Trials and Tribulations of Designing a Modelling Language

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Outline

1. Design Goals for Zinc
2. Modelling Combinatorial Optimization Problems
3. Natural Modelling
4. Extensible Modelling
5. Software Engineering
6. Practical Solver-Independence
7. Conclusion

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Problem Solving Process

- Problem
  - "Find 4 different integers between 1 and 5 that sum to 14"
- Conceptual Model
  - Precisely specify the problem without describing how to solve it
  - $S \subseteq \{1,2,3,4,5\} \wedge |S| = 4 \wedge \sum S = 14$
- Design Model
  - Correct efficient algorithm
  - Specified using some solver technology and search strategy
  - `[W,X,Y,Z] :: 1..5, alldifferent([W,X,Y,Z]), W + X + Y + Z #= 14, labeling([X,Y,Z,T])`
- Solution
  - $W = 2 \wedge X = 3 \wedge Y = 4 \wedge Z = 5 \Rightarrow S = \{2,3,4,5\}$

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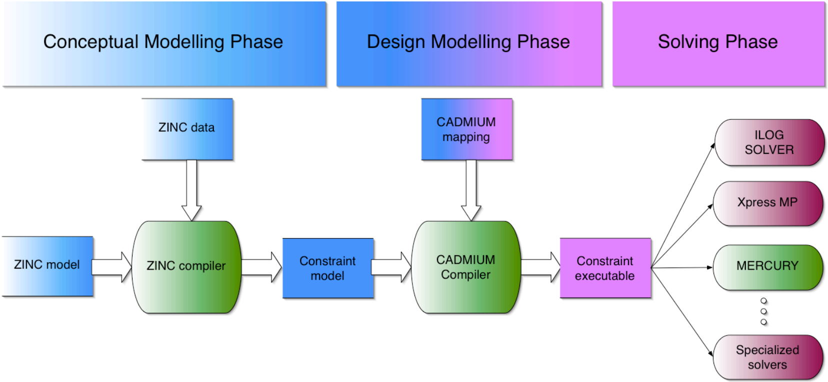
- Designed to mimic the problem solving process
- Zinc: Conceptual modelling language
- Cadmium: Mapping langauge
- Mercury: Solver backends



Design Goals for Zinc

- Natural Modelling: clear and concise high-level mathematical models
- Extensible Modelling: support modelling for a wide-variety of applications by extending the modelling language
- Software Engineering: support the development of correct and maintainable models
- Practical Solver-Independence: allow a single conceptual model to be mapped to many design models

Designing Zinc

- Three years of work
- Many refinements and clarifications
- Many Zinc models written and reviewed
- Reconsidered decisions in light of experience
- Still refinements to be made!
- Highlighting some of the important decisions made/discovered

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Modelling Combinatorial Optimization Problems

- Specification Languages
  - Generic specification languages: e.g. Z, B
    - More expressive than Zinc
    - Turing-complete, require theorem proving
    - Too powerful for combinatorial optimization
  - DIMACS SAT representation
    - Designed for solvers, not for modelling
    - Application and algorithm independent
    - Difficult to encode some problems: pigeonhole
  - MPS (lower level mathematical programming specification)
    - Again designed for solvers, not for modelling
    - Application and algorithm independent
    - Difficult to encode some problems: zebra
  - CSP $< V, D, C >$ variables V, domains D, constraints C
    - Expressive, algorithm independent
    - Limited modelling features (extensive constraint defn!)
  - No separation of data and model

Modelling Combinatorial Optimization Problems

- Modelling Languages
    - Mathematical modelling languages: MOLGEN, AMPL, GAMS
        - Support a range of mathematical programming solvers
        - Arrays of variables, iteration, separation of model and data
        - Restricted to linear arithmetic
    - Constraint logic programming languages: Eclipse, CHIP
        - Solver independence (Eclipse)
        - Full programming langugges
        - Untyped and procedural
    - Comet
        - Procedural Turing-complete language
        - Allows specification of conceptual and design models
        - Currently restricted to local search

Modelling Combinatorial Optimization Problems

- Modelling Languages
  - OPL
    - Inherits good features from math modelling langauges
    - Enumerated domains, type declarations, data structures, reification
    - Discrete and continuous variables
  - ESRA
    - Very high level: set and relation variables
    - Discrete variables only
  - Essence
    - Very high level: set, multiset, relation and function variables
    - Strongly typed
    - Discrete variables only

Modelling Combinatorial Optimization Problems

	CSP	AMPL	Comet	ECLiPSe	OPL	ESRA	ESSENCE	Zinc
Decidable	*Yes*	*Yes*	*Yes*	-	*Yes*	*Yes*	*Yes*	*Yes*
Typed	-	-	*Yes*	-	*Yes*	*Yes*	*Yes*	*Yes*
High-level	-	-	*Yes*	-	*Yes*	*Yes*	*Yes*	*Yes*
Con-types	-	-	-	-	-	-	-	*Yes*
Coercions	-	-	-	-	-	-	-	*Yes*
Extensible	-	-	*Yes*	*Yes*	-	-	-	*Yes*
Sep-model	-	*Yes*	*Yes*	-	*Yes*	*Yes*	*Yes*	*Yes*
Platforms	-	*Yes*	-	*Yes*	-	-	*Yes*	*Yes*
Domains	E	CD	DBS	CDEBS	CDE	DEBS	DEBS	CDEBS

- Decidable: all models are solvable.
- Typed: language is typed.
- High-level: admits data structures such as records and sets.
- Con-types: constraints can be associated with (all variables/values of) a type.
- Coercions: support for both overloading and type coercions.
- Extensible: core language provides extensible features.
- Sep-model: model and data can be provided separately.
- Platforms: models can be mapped to different underlying platforms.
- Domains: supported constraint domains—continuous arithmetic (C), discrete arithmetic (D), discrete symbolic (E), Booleans (B), sets (S).

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Example: Perfect Squares `perfsq.zinc`

```
type PosInt = (int: x where x > 0);
PosInt: base;
type Square = record(var 1..base: x, var 1..base: y, PosInt: size);
list of Square: squares;

predicate nonOverlap(Square: s, Square: t) =
              s.x + s.size <= t.x  \/  t.x + s.size <= s.x  \/
              s.y + s.size <= t.y  \/  t.y + s.size <= s.y;

constraint forall(s in squares) (
               s.x + s.size < base  /\  s.y + s.size < base  );
constraint forall(i, j in 1..length(squares) where i < j) (
               nonOverlap(squares[i], squares[j])         );

constraint assert(sum(s in squares)(s.size * s.size) == base*base,
               "Squares do not cover the base exactly");

solve satisfy;
output show(squares);
```

Example: Perfect Squares `perfsq.zinc`

- Constrained type item: `type PosInt = (int:  x where x > 0);`
- Parameter declaration item: `PosInt:  base;`
- Record type declaration item

  ```
  type Square = record(var 1..base:  x, var 1..base:  y,
                       PosInt:  size);
  ```
- (Array) Variable declaration item: `list of Square:  squares;`
- Predicate definition item

  ```
  predicate nonOverlap(Square: s, Square: t) =
                s.x + s.size <= t.x  \/  t.x + s.size <= s.x  \/
                s.y + s.size <= t.y  \/  t.y + s.size <= s.y;
  ```

Example: Perfect Squares `perfsq.zinc`

- Constraint items

```
constraint forall(s in squares) (
                s.x + s.size < base  /\  s.y + s.size < base  );
constraint forall(i, j in 1..length(squares) where i < j) (
                nonOverlap(squares[i], squares[j])         );
```

- Assertion

```
constraint
    assert(sum(s in squares)(s.size * s.size) == base*base,
           "Squares do not cover the base exactly");
```

- Solve item: `solve satisfy;`
- Output item: `output show(squares);`

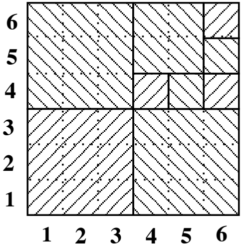
Example: Perfect Squares Data `perfsq6.data`

- Problem instance defined by separate data file
- Example

```
base = 6;
squares = [ (x:_, y:_, size:s) | s in [3,3,3,2,1,1,1,1,1] ];
```

- _ represents anonymous variable.
- Result

```
[(x:1, y:1, size:3), (x:4, y:1, size:3),
 (x:1, y:4, size:3), (x:4, y:5, size:2),
 (x:4, y:4, size:1), (x:5, y:4, size:1),
 (x:6, y:4, size:1), (x:6, y:5, size:1),
 (x:6, y:6, size:1)]
```



Types and Insts

- Types
  - Base: Booleans, integers, floats, strings
  - Constructors: sets, arrays, tuples, records, variant records, enumerated types
- Instantiations (Insts)
  - `par`: parameter — fixed by the data
  - `var`: decision variable
  - default is `par`
- Type-Inst (pairing of type and instantiation)

Design Decision: Type-Insts

- Modellers must distiguish between parameters and decision variables
- Allows: checking of parameter initialization, translation simplification
- Improves: readability, error checking

Base types

- Numbers
  - integers `23` and floats `2.3e-05`, `0.0067`
  - built-in arithmetic: `+`, `*`, `round`, ...
- Booleans
  - `true`, `false`
  - built-in operators: `\/`, `/\`, `->`, ...

Design Decision: Booleans

- Dont represent Booleans as 0..1 integers
- Better error checking, easier to map to different solvers

- Strings
  - `"one two three \n"`

Enumerated types and Sets

- Enumerated types
  - `enum Colour = {Red, Green, Blue};`
    `array[Colour,Colour] of var Colour:  Clashing;`

Design Decision: Enumerated types

- Name space for elements is global
- Type name is a set expression
- Can be declared in a model and defined in data file

  - `for(i,j in Colour)(Clashing[i,j] != i)`

Sets and Comprehensions

- Sets
  - sets literals `{1.0,-5.3}` and ranges `1..4`
  - built-in operators: `in`, `card`, `union`, `intersect`, ...
  - Only sets of `par` instantations
  - Var sets must have finite type
    - Yes: `var set of Colour`, `set of tuple(int,int)`
    - No: `set of var Colour`, `var set of tuple(int,int)`
  - set comprehensions
    - `{i * j | i,j in 1..10 where i != j }`

Design Decision: Comprehensions

- Generator sets of arrays must be `par`
- Ensures finiteness, some confusion

- No: `var set of 1..10:  p; constraint sum(i in p)(i) > 0;`
- Yes: `constraint sum(i in 1..10)(bool2int(i in p)*i) > 0;`

Arrays

- Arrays
  - Elements with any instantiation
  - Indices must be `par`, arrays are never `var`

Design Decision: Arrays

- Arrays of variable length are disallowed (finiteness)
- Multi-dimensional arrays are actually arrays indexed by tuple
- Lists are syntactic sugar for arrays

  - `for(i,j in Colour)(Clashing[(i,j)] != i)`
- Explicit and implicit indices
  - By default integral beginning at 0

  -
    ```
    array[1..3]   of int: a1 = [5, 6, 7];  % explicit-index
    array[int]    of int: a2 = [0:8, 1:9]; % implicit-index
    array[1..100] of int: a3 = [1:1, 2:2] default 0;
    ```
- array comprehensions
  - `[i * j | i,j in 1..10 where i != j ]`

Tuples, Records and Variant Records

- Tuples
  - Elements of any instantiation
  - `tuple(int,float,bool):  x = (1, 4.56, true);`
  - Field numbers: `x.3 \/ y >= 0`
- Records
  - Elements of any instantiation
  - `record(x:int, y:float, z:bool):  y = x;`
  - Field access: `y.1 >= 8`
  - Coercion from tuples of correct type.
- Variant Records
  - Non-recursive

  -
    ```
    variant_record thing = {
                        integer(int:x),
                        boolean(bool:b),
                        pair(int:x.  int:y) };
    ```

Let Expressions

- Let expressions
  - Introduce local variables and parameters
  - `let { int:x = z*z, var int:y = u*u } in x + y`

Design Decision: Let

- Let expressions in negative contexts must functionally define variables
- Otherwise: requires universal quantification

  - No: `not (let {var int:z } in x == 2 * z)`
  - Yes: `not (exists(z in 1..10)(x == 2 * z))`
  - Yes: `not (let {var int:z = floor(x / 2) } in x == 2 * z)` is equivalent to `let {var int:z = floor(x / 2) } in not(x == 2 * z)`

Items

- Assignment items
  - `x = 3;`
  - Joint variable declarations are sugar: e.g. `int:x = 3;` ⇒ `int:x; x = 3;`
- Include items
  - `include "globals.zinc"`
  - Textually insert file into incliding file

Design Decision: Items

- Items can appear in any order (helps include)
- Data files are just Zinc (complex expressions allowed)

-
```
enum Colour;
Colour: None;
int: x;
var Colour:y;
include "test.data"
constraint y != None;
```

```
enum Colour = { Empty, Red, Blue };
None = Empty;
x = card(Colour) - 1;
```

test.data

Outline

Extensible Modelling

- Rich type language
  - Define a new type for some application domain
- User-defined predicates and functions
  - Define the operations (functions) for the new type
  - Define the constraints (predicates) for the new type
- Constrained types
  - Enforce certain constraints on all members of the type

Predicates and Functions

- Predicates are simply functions with return type `var bool`
- Functions can be overloaded on type-inst
- Types can use type parameters `$T`

```
function var bool: between($T: x, $T: y, $T: z) =
    (x <= y /\ y <= z) \/ (z <= y /\ y <= x);
function par bool: between(par $T: x, par $T: y, par $T: z) =
    (x <= y /\ y <= z) \/ (z <= y /\ y <= x);
```

Second version gives more accurate type-inst on return `par bool`

Design InDecision: Overloading

- Currently body is duplicated for overloading
- Later perhaps we allow sharing of bodies

foldl and foldr

- Two higher-order built-in functions
- `foldl(fun, init, array)`
  - Apply binary function *fun* to each element in *array* starting from *init*

```
predicate        forall(array[int] of var bool:  xs) =
                        foldl('/\', true, xs);
function var int:   sum(array[int] of var int:   xs) =
                        foldl('+', 0, xs);
function var float: sum(array[int] of var float: xs) =
                        foldl('+', 0, xs);
```

Design Decision: Fold

- Provide powerful building blocks `foldl`, `foldr`
- Preferable to many built-in iteration constructs `forall`, `sum`

Reflection functions

- In order to define functions and predicates
- Reflection functions
  - `index_set`: returns index of array argument
  - e.g. `index_set_1of2`: first index of 2d array
  - `lb`: returns declared lower bound of `var int`, `var float`
  - `dom`: returns declared domain of `var int`, `var float`
  - `ub`: returns declared upper bound of `var int`, `var float`, `var set of ..`

```
%-----------------------------------------------------------%
% Requires the image of function 'x' (represented as array)
% on set of values 's' is 't'
%-----------------------------------------------------------%
predicate range(array[int] of var int:x, var set of int:s,
                var set of int:t) =
    forall(i in ub(t))(
        i in t -> exists(j in ub(s))(j in s -> x[j] == i));
```

Constrained Types

- All elements of the type must satisfy a certain constraint

```
type PosInt   = (int: i where i > 0);
type Interval = (record(var int: start, var int: end):
                          r where r.end >= r.start);
```

- ranges `1..n` are a special case
- Constraint view: syntactic sugar, e.g. `var PosInt:  y;` ⇒

```
var int:  y;
constraint y > 0;
```

- Type theoretic view: subtype not active constraint
  - should not affect execution of type correct program
- The constraint view and subtype view are incompatible!

Constrained Types Difficulties

- `predicate ge(var PosInt:x, var PosInt:y) =  x >= y;`

  ```
  var int: x = 5;  var int: y = -6;  var int: z;
  constraint ge(x,y) \/ z = 1;
  ```

- subtype view: type error!
- constraint view: `z = 1` since equivalent to

  ```
  constraint (x > 0 /\ y > 0 /\ x >= y) \/ z = 1;
  ```

- Problematic with negation!

  ```
  constraint ( ge(x,y) /\ z = 1) \/ (not ge(x,y) /\ z = 2);
  ```

  - Constraint view: $x = 5 \wedge y = -6 \wedge z = 2$
  - subtype view: type error

Design Decision: Constrained types

- Check statically whether formal types implied by actual types
- Warn if not so, and use constraint viewpoint

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Software Engineering

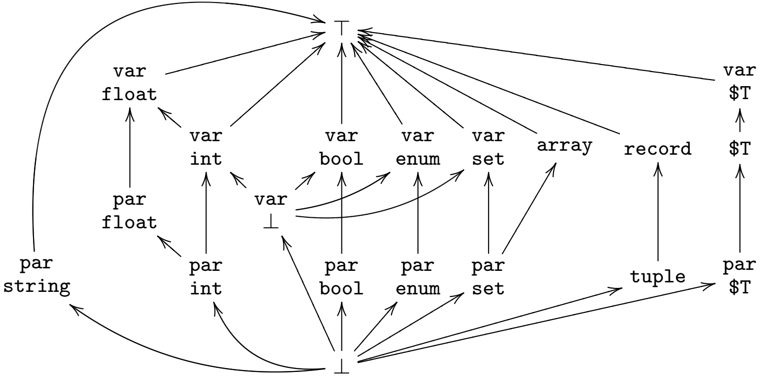
- Zinc designed to support good software engineering practices
  - Conciseness and readability
  - Strong static error checking
  - Avoiding code duplication (parametricity)
- Problem instance evaluation
  1. *model checking*: data-independent check of model
  2. *instance checking*: checking once the data is combined with the model
  3. *instance evaluaion*:
- Zinc checks errors as soon as possible in the pipeline

Design Decision: Variable Declarations

- All variables except generator variables must have a declared type-inst
- Complete inference impossible with separate data files

Type-Inst Checking

- Main static check
- Type system: Hindley-Milner with coercions and overloading
- Type-inst lattice



Type-Inst Checking

- Coercions
  - Integers can be coerced to floats
  - Sets S of type t can be coerced to arrays indexed by $0..card(s)$ of type t
  - Tuples can be coerced to records (if type appropriate)
  - `par` values can be coerced to `var` values
  - Anonymous variable `_` (type-inst `var` $\perp$) can be coerced to any `var` type-inst.
- Algorithm
  - Bottom-up inference: determine type-inst
  - Top-down: add appropriate coercions

Type-Inst Checking Example

- `array[int] of var float:  x = [1,2,_]`
- array element type: lub of `par int`, `var` $\perp$ = `var int`

```
[ coerce(par int,    var int, 1),
  coerce(par int,    var int, 2),
  coerce(var bottom, var int, _) ];
```

- Assignment means coerce `array[int] of var int` to `array[int] of var float`

```
array[int] of var float: x =
    coerce(array[int] of var int, array[int] of var float,
           [ coerce(par int,    var int, 1),
             coerce(par int,    var int, 2),
             coerce(var bottom, var int, _) ];
```

- Coercions are pushed down as far as possible

```
array[int] of var float: x =
           [ coerce(par int,    var float, 1),
             coerce(par int,    var float, 2),
             coerce(var bottom, var float, _) ];
```

Overloading

- Overloading interacts dangerously with coercion!

```
function int: f(int: x, float: y) = 0;
function int: f(float: x, int: y) = 1;
```

  Type of `f(3,3)` determines result 0 or 1!

- Type-inst checking with overloading and coercion may be expensive

```
function int:   g(int: x,   float: y);
function float: g(float: x, int: y);
```

  Checking `g(g(g(1,2),g(3,4)),g(5,g(6,7)))` is combinatorial

Design Decision: Overloading

- Overloaded versions of functions should be semantically equivalent wrt coercion: Not statically checkable
- Overloaded functions must be closed under type conjunction
- Overloaded functions must be monotonic

Array access, Domain checking, Assertions

- Array access (`var`ifying)
  - `array[int] of par int:  x = [1,2,3];`
  - `var int:i;` then `a[i]` has type-inst `var int`
- Domain checking
  - Set solvers require sets to range over finite domains
  - *finite type*: enumerated type, range, Boolean: tuples, records, sets of finite types.
- Assertions
  - Two assertion functions

    ```
    function $T: assert(par bool: c, par string: s, $T: val);
    function par bool: assert(par bool: c, par string: s);
    ```
  - Check `c`, if false print `s` else return `val` (or true)

Design Decision: Assertions

- Originally assertion item, expressions are more flexible, particularly for functions

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Practical Solver-Independence

- Zinc is designed to allow the mapping of a conceptual model to different design models.
- Features to support this are:
    - Annotations
    - Decomposable Global Constraints
    - Implementability

Annotations

- Annotations can be used to control the translation of conceptual models
- Solver dependent information (this includes search!)
- Can be ignored by a solver
- Declared with types:

```
enum SolverKind = { Lp, Ip, Fd, Sat };
annotation solver(SolverKind);
annotation bounds;
```

- Attached to expressions and items with `::`

```
var array[int] of var int: x :: bounds;
constraint all_different(x) :: solver(Fd) :: bounds;
constraint sum(i in S)(a[i] * x[i]) <= 10 :: solver(Lp);
```

Decomposable Global Constraints

- Global constraints are simply predicates
- Low-level logical definition means they can be used for any solver!
- Example: sequence constraints: in each sequence of length `k` in `x` there are between `l` and `u`, 1s:

```
predicate sequence_among(int: l, int: u, int: k,
                         array[int] of var 0..1: x) =
    let { int: n = max(index_set(x)) } in
    assert(min(index_set(x)) == 1 /\ card(index_set(x)) == n,
        "array x must be indexed 1..n",
    forall(i in 1..n-k+1)(
        among(l, u, [x[j] | j in i..i+k-1])));

predicate among(int: l, int: u, array[int] of var 0..1: x) =
    let { var int: s = sum(x) } in l <= s /\ s <= u;
```

- Supports experimentation with different decompositions, e.g. `sequence_cumul`

Implementability

Zinc is designed so that the features are mappable to modern CP solvers

- Evaluate parameters
- Determine initial domain for all decision variables
- Simplify records to tuples, flatten tuples, replace field accesses
- Replace enumerated types by integer range types
- Unfold built-in and defined predicates and functions
- Insert constraints arising from constrained types
- Lift lets to be global (rename variables)
- Simplify arrays to be one dimensional integer indexed from 0
- Translate variables sets on structured types to `var set of int`
- Reify to separate logical combinations of constraints

Implementations

- Two implementations
- Prototype Full Zinc compiler
  - 12000 Mercury LOC, 5000 C LOC
  - Generates simplifed Zinc using translation on previous slide
  - Maps simplified Zinc to Eclipse: 3 solver
    - Complete tree search with FD propagation
    - (Repair-based) Local search maintaining some hard constraints
    - Mapping to MIP and branch and bound search
- In progress "Industrial Strength" Zinc compiler
  - 25000 Mercury LOC
  - Syntax and semantics checks
  - Transformation using Cadmium of a subset of Zinc, MiniZinc, to FlatZinc
  - FlatZinc interpretable by LP/FD solvers, and Gecode, Eclipse

Outline

Conclusion

Zinc

- Allows clear and concise high-level mathematical models
- Extensible with constrained types, and user-defined predicates and functions
- Supports the development of correct and maintainable models
- Allows a single conceptual model to be mapped to many design models

Future

- Many mapping of Zinc to solvers to explore
- Specifying search in Zinc
- Direct mapping of Zinc to Mercury

TRY OUT MiniZinc!

www.g12.mu.oz.au/minizinc/