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A Decomposition-Based Heuristic for Collaborative Scheduling in a Network of Open-Pit Mines

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We consider the short-term production scheduling problem for a network of multiple open-pit mines and ports. Ore produced at each mine is transported by rail to a set of ports and blended into signature products for shipping. Consistency in the grade and quality of production over time is critical for customer satisfaction, while the maximal production of blended products is required to maximise profit. In practice, short-term schedules are formed independently at each mine, tasked with achieving the grade and quality targets outlined in a medium-term plan. However, due to uncertainty in the data available to a medium-term planner, and the dynamics of the mining environment, such targets may not be feasible in the short-term. We present a decomposition-based heuristic for this short-term scheduling problem in which the grade and quality goals assigned to each mine are collaboratively adapted – ensuring the satisfaction of blending constraints at each port, and exploiting opportunities to maximise production in the network that would otherwise be missed.

Key words: short-term open-pit mine production scheduling, hybrid optimisation, non-linear programming

1. Introduction

We consider the Multiple Mine Planning Problem (MMPP) of scheduling the production of multiple open-pit mines to supply multiple ports with ore that can be blended to form products of a desired composition. The operational objectives of the network, in the short-term, are to maximise the production of such products at each port, while maximising the utilisation of equipment at each mine (Everett 2007). A blend is characterised by its grade, denoting how much of the metal of interest it contains, and its quality, the percentage of a number of impurities in its composition. We consider the open-pit mining of mineral ores that are sold in two granularities – lump and fines – distinguished by their particle size.

A solution to the short-term MMPP schedules the movement of material, from available sources of ore and waste to appropriate destinations, at each mine, and the transport of ore between each mine and port, during each week of a 13 week horizon. We restrict our attention, in this paper, to the single time period (1 week) instantiation of the MMPP, with the full 13 week instantiation forming the basis of future work. At each mine, ore from a variety of sources is processed and blended in a stockyard, producing a consistent grade and quality of ore over the time period. Produced ore is reclaimed from this stockyard onto trains, railed to a port, and blended with ore from other mines to form desired products. An optimal solution to the MMPP requires coordination across the network of mines. The grade and quality of production at each mine must be configured to: ensure the formation of correctly blended products at each port; maximise the productivity of the mine; and maximise the tons of blended products formed across the port system.

Even in the single time period case, the MMPP is a difficult problem. Ore produced at each mine passes through two blending processes: an intermediate stage of blending in the stockyard of the mine; and the downstream blending of this material into final products. The presence of pooling behaviour in the mining supply chain introduces non-linearities into its mathematical modelling (Floudas and Aggarwal 1990, Greenberg 1995, Audet et al. 2004, Misener and Floudas 2009). The single time period, short-term MMPP can thus be modelled as a non-linear mixed integer program (MINLP), containing non-linear constraints that characterise the chemistry of production across the network of mines.

We present a non-linear mixed integer program (MINLP) modelling of the single time period, short-term MMPP. This model is a bilinear program – involving the product of two continuous variables in its constraints – similar in structure to a pooling problem (Haverly 1978, Audet et al. 2004, Meyer and Floudas 2006, Misener and Floudas 2009, Alfaki 2012). We apply various techniques to solve this MINLP, including those previously applied to pooling problems, on an 8-mine, 2-port network, constructed using data provided by an industry partner. Expressing and solving the MMPP in terms of a single MINLP proves to be inadequate: prohibitive in the time required to find high quality solutions; and ill equipped to manage increased complexity in the network and extension of the planning horizon to 13 weeks. To overcome this, we develop a decomposition-based heuristic for solving the MMPP, and compare its solutions to those obtained via the MINLP model.

Inspired by the agent-based decomposition of supply chains across a variety of domains (Shen et al. 2006, Frayet et al. 2007, Leitao 2009), we decompose the problem of scheduling the movement of material at each mine, and the transport of ore between each mine and port, into a set of smaller problems – each associated with a decision-making entity in the network: a mine, or the set of ports. This decomposition splits the problem, along its non-linear constraints, into a linear problem for each mine, and the port system.

Let $m \in \mathcal{M}$ denote a mine m in a set of mines $\mathcal{M}$, and $\pi \in \Pi$ a port π in a set of ports Π. We formulate an optimisation problem for each mine, O_m, in which a mixed integer program (MIP) is solved to determine the set of ore sources (which we call blocks) to be extracted at mine m, over the relevant time period, while maximising its productivity. We define a measure of productivity that captures production (involving the utilisation of processing equipment, plants and mills) and transportation (involving the utilisation of trucking resources). The discretisation of the material available for extraction at a mine into 'blocks' is described in detail in Section 2. Each O_m is solved to produce N solutions (or schedules), across which the chemistry of produced ore is clustered about a point, provided as input, in the space of producible grade-quality combinations. An optimisation problem for the port system, O_Π, is designed to receive, as input, N solutions to each O_m. Formulated as a MIP, a solution to O_Π characterises the flow of ore between each mine and port, and defines which of the N solutions to each O_m is to be enacted at mine m. The objective in this blending problem is to form lump and fines products at each port whose composition does not deviate from desired bounds on grade and quality, and whose sale maximises revenue – a product of the tons of each blend produced and its sale value.

We propose a heuristic in which the solving of each O_m, followed by O_Π, is iterated – yielding a sequence of improving solutions to the single period, short-term MMPP. Each solution defines a block extraction schedule to be followed at each mine, and a routing of trains from each mine to port. O_Π provides, as an output, grade and quality profiles to form the input to each O_m in the next iteration. These profiles denote the composition of the ore produced by each mine in the best solution found by O_Π across all prior iterations. Each mine is, in this way, guided toward finding solutions to its optimisation problem that allow each port to form correctly blended products, while maximising revenue.

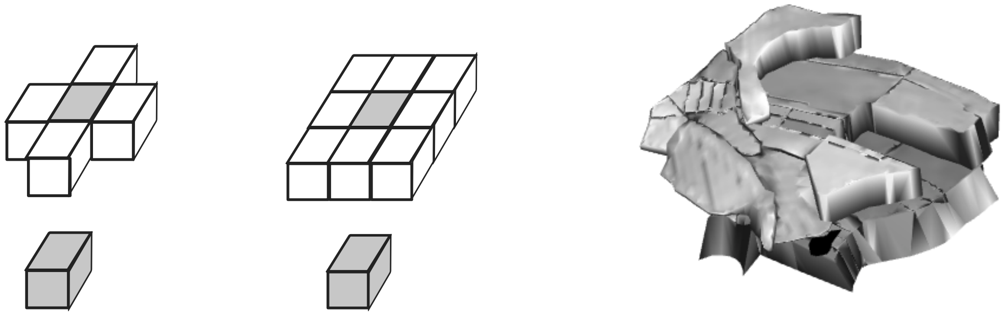
The key contribution of this paper is a novel methodology for production scheduling in supply chains with multiple producers and a downstream blending component. This type of problem appears in many domains, including: the mining of natural resources (such as iron ore and coal); the scheduling of operations in offshore oil fields (Iyer and Grossmann 1998, van den Heever and Grossmann 2000, Neiro and Pinto 2004); and production planning in natural gas supply chains (Li et al. 2011). While we concentrate on the application of scheduling in open-pit mines, our methodology is well suited to solving large-scale, combinatorially challenging scheduling problems that arise in each of these domains.

The remainder of this paper is structured as follows. In Section 2, we highlight existing work related to the MMPP. We describe the MMPP, and a set of benchmark instances, in Sections 3 and 4. In Section 5, we present a MINLP modelling of the problem, and describe a range of existing solving techniques. We follow with a description of our decomposition-based heuristic for the generation of week-long extraction plans in Section 6, outlining the conditions upon which it terminates, and presenting the MIP models underlying the mine and port optimisation problems. An evaluation of our heuristic is provided in Appendix C.

2. Background and Related Work

An open-pit mine consists of a set of pits, in which horizontal layers of material (benches) have been extracted (from the top down) to form a stepped-wall cavity (Hustrulid and Kuchta 2006). A block model divides each of these benches into a grid of equally-sized blocks, each of which is assigned an estimate of its grade and quality. Long-term (such as life-of-mine) planning at an open-pit mine determines the set of blocks in this model to be extracted, and processed, during each year of the mine's life. Precedences exist between the blocks in this model, defining which blocks must be extracted before others can be accessed. Typically, the 5 (or 9) blocks directly above each block in an orebody block model (see Figure 1a–1b) are its precedences (or predecessors), and must be extracted before it. Such precedences ensure that constraints on the slope of pit walls are respected during mining. Pit walls that are too steep are unstable, and present a risk of slope failure.

In the short-term, portions of the orebody block model(s) at each mine are aggregated into larger units, denoted blast blocks or blast regions. These regions are blasted (via explosives inserted into drill holes) to form the broken stock of the mine – ore and waste that is available and primed for extraction. Blast regions are partitioned into grade blocks – areas of waste, low grade, and high grade ore – on the basis of samples extracted from



(a) (b) (c)

Figure 1 **(a) The 5, and (b) 9, blocks above a block in a block model, and (c) a grade block model.**

drill holes. Figure 1c depicts a grade block characterisation of a portion of an orebody. Each grade block can be viewed as an aggregation of blocks in the orebody or ‘regularised’ block model of a mine. The chemistry of each grade block, however, is determined through the averaging of samples obtained via the drilling of blast blocks, rather than the averaging of less certain estimates associated with blocks in the regularised model. Typically, there is a sufficient quantity of broken stock at a mine to supply its production for 2-3 weeks.

A short-term (13 week) planner selects a number of regions (grade *and* block model blocks) in a mine to be extracted, and the destination of this material (stockpiles or processing plants), during each week of a 13 week period. Grade blocks are scheduled to be mined in the first few weeks of this period, while smaller block model blocks (characterising the portion of the mine’s orebody reachable in the planning horizon) are scheduled in the remainder. These block model blocks will be sampled, blasted, and aggregated into grade blocks before extraction. The grade, quality, and characteristics of each processed block (how a block splits into lump and fines upon processing) determines the composition of the lump and fines ore produced at the mine. This ore is railed to a set of ports, and blended with that of other mines, to form products with defined bounds on grade and impurities.

In practice, such extraction sequences are formed independently at each mine, on the basis of a two year, or medium-term, plan. This plan sets monthly grade and quality targets on mine production – assumed to be both achievable given the estimated composition of material in pit benches, and supportive of port blending constraints. These monthly targets define the chemistry of ore to be produced by a mine during each week of the 13 week horizon. The chemistry of ore available for extraction at a mine is revised through the shorter-term sampling and partitioning of blast blocks. Medium-term targets are formed on the basis more uncertain geological models, and estimated parameters characterising the availability of resources, and the production capability of a mine (Yarmuch and Ortiz 2011). In the short-term, such targets may not be achievable at one or more mine sites, during one or more weeks, jeopardising the production of blended products at each port.

In the literature, the short-term production scheduling problem at open-pit mines has not been widely considered in lieu of the medium- and long-term horizons (Newman et al. 2010). In long-term settings, geometric block models (containing on the order of a million blocks) describe the nature of each ore-body to be mined, while extraction sequences are devised to maximise the net present value (NPV) of a venture (Fricke 2006, Osanloo et al. 2008, Gleixner 2008, Newman et al. 2010, Epstein et al. 2012). The grade blocks scheduled for extraction in the short-term do not conform to a regular gridded structure. Mining precedences among blocks in the same bench become more relevant in this setting, as any extraction schedule must consider how a block can be accessed from the mining face. Espinoza et al. (2012) identify the importance of general representations of precedence in open-pit mining models, allowing the specification of any collection of blocks as the predecessors of another (in contrast to the schemes shown in Figures 1a and 1b) in the MineLib library of open-pit production scheduling problems. The predecessors of a block may vary, however, on the basis of the direction from which it is being approached. Eivazy and Askari-Nasab (2012) generate precedences *a priori* given a fixed mining direction. A MIP modelling of a short-term open-pit mine production scheduling problem is solved, in a range of scenarios, each scenario imposing a different mining direction. In contrast, we support the use of disjunctive precedences among blocks in the same bench in our MINLP modelling of the MMPP (Section 5). In this scheme, blocks that are not directly accessible from the mining face can be accessed by the removal of at least one adjacent block. Gholamnejad (2008) follow a similar approach in the specification of precedences among blocks in a regularised model (of the type shown in Figure 1a–1b), but require three contiguous neighbours of a block, on the same bench, to be removed to allow access.

NPV maximisation is replaced, in the short-term, with the objective of maximising production tons and equipment utilisation. Decisions that determine the costs of mining, such as the number of trucks (fleet size) available in each mine, are made in the medium- to long-term planning horizons. Consequently, the minimisation of operating costs is typically not relevant in the short-term. While some works consider the use of cost minimisation in the short-term scheduling of open-pit mines (see, for example, Eivazy and Askari-Nasab (2012)), the objectives of concern to our industry partner are the maximal production of correctly blended products at each port, and the maximal use of equipment at each mine.

Much existing work on the short- (and, indeed, the long-) term problem considers scheduling in single mine systems (Elbrond and Soumis 1987, Fytas et al. 1993, Chanda and Dagdelen 1995, Smith 1998, Everett 2007, Newman et al. 2007, Martinez and Newman 2012). Consideration of the influence of scheduling decisions at a single mine on its parent system, and the optimisation of such decisions in conjunction with those at other mines, are seen as unaddressed challenges in the production scheduling of open-pit mines (Espinoza et al. 2012). The presence of pooling behaviour in an open-pit supply chain of multiple mines – arising from the blending and stockpiling of ore in a stockyard at each mine (each stockyard representing a 'pool' of ore) – introduces non-linearities into a mathematical modelling of the problem. In Section 5.3, we highlight the relationship between the MMPP and the classic pooling problem (Haverly 1978, Misener and Floudas 2009). In a single mine system, no downstream blending of a mine's production with that of other mines takes place. Such a mine will have defined upper and lower bounds on the range of attributes that constitute the chemistry of produced ore, which can be formulated into linear constraints (Ramazan and Dimitrakopoulos 2004, Osanloo et al. 2008). The determination of what composition of ore each mine should produce to meet the blending requirements of each port occurs only in multiple mine optimisation.

The collaborative adjustment of grade and quality targets assigned to a set of mines, by a longer-term plan, in the generation of short-term plans, can ensure that each mine is assigned weekly goals that can be achieved while maximising both productivity (a measure of ore production and the utilisation of equipment) and the production of correctly blended products at the ports. We propose, in this paper, a decomposition-based heuristic, in which this collaborative adjustment is achieved, to form a week-long extraction plan at each mine in a multiple mine network. To the best of our knowledge, this is the first work to tackle the scheduling of production in multiple open-pit mines, where the grade and quality of ore to be produced by each mine is not known *a priori*, but determined as part of the optimisation. While there exists work in which the mine-to-port transportation problem, in a network of multiple mines and ports, is optimised (Singh et al. 2013), the production of each mine is known *a priori*, in contrast to the problem we tackle in this paper.

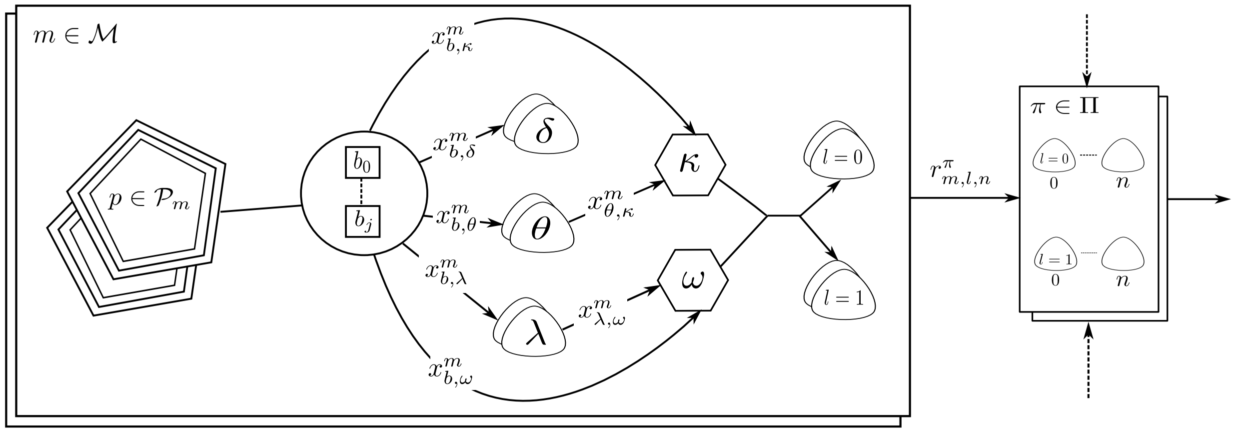


Figure 2 **Flow of material through an open-pit network of mines $\mathcal{M}$ and ports Π, where: $\mathcal{P}_m$ and $b_0 \dots b_j$ denote the set of pits at mine m and blocks within a pit; $x^m_{s,d}$ is a variable denoting the tons of material being transported between source s and destination d; δ, θ, and λ denote a waste dump, high, and low grade stockpile; l refers to a granularity of ore (lump/fines); and $r^\pi_{m,l,n}$ is a variable denoting the number of trainloads of granularity l being transported from mine m to port π to form part of product $n \in N^\pi_l$.**

3. The Multiple Mine Network

We consider a network of mines, $\mathcal{M}$, connected by rail to a port system, Π. At each mine $m \in \mathcal{M}$, ore and waste is extracted from geological regions (known as grade blocks), processed into lump (particle size of approximately 6 to 31 mm) and fines (< 6 mm) granularities, and loaded onto trains to be railed to a port $\pi \in \Pi$. Ore arriving at each port is blended onto stockpiles, from which it is loaded onto ships for delivery to customers. We present a model of this network, detail the constraints that exist on the operation of each mine and port, and define the scheduling problem that we seek to solve for a single time period. Appendix A outlines the meaning of the notation used throughout this section.

Each mine $m \in \mathcal{M}$ contains a set of pits, $\mathcal{P}_m$, and each pit $p \in \mathcal{P}_m$ contains a set of blocks, $\mathcal{B}_{m,p} \subseteq \mathcal{B}_m$, where $\mathcal{B}_m$ denotes the set of blocks available for scheduling at mine m[1]. Each block $b \in \mathcal{B}_m$ has a high ($b \in \mathcal{B}_{m,hg}$), low grade ($b \in \mathcal{B}_{m,lg}$), or waste ($b \in \mathcal{B}_{m,w}$) classification, controlling the destinations at m to which material extracted from b can be transported. Waste is hauled, by truck, to a waste dump ($\delta \in \Delta_m$). High grade ore is hauled to a dry processing plant (κ), or one of a number of high grade stockpiles ($\theta \in \Theta_m$). Low grade ore is hauled to a low grade stockpile ($\lambda \in \Lambda_m$), or a wet processing plant (ω, if one exists at m). Both forms of processing split ore into lump ($l = 0$) and fines ($l = 1$) granularities to be blended in a stockyard. The split of a block $b \in \mathcal{B}_m$ ($S_{m,b,l}$) defines the

[1] As our focus is restricted to the single time period (single week) setting, the set $\mathcal{B}_m$ contains only grade blocks.

percentage of b that will split (upon processing) into granularity $l \in \mathcal{L}$. The set of ore and waste sources at mine m is denoted $\mathcal{S}_m = \mathcal{B}_m \cup \Theta_m \cup \Lambda_m$. The set of destinations to which a source of ore or waste can be transported is denoted $\mathcal{D}_m = \{\kappa, \omega\} \cup \Delta_m \cup \Theta_m \cup \Lambda_m$. Each source $s \in \mathcal{S}_m$ has a tonnage (T^m_s) available for extraction, and a composition defined in terms of the percentage of a set $\mathcal{Q}$ of relevant elements (e.g. metal grade) in its lump and fines components ($G^m_{s,l,q}$ for $q \in \mathcal{Q}$ and $l \in \mathcal{L}$). The crushing and screening of a source $s \in \mathcal{S}_m$ results in a stream of lump and fines ore with a composition $G^m_{s,l,q}$ for $q \in \mathcal{Q}$ and $l = 0$ or 1.

A wet processing plant upgrades (increases the percentage of metal in) low grade ore. Feeds of lump and fines (resulting from a process of crushing and screening ore from a source s) are processed to separate the metal in the mineral of interest from gangue material (worthless material surrounding the metal in ore). The result is a stream of tailings (rejected material) and a concentrate. The tons of valuable metal (and other attributes) in this concentrate is a fraction of that in the input feed of fines or lump (as per a recovery factor $R^{m,\omega}_{s,l,q}$ for $q \in \mathcal{Q}$). The tons of concentrate produced is a fraction of the mass of the input feed (as per a yield factor $Y^{m,\omega}_{s,l}$). This concentrate is blended with the lump and fines produced from the dry processing of high grade ore (see Equation (4), Section 3.1).

Ore can be reclaimed (extracted) from the low and high grade stockpiles at each mine. Reclaimed low grade ore is hauled to the wet processing plant, while reclaimed high grade ore is dry processed. Processed ore from both plants is blended onto lump and fines stockpiles, from which it is transported in T_R ton trainloads to a port $\pi \in \Pi$. Trainloads of ore arriving at each port, $\pi \in \Pi$, are blended to form a set N^π_l of products of each granularity $l \in \mathcal{L}$. Each product $n \in N^\pi_l$ is associated with bounds on its grade and quality, expressed in terms of a lower ($L^{\pi,l}_{n,q}$) and upper ($U^{\pi,l}_{n,q}$) bound on the percentage of each $q \in \mathcal{Q}$.

Figure 2 depicts the flow of mined material from pit to stockyard, and from mine to port. Variables $x^m_{s,d}$ for $s \in \mathcal{S}_m$ and $d \in \mathcal{D}_m$ at mine m denote the tons of each source s extracted and hauled to each of its possible destinations d. Variable $r^\pi_{m,l,n}$ denotes the integer number of trainloads of granularity $l \in \mathcal{L}$ transported by rail from mine m to port π, to be blended into product $n \in N^\pi_l$. Capacity limits exist on the: extraction of material in each pit $p \in \mathcal{P}_m$ (C^m_p tons) on the basis of equipment location; tons of material hauled by truck (C^m_τ); tons of ore processed by the dry (C^m_κ) and wet (C^m_ω) plants; and the tons of each source $s \in \mathcal{S}_m$ available for extraction (T^m_s). Mining precedences constrain the order in which blocks can be extracted at a mine m. $\mathcal{A}^\wedge_{m,b}$ denotes the set of blocks that lie directly above b, *all* of which must be mined before b can be accessed. $\mathcal{A}^{\vee}_{m,b}$ denotes the set of blocks adjacent to b, in the same bench, only *one* of which must be mined before b can be accessed. Minimum production demands (D^m_l) exist on the quantity of each type of ore produced by each mine. The capacity of each port π constrains the quantity of ore handled (C_π), while a lower bound exists on the tons of each product formed ($D^\pi_{l,n}$ for each $n \in N^\pi_l$).

3.1. Calculating Production Tons, Quality Profiles, Productivity, and Revenue

Let $\vec{x}_m$ denote the set of variables $x^m_{s,d}$, for each $s \in \mathcal{S}_m$ and $d \in \mathcal{D}_m$ at mine $m \in \mathcal{M}$; $\vec{x}$ the set of variables $x^m_{s,d}$, for each mine m, $s \in \mathcal{S}_m$ and $d \in \mathcal{D}_m$; $\vec{r}^{\,\pi}_{l,n}$ the set of variables $r^\pi_{m,l,n}$, for each mine m, given granularity $l \in \mathcal{L}$, and product $n \in N^\pi_l$ at port $\pi \in \Pi$; $\vec{r}_\pi$ the set of variables $r^\pi_{m,l,n}$, for each mine m, granularity $l \in \mathcal{L}$, and product $n \in N^\pi_l$ at port $\pi \in \Pi$; and $\vec{r}$ the set of all $r^\pi_{m,l,n}$, for each port π, mine m, granularity $l \in \mathcal{L}$, and product $n \in N^\pi_l$.

Equation (1) defines the tons of granularity $l \in \mathcal{L}$ formed by the processing of ore from source s at mine m, $\tau^m_{s,l}(\vec{x}_m)$. The tons of each granularity produced at m, denoted $\tau^m_l(\vec{x}_m)$, is defined in Equation (2). Equation (3) defines the tons of product $n \in N^\pi_l$, $l \in \mathcal{L}$, formed at port π, given T_R tons in a train.

$$\tau^m_{s,l}(\vec{x}_m) = S_{m,s,l}\left[x^m_{s,\kappa} + x^m_{s,\omega} Y^{m,\omega}_{s,l}\right] \tag{1}$$

$$\tau^m_l(\vec{x}_m) = \sum_{s \in \mathcal{S}_m} S_{m,s,l}\left[x^m_{s,\kappa} + x^m_{s,\omega} Y^{m,\omega}_{s,l}\right] = \sum_{s \in \mathcal{S}_m} \tau^m_{s,l}(\vec{x}_m) \tag{2}$$

$$\tau^\pi_{l,n}(\vec{r}_\pi) = \sum_{m \in \mathcal{M}} r^\pi_{m,l,n} T_R \tag{3}$$

Equations (4)–(5) define the percentage of each $q \in \mathcal{Q}$: in the ore of granularity l produced by mine m, $v^m_{l,q}(\vec{x}_m)$; and in product $n \in N^\pi_l$ formed by port π, $v^\pi_{l,n,q}(\vec{x}, \vec{r}^{\,\pi}_{l,n})$.

$$v^m_{l,q}(\vec{x}_m) = \frac{\sum\limits_{s \in \mathcal{S}_m} S_{m,s,l}\, G^m_{s,l,q}\left[x^m_{s,\kappa} + x^m_{s,\omega} R^{m,\omega}_{s,l,q}\right]}{\sum\limits_{s \in \mathcal{S}_m} S_{m,s,l}\left[x^m_{s,\kappa} + x^m_{s,\omega} Y^{m,\omega}_{s,l}\right]} \tag{4}$$

$$v^\pi_{l,n,q}(\vec{x}, \vec{r}^{\,\pi}_{l,n}) = \frac{\sum\limits_{m \in \mathcal{M}} r^\pi_{m,l,n} v^m_{l,q}(\vec{x}_m) T_R}{\sum\limits_{m \in \mathcal{M}} r^\pi_{m,l,n} T_R} \tag{5}$$

Equation (6) calculates the revenue generated by the sale of ore formed across ports, $\nu(\vec{r})$. $V^\pi_{l,n}$ denotes the sale price per ton for ore of product $n \in N^\pi_l$.

$$\nu(\vec{r}) = \sum_{\pi \in \Pi} \sum_{m \in \mathcal{M}} \sum_{l \in \mathcal{L}} \sum_{n \in N_l^\pi} r^\pi_{m,l,n} T_R V^\pi_{l,n} \tag{6}$$

The total deviation in the blend of products formed across ports from their specification, denoted by bounds $[L^{\pi,l}_{n,q}, U^{\pi,l}_{n,q}]$ for all $\pi \in \Pi$, $l \in \mathcal{L}$, $n \in N^\pi_l$, and $q \in \mathcal{Q}$, is defined as:

$$\eta(\vec{x}, \vec{r}) = \sum_{\pi \in \Pi} \sum_{l \in \mathcal{L}} \sum_{n \in N_l^\pi} \sum_{q \in \mathcal{Q}} \frac{1}{\Delta_q^+} \left[\max\{0, v^\pi_{l,n,q}(\vec{x}, \vec{r}^\pi_{l,n}) - U^{\pi,l}_{n,q}, L^{\pi,l}_{n,q} - v^\pi_{l,n,q}(\vec{x}, \vec{r}^\pi_{l,n})\} \right] \tag{7}$$

where Δ_q^+ denotes a 'significant' change in the percentage of $q \in \mathcal{Q}$ in a body of ore[2]. The value of $\eta(\vec{x}, \vec{r})$ is not a percentage, but a weighted sum of percentage deviations.

We define the productivity of a mine m, $\rho_m(\vec{x}_m)$, in terms of: a weighted sum of the tons of ore, of each granularity, produced by the mine; the tons of waste extracted and transported to a dump; and the tons of ore transported to low and high grade stockpiles. Trucking resources are expected to be utilised for desirable purposes: the transportation of ore to processing plants; and the transportation of waste to a dump. The haulage of high grade ore to stockpiles is an undesirable use of resources, while the haulage of low grade ore to stockpiles is undesirable in mines that have facilities for its upgrade (i.e. it is preferable to send this material directly to the wet processing plant). Let: α_1 and α_2 denote constants such that $\alpha_1 \gg \alpha_2$; and Ψ^m_ω a binary parameter such that $\Psi^m_\omega = 1$ if mine m has the facilities to upgrade low grade ore, and $\Psi^m_\omega = 0$ otherwise. In the instance that $\Psi^m_\omega = 0$, low grade stockpiles are effectively additional dump sites. In this setting, the transport of low grade ore to these stockpiles is not viewed as an undesirable use of trucking resources.

$$\rho_m(\vec{x}_m) = \alpha_1 \sum_{l \in \mathcal{L}} \tau^m_l(\vec{x}_m) + \alpha_2 \sum_{s \in \mathcal{S}_m} \left[\sum_{\delta \in \Delta_m} x^m_{s,\delta} + (1 - 2\Psi^m_w) \sum_{\lambda \in \Lambda_m} x^m_{s,\lambda} - \sum_{\theta \in \Theta_m} x^m_{s,\theta} \right] \tag{8}$$

The measure $\rho_m(\vec{x}_m)$, in Equation (8), is a high level representation of productivity at mine m, in which the behaviour of individual pieces of equipment is not taken into account.

[2] A significant change in the percentage of a metal (such as Iron) in a body of ore may be on the order of 1%, for example, while that of a trace element may be on the order of 0.1% or less.

3.2. The Multiple Mine Planning Problem (MMPP)

Given a network of mines $\mathcal{M}$, ports Π, and parameters (of Appendix A), the MMPP is defined as finding an instantiation of variables $\vec{x} = \{x^m_{s,d} \,|\, m \in \mathcal{M}, s \in \mathcal{S}_m, d \in \mathcal{D}_m\}$ and $\vec{r} = \{r^\pi_{m,l,n} \,|\, m \in \mathcal{M}, \pi \in \Pi, l \in \mathcal{L}, n \in N^\pi_l\}$ that satisfies all relevant constraints (formalised in the MINLP of Section 5). An optimal solution to the MMPP is an instantiation of $\vec{x}$ and $\vec{r}$ for which the objective Z_{MMPP}, shown in Equation (9), is minimised. Let β_1, β_2, and β_3, denote constants such that $\beta_1 \gg \beta_2 \gg \beta_3$. Recall that: $\eta(\vec{x}, \vec{r})$ denotes a measure of the extent to which the composition of each port product deviates from desired bounds, summed over all ports $\pi \in \Pi$, and products $n \in N^\pi_l$ of each granularity $l \in \mathcal{L}$ (Equation (7)); $\nu(\vec{r})$ the revenue generated from the sale of products formed across the system of ports (Equation (6)); and $\rho_m(\vec{x}_m)$ the productivity of mine m (Equation (8)).

$$Z_{MMPP} = \min \ \beta_1 \eta(\vec{x}, \vec{r}) - \beta_2 \nu(\vec{r}) - \beta_3 \sum_{m \in \mathcal{M}} \rho_m(\vec{x}_m) \tag{9}$$

An $\eta(\vec{x}, \vec{r})$ of 0 indicates that the blending constraint set, below, is satisfied at each port $\pi \in \Pi$ over the relevant time period, where $v^\pi_{l,n,q}(\vec{x}, \vec{r}^{\,\pi}_{l,n})$ is defined as in Equation (5).

$$\forall \pi \in \Pi, l \in \mathcal{L}, n \in N^\pi_l, q \in \mathcal{Q} \quad L^{\pi,l}_{n,q} \le v^\pi_{l,n,q}(\vec{x}, \vec{r}^{\,\pi}_{l,n}) \le U^{\pi,l}_{n,q} \tag{10}$$

Products formed at port whose composition deviates from desired bounds typically cannot be sold, except in small quantities, or incur large penalties and loss of reputation.

3.3. Assumptions

We make a number of simplifying assumptions in our modelling of the MMPP. We assume that: waste dumps at each mine have an infinite capacity; the capacity of the rail network is infinite; and material can be both deposited on, and extracted from, a stockpile at a mine over the course of the scheduling horizon, but that only material already on the stockpile at the beginning of the horizon can be reclaimed (we do not consider blending on low and high grade stockpiles at each mine). In practice, each mine is tasked with producing a consistent blend of ore, to be loaded onto arriving and departing trains, over the course of a week-long horizon. We consider a simplified setting in which the average composition of lump and fines produced at a mine m forms the composition of each train departing m to a port. As a topic of future work, we intend to incorporate this blend consistency requirement, and additional practical mining constraints, such as: the feasibility (and desirability) of equipment movement within a pit; minimum bounds on the tons of material left un-mined in a grade block; a bound on available trucking hours (in place of a haulage capacity in tons); and constraints involving the rail network. We assume that an incorrectly blended product produced at a port cannot be sold (no revenue is gained). Hence, we do not model financial penalties for blend deviations or reputation loss, but rather force this deviation to 0 by pushing the blending constraints of Equation (10) into the objective of Equation (9) via the use of a penalty term $\beta_1 \eta(\vec{x}, \vec{r})$, $\beta_1 \gg 1$. In our experience, models generated to represent the MMPP can be solved more efficiently in this setting.

4. An 8-mine, 2-port network

We have constructed a test suite with which to evaluate our decomposition-based heuristic, and contrast its performance with alternative solution methods. These tests define an 8-mine, 2-port network, characterised using data provided by an industry partner. This network represents a currently operating system of open-pit mines that produce over 200 million tons of ore annually. In each test case, we provide each mine with: a set of grade blocks available for extraction, listing their grade, quality profile, and tonnage; the mining precedences that exist between blocks; compositions and sizes for each high and low grade stockpile; and a limit on the tons of material extracted in each pit, and hauled mine-wide.

Test cases have been generated using historical block extraction data obtained for each mine. This data lists the set of grade blocks that have been defined by geologists at each mine, over the period of a year, and the dates by which they have been extracted. Each test case has been generated by selecting a date in the year long period covered by the historical block extraction data, and determining the state of each mine (the grade blocks available for extraction) at this time point. The number of grade blocks available for scheduling at each mine, across the test suite, ranges from 34 to 297. Haulage capacities at each mine, minimum production requirements, port throughput capacities, and blend requirements at each port are fixed across all test cases. In each test, each port produces one product of each granularity ($|N_l^\pi| = 1$ for all $\pi \in \Pi$ and $l \in \mathcal{L}$).

All evaluations presented in this paper have been conducted on a 2.40 GHz Intel Xeon CPU with 8 GB RAM.

5. A MINLP Formulation

We introduce variables $v^m_{l,q}$ and τ^m_l to denote the percentage of attribute $q \in \mathcal{Q}$ in granularity l at the stockyard of mine $m \in \mathcal{M}$, and the tons of granularity $l \in \mathcal{L}$ produced at m, respectively. This allows us to express the total deviation between the achieved composition of each port product and its desired bounds, $\eta(\vec{x}, \vec{r})$ in Equation (7), in a form that can be linearised, and in addition, reduce the number of bilinear terms in the model.

5.1. The Objective

We derive a linearised approximation of Z_{MMPP} in Equation (9) to form the objective of the MINLP. Z_{MMPP} seeks to minimise the total deviation between port product composition and desired bounds, $\eta(\vec{x}, \vec{r})$, as defined in Equation (7). The presence of $v^{\pi}_{l,n,q}(\vec{x}, \vec{r}^{\pi}_{l,n})$, the percentage of $q \in \mathcal{Q}$ in product $n \in N^{\pi}_l$ formed by port π, defined in Equation (5), introduces a non-linear term into the computation of $\eta(\vec{x}, \vec{r})$. We express the bounds $[L^{\pi,l}_{n,q}, U^{\pi,l}_{n,q}]$ on the percentage of each $q \in \mathcal{Q}$ in product $n \in N^{\pi}_l$, in terms of tons. The tons of attribute $q \in \mathcal{Q}$ in product $n \in N^{\pi}_l$ is computed as shown in Equation (11). The variable $v^m_{l,q}$, introduced above, is used to denote the percentage of $q \in \mathcal{Q}$ in ore of granularity $l \in \mathcal{L}$ produced at mine m. Each $r^{\pi}_{m,l,n} v^m_{l,q}$ is the product of an integer and continuous variable, which can be expanded into a sum over products of binary and continuous variables. Each $br^{\pi,j}_{m,l,n}$ is a binary variable whose value is 1 if and only if j trains of granularity l from mine m are scheduled to form part of product $n \in N^{\pi}_l$ at port π. $U_{m,l}$ denotes the maximum number of trainloads of granularity l producible at mine m during the scheduling horizon, and ranges from 2 to 28 across the network of mines in our network (Section 4). Each $br^{\pi,j}_{m,l,n}\, v^m_{l,q}$ is the product of a binary and continuous variable, linearisable via standard techniques.

$$\tau^{\pi}_{l,n,q}(\vec{r}^{\pi}_{l,n}) = \sum_{m \in \mathcal{M}} r^{\pi}_{m,l,n}\, v^m_{l,q}\, T_R = \sum_{m \in \mathcal{M}} \sum_{j=0}^{U_{m,l}} j\, br^{\pi,j}_{m,l,n}\, v^m_{l,q}\, T_R \tag{11}$$

Equation (12) defines our linearised $\eta(\vec{x}, \vec{r})$, denoted $\eta'(\vec{x}, \vec{r})$. We compare the tons of attribute $q \in \mathcal{Q}$ in each product $n \in N^{\pi}_l$ to a lower and upper bound defined by the multiplication of $L^{\pi,l}_{n,q}$ and $U^{\pi,l}_{n,q}$ with the tons of product n formed by port π, $\tau^{\pi}_{l,n}(\vec{r}_{\pi})$. The two alternative measures are not equivalent, but both provide an indication of the extent of deviation between the achieved composition of each port product and its desired bounds.

$$\eta'(\vec{x},\vec{r}) = \sum_{\pi\in\Pi}\sum_{l\in\mathcal{L}}\sum_{n\in N_l^\pi}\sum_{q\in\mathcal{Q}} \frac{1}{\Delta_q^+} \max\{0, \tau_{l,n,q}^{\pi}(\vec{r}_{l,n}^{\pi}) - U_{n,q}^{\pi,l}\tau_{l,n}^{\pi}(\vec{r}_\pi)\} + \sum_{\pi\in\Pi}\sum_{l\in\mathcal{L}}\sum_{n\in N_l^\pi}\sum_{q\in\mathcal{Q}} \frac{1}{\Delta_q^+} \max\{0, L_{n,q}^{\pi,l}\tau_{l,n}^{\pi}(\vec{r}_\pi) - \tau_{l,n,q}^{\pi}(\vec{r}_{l,n}^{\pi})\} \tag{12}$$

Expressing Z_{MMPP} in terms of the deviation measure $\eta'(\vec{x},\vec{r})$ yields the following linear objective function, denoted Z'_{MMPP}. The constants β_1, β_2, and β_3, and the expressions $\nu(\vec{r})$, and $\rho_m(\vec{x}_m)$, are defined as in Section 3.2.

$$Z'_{MMPP} = \min\ \beta_1\eta'(\vec{x},\vec{r}) - \beta_2\nu(\vec{r}) - \beta_3\sum_{m\in\mathcal{M}}\rho_m(\vec{x}_m) \tag{13}$$

5.2. Constraints

Constraints (14)–(15) enforce minimum production demands at: each mine $m \in \mathcal{M}$, denoted D_l^m for each granularity $l \in \mathcal{L}$; and port $\pi \in \Pi$, denoted $D_{l,n}^{\pi}$ for each product $n \in N_l^\pi$, $l \in \mathcal{L}$. Constraint (16) ensures that the tons of each granularity railed from a mine m, to the set of ports, is no more than what has been produced.

$$\tau_l^m \geq D_l^m \qquad \forall\ m\in\mathcal{M}, l\in\mathcal{L}, \tag{14}$$

$$\sum_{m\in\mathcal{M}} T_R\, r_{m,l,n}^{\pi} \geq D_{l,n}^{\pi} \qquad \forall\ \pi\in\Pi, l\in\mathcal{L}, n\in N_l^\pi, \tag{15}$$

$$\sum_{\pi\in\Pi}\sum_{n\in N_l^\pi} T_R\, r_{m,l,n}^{\pi} \leq \tau_l^m \qquad \forall\ m\in\mathcal{M}, l\in\mathcal{L}, \tag{16}$$

The reclamation and placement of material from, and onto, high and low grade stockpiles at a mine is restricted by stockpile capacity C_s^m (Constraint (17)), and the quantity of material on the stockpile, T_s^m, at the start of the scheduling horizon (Constraint (18)).

$$T_s^m - x_{s,\kappa}^m - x_{s,\omega}^m + \sum_{b\in\mathcal{B}_m} x_{b,s}^m \leq C_s^m \qquad \forall\ m\in\mathcal{M}, s\in\Theta_m\cup\Lambda_m, \tag{17}$$

$$x_{s,\kappa}^m + x_{s,\omega}^m \leq T_s^m \qquad \forall\ m\in\mathcal{M}, s\in\Theta_m\cup\Lambda_m, \tag{18}$$

Constraints (19)–(22) ensure that: material moved from each mine pit, $p\in\mathcal{P}_m$, is limited by an extraction capacity, C_p^m; material hauled at the mine is limited by a trucking capacity,

C_τ^m; the processing of ore in the dry and wet plants is within capacity, C_d^m for $d \in \{\kappa, \omega\}$; and the tons of ore railed to each port π is limited by its capacity, C_π.

$$\sum_{b\in\mathcal{B}_p}\sum_{d\in\mathcal{D}_m} x_{b,d}^m \le C_p^m \qquad \forall\, m\in\mathcal{M}, p\in\mathcal{P}_m, \tag{19}$$

$$\sum_{s\in\mathcal{S}_m}\sum_{d\in\mathcal{D}_m} x_{s,d}^m \le C_\tau^m \qquad \forall\, m\in\mathcal{M}, \tag{20}$$

$$\sum_{s\in\mathcal{S}_m} x_{s,d}^m \le C_d^m \qquad \forall\, m\in\mathcal{M}, d\in\{\kappa,\omega\}, \tag{21}$$

$$\sum_{m\in\mathcal{M}}\sum_{l\in\mathcal{L}}\sum_{n\in N_l^\pi} T_R\, r_{m,l,n}^\pi \le C_\pi \qquad \forall\, \pi\in\Pi, \tag{22}$$

Constraints (23)–(24) place bounds on the total material extracted from each grade block, linking variables $x_{b,d}^m$ for $b\in\mathcal{B}_m$ and $d\in\mathcal{D}_m$ to the binary $y_{m,b}^\sigma$ (1 if the mining of b is scheduled) and $y_{m,b}^\tau$ (1 if b is scheduled to be entirely extracted). Note that T_b^m denotes the tons of material remaining in block $b\in\mathcal{B}_m$ at the start of the scheduling horizon. Vertical and disjunctive block precedences are respectively expressed in Constraints (25)–(26).

$$\sum_{d\in\mathcal{D}_m} x_{b,d}^m \le T_b^m\, y_{m,b}^\sigma \qquad \forall\, m\in\mathcal{M}, b\in\mathcal{B}_m, \tag{23}$$

$$\sum_{d\in\mathcal{D}_m} x_{b,d}^m \ge T_b^m\, y_{m,b}^\tau \qquad \forall\, m\in\mathcal{M}, b\in\mathcal{B}_m, \tag{24}$$

$$y_{m,b'}^\tau \ge y_{m,b}^\sigma \qquad \forall\, m\in\mathcal{M}, b\in\mathcal{B}_m, b'\in\mathcal{A}_{m,b}^\wedge, \tag{25}$$

$$\sum_{b'\in\mathcal{A}_{m,b}^\vee} y_{m,b'}^\tau \ge y_{m,b}^\sigma \qquad \forall\, m\in\mathcal{M}, b\in\mathcal{B}_m, \tag{26}$$

Constraint (26) supports the scheduling of drop cuts at each mine m. A drop cut occurs when a set of contiguous (connected) blocks $\mathcal{B}'_m \subset \mathcal{B}_m$, each of which lies on a single bench (horizontal slice of earth), is extracted, despite no block in $\mathcal{B}'_m$ being immediately accessible on the mining face. A block $b'\in\mathcal{B}'_m$ lies on a mining face if $|\mathcal{A}_{m,b'}^\vee| = 0$ (no blocks adjacent to b' need to be removed before b' is accessed). We can ensure that such sets of contiguous blocks, $\mathcal{B}'_m$, are extracted only if there exists a $b'\in\mathcal{B}'_m$ for which $|\mathcal{A}_{m,b'}^\vee| = 0$, avoiding the scheduling of drop cuts, via Constraint (27). We define $\mathcal{P}'(\mathcal{B}_m)$ as the set of all contingous sets of blocks $\mathcal{B}'_m \subset \mathcal{B}_m$ for which $\nexists b'\in\mathcal{B}'_m.\ |\mathcal{A}_{m,b'}^\vee| = 0$; and $\mathcal{N}(\mathcal{B}_m, \mathcal{B}'_m)$ as the set of blocks $b''\in\mathcal{B}_m\backslash\mathcal{B}'_m$ for which $\exists b'\in\mathcal{B}'_m\ .\ (b', b'')\in\mathcal{A}_{m,b'}^\vee$ (ie. the 'neighbours' of set $\mathcal{B}'_m$).

$$\sum_{b'' \in \mathcal{N}(\mathcal{B}_m, \mathcal{B}'_m)} y^{\tau}_{m,b''} \geq \frac{1}{|\mathcal{B}'_m|} \sum_{b' \in \mathcal{B}'_m} y^{\sigma}_{m,b'} \qquad \forall m \in \mathcal{M}, \mathcal{B}'_m \in \mathcal{P}'(\mathcal{B}_m) \tag{27}$$

The set of constraints defined in Equation (27) is too large to be added to the MINLP formulation of the MMPP in its entirety. We use a separation algorithm to detect the presence of drop cuts, in the form of a contiguous set of blocks $\mathcal{B}'_m$, in any solution to the MINLP. Selected instances of Constraint (27) are consequently added to the model as cuts. For brevity, a detailed description of this procedure is omitted from the paper.

Variables $v^m_{l,q}$ and τ^m_l are defined in Constraints (28)–(29). The number of bilinear terms in the model, arising in Constraint (28), is $|\mathcal{M}||\mathcal{L}||\mathcal{Q}|$.

$$v^m_{l,q} \tau^m_l - \sum_{s \in \mathcal{S}_m} S_{m,s,l} G^m_{s,l,q} \left[x^m_{s,\kappa} + x^m_{s,\omega} R^{m,\omega}_{s,l,q} \right] = 0 \qquad \forall\ m \in \mathcal{M}, l \in \mathcal{L}, q \in \mathcal{Q}, \tag{28}$$

$$\tau^m_l - \sum_{s \in \mathcal{S}_m} S_{m,s,l} \left[x^m_{s,\kappa} + x^m_{s,\omega} Y^{m,\omega}_{s,l} \right] = 0 \qquad \forall\ m \in \mathcal{M}, l \in \mathcal{L}, q \in \mathcal{Q}, \tag{29}$$

Constraints (30)–(34) prevent the movement of ore at each mine $m \in \mathcal{M}$ between invalid source $s \in \mathcal{S}_m$ and destination $d \in \mathcal{D}_m$ pairs.

$$x^m_{s,\kappa} = 0 \qquad \forall\ m \in \mathcal{M}, s \in \mathcal{S}_m \backslash \{\mathcal{B}_{m,hg} \cup \Theta_m\}, \tag{30}$$

$$x^m_{s,\omega} = 0 \qquad \forall\ m \in \mathcal{M}, s \in \mathcal{S}_m \backslash \{\mathcal{B}_{m,lg} \cup \Lambda_m\}, \tag{31}$$

$$x^m_{s,\delta} = 0 \qquad \forall\ m \in \mathcal{M}, s \in \mathcal{S}_m \backslash \mathcal{B}_{m,w}, \delta \in \Delta_m, \tag{32}$$

$$x^m_{s,\lambda} = 0 \qquad \forall\ m \in \mathcal{M}, s \in \mathcal{S}_m \backslash \mathcal{B}_{m,lg}, \lambda \in \Lambda_m, \tag{33}$$

$$x^m_{s,\theta} = 0 \qquad \forall\ m \in \mathcal{M}, s \in \mathcal{S}_m \backslash \mathcal{B}_{m,hg}, \theta \in \Theta_m, \tag{34}$$

Constraints (35)–(37) restrict the values of: variables $x^m_{s,d}$, τ^m_l, and $v^m_{l,q}$, to non-negative reals; indicators $y^{\tau}_{m,b}$ and $y^{\sigma}_{m,b}$ to binary values; and variables $r^{\pi}_{m,l,n}$ to non-negative integers.

$$x^m_{s,d}, \tau^m_l, v^m_{l,q} \in \mathbf{R}^+ \cup \{0\} \qquad \forall\ m \in \mathcal{M}, s \in \mathcal{S}_m, d \in \mathcal{D}_m, \tag{35}$$

$$y^{\tau}_{m,b}, y^{\sigma}_{m,b} \in \{0, 1\} \qquad \forall\ m \in \mathcal{M}, b \in \mathcal{B}_m, \tag{36}$$

$$r^{\pi}_{m,l,n} \in \mathbf{Z}^+ \cup \{0\} \qquad \forall\ m \in \mathcal{M}, \pi \in \Pi, l \in \mathcal{L}, n \in N^{\tau}_l. \tag{37}$$

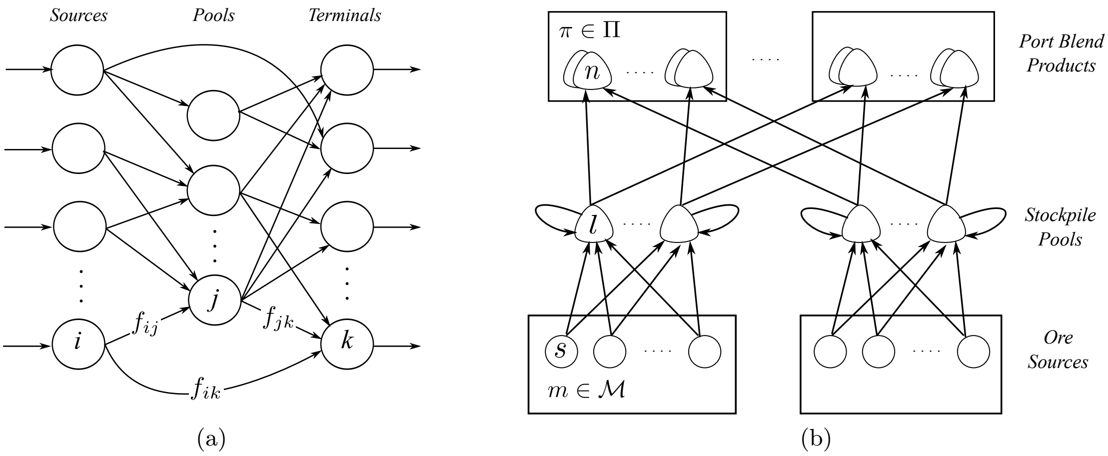


Figure 3 (a) An example of a pooling problem, and (b) the MMPP formulated as a pooling problem.

5.3. Bilinearity and the Pooling Problem

The structure of the MMPP is similar to that of a pooling problem. The pooling problem (Haverly 1978) models the blending of materials in a feed forward network of source nodes, intermediate blending pools, and terminal or product nodes (Figure 3a). Material streams, with defined quality attributes, flow along arcs in the network: from source nodes into blending pools; from blending pools into one of a number of terminal nodes; and from source nodes into terminals. The flow from, and to, sources, pools, and terminals, is limited by network capacities, while conservation constraints ensure that the quality of each stream leaving a blending pool is that of the combined quality of streams entering it. Optimisation of the pooling network determines the rate of flow along each arc, such that profit is maximised in the formation of blended products at terminals, and market demands on their quality are satisfied (Misener and Floudas 2009). The pooling problem arises in various domains, including: the refinement of oil and fuel (Amos et al. 1997); the transportation of natural gas (Romo et al. 2009); and waste water treatment (Misener and Floudas 2010).

The optimisation of our multiple mine network can be viewed, on a conceptual level, as a kind of pooling problem, with: each source of ore at each mine m, $s \in \mathcal{S}_m$, denoting a source node; stockpiles of lump and fines ore at each mine denoting blending pools; and the blended products formed at each port denoting terminals (Figure 3b). Ore flowing from a stockpile pool to port product nodes need not balance with that flowing into the pool as in a traditional pooling network – some material may remain stockpiled at each mine. Instances of the pooling problem in the blending of oil, water, and gas, are problems different to the MMPP. However, these problems can all be modelled as a MINLP with bilinear terms characterising the composition of a blend of material from various sources.

5.4. Solving MINLPs with Bilinear Terms

We consider several approaches for the solution of MINLPs with bilinear terms. Much work in this space has concentrated on the generation of tight lower bounds (for MINLPs with a minimisation objective) for use in a branch and bound algorithm. Most popular are linear (McCormick 1976, Al-Khayyal and Falk 1983) and piecewise-linear (Meyer and Floudas 2006, Bergamini et al. 2008, Wicaksono and Karimi 2008, Gounaris et al. 2009, Hasan and Karimi 2010) relaxations. A linear relaxation of a MINLP with bilinear terms can be obtained by replacing each of these terms with its convex envelope (McCormick 1976). Piecewise-linear relaxations partition the domain of one or both variables in each bilinear term into segments of uniform or varying length, generating a linear relaxation of the term in each of these segments. Gounaris et al. (2009) presents and computationally compares a range of such relaxations. Adhya et al. (1999) alternatively solves the Lagrangian dual of a bilinear program (BLP) for the determination of lower bounds during branch and bound.

A range of decomposition-based approaches split a MINLP (or NLP) into two subproblems, a primal and a dual (or master) problem, and apply Generalised Benders' Decomposition (Geoffrion 1972) to search for a global optimal solution (Floudas et al. 1989, Floudas and Aggarwal 1990, Floudas and Visweswaran 1990, Visweswaran and Floudas 1993). The primal problem is the original MINLP with fixed values for a set of complicating variables – variables that reduce the MINLP to a MIP when fixed. The master problem is the Lagrangian dual of the primal – its solution providing a lower bound on the global optimum; and values for the complicating variables of the non-linear problem. A solution to the primal problem provides an upper bound on this optimum, constraints (or cuts) to add to the master problem, and values for its Lagrangian multipliers. Algorithms that employ this decomposition, iterate between the solving of the primal and master problems, and terminate at a global optimum when the discovered upper and lower bounds converge.

Kolodziej and Grossmann (2012), Kolodziej et al. (2013) and Pham et al. (2009) present algorithms for the solution of multi-period blending problems, expressed as MINLPs with bilinear terms, that perform a similar iteration over upper and lower bounding subproblems. The original MINLP is transformed into a MIP via the discretisation of the domain of the complicating variables (a set containing one variable from each bilinear term). These variables can be assigned only one of a finite set of values, yielding a problem whose feasible region is smaller than that of the MINLP. The solution of the resulting MIP provides an upper bound on the global optimum of the MINLP (under the assumption that its objective is to be minimised). A piecewise-linear relaxation of the the MINLP yields a lower bounding problem. Kolodziej and Grossmann (2012) and Kolodziej et al. (2013) define several global optimisation methods in which the solving of these two problems is iterated in the search for a global optimum. Pham et al. (2009) present a heuristic, for bilinear programs (BLPs) with maximisation objectives, that combines iterative partitioning of the domain of bilinear variables, and the solving of lower (via discretisation) and upper (via linear relaxation) bounding problems to prune partitions from consideration.

Audet et al. (2004) present an iterative heuristic (ALT) for solving general BLPs, in which a series of LPs are generated by alternately fixing two sets of variables. These two sets denote the set of x and y variables that appear in each bilinear term, xy. Given an initial feasible value for each x variable, the solution of the LP obtained by fixing each x to its initial value yields a set of feasible values for each y variable. The fixing of each y to its value in this LP solution, yields another LP, whose solution provides new instantiations for each x. Repeating this process of variable-fix-and-solve until the values of our x or y variables converge to a fixed point in successive solves, produces a local optimum.

Successive linear programming (SLP), in which the non-linear terms in a MINLP are replaced by their linear Taylor expansion (about a base point), has achieved some success when applied to pooling problems (Palacios-Gomez et al. 1982, Baker and Lasdon 1985, Sarker and Gunn 1997). An initial feasible solution to a MINLP with bilinear terms forms a base point about which the linear Taylor expansion of each term is obtained. The solution of the resulting MIP is consequently used as the base point about which a new MIP is generated, again replacing each bilinear term with its linear Taylor expansion. This iterative process continues until we converge to a fixed point, forming our MINLP solution.

In Section C.1 we solve a series of linear relaxations of the MINLP generated in each of our benchmark tests. We first replace each bilinear term with its convex envelope (McCormick 1976) to obtain a lower bound on the objective in each test. We additionally generate and solve several piecewise-linear relaxations (Gounaris et al. 2009), of increasing fidelity, of the model. Due to discrepancies between the evaluation of port product composition in these relaxed models, and their actual composition, port products were not correctly blended in the obtained solutions. We use the magnitude of these discrepancies to narrow the bounds describing desired product composition, and resolve the piecewise-linear relaxed models. The composition of port products in the resulting solutions lie within the original bounds. Lower bounds obtained on the MINLP objective, and the quality of solutions found via the use of piecewise-linear relaxation and the ALT heuristic (Section C.3), are used to evaluate our decomposition-based heuristic in Appendix C. Solving our MINLP using the branch-and-bound-based Couenne (Belotti et al. 2009) and Bonmin (Bonami et al. 2008) solvers[3] did not provide solutions within a 12 hour time frame. The SLP heuristic, implemented as in Baker and Lasdon (1985), could not form solutions in which port products were correctly blended, in any of our tests, with deviations in metal percentage of up to 2% from desired bounds present in the solution set. These results have been omitted from the paper.

6. A Decomposition-Based Heuristic

We decompose the MMPP into a set of sub-problems, consisting of: an optimisation problem, O_m, to be solved on behalf of each mine $m \in \mathcal{M}$; and an optimisation problem, O_Π, to be solved on behalf of the system of ports, Π. We describe how the input and output of this set of problems is used, in an iterative heuristic, to find a monotonically improving sequence of solutions to the MMPP. Each of these solutions defines a value for each variable in the set $\vec{x} \cup \vec{r}$, where: $\vec{x} = \{x^m_{s,d} \,|\, m \in \mathcal{M}, s \in \mathcal{S}_m, d \in \mathcal{D}_m\}$ characterises the flow of ore and waste between sources and destinations at each mine; and $\vec{r} = \{r^\pi_{m,l,n} \,|\, m \in \mathcal{M}, \pi \in \Pi, l \in \mathcal{L}, n \in N^\pi_l\}$ characterises the railing of ore between each mine and port. Each such solution satisfies the constraints, and represents a feasible solution, of our MINLP model of the MMPP in Section 5. Our decomposition-based heuristic finds solutions to the MMPP whose quality (evaluation of the MINLP objective Z'_{MMPP} in Equation (13) with respect to the values of variables $\vec{x} \cup \vec{r}$ in each solution) is competitive with that of the best performing alternatives in Section 5. Moreover, our heuristic discovers a solution in a fraction of the time used by these alternatives to find a solution of comparable quality.

Sections 6.1 and 6.2 describe the mine- and port-side optimisation problems that form the basis of an iterative heuristic, outlined in Section 6.3 and summarised in Listing 1.

[3] The simple branch-and-bound algorithm, with increased values for the num_resolve_at_root and num_resolve_at_node options, was used when solving with Bonmin – as recommended for non-convex MINLPs.

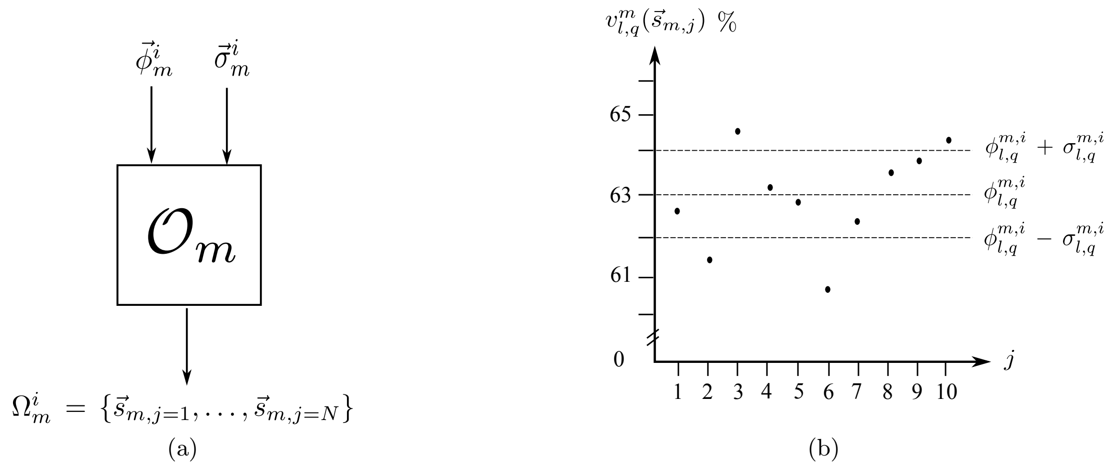


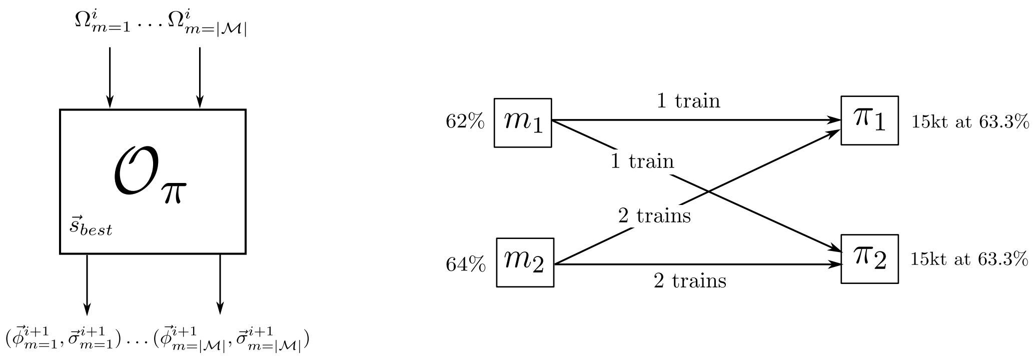
Figure 4 **(a) Each mine-side optimisation problem, O_m, takes as input a grade and quality target, $\vec{\phi}^i_m$, and a set of standard deviations, $\vec{\sigma}^i_m$, producing N productivity-maximising schedules for mine m as an output. (b) A plot of the percentage of attribute q in ore produced by mine m in each schedule $\vec{s}_{m,j}$ ($v^m_{l,q}(\vec{s}_{m,j})$) formed by a solve of problem O_m, given the target $\vec{\phi}^i_m$ and standard deviation $\vec{\sigma}^i_m$ as input.**

6.1. The O_m Problem

Each O_m is formulated to find, in each iteration i of the heuristic, a set of N schedules, denoted Ω^i_m, available for implementation at mine m over the scheduling horizon. Each schedule $\vec{s}_m \in \Omega^i_m$ instantiates the variables in the set $\vec{x}_m = \{x^m_{s,d} \,|\, s \in \mathcal{S}_m, d \in \mathcal{D}_m\}$, characterising the flow of ore and waste between each source and destination at m. The result of a schedule $\vec{s}_m$ is the production of a quantity of ore of each granularity $l \in \mathcal{L}$, denoted $\tau^m_l(\vec{s}_m)$, whose composition is defined in terms of the percentage of each attribute $q \in \mathcal{Q}$, denoted $v^m_{l,q}(\vec{s}_m)$. The value of each variable $x^m_{s,d} \in \vec{x}_m$ in $\vec{s}_m$ is denoted $x^m_{s,d}(\vec{s}_m)$.

The input to O_m, in each iteration i, is a grade and quality target $\vec{\phi}^i_m = \{\phi^{m,i}_{l,q} | \forall l \in \mathcal{L}, q \in \mathcal{Q}\}$, defining the expected composition of the ore to be produced by m, and a set of standard deviations $\vec{\sigma}^i_m = \{\sigma^{m,i}_{l,q} | \forall l \in \mathcal{L}, q \in \mathcal{Q}\}$. The objective of O_m is to form a schedule set Ω^i_m for which: the productivity of m is maximised; and the composition of ore produced in each schedule lies in a normal distribution with mean $\vec{\phi}^i_m$ and standard deviation $\vec{\sigma}^i_m$ (see Figure 4). The productivity of a mine m, given an instantiation of $\vec{x}_m$, is calculated as per Equation (8). The productivity of m in schedule $\vec{s}_m$ is denoted $\rho(\vec{s}_m)$.

Example 6.1 *Consider a mine m that produces a single granularity of ore l. The composition of this ore is characterised by a single quality attribute q, denoting metal grade. O_m is given a target of 63% metal, with a standard deviation of 1%, as input in iteration i. Let $N = 10$. Figure 4b plots the percentage of metal in the ore produced by m in each of the 10*



(a) (b)

Figure 5 **(a)** O_Π**: is given a schedule set** Ω^i_m **by each** O_m**; selects a schedule in each** $\Omega^i_m \cup \{\vec{s}_{best,m}\}$ **to be enacted; and routes trains of ore from each mine to port, forming a solution** $\vec{s}_i$ **to the MMPP.** O_Π **produces a grade and quality target** $\vec{\phi}^{i+1}_m$ **and standard deviation** $\vec{\sigma}^{i+1}_m$ **to be given to each** O_m **in iteration** $i+1$.

schedules in a possible solution of O_m. The schedules formed by O_m are distinguished on the horizontal axis of the plot (with index j). The vertical axis denotes metal percentage.

A formulation of O_m as a MIP is presented in Section 6.4.

6.2. The O_Π Problem

The port-side optimisation problem O_Π is formulated to: accept a schedule set, Ω^i_m, from each O_m in each iteration i; select *one* schedule from each Ω^i_m, denoted $\Pi(\Omega^i_m)$, to be implemented at mine m; and determine the number of trainloads of ore, of each granularity $l \in \mathcal{L}$, from each mine, that will be railed to a port π to form part of a product $n \in N^\pi_l$. A solution to O_Π, denoted $\vec{s}_i$, instantiates each variable in the set $\vec{x} \cup \vec{r}$. Recall that $\vec{x} = \{x^m_{s,d} \,|\, s \in \mathcal{S}_m, d \in \mathcal{D}_m\}$ defines the flow of material from source to destination at each mine, while $\vec{r} = \{r^\pi_{m,l,n} \,|\, m \in \mathcal{M}, \pi \in \Pi, l \in \mathcal{L}, n \in N^\pi_l\}$ defines the flow of ore between each mine, port, and port product. The selection of a schedule to be enacted at each mine instantiates the variable set $\vec{x}$, while the routing of trains between each mine and port, and the selection of a product to which they will contribute, instantiates the variable set $\vec{r}$. The value of each variable $x^m_{s,d} \in \vec{x}$ in solution $\vec{s}_i$ is denoted $x^m_{s,d}(\vec{s}_i)$. The value of each variable $r^\pi_{m,l,n} \in \vec{r}$ in solution $\vec{s}_i$ is denoted $r^\pi_{m,l,n}(\vec{s}_i)$.

The objective of O_Π is to select a schedule to be followed at each mine, and organise the transport of ore produced in those schedules from mine to port, and port product, such that: the deviation between the composition of each port product and its desired bounds is minimised (as a first priority); the revenue generated from the sale of such products is maximised (as a second priority); and the productivity of each mine is maximised (as a third priority). O_Π evaluates a solution $\vec{s}_i$ by computing the value of the MINLP objective Z'_{MMPP} in Equation (13) with respect to the instantiation of variables $\vec{x}$ and $\vec{r}$ in $\vec{s}_i$.

O_Π maintains a record of the best solution it has found over the course of the heuristic, denoted $\vec{s}_{best}$. This solution is replaced with $\vec{s}_i$ if and only if $\vec{s}_i$ has a lower objective value. O_Π produces, as output, a grade and quality target $\vec{\phi}_m^{i+1}$ and standard deviation $\vec{\sigma}_m^{i+1}$ to be given to each O_m, as input, in iteration $i+1$ (see Figure 5). The manner in which each $\vec{\phi}_m^{i+1}$ and $\vec{\sigma}_m^{i+1}$ is formed, and the purpose of this feedback, is described in Section 6.3.

To ensure the generation of a monotonically improving (in objective value) sequence of solutions to the MMPP, we alter our earlier description of O_Π's behaviour as follows. Given a set of schedules, Ω_m^i, from each O_m in iteration i, O_Π selects one schedule from each $\Omega_m^i \cup \{\vec{s}_{best,m}\}$, denoted $\Pi(\Omega_m^i \cup \{\vec{s}_{best,m}\})$, to be implemented at mine m, where $\vec{s}_{best,m}$ denotes the schedule assigned to m in the best found solution $\vec{s}_{best}$. The objective value of the solution formed by O_Π in iteration i will therefore be at least as good as that of $\vec{s}_{best}$.

Example 6.2 *Consider a system of two mines, m_1 and m_2. $\mathcal{O}_{m_1}$ and $\mathcal{O}_{m_2}$ have each formed two schedules to be presented to O_Π in iteration i. These schedules are denoted $\Omega_{m_1}^i = \{\vec{s}_{m_1,1}, \vec{s}_{m_1,2}\}$ and $\Omega_{m_2}^i = \{\vec{s}_{m_2,1}, \vec{s}_{m_2,2}\}$. Each mine produces ore of a single granularity l, characterised by a single quality attribute q, denoting metal grade. Schedules $\vec{s}_{m_1,1}$ and $\vec{s}_{m_1,2}$ produce 10kt and 15kt at a grade of 62% and 60%, respectively. Schedules $\vec{s}_{m_2,1}$ and $\vec{s}_{m_2,2}$ produce 15kt and 20kt at a grade of 61% and 64%, respectively. Each train transports 5kt of ore between a mine and one of two ports, π_1 and π_2, each of which produces a single product of granularity l. In Figure 5b, O_Π has selected: schedule $\vec{s}_{m_1,1}$ and $\vec{s}_{m_2,2}$ to be implemented at mines m_1 and m_2; 1 train of ore to be routed from mine m_1 to each port; and 2 trains of ore to be routed from mine m_2 to each port. In the MMPP solution formed by O_Π, $\vec{s}_i$, 15kt of blended ore, with a metal grade of 63.3%, is formed at both ports.*

A formulation of O_Π as a MIP is presented in Section 6.5.

6.3. The Heuristic

Our decomposition-based heuristic (Listing 1) repeats a two-stage process – the solving of each O_m followed by O_Π – in a sequence of iterations. Each iteration i results in a solution $\vec{s}_i$ to the MMPP. Let: $\vec{\phi}_m^1 = \Xi_m$ and $\vec{\sigma}_m^1 = \vec{\sigma}^+ = \{\sigma_{l,q}^+ = \Delta_q^+ | \forall l \in \mathcal{L}, q \in \mathcal{Q}\}$, for each mine m, where Ξ_m denotes the grade and quality target assigned to m, by a longer-term (two year) plan, and Δ_q^+ a significant change in the percentage of $q \in \mathcal{Q}$ in a volume of ore.

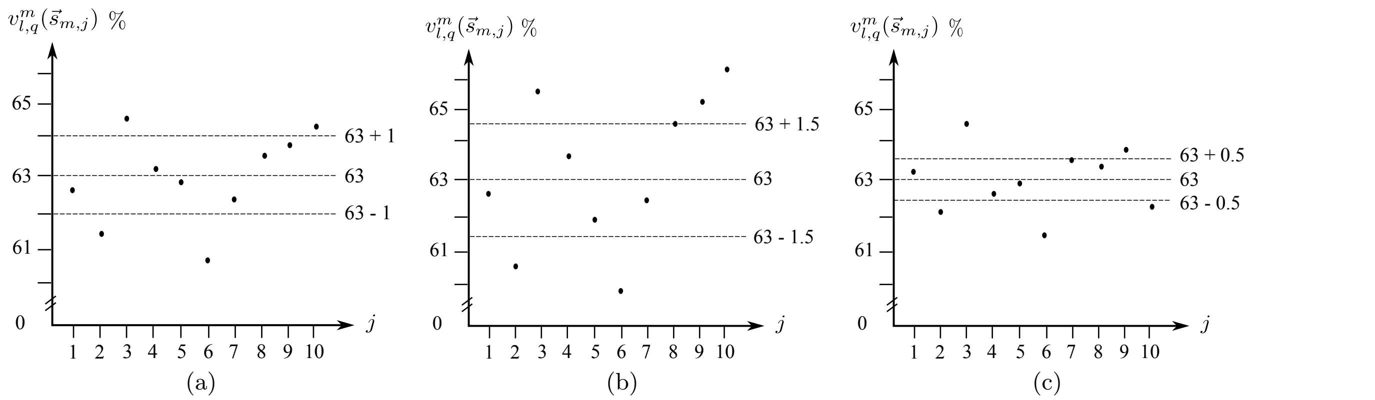


Figure 6 **A mine-side optimiser O_m forms a set of $N = 10$ schedules for a mine m, producing ore of a single granularity l, characterised by a single quality attribute q, given varying $\vec{\phi}^i_m$ and $\vec{\sigma}^i_m$ in iteration i: (a) $\vec{\phi}^i_m = \{63\}$ and $\vec{\sigma}^i_m = \{1\}$; (b) $\vec{\phi}^i_m = \{63\}$ and $\vec{\sigma}^i_m = \{1.5\}$; and (c) $\vec{\phi}^i_m = \{63\}$ and $\vec{\sigma}^i_m = \{0.5\}$.**

The set of standard deviations given to each mine in this first iteration, $\vec{\sigma}^1_m$, is designed to promote a substantial degree of diversity in the composition of produced ore, across the set of schedules formed by O_m. A set of larger standard deviations will result in schedules for which the composition of produced ore exhibits a greater range of values, in each attribute, across the schedule set. A smaller $\vec{\sigma}^1_m$ will result in the formation of schedules for which the composition of produced ore is more tightly clustered about $\vec{\phi}^i_m$ (see Figure 6).

A solution to each O_m, in iteration i, is a set of N schedules for mine m, Ω^i_m, to be implemented over the relevant scheduling horizon (Step 7). O_Π receives as input the set Ω^i_m from each m. O_Π maintains a record of the best solution, $\vec{s}_{best}$, it has found to the MMPP over all prior iterations. In the first iteration, this record is empty. O_Π selects: one schedule in the set $\Omega^i_m \cup \{\vec{s}_{best,m}\}$ to be enacted at mine m (Step 8), where $\vec{s}_{best,m}$ is the schedule assigned to m in the solution $\vec{s}_{best}$; and the number of trains of ore, of each granularity $l \in \mathcal{L}$, produced by m in that schedule to form part of each product $n \in N^\pi_l$, at each port $\pi \in \Pi$. Let $Z'_{MMPP}(\vec{s}_i)$ denote the value of objective Z'_{MMPP} (Equation (13)) in solution $\vec{s}_i$. O_Π replaces $\vec{s}_{best}$ with $\vec{s}_i$ if and only if $Z'_{MMPP}(\vec{s}_i) < Z'_{MMPP}(\vec{s}_{best})$ (Step 9).

O_Π provides each O_m with feedback in the form of a grade and quality target $\vec{\phi}^{i+1}_m$, and a set of standard deviations $\vec{\sigma}^{i+1}_m$, as its input in iteration $i+1$ (Step 10). The role of this feedback is to guide each O_m toward the presentation of schedules that allow O_Π to form a solution that improves upon the current best, $\vec{s}_{best}$. Table 1 defines the three heuristic rules by which $\vec{\phi}^{i+1}_m$ and $\vec{\sigma}^{i+1}_m$ are generated for each mine m. Each rule is defined in terms of a set of conditions on the solution $\vec{s}_i$ formed by O_Π, and a set of equations that define $\vec{\phi}^{i+1}_m$ and $\vec{\sigma}^{i+1}_m$ at each mine if those conditions are satisfied. More sophisticated techniques for

Listing 1 A decomposition-based heuristic for the MMPP, where: Δ_q^+ denotes a significant change in $q \in \mathcal{Q}$ percentage; and Ξ_m a longer-term (two year) grade and quality target assigned to mine $m \in \mathcal{M}$.

$\vec{s}_{best} \leftarrow \emptyset$
$\vec{\sigma}^+ \leftarrow \{\sigma_{l,q}^+ = \Delta_q^+ | \forall l \in \mathcal{L}, q \in \mathcal{Q}\}$
$\vec{\sigma}^- \leftarrow \{\sigma_{l,q}^- = \Delta_q^- | \forall l \in \mathcal{L}, q \in \mathcal{Q}\}$
$i \leftarrow 1$
Initialise expected mine targets and standard deviation sets: $\vec{\phi}_m^i \leftarrow \Xi_m$ and $\vec{\sigma}_m^i \leftarrow \vec{\sigma}^+$.
repeat
    Solve each O_m to find N schedules for mine m, Ω_m^i, producing ore whose composition is normally distributed about $\vec{\phi}_m^i$ with standard deviation $\vec{\sigma}_m^i$.
    Solve O_Π given sets $\Omega_m^i \cup \{\vec{s}_{best,m}\}$ from each $m \in \mathcal{M}$, where $\vec{s}_{best,m} \in \vec{s}_{best}$ is the schedule to be enacted by m in the best solution found thus far. Select a schedule to be enacted at each mine, and a routing of trainloads of ore from each mine to port, forming a solution $\vec{s}_i$ to the MMPP.
    Update best solution $\vec{s}_{best}$ if and only if $Z'_{MMPP}(\vec{s}_i) < Z'_{MMPP}(\vec{s}_{best})$.
    Generate feedback to each O_m by adapting $\vec{\phi}_m^i$ and $\vec{\sigma}_m^i$ to form $\vec{\phi}_m^{i+1}$ and $\vec{\sigma}_m^{i+1}$.
    $i \leftarrow i + 1$
until $[Z'_{MMPP}(\vec{s}_i) \geq Z'_{MMPP}(\vec{s}_{best}) \wedge \nexists m \in \mathcal{M}.\ \vec{\sigma}_m^i \neq \vec{\sigma}^-] \vee i > MAX_{iterations}$
return $\vec{s}_{best}$

adapting the targets and standard deviations assigned to each mine are certainly possible, however these simple rules were found to perform well in computational experiments.

The first rule in Table 1 states that if O_Π does not find a solution better than $\vec{s}_{best}$ in iteration i, the grade and quality targets assigned to each mine remain the same, $\vec{\phi}_m^{i+1} = \vec{\phi}_m^i$, but its assigned set of standard deviations is reduced by a pre-determined factor γ, $\vec{\sigma}_m^{i+1} = \gamma\, \vec{\sigma}_m^i$, where $0 < \gamma < 1$. The assumption is that as target $\vec{\phi}_m^i$ is produced by mine m in the current best solution, $\vec{s}_{best}$, there may be a target in the neighbourhood of $\vec{\phi}_m^i$ that, if produced, will yield an improved solution. As such a schedule was not formed by O_m in iteration i, it may be the case that it was concentrating on achieving too large a spread in the composition of produced ore about $\vec{\phi}_m^i$. Reducing each $\vec{\sigma}_m^i$ forces each mine to propose schedules for which the composition of produced ore is more tightly clustered about $\vec{\phi}_m^i$.

The second and third rules in Table 1 are implemented when a new $\vec{s}_{best}$ is discovered by the port-side optimiser in an iteration i. In both rules, the grade and quality target assigned to each mine m, in iteration $i + 1$, is equal to the composition of ore produced by m in solution $\vec{s}_i$, $\vec{\phi}_m^{i+1} = \{v_{l,q}^m(\vec{s}_i)\, |\, \forall\ l \in \mathcal{L}, q \in \mathcal{Q}\}$. The assumption is that as each target $\vec{\phi}_m^{i+1}$ is produced by mine m in what is now the current best solution, $\vec{s}_i$, there may be a target in a neighbourhood of each $\vec{\phi}_m^{i+1}$ that, if produced by m, will improve upon $\vec{s}_i$.

#	Condition	Feedback	
1	$Z'_{MMPP}(\vec{s}_i) \geq Z'_{MMPP}(\vec{s}_{best})$	$\vec{\phi}^{i+1}_m = \vec{\phi}^i_m,$	$\forall\ m \in \mathcal{M}$
		$\vec{\sigma}^{i+1}_m = max(\vec{\sigma}^-, \gamma\,\vec{\sigma}^i_m),$	$\forall\ m \in \mathcal{M}$
2	$Z'_{MMPP}(\vec{s}_i) < Z'_{MMPP}(\vec{s}_{best})$	$\vec{\phi}^{i+1}_m = \{v^m_{l,q}(\vec{s}_i) \mid \forall\ l \in \mathcal{L}, q \in \mathcal{Q}\},$	$m \in \mathcal{M}$
	$\exists l \in \mathcal{L}, q \in \mathcal{Q}. \lvert v^m_{l,q}(\vec{s}_i) - \phi^{m,i}_{l,q} \rvert > \sigma^{m,i}_{l,q}$	$\vec{\sigma}^{i+1}_m = min(\vec{\sigma}^+, \frac{\vec{\sigma}^i_m}{\gamma}),$	$m \in \mathcal{M}$
3	$Z'_{MMPP}(\vec{s}_i) < Z'_{MMPP}(\vec{s}_{best})$	$\vec{\phi}^{i+1}_m = \{v^m_{l,q}(\vec{s}_i) \mid \forall\ l \in \mathcal{L}, q \in \mathcal{Q}\},$	$m \in \mathcal{M}$
	$\nexists l \in \mathcal{L}, q \in \mathcal{Q}. \lvert v^m_{l,q}(\vec{s}_i) - \phi^{m,i}_{l,q} \rvert > \sigma^{m,i}_{l,q}$	$\vec{\sigma}^{i+1}_m = \vec{\sigma}^i_m,$	$m \in \mathcal{M}$

Table 1 Rules defining the targets and standard deviations provided to each O_m as input in iteration $i+1$, where: $\vec{\sigma}^-$ and $\vec{\sigma}^+$ denote lower and upper bounds on the size of each $\vec{\sigma}^i_m$; $\vec{s}_{best}$ denotes the best solution found by the heuristic; $\vec{s}_i$ denotes the solution found by the heuristic in iteration i; $v^m_{l,q}(\vec{s}_i)$ denotes the percentage of attribute $q \in \mathcal{Q}$ in the ore of granularity $l \in \mathcal{L}$ produced by mine m in solution $\vec{s}_i$; $\phi^{m,i}_{l,q} \in \vec{\phi}^i_m$; and $\sigma^{m,i}_{l,q} \in \vec{\sigma}^i_m$.

If the schedule selected for mine m produces ore of a composition that is sufficiently distant from its target $\vec{\phi}^i_m$, the set of standard deviations assigned to m is increased by a pre-determined factor γ, $\vec{\sigma}^{i+1}_m = \frac{\vec{\sigma}^i_m}{\gamma}$, where $0 < \gamma < 1$ (rule 2). The assumption is that any reduction in the size of the standard deviations assigned to mine m in prior iterations, restricting the diversity of the schedules proposed by O_m, may have been premature. Increasing $\vec{\sigma}^i_m$ forces mine m to propose schedules for which the composition of produced ore is more widely spread about its new target $\vec{\phi}^{i+1}_m$. If the schedule selected for mine m in $\vec{s}_i$ produces ore of a composition that is sufficiently close to its target $\vec{\phi}^i_m$, the set of standard deviations assigned to m does not change, $\vec{\sigma}^{i+1}_m = \vec{\sigma}^i_m$ (rule 3).

Standard deviation vectors are bounded above and below by $\vec{\sigma}^+$ and $\vec{\sigma}^-$. Recall that $\vec{\sigma}^+ = \{\sigma^+_{l,q} = \Delta^+_q \mid \forall\ l \in \mathcal{L}, q \in \mathcal{Q}\}$, where Δ^+_q defines a unit of significant change in the percentage content of $q \in \mathcal{Q}$ in a volume of ore. We define the minimum bound on standard deviations as $\vec{\sigma}^- = \{\sigma^-_{l,q} = \Delta^-_q \mid \forall\ l \in \mathcal{L}, q \in \mathcal{Q}\}$, where Δ^-_q defines a unit of insignificant change in the percentage content of attribute $q \in \mathcal{Q}$ in a volume of ore.

The heuristic is terminated in iteration i if O_Π fails to find a solution $\vec{s}_i$ such that $Z'_{MMPP}(\vec{s}_i) < Z'_{MMPP}(\vec{s}_{best})$, and each $\vec{\sigma}^i_m$ equals $\vec{\sigma}^-$, or a limit on the number of executions of the feedback loop, $MAX_{iterations}$, has been reached (Step 12). Across each of the computational tests in Appendix C, the heuristic has terminated within 100 iterations. While there are no theoretical guarantees that the heuristic will discover a local or global optimum to the MMPP, it does, in practice, find near-optimal solutions.

6.4. Optimisation at the Mines: A MIP Model

We model O_m, for each $m \in \mathcal{M}$, in terms of a MIP. Maximisation of productivity at m, as per Equation (38), forms the objective. A set of ranges, $[L^m_{l,q}, U^m_{l,q}]$ for each $l \in \mathcal{L}$ and $q \in \mathcal{Q}$, constrain the blend of ore produced at the mine over the course of the scheduling horizon, where $L^m_{l,q}$ and $U^m_{l,q}$ denote a lower and upper bound on the percentage of $q \in \mathcal{Q}$ in the ore of granularity $l \in \mathcal{L}$ produced at m. These ranges are varied, and the MIP, shown below, is solved to produce a set of N schedules for mine m. We explain, in the proceeding paragraphs, how this set is generated so that the composition of ore produced across schedules forms a normal distribution with a mean $\vec{\phi}_m$ and standard deviation $\vec{\sigma}_m$.

All notation is explained in Appendices A and B, while $\tau^m_l(\vec{x}_m)$, and $v^m_{l,q}(\vec{x}_m)$, are defined in Equations (2), and (4). Recall that $\vec{x}_m$ denotes the set $\{x^m_{s,d} | \forall s \in \mathcal{S}_m, d \in \mathcal{D}_m\}$. We have found, via experimentation, that the decomposition-based heuristic performs best if, in the computation of a mines productivity, the production of each granularity is weighted according to the expected value of the port products it is likely to contribute to[4]. For example, lump products are typically sold at a higher price, per ton, than fines due to their (typically) higher metal content. Let W_l denote a priority weighting assigned to the production of granularity $l \in \mathcal{L}$ at each mine. Our expression for the productivity of a mine m, denoted $\rho_m(\vec{x}_m)$, in Equation (8) is altered as shown in Equation (38), to form $\rho^*_m(\vec{x}_m)$, where: α_1 and α_2 denote constants such that $\alpha_1 \gg \alpha_2$; and Ψ^m_ω a binary parameter such that $\Psi^m_\omega = 1$ if mine m has the facilities to upgrade low grade ore ($\Psi^m_\omega = 0$, otherwise).

$$\rho^*_m(\vec{x}_m) = \alpha_1 \sum_{l \in \mathcal{L}} W_l \, \tau^m_l(\vec{x}_m) + \alpha_2 \sum_{s \in \mathcal{S}_m} \left[\sum_{\delta \in \Delta_m} x^m_{s,d} + (1 - 2\Psi^m_w) \sum_{\lambda \in \Lambda_m} x^m_{s,d} - \sum_{\theta \in \Theta_m} x^m_{s,d} \right] \tag{38}$$

A solution to the following MIP represents a single schedule available for implementation at mine $m \in \mathcal{M}$.

$$\begin{aligned}
\max \quad & \rho^*_m(\vec{x}_m) & & \\
\text{subject to} \quad & \tau^m_l(\vec{x}_m) \geq D^m_l & \forall\, l \in \mathcal{L}, & \quad (39) \\
& L^m_{l,q} \leq v^m_{l,q}(\vec{x}_m) \leq U^m_{l,q} & \forall\, q \in \mathcal{Q}, & \quad (40)
\end{aligned}$$

[4] This change was not found to yield an improvement in the solutions found by any of the approaches in Section 5.

Listing 2 Generation of clustered bounds on the blend of produced ore at mine $m \in \mathcal{M}$.

1: **for** each $l \in \mathcal{L}$ and $q \in \mathcal{Q}$ **do**
2: $\Delta_N \leftarrow \text{RANDNORMAL}(0, \sigma_{l,q} \in \vec{\sigma}_m)$
3: $L^m_{l,q} \leftarrow \phi_{l,q} + \Delta_N - \sigma_{l,q}$
4: $U^m_{l,q} \leftarrow \phi_{l,q} + \Delta_N + \sigma_{l,q}$
5: **end for**

$$x^m_{s,d} \in \mathbf{R}^+ \cup \{0\} \qquad \forall\ s \in \mathcal{S}_m, d \in \mathcal{D}_m, \quad (41)$$

Constraints (17)–(21), (23)–(27), (30)–(34), and
(36) from the MINLP of Section 5 for mine m.

Constraint (39) places a minimum bound on production at mine m. Constraint (40) restricts the composition of the lump and fines ore produced by m, such that $v^m_{l,q}(\vec{x}_m)$ lies within $[L^m_{l,q}, U^m_{l,q}]$. The remaining constraints form a subset of the MINLP in Section 5. Constraint (27) of the MINLP is implemented in the form of a separation algorithm.

To generate N schedules for mine m, across which the grade and quality of produced ore is normally distributed about a target $\vec{\phi}_m$, with a standard deviation $\vec{\sigma}_m$, the solving of the above MIP is repeated with a varying sequence of bounds on the percentage of each $q \in \mathcal{Q}$ in ore of each granularity $l \in \mathcal{L}$. This MIP is solved until N distinct schedules are discovered, or a pre-defined limit on the number of solves has been reached. Each set of bounds in this sequence, $[L^m_{l,q}, U^m_{l,q}]$ for each $l \in \mathcal{L}$ and $q \in \mathcal{Q}$, is formed as described in Listing 2. A normally distributed random value Δ_N, for each $l \in \mathcal{L}$ and $q \in \mathcal{Q}$, is generated from a distribution with mean 0 and standard deviation $\sigma_{l,q} \in \vec{\sigma}_m$ (Step 2). The percentage of each $q \in \mathcal{Q}$ in ore of granularity $l \in \mathcal{L}$ produced by the mine is constrained to lie between $\phi_{l,q} + \Delta_N - \sigma_{l,q}$ and $\phi_{l,q} + \Delta_N + \sigma_{l,q}$, where $\phi_{l,q} \in \vec{\phi}_m$ (Steps 3 and 4).

6.5. Blending at the Ports: A MIP Model

Recall that each mine $m \in \mathcal{M}$ has (up to) N possible outputs – resulting in $N+1$ blends of lump and fines ore available for transportation to a port – as defined in the set of solutions $\Omega_m \cup \{\vec{s}_{best,m}\}$ to each O_m, where $\vec{s}_{best,m} \in \vec{s}_{best}$. The j^{th} schedule available for selection at mine m is denoted $\vec{s}_{m,j} \in \Omega_m \cup \{\vec{s}_{best,m}\}$. Only one schedule formed by each O_m can be enacted. Consequently, ore railed from each mine m must originate from only one $\vec{s}_{m,j}$.

Let integer variable $r^{\pi}_{m,l,n,j}$ denote the number of trainloads of granularity $l \in \mathcal{L}$, formed by mine m in schedule $\vec{s}_{m,j} \in \Omega_m \cup \{\vec{s}_{best,m}\}$, delivered to port π to form part of product

$n \in N_l^\pi$. Binary variables $o_{m,j}$ denote which schedule $\vec{s}_{m,j} \in \Omega_m \cup \{\vec{s}_{best,m}\}$, for each mine m, has been selected ($o_{m,j} = 1$) for implementation ($o_{m,j} = 0$ otherwise). As in the MINLP of Section 5, the objective of the port-side MIP is to minimise deviation in the composition of products formed at each port π from desired bounds, $[L_{n,q}^{\pi,l}, U_{n,q}^{\pi,l}]$ for each $n \in N_l^\pi$, $l \in \mathcal{L}$, and $q \in \mathcal{Q}$, as a first priority, while maximising revenue achieved via the sale of such products and the productivity of each mine, as second and third priorities, respectively.

Let $N_m = |\Omega_m \cup \{\vec{s}_{best,m}\}|$, and $\vec{\Omega} = \{\Omega_m \cup \{\vec{s}_{best,m}\} | \forall m \in \mathcal{M}\}$. Moreover, let $\vec{r}'$, $\vec{r}_{l,n}^{\pi}{}'$, and $\vec{o}$ denote the variable sets: $\vec{r}' = \{r_{m,l,n,j}^{\pi} | \forall \pi \in \Pi, m \in \mathcal{M}, l \in \mathcal{L}, n \in N_l^\pi, 1 \le j \le N_m\}$; $\vec{r}_{l,n}^{\pi}{}' = \{r_{m,l,n,j}^{\pi} | \forall m \in \mathcal{M}, 1 \le j \le N_m\}$; and $\vec{o} = \{o_{m,j} | \forall m \in \mathcal{M}, 1 \le j \le N_m\}$. Recall that: the tons of granularity $l \in \mathcal{L}$ produced by mine m in a schedule $\vec{s}_{m,j}$ is denoted $\tau_l^m(\vec{s}_{m,j})$; the percentage of $q \in \mathcal{Q}$ in the ore of granularity $l \in \mathcal{L}$ produced by m in $\vec{s}_{m,j}$ is denoted $v_{l,q}^m(\vec{s}_{m,j})$; and the productivity of mine m in $\vec{s}_{m,j}$ is denoted $\rho_m(\vec{s}_{m,j})$. Each of $\tau_l^m(\vec{s}_{m,j})$, $v_{l,q}^m(\vec{s}_{m,j})$, and $\rho_m(\vec{s}_{m,j})$ are constants in the port-side MIP model. We define: the revenue generated by the sale of products formed across ports as $\nu'(\vec{r}')$ in Equation (42); the tons of product $n \in N_l^\pi$ formed at port π as $\tau_{l,n}^{\pi}{}'(\vec{r}')$ in Equation (43); the tons of attribute $q \in \mathcal{Q}$ in product $n \in N_l^\pi$ formed at port π as $\tau_{n,q}^{\pi,l}{}'(\vec{\Omega}, \vec{r}_{l,n}^{\pi}{}')$ in Equation (44); and the total deviation between the composition of products, across all ports, and desired bounds as $\eta'(\vec{\Omega}, \vec{r}')$ in Equation (45). $V_{l,n}^\pi$ denotes the sale price, per ton, of product $n \in N_l^\pi$.

$$\nu'(\vec{r}') = \sum_{\pi \in \Pi} \sum_{m \in \mathcal{M}} \sum_{l \in \mathcal{L}} \sum_{n \in N_l^\pi} \sum_{j=1}^{N_m} r_{m,l,n,j}^{\pi} T_R V_{l,n}^{\pi} \tag{42}$$

$$\tau_{l,n}^{\pi}{}'(\vec{r}') = \sum_{m \in \mathcal{M}} \sum_{j=1}^{N_m} r_{m,l,n,j}^{\pi} T_R \tag{43}$$

$$\tau_{n,q}^{\pi,l}{}'(\vec{\Omega}, \vec{r}_{l,n}^{\pi}{}') = \sum_{m \in \mathcal{M}} \sum_{j=1}^{N_m} r_{m,l,n,j}^{\pi}\, v_{l,q}(\vec{s}_{m,j})\, T_R \tag{44}$$

$$\begin{aligned} \eta'(\vec{\Omega}, \vec{r}') = &\sum_{\pi \in \Pi} \sum_{l \in \mathcal{L}} \sum_{n \in N_l^\pi} \sum_{q \in \mathcal{Q}} \frac{1}{\Delta_q^+} \max\{0, \tau_{n,q}^{\pi,l}{}'(\vec{\Omega}, \vec{r}_{l,n}^{\pi}{}') - U_{n,q}^{\pi,l} \tau_{l,n}^{\pi}{}'(\vec{r}')\} \\ &+ \sum_{\pi \in \Pi} \sum_{l \in \mathcal{L}} \sum_{n \in N_l^\pi} \sum_{q \in \mathcal{Q}} \frac{1}{\Delta_q^+} \max\{0, L_{n,q}^{\pi,l} \tau_{l,n}^{\pi}{}'(\vec{r}') - \tau_{n,q}^{\pi,l}{}'(\vec{\Omega}, \vec{r}_{l,n}^{\pi}{}')\} \end{aligned} \tag{45}$$

The following MIP describes the mine-to-port transportation and blending problem, O_Π, where: β_1,β_2, and β_3 are constants such that $\beta_1 \gg \beta_2 \gg \beta_3$.

$$\min \quad \beta_1 \eta'(\vec{\Omega}, \vec{r}') - \beta_2 \nu'(\vec{r}') - \beta_3 \sum_{m \in \mathcal{M}} \sum_{j=1}^{N_m} o_{m,j}\, \rho_m(\vec{s}_{m,j})$$

$$\text{subject to} \quad \sum_{m \in \mathcal{M}} \sum_{j=1}^{N_m} r^{\pi}_{m,l,n,j} T_R \geq D^{\pi}_{l,n} \qquad \forall\ \pi \in \Pi, l \in \mathcal{L}, n \in N^{\pi}_l \tag{46}$$

$$\sum_{m \in \mathcal{M}} \sum_{l \in \mathcal{L}} \sum_{n \in N^{\pi}_l} \sum_{j=1}^{N_m} r^{\pi}_{m,l,n,j} T_R \leq C_{\pi} \qquad \forall\ \pi \in \Pi, \tag{47}$$

$$\sum_{\pi \in \Pi} \sum_{n \in N^{\pi}_l} r^{\pi}_{m,l,n,j} T_R \leq o_{m,j}\, \tau^{m}_l(\vec{s}_{m,j}) \qquad \forall\ m \in \mathcal{M}, \vec{s}_{m,j} \in \Omega_m \cup \{\vec{s}_{best,m}\}, l \in \mathcal{L}, \tag{48}$$

$$\sum_{j=1}^{N_m} o_{m,j} = 1 \qquad \forall\ m \in \mathcal{M}, \tag{49}$$

$$r^{\pi}_{m,l,n,j} \in \mathbf{R}^{+} \cup \{0\} \qquad \forall\ \pi \in \Pi, l \in \mathcal{L}, n \in N^{\pi}_l, m \in \mathcal{M}, \quad 1 \leq j \leq N_m, \tag{50}$$

$$o_{m,j} \in \{0,1\} \qquad \forall\ m \in \mathcal{M}, 1 \leq j \leq N_m. \tag{51}$$

Constraint (46) places a lower bound on the tons of product $n \in N^{\pi}_l$ of granularity $l \in \mathcal{L}$ produced at port $\pi \in \Pi$. The tons of ore transported to a port is limited by its capacity (Constraint (47)). Constraint (48) constrains the value of each binary indicator, $o_{m,j}$, to 1 if solution $\vec{s}_{m,j} \in \Omega_m \cup \{\vec{s}_{best,m}\}$ is selected to be enacted at mine $m \in \mathcal{M}$, and places an upper bound on the tons of ore transported from each mine to the set of ports (to that produced by m in the selected $\vec{s}_{m,j}$). Constraint (49) ensures that only one $\vec{s}_{m,j} \in \Omega_m \cup \{\vec{s}_{best,m}\}$, for each $m \in \mathcal{M}$, is selected to be implemented at mine m.

7. Computational Results

We have used our decomposition-based heuristic to solve each test case described in Section 4, generated for our 8-mine, 2-port network. IBM CPLEX 12.5 was used to solve all MIPs. Appendix C records the results of the decomposition-based heuristic for varying combinations of parameters N and γ, averaged over 10 runs, each initialised with a different random seed. We describe the method by which we obtain lower bounds on the MINLP objective Z'_{MMPP} in each test (Section C.1). Sections C.2 and C.3 evaluate our heuristic with respect to alternative solution methods, namely: piecewise-linear relaxation (Gounaris et al. 2009); and the ALT heuristic (Audet et al. 2004). These results demonstrate that our heuristic finds solutions equally as good, or better, than the considered alternatives, in orders of magnitude less time, on a majority of tests.

8. Concluding Remarks

We have described a short-term, multiple mine and port, open-pit production scheduling problem (MMPP). We have presented a decomposition-based heuristic, in which this scheduling problem is solved, in the single time period case, through the interaction of a set of optimisation problems – one for each mine, and the system of ports. A solution to the optimisation problem at each mine defines the movement of ore and waste from grade blocks and stockpiles, to dumps, stockpiles and processing plants. In an iterative process, the schedules formed in each of these mine-side optimisations are provided as input to a port-side blending problem, the solution of which selects a schedule to be enacted at each mine, and defines the movement of ore between each mine and port. The composition of ore produced at each mine, across the schedules formed by the mine-side optimisation, is guided by the port-side schedule selections made in prior iterations, encouraging the formation of schedules that allow the ports to maximise their production of correctly blended products.

We have evaluated this heuristic on a suite of test cases generated for an 8-mine, 2-port network, using data provided by an industry partner – contrasting its performance with a range of solvers for a MINLP modelling of the problem. The presented decomposition-based heuristic was found to find solutions of higher quality, on a subset of test cases, than the alternatives in Section 5. Each alternative was afforded 12 hours, for each test case, in which to find a solution. Where the heuristic did not find a solution higher in quality than that found by an alternative, it returned a good quality solution for which the alternative required orders of magnitude more time, relative to the heuristic run time, to match. Overall our decomposition-based heuristic approach provides a highly competitive solution to the short-term multiple port and mine open-pit production scheduling problem.

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Appendix A: Modelling Notation

Sets and Indices

$m, \mathcal{M}$	mines
π, Π	ports
$p, \mathcal{P}_m$	pits
$b, \mathcal{B}_{m,p}, \mathcal{B}_m$	blocks in pit $p \in \mathcal{P}_m$, at $m \in \mathcal{M}$, and grade blocks $b \in \mathcal{B}_m$, at m
$l, \mathcal{L}$	granularities denoting lump and fines $\mathcal{L} = \{0,1\}$
$\mathcal{B}_{hg}, \mathcal{B}_{lg}, \mathcal{B}_w$	high, low grade, and waste blocks at mine m
δ, Δ_m	waste dumps at $m \in \mathcal{M}$
λ, Λ_m	low grade stockpiles at $m \in \mathcal{M}$
θ, Θ_m	high grade stockpiles at $m \in \mathcal{M}$
$q, \mathcal{Q}$	grade and quality attributes
κ, ω	dry/wet processing plant
$s, \mathcal{S}_m$	material sources at $m \in \mathcal{M}$, $\mathcal{S}_m = \{\mathcal{B}_m \cup \Lambda_m \cup \Theta_m\}$
$d, \mathcal{D}_m$	material destinations at $m \in \mathcal{M}$, $\mathcal{D}_m = \{\Delta_m \cup \Lambda_m \cup \Theta_m \cup \{\kappa, \omega\}\}$
n, N_l^π	products of granularity $l \in \mathcal{L}$ to be formed by port π

Parameters

Δ_q^+	significant change in $q \in \mathcal{Q}$ percentage
Δ_q^-	insignificant change in $q \in \mathcal{Q}$ percentage
$G_{s,l,q}^m$	percentage of $q \in \mathcal{Q}$ in granularity $l \in \mathcal{L}$ within $s \in \mathcal{S}_m$ at $m \in \mathcal{M}$
$L_{l,q}^m$	lower bound on $q \in \mathcal{Q}$ in granularity $l \in \mathcal{L}$ produced at m
$U_{l,q}^m$	upper bound on $q \in \mathcal{Q}$ in granularity $l \in \mathcal{L}$ produced at m

$L^{\pi}_{l,n,q}$	lower bound on $q \in \mathcal{Q}$ in product $n \in N^{\pi}_{l}$ produced at π
$U^{\pi}_{l,n,q}$	upper bound on $q \in \mathcal{Q}$ in product $n \in N^{\pi}_{l}$ produced at π
$R^{m,\omega}_{s,l,q}$	Percentage of $q \in \mathcal{Q}$ in granularity $l \in \mathcal{L}$ in $s \in \mathcal{S}_m$ recovered after wet processing at $m \in \mathcal{M}$
$Y^{m,\omega}_{s,l}$	Percentage of granularity $l \in \mathcal{L}$ in $s \in \mathcal{S}_m$ recovered after wet processing at $m \in \mathcal{M}$
$S_{m,s,l}$	percentage of granularity $l \in \mathcal{L}$ (split) in $s \in \mathcal{S}_m$ at $m \in \mathcal{M}$
T^{m}_{s}	tonnage of $s \in \mathcal{S}_m$ available for extraction at $m \in \mathcal{M}$
$\mathcal{A}^{\wedge}_{m,b}$	mining precedences of $b \in \mathcal{B}_m$, *all* of which must be mined before b
$\mathcal{A}^{\vee}_{m,b}$	mining precedences of $b \in \mathcal{B}_m$, *one* of which must be mined before b
D^{d}_{l}	minimum demand on $l \in \mathcal{L}$ production at $d \in \{m, \pi\}$
C^{m}_{p}	maximum tons extractable from pit $p \in \mathcal{P}_m$ at $m \in \mathcal{M}$
C^{m}_{d}	processing capacity (tons) at plant $d \in \{\kappa, \omega\}$ at $m \in \mathcal{M}$
C_{π}	capacity (throughput) at $\pi \in \Pi$
T_R	assumed fixed tonnage of each train
C^{m}_{τ}	maximum tons transportable by trucking resources at $m \in \mathcal{M}$, over the scheduling horizon
C^{m}_{s}	capacity (tons) of stockpile $s \in \Theta_m \cup \Lambda_m$ at $m \in \mathcal{M}$
$V^{\pi}_{l,n}$	price per ton for ore of product $n \in N^{\pi}_{l}$ formed by π
$L^{\pi,l}_{n,q}, U^{\pi,l}_{n,q}$	lower and upper bound on attribute $q \in \mathcal{Q}$ in product $n \in N^{\pi}_{l}$
$D^{m}_{l}, D^{\pi}_{l,n}$	production demand for granularity l at mine m, and product $n \in N^{\pi}_{l}$ at port π
Ψ^{m}_{ω}	binary, value of 1 if mine m has a wet processing plant
$U_{m,l}$	Maximum trainloads of granularity l that can be railed from mine m to the set of ports

Decision variables

$x^{m}_{s,d}$	tons of source $s \in \mathcal{S}_m$ sent to destination $d \in \mathcal{D}_m$ at $m \in \mathcal{M}$
$r^{\pi}_{m,l,n}$	trainloads of granularity $l \in \mathcal{L}$ railed from $m \in \mathcal{M}$ to $\pi \in \Pi$ to form part of product $n \in N^{\pi}_{l}$
$y^{\sigma}_{m,b}$	binary variable, 1 if $b \in \mathcal{B}_m$ is to be extracted
$y^{\tau}_{m,b}$	binary variable, 1 if $b \in \mathcal{B}_m$ is to be completely extracted
$br^{\pi,j}_{m,l,n}$	binary variable, 1 if j trains of granularity l are railed to π to form part of product $n \in N^{\pi}_{l}$
$v^{m}_{l,q}$	percentage of attribute q in granularity l produced by mine m
τ^{m}_{l}	tons of granularity l produced by mine m
$\vec{x}_m$, $\vec{x}$	the set $\{x^{m}_{s,d} \mid \forall s \in \mathcal{S}_m, d \in \mathcal{D}_m\}$ and $\{x^{m}_{s,d} \mid \forall s \in \mathcal{S}_m, d \in \mathcal{D}_m, m \in \mathcal{M}\}$
$\vec{r}^{\,\pi}_{l,n}$, $\vec{r}_{\pi}$	the set $\{r^{\pi}_{m,l,n} \mid \forall m \in \mathcal{M}\}$ and $\{r^{\pi}_{m,l,n} \mid \forall m \in \mathcal{M}, l \in \mathcal{L}\}$
$\vec{r}$	the set $\{r^{\pi}_{m,l,n} \mid \forall m \in \mathcal{M}, l \in \mathcal{L}, \pi \in \Pi\}$

Functions

$\tau^{m}_{s,l}(\vec{x}_m)$	tons of granularity $l \in \mathcal{L}$ produced from $s \in \mathcal{S}_m$ at $m \in \mathcal{M}$
$\tau^{m}_{l}(\vec{x}_m)$	tons of granularity $l \in \mathcal{L}$ produced at $m \in \mathcal{M}$
$v^{m}_{l,q}(\vec{x}_m)$	percentage of each $q \in \mathcal{Q}$ in ore of granularity $l \in \mathcal{L}$ produced at $m \in \mathcal{M}$
$v^{\pi}_{l,n,q}(\vec{x}, \vec{r}^{\,\pi}_{l,n})$	percentage of each $q \in \mathcal{Q}$ in product $n \in N^{\pi}_{l}$ produced at $\pi \in \Pi$
$\nu(\vec{r})$	revenue generated by the sale of ore products across the port system
$\rho_m(\vec{x}_m)$	productivity of mine $m \in \mathcal{M}$
$\eta(\vec{x}, \vec{r}), \eta'(\vec{x}, \vec{r})$	Non-linear ($\eta(\vec{x}, \vec{r})$) and linear ($\eta'(\vec{x}, \vec{r})$) expressions defining the extent of deviation between port product compositions and desired bounds

Appendix B: Decomposition-Based Heuristic

Sets and Indicies

i	iteration
$\vec{\phi}^{\,i}_{m}$	grade and quality target assigned to mine m in iteration i
$\vec{\sigma}^{\,i}_{m}$	standard deviations with which O_m generates a set of schedules for mine m
$\vec{s}_{best}$	best solution found by heuristic
$\vec{s}_i$	solution found by heuristic in iteration i

$\vec{s}_{best,m}$	schedule for mine m in the best found solution $\vec{s}_{best}$
$\vec{s}_m$	a schedule for mine m produced by O_m
Ω_m^i	set of schedules produced by O_m for mine m in iteration i

Parameters

γ	factor by which to increase or reduce a set of standard deviations, $0 < \gamma < 1$
N	number of schedules formed by each O_m in each iteration i
Ξ_m	grade and quality target assigned to mine m in a two year plan
$\vec{\sigma}_m^+$	$\vec{\sigma}^+ = \{\sigma_{l,q}^+ = \Delta_q^+ \vert \forall l \in \mathcal{L}, q \in \mathcal{Q}\}$
$\vec{\sigma}_m^-$	$\vec{\sigma}^- = \{\sigma_{l,q}^- = \Delta_q^- \vert \forall l \in \mathcal{L}, q \in \mathcal{Q}\}$
W_l	priority weighting given to the production of granularity $l \in \mathcal{L}$ in each mine
$MAX_{iterations}$	maximum number of iterations of the heuristic performed before termination

MIP for $\mathcal{O}_m$

Δ_N	a random value generated from a normal distribution
$\rho^*(\vec{x}_m)$	productivity of mine m computed with priority weightings assigned to the production of each granularity l

MIP for $\mathcal{O}_\Pi$

$\Pi(\Omega_m)$	the schedule selected to be enacted at mine m by O_Π
$\vec{s}_{m,j}$	the j^{th} schedule in the set Ω_m^i available for selection at mine m
$o_{m,j}$	binary variable, 1 if O_Π selects the j^{th} schedule in set Ω_m^i to be enacted at mine m
$\vec{o}$	$\vec{o} = \{o_{m,j} \vert \forall m \in \mathcal{M}, 1 \leq j \leq N_m\}$
N_m	$N_m = \vert\Omega_m \cup \{\vec{s}_{best,m}\}\vert$, the number of schedules for mine m available to O_Π for selection
$r_{m,l,n,j}^\pi$	trainloads of granularity l, produced in the j^{th} schedule available at mine m, railed to port π to form part of product $n \in N_l^\pi$
$\vec{\Omega}$	$\vec{\Omega} = \{\Omega_m \cup \{\vec{s}_{best,m}\} \vert \forall m \in \mathcal{M}\}$
$\vec{r}'$	$\vec{r}' = \{r_{m,l,n,j}^\pi \vert \forall \pi \in \Pi, m \in \mathcal{M}, l \in \mathcal{L}, n \in N_l^\pi, 1 \leq j \leq N_m\}$
$\vec{r}_{l,n}^{\pi\,\prime}$	$\vec{r}_{l,n}^{\pi\,\prime} = \{r_{m,l,n,j}^\pi \vert \forall m \in \mathcal{M}, 1 \leq j \leq N_m\}$
$\nu'(\vec{\Omega}, \vec{r}')$	total revenue achieved via the sale of port products
$\tau_{l,n}^{\pi\,\prime}(\vec{r}')$	tons of product $n \in N_l^\pi$ formed at port π
$\tau_{n,q}^{\pi,l\,\prime}(\vec{\Omega}, \vec{r}_{l,n}^{\pi\,\prime})$	tons of attribute q in product $n \in N_l^\pi$ formed at port π
$\eta'(\vec{\Omega}, \vec{r}')$	total deviation between port product compositions and desired bounds

Appendix C: Computational Results

We have used our decomposition-based heuristic to solve each test case described in Section 4, generated for our 8-mine, 2-port network. IBM CPLEX 12.5 was used to solve all MIPs.

Table 4 records the results of the decomposition-based heuristic, averaged over 10 seeded runs on each of our benchmark tests, with: $N = 10, 15$, and 20; $\gamma = 0.75$; and priority weightings $W_{l=0} = 0.6$ and $W_{l=1} = 0.4$ assigned to lump and fines production at each mine. Table 5 records the results of our heuristic with $N = 10$, and varying γ. We record, for the best solution found by the heuristic, $\vec{s}_{best}$: the elapsed time to termination (s); revenue achieved via the sale of products formed at each port (\$); the total utilisation of trucking resources, and the dry and wet processing plants (stated as a percentage of total haulage capacity across the set of mines); the total percentage (%) of (network-wide) haulage capacity spent on undesirable stockpiling across all mines; the maximum deviation (%) from desired bounds present in port products formed across the 10 seeded runs (deviation in metal grade is listed separately from that in other attributes); and the

Table 4 Best solution $\vec{s}_{best}$ found by our heuristic for $N = 10, 15, 20$, and $\gamma = 0.75$, in each test #, recording: elapsed time to completion of solve (s); revenue achieved (\$); the total utilisation of trucks, and processing plants (% of network-wide capacity); the total percentage (%) of (network-wide) haulage capacity spent on undesirable stockpiling; the max deviation (%) from desired bounds (on metal grade, and other attributes) present in port products across 10 seeded runs; and the gap (%) between $Z'_{MMPP}(\vec{s}_{best})$ and the best known lower bound. Quantities have been averaged over 10 seeded runs, with the average (μ) and standard deviation (σ) recorded.

$N = 10$, $\gamma = 0.75$					Utilisation (over all mines) (%)								Deviation (%)		Gap to (%)	
#	Time (s)		Revenue (\$)		Trucking		Dry		Wet		Stockpiling		Metal	Other	MINLP$_{lb}$	
	μ_T	σ_T	μ_R	σ_R	μ_K	σ_K	μ_D	σ_D	μ_W	σ_W	μ_S	σ_S			μ_G	σ_G
1	201	18.56	318297600	226800	99.95	0.01	100	0.95	100	0	2.70	0.05	0	0	0.98	0.01
2	360	44.17	321132600	226800	97.93	0.03	99.23	0.92	100	0	3.22	0.08	0	0	0.09	0.01
3	271	30.93	317957400	226800	98.32	0.06	100	0.92	100	0	4.08	0.10	0	0	1.08	0.01
4	333	46.03	316426500	380355	98.77	0.11	95.90	0.85	100	0	3.55	0.06	0	0	1.56	0.01
5	358	35.70	317277000	0	97.59	0.09	97.95	0.86	100	0	4.95	0.08	0	0	1.29	0
6	310	66.78	319091400	941971	99.50	0.02	99.60	0.89	100	0	5.47	0.09	0	0	0.73	0.03
7	350	63.84	316710000	253570	99.22	0.09	97.44	0.85	100	0	5.50	0.12	0	0	1.47	0.01
8	363	19.68	321246000	0	99.73	0.02	99.24	0.87	100	0	6.05	0.11	0	0	0.06	0
9	159	21.16	316993500	380355	99.84	0.02	100	0.92	100	0	4.05	0.07	0	0	1.38	0.01
10	319	61.08	321132600	226799	99.13	0.02	99.01	0.91	100	0	3.78	0.05	0	0	0.09	0.01
11	363	57.86	318354300	534906	99.84	0.01	100	0.94	100	0	2.89	0.09	0	0	0.96	0.02
12	222	44.49	317617200	1322459	99.73	0.02	97.66	0.88	100	0	3.76	0.06	0	0	1.19	0.04
13	250	38.55	319148100	259832	99.23	0.03	99.99	0.95	100	0	2.76	0.07	0	0	0.71	0.01
14	177	32.18	318581100	442841	99.53	0.03	100	0.98	100	0	1.21	0.05	0	0	0.89	0.01
15	428	73.59	317163600	833316	99.82	0.04	99.73	0.92	100	0	3.90	0.05	0	0	1.33	0.03
16	230	20.84	320962500	457130	99.06	0.06	99.32	0.87	100	0	5.98	0.07	0	0	0.15	0.01
17	220	27.57	321246000	0	99.83	0.02	99.44	0.93	100	0	2.96	0.11	0	0	0.06	0
18	195	18.71	321246000	0	99.71	0.03	97.16	0.89	94.33	1.70	2.56	0.04	0	0	0.06	0
19	227	19.35	321246000	0	99.35	0.05	99.77	0.92	100	0	3.62	0.07	0	0	0.06	0
20	456	63.52	313351200	1315828	97.47	0.06	93.15	0.77	99.96	0.01	5.67	0.04	0	0	2.52	0.04
$N = 15, \gamma = 0.75$																
1	282	39.88	318637800	277772	99.43	0.08	100	0.94	100	0	2.87	0.05	0	0	0.87	0.01
2	435	36.12	321246000	0	97.61	0.10	98.97	0.91	100	0	3.41	0.06	0	0	0.06	0
3	368	11.14	318297600	226800	98.22	0.03	100	0.91	100	0	4.52	0.07	0	0	0.98	0.01
4	393	49.36	316256400	941971	98.78	0.08	96.33	0.86	100	0	3.66	0.06	0	0	1.61	0.03
5	535	38.37	317220300	170100	97.91	0.04	98.67	0.87	100	0	5.14	0.06	0	0	1.31	0.01
6	383	65.74	320395500	1022173	99.22	0.04	97.68	0.85	100	0	5.60	0.06	0	0	0.32	0.03
7	462	77.65	317277000	760710	98.60	0.12	96.79	0.85	100	0	4.80	0.09	0	0	1.29	0.02
8	413	42.62	321246000	0	99.72	0.04	99.57	0.87	100	0	6.20	0.11	0	0	0.06	0
9	247	38.01	317390400	424303	99.67	0.04	100	0.92	100	0	3.92	0.05	0	0	1.26	0.01
10	348	43.16	321246000	0	99.01	0.04	98.12	0.89	100	0	3.98	0.10	0	0	0.06	0
11	488	52.84	318581100	363057	99.80	0.02	100	0.95	100	0	2.47	0.06	0	0	0.89	0.01
12	361	60.82	318581100	1015864	99.79	0.00	96.56	0.86	100	0	3.58	0.05	0	0	0.89	0.03
13	324	47.51	319091400	226799	99.25	0.05	100	0.95	100	0	2.75	0.08	0	0	0.73	0.01
14	280	37.78	319091400	424303	99.70	0.05	100	0.97	100	0	1.31	0.05	0	0	0.73	0.01
15	494	90.89	317787300	692111	99.86	0.02	99.72	0.93	100	0	3.32	0.09	0	0	1.14	0.02
16	308	28.44	321132600	340200	98.82	0.05	98.95	0.87	100	0	5.85	0.07	0	0	0.09	0.01
17	260	18.58	321246000	0	99.91	0.01	99.60	0.93	100	0	3.32	0.12	0	0	0.06	0
18	232	11.76	321246000	0	99.71	0.02	97.30	0.90	100	0	2.41	0.06	0	0	0.06	0
19	285	20.98	321246000	0	99.57	0.03	99.41	0.92	100	0	3.71	0.06	0	0	0.06	0
20	522	69.28	313804800	1437059	98.01	0.07	92.71	0.76	100	0	5.77	0.07	0	0	2.37	0.04
$N = 20, \gamma = 0.75$																
1	363	66.95	318581100	259832	99.81	0.02	100	0.95	100	0	2.74	0.05	0	0	0.89	0.01
2	523	55.34	321246000	0	97.64	0.05	98.65	0.90	100	0	3.70	0.08	0	0	0.06	0
3	444	33.52	318297600	226800	98.07	0.06	100	0.90	100	0	4.95	0.11	0	0	0.98	0.01
4	458	73.17	316426500	923009	99.14	0.04	96.10	0.85	100	0	3.72	0.05	0	0	1.56	0.03
5	701	108.19	317277000	0	98.08	0.03	99.69	0.90	100	0	4.69	0.04	0	0	1.29	0
6	455	38.72	321189300	170099	99.40	0.03	96.32	0.83	100	0	5.30	0.08	0	0	0.08	0.01
7	581	143.39	317560500	957206	99.23	0.07	96.60	0.84	100	0	5.06	0.07	0	0	1.21	0.03
8	497	49.23	321246000	0	99.88	0.02	99.47	0.86	100	0	6.58	0.09	0	0	0.06	0
9	322	28.94	317560500	380355	99.69	0.04	100	0.93	100	0	3.75	0.06	0	0	1.21	0.01
10	396	63.20	321246000	0	99.07	0.04	97.58	0.88	100	0	3.71	0.05	0	0	0.06	0
11	584	94.76	318751200	453600	99.75	0.05	100	0.95	100	0	2.70	0.05	0	0	0.84	0.01
12	490	63.72	319374900	623700	99.75	0.00	95.19	0.83	100	0	4.02	0.05	0	0	0.64	0.02
13	401	50.52	319318200	277772	99.35	0.05	100	0.95	100	0	2.51	0.06	0	0	0.66	0.01
14	343	35.62	319148100	363057	99.60	0.04	100	0.97	100	0	1.58	0.08	0	0	0.71	0.01
15	596	68.63	317957400	424303	99.83	0.02	99.70	0.93	100	0	3.24	0.04	0	0	1.08	0.01
16	344	30.74	321246000	0	98.86	0.05	98.68	0.87	100	0	5.45	0.05	0	0	0.06	0
17	299	10.67	321246000	0	99.79	0.01	99.67	0.92	100	0	3.65	0.12	0	0	0.06	0
18	269	13.24	321246000	0	99.54	0.03	97.47	0.91	100	0	2.32	0.08	0	0	0.06	0
19	326	11.09	321246000	0	99.49	0.05	99.48	0.92	100	0	3.73	0.06	0	0	0.06	0
20	631	70.57	314158500	633925	98.13	0.07	92.52	0.75	100	0	5.65	0.06	0	0	2.26	0.02

gap (%) between $Z'_{MMPP}(\vec{s}_{best})$ and the best lower bound discovered in Section C.1. Quantities have been averaged over 10 seeded runs, with the average (μ) and standard deviation (σ) recorded.

Table 5 Best solution $\vec{s}_{best}$ found by heuristic for $\gamma = 0.25, 0.50$, and $N = 10$. Columns are defined as in Table 4. Quantities have been averaged over 10 seeded runs, with the average (μ) and standard deviation (σ) recorded.

$N=10,\ \gamma=0.25$					Utilisation (over all mines) (%)								Deviation (%)		Gap to (%)	
#	Time (s)		Revenue ($)		Trucking		Dry		Wet		Stockpiling		Metal	Other	MINLP$_{lb}$	
	μ_T	σ_T	μ_R	σ_R	μ_K	σ_K	μ_D	σ_D	μ_W	σ_W	μ_S	σ_S			μ_G	σ_G
1	98	17.12	318127500	283500	99.80	0.02	100	0.95	100	0	2.56	0.05	0	0	1.03	0.01
2	202	52.71	320225400	1262768	98.14	0.05	99.15	0.91	100	0	3.79	0.05	0	0	0.38	0.04
3	135	25.37	317673900	363057	98.26	0.05	99.79	0.92	100	0	4.08	0.08	0	0	1.17	0.01
4	162	38.17	315235800	1416365	98.93	0.06	95.43	0.84	99.64	0.11	3.70	0.05	0	0	1.93	0.04
5	235	41.38	316880100	510300	97.85	0.04	97.51	0.85	100	0	5.36	0.05	0	0	1.42	0.02
6	167	44.46	319318200	955525	98.77	0.12	98.95	0.88	100	0	5.26	0.07	0	0	0.66	0.03
7	208	34.04	316653300	305338	99.33	0.09	96.80	0.83	100	0	5.72	0.13	0	0	1.49	0.01
8	188	19.79	321019200	277772	99.66	0.03	99.11	0.87	100	0	5.97	0.11	0	0	0.13	0.01
9	79	21.98	316766700	396900	99.84	0.02	100	0.92	100	0	4.18	0.05	0	0	1.45	0.01
10	192	63.47	320679000	439196	99.01	0.10	99.60	0.92	100	0	3.73	0.09	0	0	0.24	0.01
11	160	39.51	317844000	439196	99.73	0.02	100	0.94	100	0	3.04	0.06	0	0	1.12	0.01
12	111	35.63	317220300	996695	99.78	0.01	97.30	0.88	100	0	3.65	0.05	0	0	1.31	0.03
13	122	31.35	318807900	259832	99.01	0.11	99.96	0.94	100	0	2.93	0.08	0	0	0.82	0.01
14	82	20.78	318411000	0	99.48	0.04	100	0.98	100	0	1.24	0.04	0	0	0.94	0
15	252	54.81	316539900	983708	99.94	0.01	99.33	0.91	100	0	4.03	0.06	0	0	1.52	0.03
16	123	12.06	320679000	760710	99.02	0.04	99.46	0.88	100	0	5.92	0.07	0	0	0.24	0.02
17	92	14.90	321246000	0	99.83	0.02	99.44	0.93	100	0	3.00	0.10	0	0	0.06	0
18	92	13.52	321246000	0	99.62	0.03	97.16	0.90	94.33	1.70	2.55	0.03	0	0	0.06	0
19	94	13.68	321246000	0	99.35	0.05	99.77	0.92	100	0	3.62	0.07	0	0	0.06	0
20	267	84.52	290417400	45159371	97.78	0.08	92.34	0.75	99.94	0.01	6.02	0.07	0.02	0	>100	–
$N=10,\ \gamma=0.50$																
1	133	34.53	318240900	259832	99.56	0.04	100	0.95	100	0	2.62	0.06	0	0	0.99	0.01
2	256	29.16	320962500	283500	97.32	0.15	98.93	0.92	100	0	3.17	0.07	0	0	0.15	0.01
3	173	29.34	317900700	305338	98.04	0.06	100	0.91	100	0	4.45	0.10	0	0	1.10	0.01
4	230	55.80	315576000	1216079	98.83	0.11	96.47	0.86	100	0	3.78	0.07	0	0	1.82	0.04
5	231	41.04	316880100	259832	97.56	0.08	97.41	0.85	100	0	5.14	0.06	0	0	1.42	0.01
6	210	69.28	318978000	760710	99.35	0.03	99.85	0.89	100	0	5.31	0.09	0	0	0.76	0.02
7	251	41.48	316710000	253570	98.98	0.09	97.00	0.83	100	0	5.86	0.12	0	0	1.47	0.01
8	224	16.10	321189300	170099	99.68	0.03	98.95	0.86	100	0	6.24	0.10	0	0	0.08	0.01
9	98	16.95	316823400	340200	99.59	0.05	100	0.92	100	0	4.02	0.07	0	0	1.43	0.01
10	230	49.37	320849100	259832	99.22	0.02	99.61	0.91	100	0	4.25	0.06	0	0	0.18	0.01
11	197	27.79	317787300	396900	99.81	0.02	99.99	0.95	100	0	2.66	0.06	0	0	1.14	0.01
12	157	38.21	317560500	957206	99.78	0.00	97.34	0.88	100	0	3.79	0.05	0	0	1.21	0.03
13	151	25.77	318807900	259832	98.79	0.12	100	0.95	100	0	2.69	0.09	0	0	0.82	0.01
14	105	14.99	318467700	170100	99.53	0.03	100	0.98	100	0	1.15	0.04	0	0	0.92	0.01
15	283	37.08	316596600	941971	99.95	0.01	99.52	0.93	100	0	3.30	0.06	0	0	1.51	0.03
16	151	10.48	320735700	737100	98.91	0.05	99.82	0.88	100	0	5.96	0.07	0	0	0.22	0.02
17	124	17.42	321246000	0	99.83	0.02	99.44	0.93	100	0	2.96	0.11	0	0	0.06	0
18	117	14.42	321246000	0	99.71	0.03	97.16	0.89	94.33	1.70	2.60	0.05	0	0	0.06	0
19	124	16.55	321246000	0	99.35	0.05	99.77	0.92	100	0	3.62	0.07	0	0	0.06	0
20	248	76.29	276115500	58427971	97.51	0.09	91.02	0.73	100	0	5.86	0.11	0.04	0	>100	–

Increasing N, the number of schedules formed during the solve of each O_m in each iteration of the heuristic, and γ, altering the degree to which the standard deviations given to each O_m as input are increased or decreased (a larger γ results in smaller changes), improves, in general, the quality of solutions found by the heuristic. The heuristic is successful, across all tested combinations of the N and γ parameters, at discovering near optimal solutions to the MMPP – with gaps of less than 2% achieved (in all but one test case) between $Z'_{MMPP}(\vec{s}_{best})$ and its best known lower bound. For $N = 10{,}15{,}20$ and $\gamma = 0.50{,}0.75$, gaps of less than 1% are reported in a majority of test cases. Decreasing γ results in the heuristic performing less iterations, reducing the time it takes to solve, but limiting its opportunities to improve the quality of its current best found solution.

We have evaluated the extent to which our choice of port-to-mine feedback (see Table 1) improves the performance of our heuristic by considering two alternative schemes. The first, denoted $R2$, replicates our existing rules but does not increase the standard deviations provided to each mine at any stage. The second, denoted $R3$, replicates $R2$, but reduces these standard deviations only after two consecutive iterations have failed to yield an improved $\vec{s}_{best}$. For $N = 10$ and $\gamma = 0.75$, we have found that, relative to our existing rules, $R2$ results in similar heuristic solve times, but lower quality solutions, on a majority of tests. $R3$ results in solutions that are slightly higher in quality than those of Table 4, on a majority of tests, but increases heuristic solve time by almost 200s on average. For brevity, the full results of this evaluation have been omitted from this appendix.

C.1. Generation of lower bounds

We find lower bounds on the value of Z'_{MMPP}, in each test, via the use of linear (McCormick 1976) and piecewise-linear (Gounaris et al. 2009) relaxations of our non-linear model. We first relax each bilinear term, $v^m_{l,q}\tau^m_l$, in the MINLP of Section 5 with its convex envelope (McCormick 1976). Default optimality tolerances could not be reached, in any test case, when the resulting MIP was solved. In each test, a gap of 0.06% was achieved, with respect to a lower bound obtained via an LP relaxation of the MIP (after 12 hours of solving). The average deviation between desired bounds on the percentage of metal in each lump and fines port product, and its actual composition, across the solutions of the relaxed model, was 0.56% and 0.16% (with standard deviations of 0.27 and 0.20). The maximum deviations in metal percentage, across all tests, were 1.14% and 0.81% in the lump and fines products formed across the port system. To generate relaxations of greater fidelity, we linearise each bilinear term, $v^m_{l,q}\tau^m_l$ for $m \in \mathcal{M}$, $l \in \mathcal{L}$, and $q \in \mathcal{Q}$, by partitioning the domain of the τ^m_l variable into $N_\tau = 2, 5, 10$, and 20, intervals. We reformulate each τ^m_l as shown in Equations (52)–(55).

$$\tau^m_l = D^m_l + \sum_{j=0}^{N_\tau} j\,\Delta\tau^m_l\,\hat{\tau}^m_{l,j} + \Delta\tau^m_l\,\tilde{\tau}^m_l, \quad \Delta\tau^m_l = \frac{U^m_l - D^m_l}{N_\tau} \qquad \forall\, m \in \mathcal{M}, l \in \mathcal{L} \tag{52}$$

$$0 \le \tilde{\tau}^m_l \le 1 \qquad \forall\, m \in \mathcal{M}, l \in \mathcal{L} \tag{53}$$

$$\hat{\tau}^m_{l,j} \in \{0,1\} \qquad \forall\, j = 0 \ldots N_\tau, m \in \mathcal{M}, l \in \mathcal{L} \tag{54}$$

$$\sum_{j=0}^{N_\tau} \hat{\tau}^m_{l,j} = 1 \qquad \forall\, m \in \mathcal{M}, l \in \mathcal{L} \tag{55}$$

The binary variable $\hat{\tau}^m_{l,j}$ forms part of an SOS1 constraint (Equation (55)), and is active ($\hat{\tau}^m_{l,j} = 1$) only when variable τ^m_l lies between the value $D^m_l + j\,\Delta\tau^m_l$ and $D^m_l + (j+1)\,\Delta\tau^m_l$, where U^m_l denotes the maximum tons of granularity $l \in \mathcal{L}$ producible by m. The variable $\tilde{\tau}^m_l$ forms part of a slack term, allowing the value of each τ^m_l to lie between the discrete points in its domain characterised by $D^m_l + j\,\Delta\tau^m_l$ for $j = 0 \ldots N_\tau$.

We substitute the expression in Equation (52) for τ^m_l in each of the bilinear terms in our MINLP. The terms $\hat{\tau}^m_{l,j}\,v^m_{l,q}$ and $\tilde{\tau}^m_l v^m_{l,q}$ appearing in Equation (56) are replaced with variables $w^m_{l,j,q} = \hat{\tau}^m_{l,j}\,v^m_{l,q}$ and $\tilde{v}^m_{l,q} = \tilde{\tau}^m_l v^m_{l,q}$, yielding Equation (57). Each $w^m_{l,j,q}$ is constrained as shown in Equations (58)–(61). Variable $\tilde{v}^m_{l,q}$ is constrained as shown in Equations (62)–(65), where $L^m_{l,q}$ and $U^m_{l,q}$ denote lower and upper bounds on the domain of variable $v^m_{l,q}$.

$$v^m_{l,q}\tau^m_l = D^m_l v^m_{l,q} + \sum_{j=0}^{N_\tau} j\,\Delta\tau^m_l\,\hat{\tau}^m_{l,j}\,v^m_{l,q} + \Delta\tau^m_l\,\tilde{\tau}^m_l v^m_{l,q} \qquad \forall\, m \in \mathcal{M}, l \in \mathcal{L}, q \in \mathcal{Q} \tag{56}$$

$$v^m_{l,q}\tau^m_l = D^m_l v^m_{l,q} + \sum_{j=0}^{N_\tau} j\,\Delta\tau^m_l\,w^m_{l,j,q} + \Delta\tau^m_l\,\tilde{v}^m_{l,q} \qquad \forall\, m \in \mathcal{M}, l \in \mathcal{L}, q \in \mathcal{Q} \tag{57}$$

$$w^m_{l,j,q} \le U^m_{l,q}\hat{\tau}^m_l \qquad \forall\, m \in \mathcal{M}, l \in \mathcal{L}, q \in \mathcal{Q} \tag{58}$$

$$w^m_{l,j,q} \geq L^m_{l,q}\hat{\tau}^m_l \qquad \forall\, m \in \mathcal{M}, l \in \mathcal{L}, q \in \mathcal{Q} \qquad (59)$$

$$w^m_{l,j,q} \leq v^m_{l,q} + L^m_{l,q}(1-\hat{\tau}^m_l) \qquad \forall\, m \in \mathcal{M}, l \in \mathcal{L}, q \in \mathcal{Q} \qquad (60)$$

$$w^m_{l,j,q} \geq v^m_{l,q} - U^m_{l,q}(1-\hat{\tau}^m_l) \qquad \forall\, m \in \mathcal{M}, l \in \mathcal{L}, q \in \mathcal{Q} \qquad (61)$$

$$\tilde{v}^m_{l,q} \leq U^m_{l,q}\tilde{\tau}^m_l \qquad \forall\, m \in \mathcal{M}, l \in \mathcal{L}, q \in \mathcal{Q} \qquad (62)$$

$$\tilde{v}^m_{l,q} \geq L^m_{l,q}\tilde{\tau}^m_l \qquad \forall\, m \in \mathcal{M}, l \in \mathcal{L}, q \in \mathcal{Q} \qquad (63)$$

$$\tilde{v}^m_{l,q} \geq U^m_{l,q}\tilde{\tau}^m_l + v^m_{l,q} - U^m_{l,q} \qquad \forall\, m \in \mathcal{M}, l \in \mathcal{L}, q \in \mathcal{Q} \qquad (64)$$

$$\tilde{v}^m_{l,q} \leq L^m_{l,q}\tilde{\tau}^m_l + v^m_{l,q} \qquad \forall\, m \in \mathcal{M}, l \in \mathcal{L}, q \in \mathcal{Q} \qquad (65)$$

The maximum deviation in metal percentage, from desired bounds, across all port products, was found to be 1.02%, 0.69%, 0.36%, and 0.16%, respectively, in solutions to the models generated with $N_\tau = 2, 5, 10$, and 20. All MIP models generated to approximate the MINLP could not be solved to default optimality tolerances in any of the 20 tests, in a 12 hour period. Lower bounds obtained from the LP relaxation of each of these MIPs (after 12 hours of solving) have been used to assess the quality of solutions found by our heuristic in Tables 4–5.

C.2. Piecewise-linear relaxations (PLR)

To determine whether piecewise-linear relaxation is capable of finding high quality solutions to the MMPP, in which port products are correctly blended, we re-solve the $N_\tau = 10$, and 20 relaxed models (generated in Section C.1) with narrowed bounds on each attribute $q \in \mathcal{Q}$. Each set of bounds is narrowed to offset the maximum deviations incurred on the relevant attribute in the solutions to each model. Each model was able to produce solutions in which no deviation existed between port product composition and the original bounds.

Table 6 records for the best solution (*best*) found, in each test: the elapsed time (s) to the completion of solve ('–' denotes that default optimality tolerances were not reached in a 12 hour period); the elapsed time (s) to the discovery of *best*; the total revenue achieved ($) via the sale of ore products formed across the port system; the value of N_τ which generated the best solution for the test case; the total utilisation of trucking resources, and the dry and wet processing plants (% of network-wide capacity); the total percentage of network-wide haulage capacity spent on undesirable stockpiling; and the gap (%) between the objective value of *best* and the best known lower bound on Z'_{MMPP} for the test case.

We compare the results of the piecewise-linear relaxed (PLR) solver with those obtained by our heuristic, using both the worst and best performing combination of N, and γ, parameters: $N = 10$, $\gamma = 0.25$; and $N = 20$, $\gamma = 0.75$, respectively. As we perform 10 seeded runs of our heuristic on each test, and average the results of those runs in Tables 4 and 5, we use the worst performing run (producing the highest value for Z'_{MMPP}) obtained for each test and $N - \gamma$ parameter combination in our comparison. The final six columns of Table 6 denote: the gap (%) between $Z'_{MMPP}(\vec{s}_{best})$, where $\vec{s}_{best}$ is the solution found by our heuristic for the given $N - \gamma$ combination, and the best known lower bound; the elapsed time (s) at which the heuristic discovered this solution; and the time required by the PLR solver to find a solution of equivalent quality (a '–' in the PLR column indicates that the PLR solver did not find such a solution in a 12 hour timeframe).

Table 6 Comparison of piecewise-linear relaxation (PLR) and our heuristic. For the best solution $best$ found by PLR, we record for each test #: elapsed time (s) to completion of solve ('–' denotes that default optimality tolerances were not reached in 12hrs); elapsed time (s) to discovery of $best$; revenue from correctly blended port products ($); the N_τ value used to generate each solution; utilisation of trucks, and dry/wet processing plants (% of network-wide capacity); percentage of network-wide haulage capacity spent on undesirable stockpiling; and the gap (%) between $Z'_{MMPP}(best)$ and the best known lower bound. Columns 11-16 compare PLR and our heuristic. Given $N=10$, $\gamma=0.25$, and $N=20$, $\gamma=0.75$, we record for $\vec{s}_{best}$ in each test #: the gap between $Z'_{MMPP}(\vec{s}_{best})$ and the best known lower bound (Gap, %); heuristic (elapsed) solve time (Time, s); and the elapsed time (s) taken by PLR (PLR, s) to find an equally good solution ('–' indicates that no such solution was found). Differences in mine productivity across solutions are not evident in gaps rounded to two decimal places. In #1 and 7, the heuristic finds a better solution than PLR, despite both achieving gaps of 1.12 and 1.47, respectively.

	Solve	Best	Revenue	N_τ	Utilisation (%)				Gap to MINLP$_{lb}$	$N=10, \gamma=0.25$			$N=20, \gamma=0.75$		
#	(s)	(s)	($)		Trucking	Dry	Wet	Stockpiling	(%)	Gap (%)	Time (s)	PLR (s)	Gap (%)	Time (s)	PLR (s)
1	–	42793	317844000	10	98.74	99.28	100	1.49	1.12	1.12	89	–	0.94	340	–
2	–	41042	319545000	20	98.32	100	100	2.24	0.59	1.29	139	38977	0.06	554	–
3	–	30584	317844000	20	98.90	100	100	2.22	1.12	1.47	110	1390	1.12	357	30584
4	–	42696	316143000	20	98.73	96.98	100	3.09	1.65	2.70	166	39621	2.35	352	39621
5	–	40379	316710000	20	98.40	99.24	100	3.44	1.47	1.82	235	39053	1.29	565	–
6	–	42793	319545000	20	99.40	100	100	3.46	0.59	0.94	119	39177	0.24	501	–
7	–	41502	316710000	20	99.52	97.02	100	3.87	1.47	1.65	234	40245	1.47	390	–
8	–	39323	321246000	20	99.45	100	100	2.64	0.06	0.24	198	38879	0.06	448	39323
9	–	41753	316710000	10	100	100	100	2.31	1.47	1.65	50	41528	1.47	265	41753
10	–	41798	321246000	20	99.43	100	100	3.61	0.06	0.59	110	39432	0.06	312	41371
11	–	42680	318411000	20	99.65	100	100	2.64	0.94	1.47	130	39590	1.12	407	41583
12	–	41529	317844000	20	99.34	97.58	100	2.49	1.12	1.65	130	38832	0.94	336	–
13	–	40835	318411000	20	99.28	100	100	2.60	0.94	0.94	90	40835	0.76	383	–
14	–	40679	318978000	20	99.89	100	100	0.77	0.76	0.94	88	1123	0.94	250	1123
15	–	42052	317844000	20	99.90	99.75	100	1.85	1.12	1.82	236	39176	1.29	678	41004
16	–	43075	321246000	20	98.35	99.96	100	5.19	0.06	0.76	100	39046	0.06	343	41359
17	–	3346	321246000	20	100	100	100	0.35	0.06	0.06	88	3346	0.06	295	3346
18	–	2405	319545000	10	100	98	100	1.59	0.59	0.06	101	–	0.06	256	–
19	–	1027	321246000	20	100	100	100	1.03	0.06	0.06	79	679	0.06	328	679
20	–	43089	301428000	20	98.12	87.15	100	6.26	6.22	>100	148	39074	2.71	692	–

In tests 1 and 7, for $N=10, \gamma=0.25$ and $N=20, \gamma=0.75$, respectively, the gap between the objective of solutions found by the heuristic and the PLR solver, to the best known lower bounds, appears to be the same, at 1.12 and 1.47. The total productivity of the mine network is higher, however, in the heuristic solutions – the scaling that exists between port product deviation, revenue, and productivity, in Z'_{MMPP}, results in productivity changes equating to small differences in gap values, not evident when rounded to two decimal places.

Table 6 shows that, for $N=20$ and $\gamma=0.75$, our heuristic discovers solutions equally as good, or better, than the PLR solver, in orders of magnitude less time, on a majority of tests (16/20). For the worst performing parameter combination of $N=10$ and $\gamma=0.25$, the PLR solver finds higher quality solutions in a majority of tests (16/20), but requires, in 14 of the 20 tests, orders of magnitude more time to do so. The PLR solver is consequently not a viable alternative – it rarely displays good performance, and requires knowledge of the extent to which bounds on port product composition should be narrowed.

C.3. The ALT Heuristic

The ALT heuristic generates and solves a series of linear programs (LPs), by alternately fixing each set of variables that appear in the bilinear constraints of a general BLP (Audet et al. 2004). We first fix the $v^m_{l,q}$ variable in each bilinear term, $v^m_{l,q}\tau^m_l$, of our MINLP to its instantiation in the solution to the envelope-based relaxation of Section C.1. We solve the resulting MIP to obtain a set of values for each τ^m_l variable. These

Table 7 Comparison of ALT and our heuristic. For the best solution $best$ found by ALT in each test #, we record: the elapsed time (s) to the discovery of $best$, and convergence ('–' indicates that convergence did not occur in 12hrs); revenue from correctly blended port products ($); time limit (s) on each MIP solve; utilisation of trucks, and dry/wet processing plants (% of network-wide capacity); percentage of network-wide haulage capacity spent on undesirable stockpiling; and the gap (%) between $Z'_{MMPP}(best)$ and the best known lower bound. Columns 11-16 compare ALT and our heuristic. Given $N = 10$, $\gamma = 0.25$, and $N = 20$, $\gamma = 0.75$, we record for the lowest quality $\vec{s}_{best}$ found across all seeded runs of each test #: the gap between $Z'_{MMPP}(\vec{s}_{best})$ and the best known lower bound (Gap, %); the elapsed time (Time, s) taken by our heuristic to solve; and the elapsed time (ALT, s) taken by ALT to find an equally good solution ('–' indicates that no such solution was found).

									Gap to	$N=10,\gamma=0.25$			$N=20,\gamma=0.75$		
	Best	Converges	Revenue	MIP_L	Utilisation (%)				MINLP_{lb}	Gap	Time	ALT	Gap	Time	ALT
#	(s)	(s)	($)	(s)	Trucking	Dry	Wet	Stockpiling	(%)	(%)	(s)	(s)	(%)	(s)	(s)
1	36000	–	318978000	500	99.47	99.19	100	0.67	0.76	1.12	193	1000	0.94	340	1000
2	24500	–	321246000	500	98.40	99.98	100	1.86	0.06	0.24	282	1000	0.06	554	24500
3	22000	–	318411000	1000	98.58	100	100	1.06	0.94	1.12	323	12000	1.12	357	12000
4	34000	–	315009000	500	97.29	98.83	100	3.92	2.00	1.65	246	–	2.35	352	6500
5	12000	26000	316710000	500	98.63	99.79	100	1.96	1.47	1.29	379	–	1.29	565	–
6	6000	16000	319545000	500	99.26	98.83	100	2	0.59	0.94	290	1000	0.24	501	–
7	41000	–	316710000	500	99.87	97.89	100	2.63	1.47	1.65	284	41000	1.47	390	41000
8	30000	–	320679000	1000	99.59	97.8	100	2.67	0.24	0.06	390	–	0.06	448	–
9	2000	30000	317844000	500	99.88	100	100	1.19	1.12	1.47	142	1000	1.47	265	1000
10	37000	–	320679000	1000	99.25	100	100	1.35	0.24	0.24	390	37000	0.06	312	–
11	17000	38000	319545000	1000	99.79	100	100	0.86	0.59	1.12	315	3000	1.12	407	3000
12	8000	–	320679000	1000	99.76	100	100	3.19	1.65	1.65	166	8000	0.94	336	–
13	14000	–	318978000	1000	100	100	100	1.72	0.76	0.76	170	14000	0.76	383	14000
14	22000	–	318411000	500	100	100	100	0	0.94	1.12	154	22000	0.94	250	22000
15	2000	12000	315576000	1000	100	99.75	100	2.42	1.82	1.82	311	2000	1.29	678	–
16	6000	–	321246000	1000	98.81	99.89	100	2.78	0.06	0.41	263	6000	0.06	343	6000
17	6000	–	321246000	1000	100	100	100	0.28	0.06	0.06	196	3000	0.06	295	3000
18	6100	11800	320679000	100	100	98.29	100	1.71	0.24	0.06	203	–	0.06	256	–
19	8000	–	320679000	1000	99.25	99.94	100	0.54	0.24	0.06	204	–	0.06	328	–
20	10642	11142	196020000	500	97.41	97.55	100	4.60	> 100	3.63	382	–	2.71	692	–

values are then used to fix each τ_l^m variable, and solve for a new instantiation of each $v_{l,q}^m$. This process of alternate variable fixing repeats until two successive iterations of the heuristic yield equal (to a tolerance) values for either of the $v_{l,q}^m$ or τ_l^m variable sets. On our set of benchmark tests, the MIP generated from fixing each $v_{l,q}^m$ to its first value could not be solved to default optimality tolerances within a 12 hour period. We have run a variation of the ALT algorithm in which each MIP solve is given a time limit. The best solution found in that time limit is used to obtain new instantiations of the $v_{l,q}^m$ and τ_l^m variable sets. In this setting, convergence to a local optimum is no longer guaranteed, and the MINLP objective value in successive solutions may not monotonically improve. This modified ALT heuristic has been applied to each of our benchmark tests, and the best solution found over all iterations, until convergence or the 12 hour cut-off point is reached, recorded.

We have applied modified ALT with a MIP time limit of 100, 500, and 1000 seconds. We record, in Table 7, for the best solution $best$ found in each test: the elapsed time (s) to discovery, and convergence; the MIP solve limit (s) used to generate the best solution for the test; the revenue ($) achieved via the sale of ore products; the utilisation of trucking resources, and the dry and wet processing plants (% of network-wide capacity); the percentage of network-wide haulage capacity spent on undesirable stockpiling; and the gap (%) between $Z'_{MMPP}(best)$ and the best known lower bound on Z'_{MMPP} for the test case. The final six columns of Table 7 denote: the gap (%) between $Z'_{MMPP}(\vec{s}_{best})$, where $\vec{s}_{best}$ is the solution found by our heuristic for the given $N - \gamma$ combination, and the best known lower bound; the elapsed time (s) at which the heuristic discovered this solution; and the time required by the ALT solver to find a solution of equivalent quality (a '–' in the ALT column indicates that ALT did not find such a solution in a 12 hour timefame).

Table 7 shows that, for both $N - \gamma$ combinations, our heuristic discovers solutions equally as good, or better, than ALT, on a majority of tests (15/20 for $N = 20$, $\gamma = 0.25$, and 11/20 for $N = 10$, $\gamma = 0.25$). The performance of ALT, across the tests, is inconsistent, often requiring orders of magnitude more time, than our heuristic, to discover solutions of comparable quality. Moreover, Table 7 shows that ALT was unable to converge in a reasonable timeframe. This lack of convergence arises as a result of the time limit imposed on each MIP solve, preventing it from being solved to optimality.