

3. STRESSES IN THE GROUND

3.1 STRESSES IN DRY SOIL

Let A in Fig. 3.1 (a) represent a small cubical shaped element of soil at a depth z in an extensive uniform soil deposit in which the ground surface is horizontal and which has been formed by the gradual accretion of material on the ground surface. Because the soil deposit is extensive (by comparison with distance z) in the horizontal direction the stresses on element A will be identical with the stresses on an adjacent element at the same depth below the ground surface. This means that there cannot be any shear stresses existing on the vertical or horizontal planes which bound element A. In other words the vertical stress (σ_v) and horizontal stress (σ_H) are principal stresses.

The vertical stress on element A can be determined simply from the mass of the overlying material. If ρ_d represents the density of the soil, the vertical stress is

$$\sigma_v = \rho_d g z \quad (3.1)$$

The horizontal stress is customarily expressed as a proportion of the vertical stress

$$\sigma_H = K'_o \sigma_v = K'_o \rho_d g z \quad (3.2)$$

where K'_o = coefficient of earth pressure at rest **in terms of effective stresses** (see equation (3.7)).

This coefficient contains the words “at rest” since the soil was deposited under conditions of zero horizontal strain. In other words, because of the large lateral extent of the soil deposit, the vertical planes on any soil element A do not experience any lateral movement as the stresses increase as a consequence of the accretion of material on the ground surface. The distributions of σ_v and σ_H as a function of depth are illustrated in Fig. 3.1 (b).

The value of K'_o in equation (3.2) can be determined if the soil is assumed to behave as an elastic solid, as follows:

$$\epsilon_H = \frac{1}{E} (\sigma_H - \nu \sigma_v - \nu \sigma_H) = 0$$

where ϵ_H = horizontal strain
 E, ν = elastic parameters for the soil

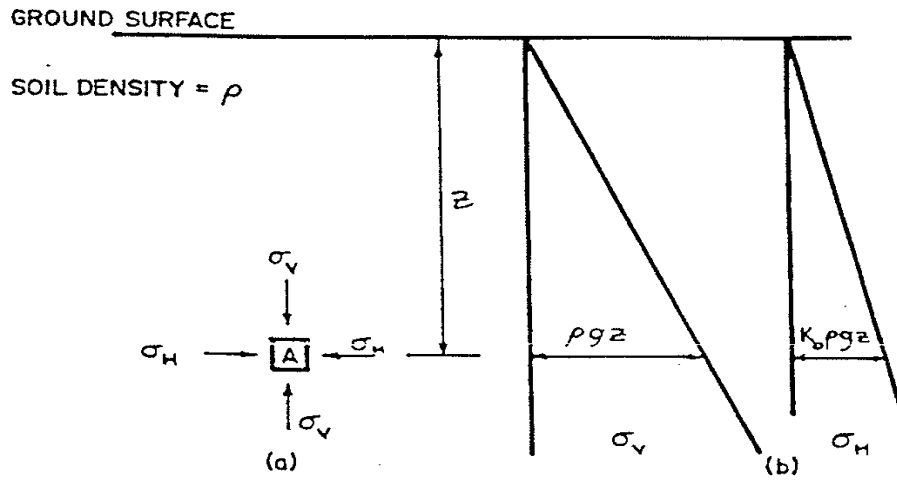


Fig. 3.1 Stresses in Dry Soil

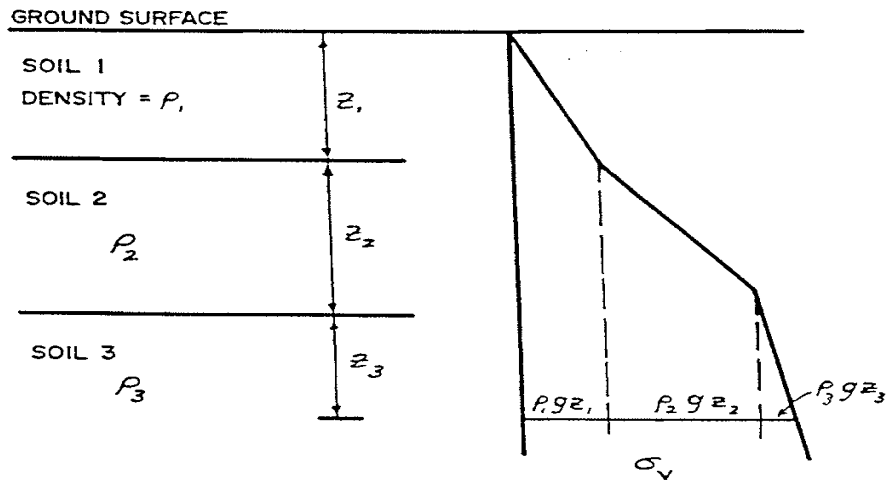


Fig. 3.2 Stresses in a Layered Deposit

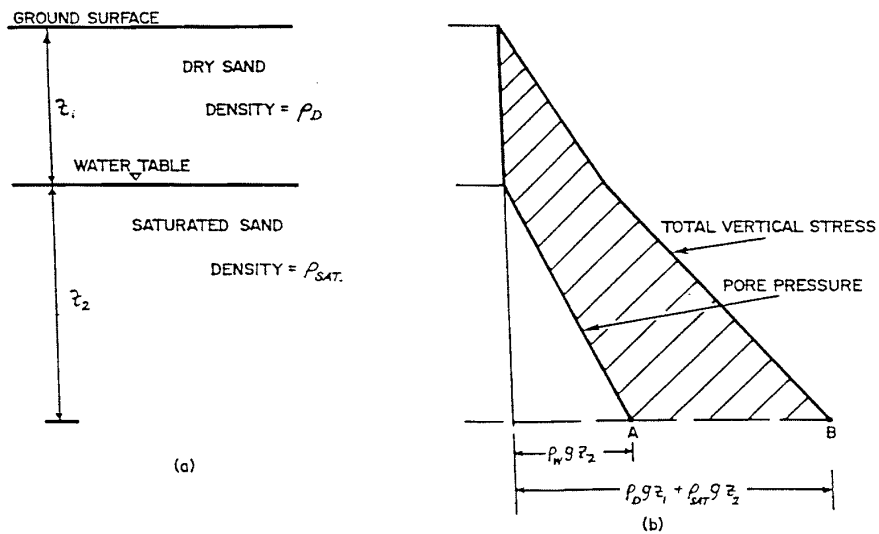


Fig. 3.3 Stresses in a Saturated Soil

$$\therefore \sigma_H - \nu\sigma_v - \nu\sigma_H = 0$$

$$\sigma_H (1 - \nu) = \nu\sigma_v$$

$$\therefore \frac{\sigma_H}{\sigma_v} = K'_o = \frac{\nu}{1-\nu} \quad (3.3)$$

which indicates that K'_o varies from 0 to 1.0 as the Poisson's ratio varies from 0 to 0.5.

The assumption of elastic behaviour for many soils may be an unrealistic idealization so the value of K'_o should be determined experimentally. In the laboratory, K'_o can be determined by applying a vertical stress σ_v to a soil sample while preventing all horizontal movement. The value of the horizontal stress (σ_H) required to prevent this movement is measured. The value of K'_o is then calculated from the measured value of σ_H and the applied value of σ_v .

The stresses in a deposit consisting of layers of soil having different densities may be determined by a simple extension of the technique described above. Referring to Fig. 3.2:

$$\begin{aligned} \text{vertical stress at depth } z_1 &= \rho_1 g z_1 \\ \text{vertical stress at depth } (z_1 + z_2) &= \rho_1 g z_1 + \rho_2 g z_2 \\ \text{vertical stress at depth } (z_1 + z_2 + z_3) &= \rho_1 g z_1 + \rho_2 g z_2 + \rho_3 g z_3 \end{aligned}$$

3.2. PRINCIPLE OF EFFECTIVE STRESS

Fig. 3.3 (a) represents a cross section through an extensive deposit of sand. The water table is present at a depth of z_1 below the ground surface. The sand below the water table is saturated and that above the water table is dry. It is assumed that the capillary rise in this soil is zero. The total vertical stress may be calculated in a similar way to that described in section 3.1 by using the density ρ_d above the water table and ρ_{sat} below the water table. This leads to an expression for the total vertical stress (σ_v) at a depth of $(z_1 + z_2)$ below the ground surface of

$$\sigma_v = \rho_d g z_1 + \rho_{sat} g z_2 \quad (3.4)$$

Because of the presence of the water table a hydrostatic pressure will also be present in the pore water. This pore pressure (u) is zero at the water table and will increase linearly with increasing depth, the value at a depth of z_2 below the water table being

$$u = \rho_w g z_2 \quad (3.5)$$

The values of the vertical stress, σ_v and pore pressure u (sometimes called neutral stress) at various depths are illustrated in the same diagram in Fig. 3.3 (b).

The difference between the total stress (σ) and the pore pressure (u) in a saturated soil has been defined by Terzaghi as the effective stress (σ'). (Terzaghi, 1936, and Skempton, 1960).

$$\sigma' = \sigma - u \quad (3.6)$$

The implications of this definition are among the most important in soil mechanics. Changes in soil properties are governed by changes in the effective stress and not solely by changes in total stress or changes in pore pressure. Unlike the pore pressure, the effective stress and total stress are tensors. This means that direction as well as magnitude must be specified.

The effective vertical stress (σ'_v) at a depth of ($z_1 + z_2$) below the ground surface (Fig. 3.3) may be found from equations (3.4) and (3.5), as follows

$$\begin{aligned} \sigma'_v &= \sigma_v - u \\ &= \rho_d g z_1 + \rho_{sat} g z_2 - \rho_w g z_2 \\ &= \rho_d g z_1 + (\rho_{sat} - \rho_w) g z_2 \\ &= \rho_d g z_1 + \rho_b g z_2 \end{aligned}$$

where $\rho_b = \rho_{sat} - \rho_w$ and is called the buoyant density or submerged density.

This value of the effective vertical stress, σ'_v is represented by the line AB (stress at a depth of ($z_1 + z_2$) below the ground surface) in Fig. 3.3 (b) and the distribution of the effective vertical stress as a function of depth is shown by the shaded area in this figure. For depths of less than z_1 below the ground surface the total and effective stresses are equal since the pore pressure is zero.

The effective horizontal stress σ'_H in an extensive soil deposit may be found from equations (3.2) and (3.6) with one important modification. When pore pressures are present the stresses in equation (3.2) must be effective stresses. Note that the value of the **coefficient of earth pressure at rest is defined in terms of effective stresses and not total stresses.**

$$\begin{aligned} \sigma'_H &= K'_o \sigma'_v \\ &= K'_o (\sigma_v - u) \end{aligned} \quad (3.7)$$

For the case illustrated in Fig. 3.3 the value of the horizontal effective stress at a depth of $(z_1 + z_2)$ below the ground surface is

$$\sigma'_H = K'_o (\rho_d g z_1 + \rho_b g z_2)$$

EXAMPLE

A reservoir of water 10m deep is underlain by an extensive sand deposit. Referring to Fig. 3.4 determine the change in effective vertical stress at point P if the water depth in the reservoir is increased to 18m.

This problem will be solved in two ways:

- (a) the initial effective vertical stress σ'_i at point P may be found from the initial total vertical stress σ_i and the initial pore pressure u_i

$$\begin{aligned} \sigma'_i &= \sigma_i - u_i \\ \sigma_i &= 1000 \times 9.81 \times 10 + 2100 \times 9.81 \times 6 \quad \text{N/m}^2 \\ &= 98.1 + 123.6 \quad \text{kN/m}^2 = 221.7 \quad \text{kN/m}^2 \\ u_i &= 1000 \times 9.81 \times 16 \quad \text{N/m}^2 = 157.0 \quad \text{kN/m}^2 \\ \therefore \sigma'_i &= 221.7 - 157.0 = 64.7 \quad \text{kN/m}^2 \end{aligned}$$

Similarly the final effective vertical stress σ'_f may be found

$$\begin{aligned} \sigma'_f &= (1000 \times 9.81 \times 18 + 2100 \times 9.81 \times 6) - (1000 \times 9.81 \times 24) \\ &= 176.6 + 123.6 - 235.5 \quad \text{kN/m}^2 = 64.7 \quad \text{kN/m}^2 \end{aligned}$$

Since the initial and final values of effective vertical stress are the same, the change in effective vertical stress is zero.

- (b) The initial effective vertical stress may be calculated from the buoyant density ρ_b of the sand:

$$\begin{aligned} \rho_b &= \rho_{\text{sat}} - \rho_w \\ &= 2100 - 1000 = 1100 \quad \text{kg/m}^3 \end{aligned}$$

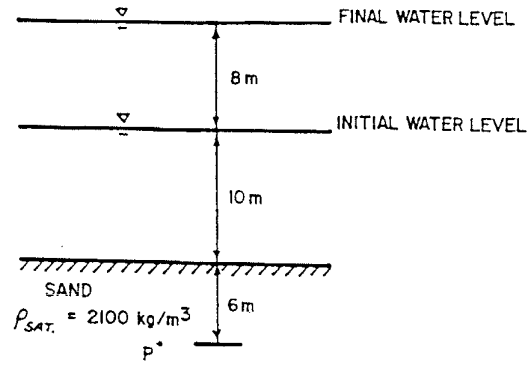


Fig. 3.4

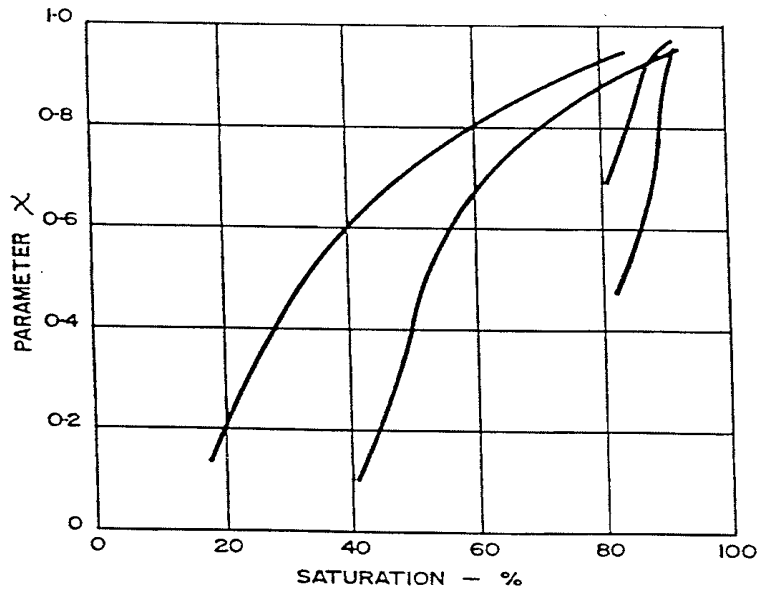


Fig. 3.5 Measured χ Values for Four Clay Soils

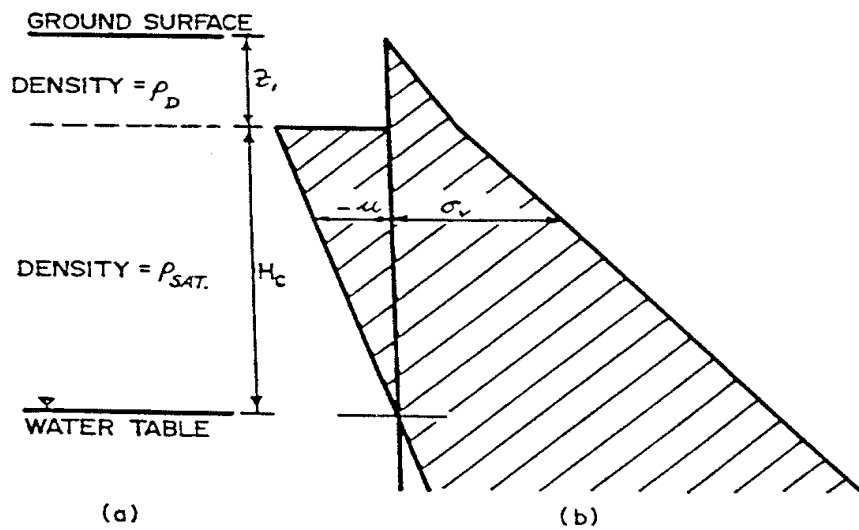


Fig. 3.6 Stresses in the Capillary Zone

$$\begin{aligned}\therefore \sigma_1' &= 1100 \times 9.81 \times 6 \text{ N/m}^2 \\ &= 64.7 \text{ kN/m}^2\end{aligned}$$

which is the same answer as in (a). When doing the calculation of effective stress in this way, it is clear that in the calculation of the final effective stress none of the terms in the equation has changed - the buoyant density remains the same and the depth of submerged sand of 6m does not alter. Consequently there can be no change in the effective stress as the reservoir depth increases.

In this problem the total stress and the pore pressure increased by exactly the same amount as the water depth increased. As long as the sand remains submerged this equality is maintained for any increase or decrease in water depth.

3.3 EFFECTIVE STRESS IN PARTLY SATURATED SOIL

Equation (3.6) cannot be used to determine effective stress in partly saturated soil because there are now two pore pressures instead of one - the pore water pressure (u_w) and the pore air pressure (u_a). From what has been said in chapter 2 it is clear that, in general, these two pore pressures will be unequal.

A definition of effective stress in partly saturated soils has been proposed by Bishop (1959) (see also Bishop, Alpan, Blight and Donald, 1960).

$$\sigma' = \sigma - u_a + \chi (u_a - u_w) \quad (3.8)$$

where χ is a parameter which may vary from 0 to 1.

When $\chi = 1$ the soil is saturated and equation (3.8) simplifies to

$$\sigma' = \sigma - u_w \quad (3.6)$$

When $\chi = 0$ the soil is dry and equation (3.8) simplifies to the same form as equation (3.6)

$$\sigma' = \sigma - u_a$$

The parameter χ has been found to depend mainly on the degree of saturation as illustrated in Fig. 3.5 but it has been suggested that it also depends on soil structure, the cycle of wetting and drying or stress change leading to the particular value of degree of saturation at which χ is measured. Some aspects of the χ concept remain to be explained since some measurements have yielded values outside of the expected range of 0 to 1.

Around 1960 several other effective stress equations for unsaturated soils were proposed, such as Aitchison (1961) and Jennings (1961). Common to all equations was the incorporation of a soil parameter characteristic of the soil behaviour. These soil parameters have been virtually impossible to evaluate uniquely and difficult to apply to practical problems (for example, see Burland (1965)). As described by Fredlund and Morgenstern (1977)) and more recently by Fredlund (1985), there has been an increased tendency to uncouple the terms in the effective stress equation, $(\sigma - u_a)$ and $(u_a - u_w)$, and treat the stress variables independently.

3.4 STRESSES IN THE CAPILLARY ZONE

Fig. 3.6 (a) represents a soil deposit in which the height of capillary rise H_c is less than the depth of the water table below the ground surface. To simplify the calculation it will be assumed that the soil is saturated within the capillary zone and completely dry above the height of the capillary rise.

The total vertical stress may be determined from an expression similar to equation (3.4). At a depth of z_1 , below the ground surface

$$\sigma_v = \rho_d g z_1$$

and at the water table elevation

$$\sigma_v = \rho_d g z_1 + \rho_{sat} g H_c$$

The distribution of σ_v as a function of depth is shown in Fig. 3.6 (b).

The pore pressure is zero down to a depth of z_1 below the ground surface. At this depth there is a change in pore pressure in the capillary zone to a value of

$$u = -\rho_w g H_c$$

and the pore pressure increases to zero at the water table elevation as shown in Fig. 3.6 (b).

Within the capillary zone the soil is saturated so the principle of effective stress as expressed in equation (3.6) may be applied

$$\sigma_v' = \sigma_v - u \quad (3.6)$$

At a depth of z_1 , below the ground surface

$$\begin{aligned}\sigma'_v &= \rho_d g z_1 - (-\rho_w g H_c) \\ &= \rho_d g z_1 + \rho_w g H_c\end{aligned}$$

and at the water table elevation

$$\sigma'_i = \sigma_v = \rho_D g z_1 + \rho_{sat} g H_c$$

The distribution of effective vertical stress is shown by the shaded area in Fig. 3.6 (b).

3.5 NON-PERPENDICULAR STRESSES

In the cases of saturated soil examined previously in this chapter the total stress and the pore pressure have been acting in the same direction so there has been no confusion over the use of the effective stress equation (3.6). It has already been stated that the total stress is a tensor so its direction must be taken into account. On the other hand, pore pressure is not a tensor and at any point it acts equally in all directions.

In Fig. 3.7 (a) BC represents a plane in a mass of saturated soil. The total stress (σ) acts on this plane at an angle θ to the normal but the pore pressure (u) acts at right angles to the plane. In this case the effective stress cannot be determined simply by subtracting u from σ . It is necessary to consider the two components of the total stress σ , one perpendicular to the plane and the other parallel to it as shown in Fig. 3.7 (b). The former component, the total normal stress (σ_n) is found by resolution in the normal direction.

$$\sigma_n = \sigma \cos \theta$$

Similarly the component parallel to the plane, the shear stress (τ) is

$$\tau = \sigma \sin \theta$$

Now the effective normal stress (σ'_n) can be found by subtracting the pore pressure from the total normal stress

$$\sigma'_n = \sigma_n - u$$

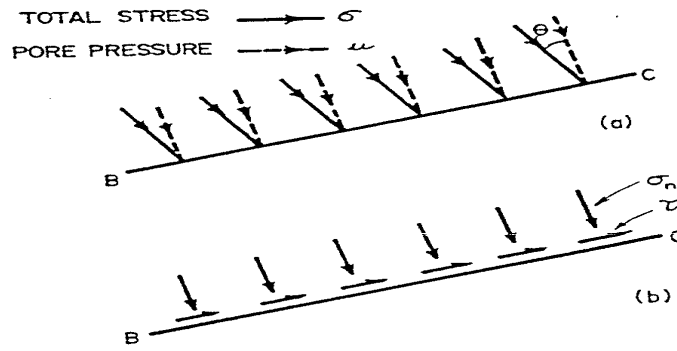


Fig. 3.7 Stresses Acting on a Plane

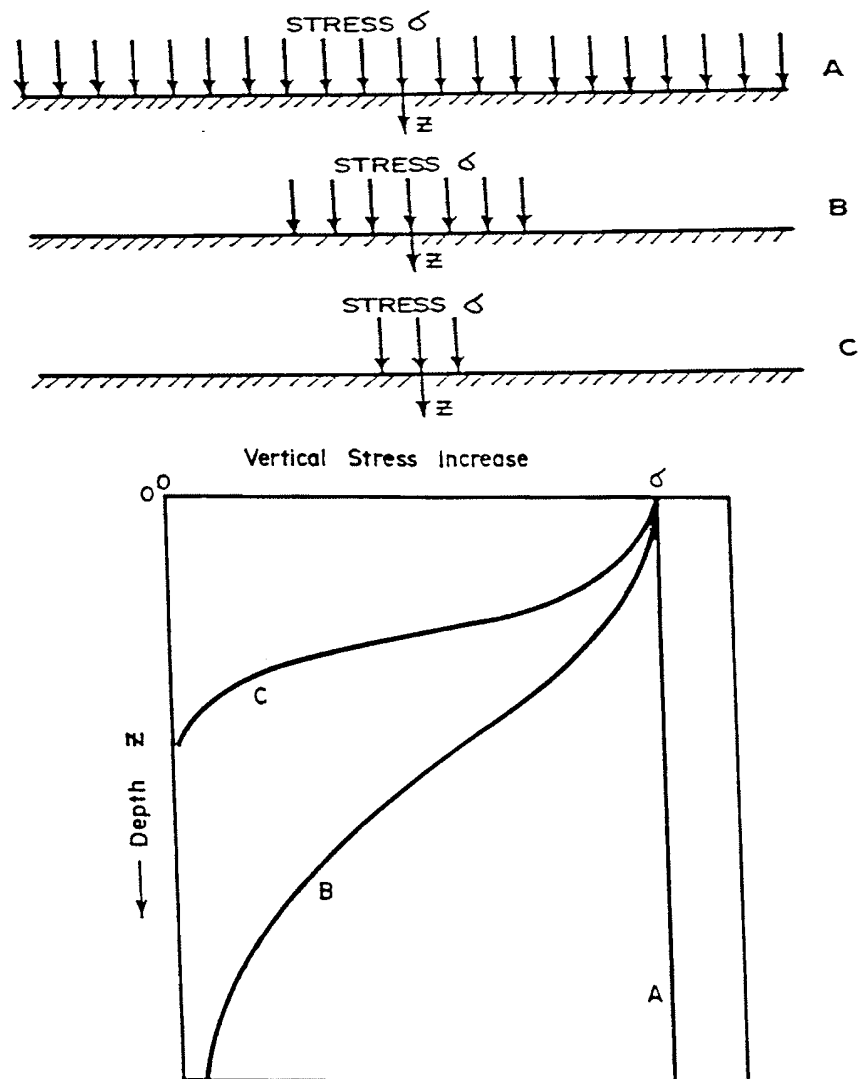


Fig. 3.8 Effect of Lateral Extent of Surface Load on Stress Change in the Ground

and the shear stress is unaffected by the pore pressure. The components of the total stress acting on plane BC are

- (a) an effective stress (σ'_n) acting normal to the plane
- (b) a pore pressure (u) acting normal to the plane, and
- (c) a shear stress (τ) acting parallel to the plane.

3.6 TRANSMISSION OF STRESS INTO THE GROUND

In earlier discussion in this chapter the stresses in the ground were produced by self weight of the soil. In this section the discussion will concentrate on stress changes produced by loads (forces and stresses) applied at the ground surface. In the case of uniform vertical stresses applied over a large area of the ground surface, the vertical stress changes at all depths in the ground are equal to the vertical stress applied at the ground surface as illustrated by case A in Fig. 3.8. In most soil engineering problems the load on the ground surface does not have a large lateral extent. In this case the stress change in the ground does not remain constant but decreases with increasing depth below the ground surface as shown in case B in Fig. 3.8. In the case of a surface loading of very limited lateral extent (case C in Fig. 3.8) a negligible stress change is experienced at great depths.

Quantitatively the stress changes at various depths are often calculated on the assumption that the solid behaves as a homogeneous, isotropic elastic solid. For such a solid, Boussinesq, in 1885 produced a solution for a concentrated load Q on the surface of the elastic solid. Using the symbols in Fig. 3.9 (a) his solution for the vertical stress change at any point was

$$\sigma_v = \frac{3Q \cos^5 \theta}{2\pi z^2} = \frac{3Q}{2\pi} \cdot \frac{z^3}{(r^2 + z^2)^{5/2}} = \frac{Q}{z^2} I_1 \quad (3.9)$$

where
$$I_1 = \frac{3}{2\pi} ((r/z)^2 + 1)^{-5/2}$$

This equation and Table 3.1 indicate the way in which the vertical stress change decreases with increasing distance from the concentrated load. This is illustrated in Fig. 3.9 (b).

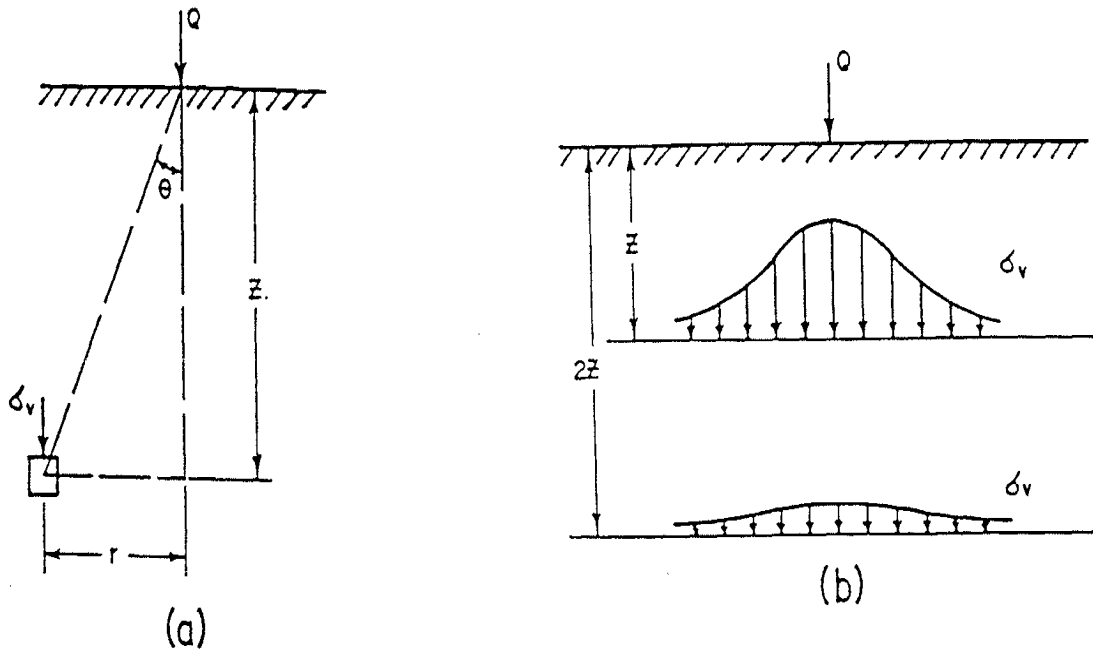


Fig. 3.9 Stresses Produced by a Concentrated Load at the Ground Surface

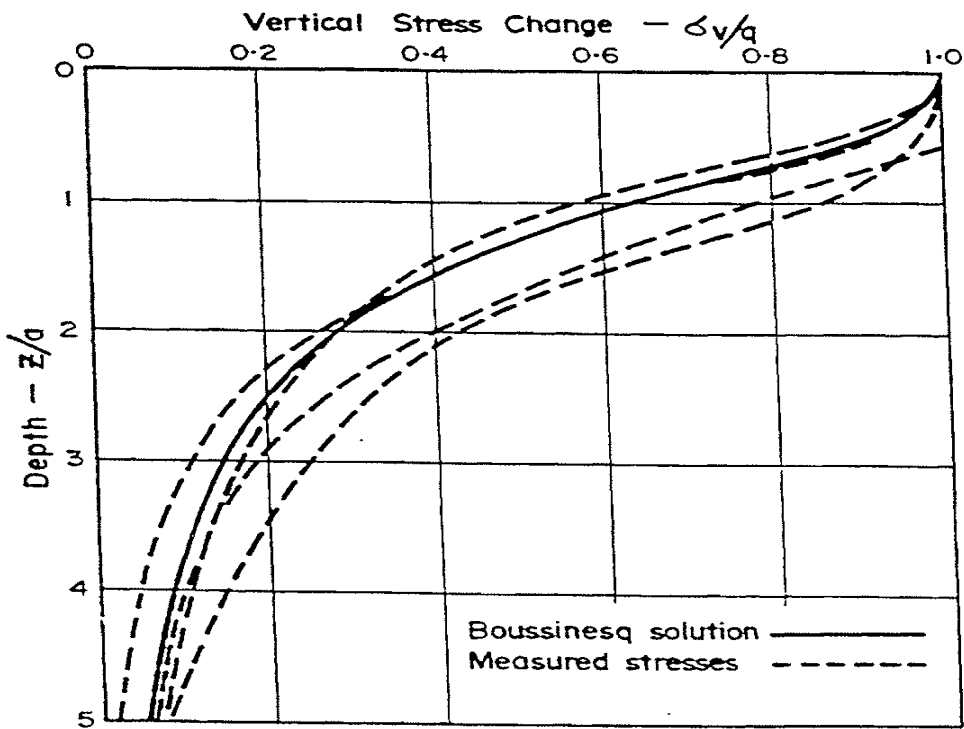


Fig. 3.10 Measured and Calculated Stresses for a Circular Loaded Area on Sand

TABLE 3.1
VALUES OF I_1 (equation (3.9))

(r/z)	I_1	(r/z)	I_1
0	0.4775	0.9	0.1083
0.1	0.4657	1.0	0.0844
0.2	0.4329	1.5	0.0251
0.3	0.3849	1.75	0.0144
0.4	0.3295	2.0	0.0085
0.5	0.2733	2.5	0.0034
0.6	0.2214	3.0	0.0015
0.7	0.1762	4.0	0.0004
0.8	0.1386	5.0	0.0001

For a pressure applied over a finite area on the surface a solution could be developed from equation (3.9) by assuming that the area is subdivided into a number of small areas with a concentrated load acting at the centre of each. For example, for a circular area of radius a and a uniform pressure of q at the surface, the expression for vertical stress change, σ_v at any depth z beneath the centre of the circular area is

$$\sigma_v = q \left(1 - \frac{z^3}{(a^2 + z^2)^{3/2}} \right) \quad (3.10)$$

The way in which σ_v varies with (z/a) is given in Table 3.2.

TABLE 3.2
VARIATIONS IN (σ_v/q) WITH (z/a) (equation (3.10))

(z/a)	(σ_v/q)	(z/a)	(σ_v/q)
0	1.0000	1.5	0.4240
0.05	0.9998	2.0	0.2845
0.10	0.9990	2.5	0.1996
0.2	0.9925	3.0	0.1436
0.5	0.9106	4.0	0.0869
1.0	0.6465	5.0	0.0571

Measurements in sandy soils indicates that the Boussinesq solution (equation (3.10)) is a reasonable approximation for the vertical stresses as illustrated by Fig. 3.10. Unfortunately, the

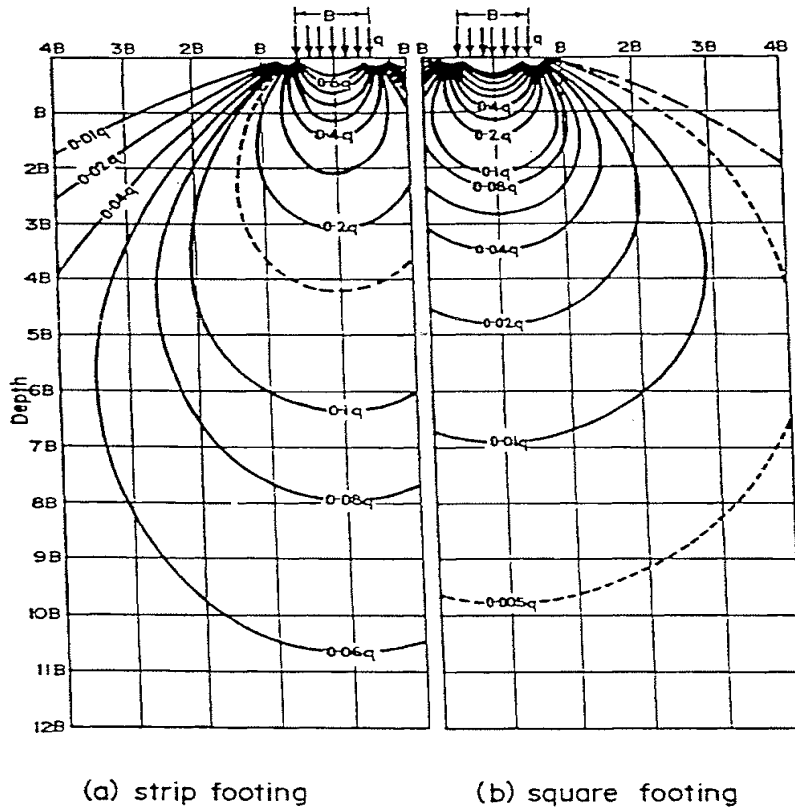


Fig. 3.11 Contours of Equal Vertical Stress (Boussinesq Solution)

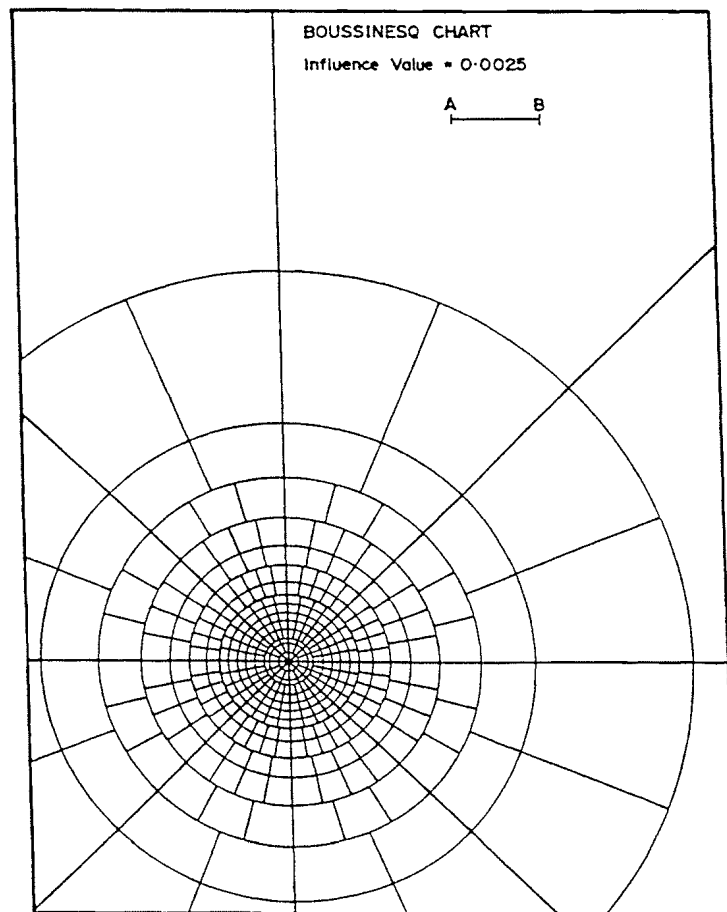


Fig. 3.12 Chart for Determination of Vertical Stress

level of agreement for measured and calculated radial and tangential stresses is much poorer than that indicated in Fig. 3.10 for vertical stresses.

Elastic solutions are also available for loaded areas of different shapes. (See Fadum (1941), Scott (1963) and Harr (1966)). Fig. 3.11 illustrates the patterns of vertical stress (isobars) produced by a uniformly loaded strip footing (infinitely long loaded area of width B) and a uniformly loaded square footing. This figure shows that the vertical stress change penetrates deeper into the ground in the case of a strip footing compared with the case of a square footing.

For loaded areas of irregular shape the simplest means of calculating the vertical stress change at any depth is by means of a numerical integration procedure for equation (3.9) by means of influence charts of the type proposed by Newmark (1942). Such a chart is shown in Fig. 3.12. The stress is determined by carrying out the following steps:

- (a) sketch the outline of the loaded area to scale such that the length AB in Fig. 3.12 is equal to the depth at which the stress is to be calculated,
- (b) the sketch is placed on the chart with the centre of the concentric circles coincident with the horizontal location for which the stress is to be calculated,
- (c) count the number of chart subdivisions (influence areas) enclosed within the sketch of the loaded area,
- (d) calculate the stress change as follows:

$$\text{stress change } \sigma_v = \begin{array}{l} \text{number of influence areas} \\ \times \text{influence value for the chart} \\ \times \text{pressure acting on loaded area.} \end{array}$$

In addition to what has been mentioned above, there is now a wide variety of solutions available for stresses in non homogeneous and anisotropic materials. (Gerrard and Wardle (1973), Harrison et al (1972)).

A simple but approximate method is sometimes used for calculating the stress change at various depths as a result of the application of a pressure at the ground surface. This method is illustrated in Fig. 3.13 in which a surface pressure (σ_v) acts on a strip footing of width B. The vertical stress change at any depth (z) is calculated by assuming that the surface force is uniformly distributed over an imaginary footing having a width of (B + z). That is, the transmission of stress is assumed to follow outward fanning lines at a slope of 1 horizontal to 2 vertical.

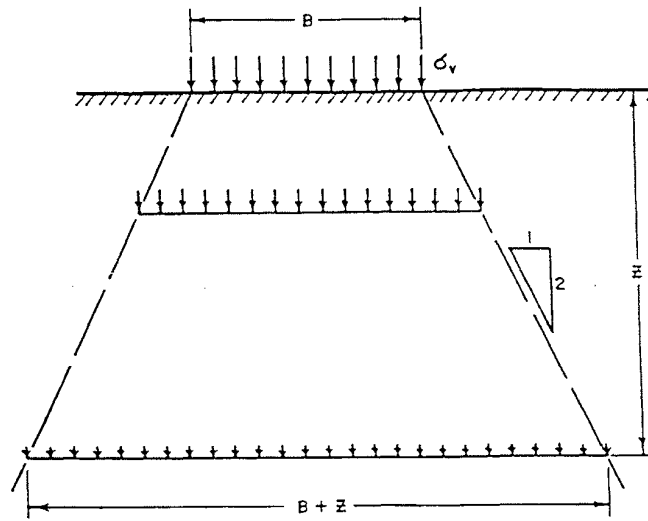


Fig. 3.13 Stress Transmission by Means of the 2:1 Distribution

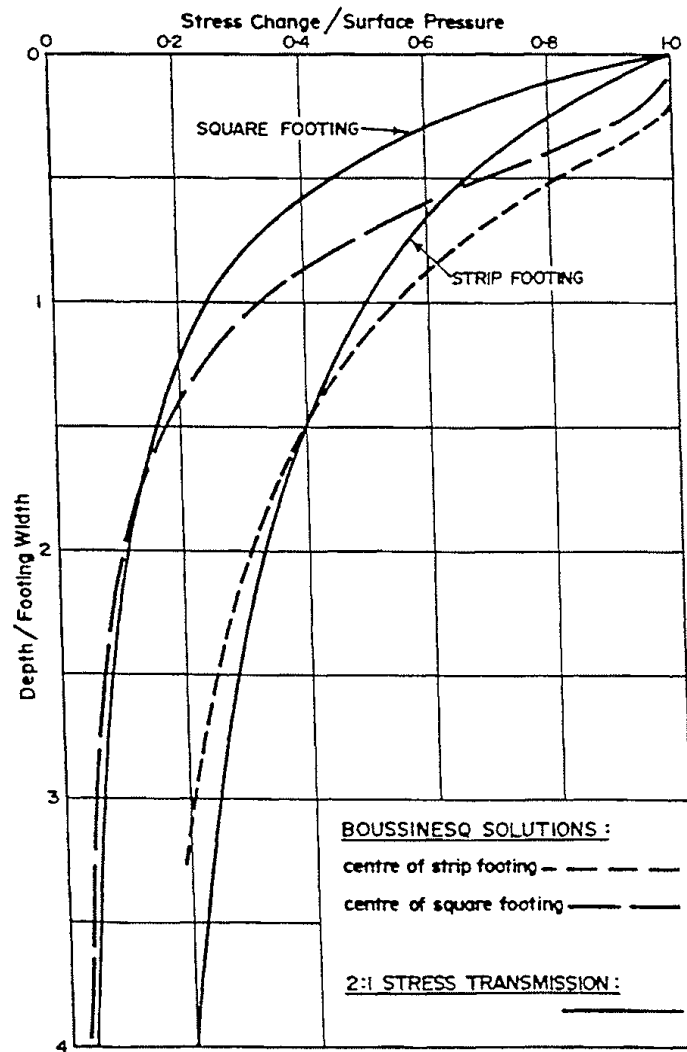


Fig. 3.14 Stress Changes due to a Surface Pressure

For a strip footing of width B the vertical stress change ($\Delta\sigma_z$) at any depth z is

$$\Delta\sigma_z = \frac{\sigma_v B}{(B + z)} = \frac{\sigma_v}{(1 + z/B)} \quad (3.11)$$

For a square footing of width B the vertical stress change ($\Delta\sigma_z$) at any depth z is

$$\Delta\sigma_z = \frac{\sigma_v B^2}{(B + z)^2} = \frac{\sigma_v}{(1 + z/B)^2} \quad (3.12)$$

The stress changes according to the 2:1 transmission in equations (3.11) and (3.12) are compared with the elastic solutions in Fig. 3.14. This figure shows that the 2:1 stress transmission results in an under-estimate of the stress change beneath the centre of the loaded area at shallow depths. However the level of agreement is considerably improved if the average stress change beneath the whole of the loaded area is considered.

EXAMPLE

Using the 2:1 stress distribution calculate the vertical stress change at a depth of 5m for the following cases:

- (a) a stress of 100kPa on a circular surface footing 2m in diameter.
- (b) a load of 10kN on a surface rectangular footing 2m x 3m.
- (c) a stress of 200kPa on a surface strip footing 1m wide.

(a) Surface load = $100 \times \pi \times 2^2/4$
 stress change at 5m depth

$$\Delta\sigma_v = \frac{\text{surface load}}{\pi \times 7^2/4} = 8.2 \text{ kPa}$$

(b) Stress change
$$\Delta\sigma_v = \frac{10}{(2 + 5)(3 + 5)}$$

$$= 0.18 \text{ kPa}$$

(c) Stress change
$$\Delta\sigma_v = 200 \times 1/(1 + 5)$$

$$= 33.3 \text{ kPa}$$

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