), and $\mathtt{e_i} = 1/(\mathtt{e_{i-1}}) - \mathtt{q_i}$ (**line 21**). The loop compares each new $\mathtt{q_i}$ to the current maximum, `maxv` (**line 22**), and if $\mathtt{q_i} > \mathtt{maxv}$ then it sets $\mathtt{maxi} = \mathtt{i}$ (**lines 23-24**).

Having computed `n` partial quotients, and knowing that `maxi` contains the index of the largest $\mathtt{q_i}$, the procedure calculates $cf(\mathtt{q_0}, \mathtt{q_1}, \ldots, \mathtt{q_{maxi-1}})$ by using a simple iterative loop (**lines 35-38**), and returns the computed fraction (**line 40**). In case that `maxi`=0, the procedure simply returns $\mathtt{q_0}$ (**line 31**).

Precision used

Obviously, the choice of precision is pretty much subjective. I decided that 50 decimal digits was a good enough precision, with a reasonable computation time. Also, 50 digits was a limit to how many characters could be printed in one line of text. Furthermore, such a precision allowed me to identify errors in my code, since smaller precisions would hide such errors.

Goodenss of approximation

We can calculate the goodness of an approximation p/q by comparing the error margin $\varepsilon = a - p/q$ to the value $1/q^2$. In fact, we would like the fraction $\frac{\varepsilon}{1/q^2}$ to be as close to 0 as possible. By running my tests, I observed that most approximations satisfied the equation length(q) / decimal digits of accuracy ≈ 2. It seemed that good approximations would have a higher proportion than 2.

Keeping all the above in ming, here are some good approximations that I achieved by doing the coursework:

- $17^{1/25} \approx 28/25$ with 6 decimal digits of accuracy.
- $17^{1/48} \approx 820/773$ with 7 decimal digits of accuracy.
- $17^{1/57} \approx 1093/1040$ with 8 decimal digits of accuracy.
- $17^{1/68} \approx 49/47$ with 5 decimal digits of accuracy.
- $17^{1/69} \approx 174/167$ with 6 decimal digits of accuracy.