Discrete Optimization for Multi-Agent Path Finding

Peter J. Stuckey

Pierre Le Bodic, Graeme Gange, Daniel Harabor, Edward Lam, Jiaoyang Li, Sven Koenig

MONASH University

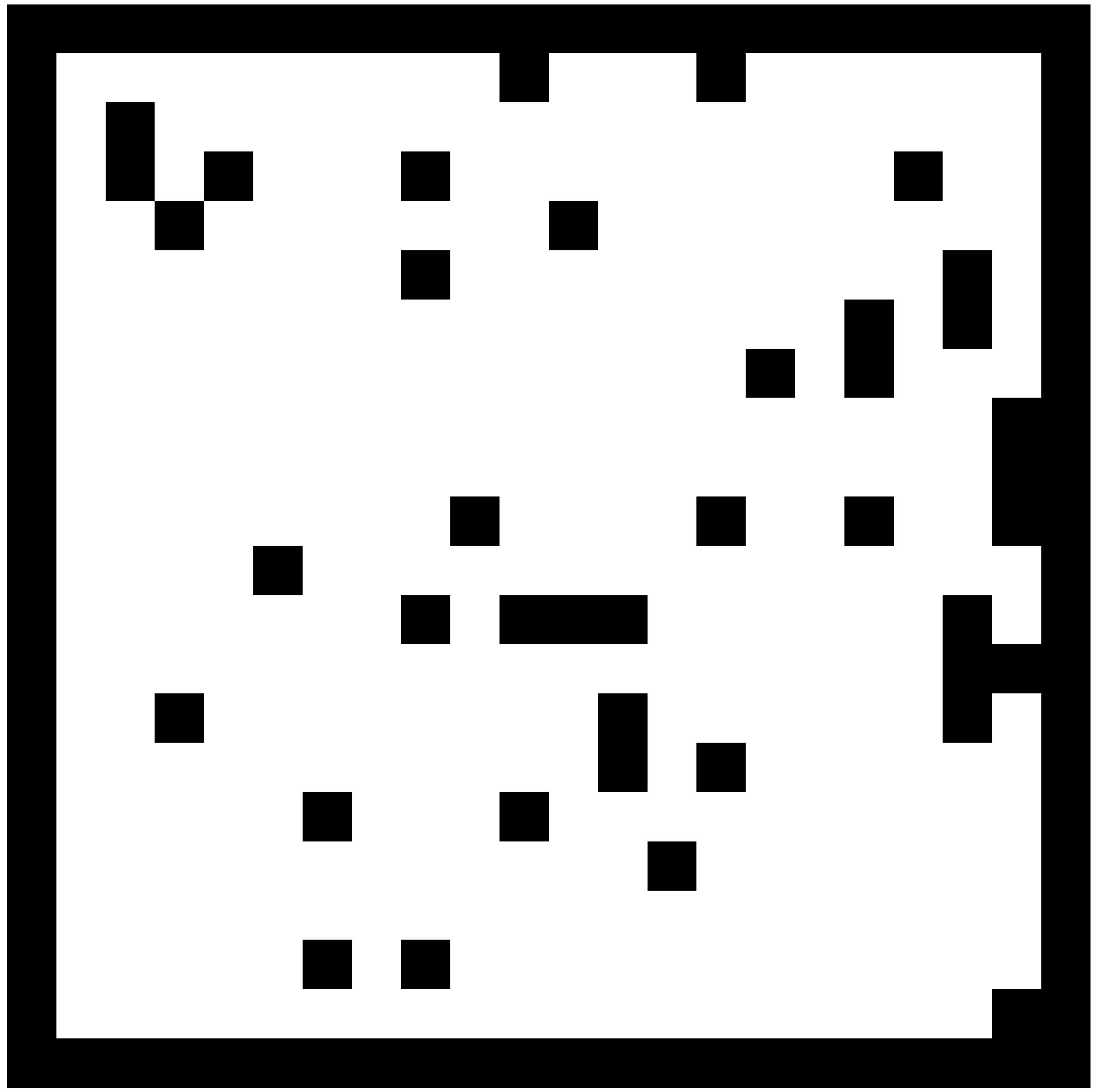
DATA 61 CSIRO

Outline

- Multi Agent Path Finding (MAPF)
  - Why should we be interested in this problem
  - MAPF Solving Methods
- CP for MAPF: Lazy CBS
  - CBS Weaknesses
  - Lazy CBS
  - Experiments
- MIP for MAPF: Branch and Cut and Price (BCP)
  - MIP Background
  - A MIP model of MAPF
  - Path Planning in BCP
  - Experiments
- Conclusion: What do Path Planning and Discrete Optimization have to say to each other

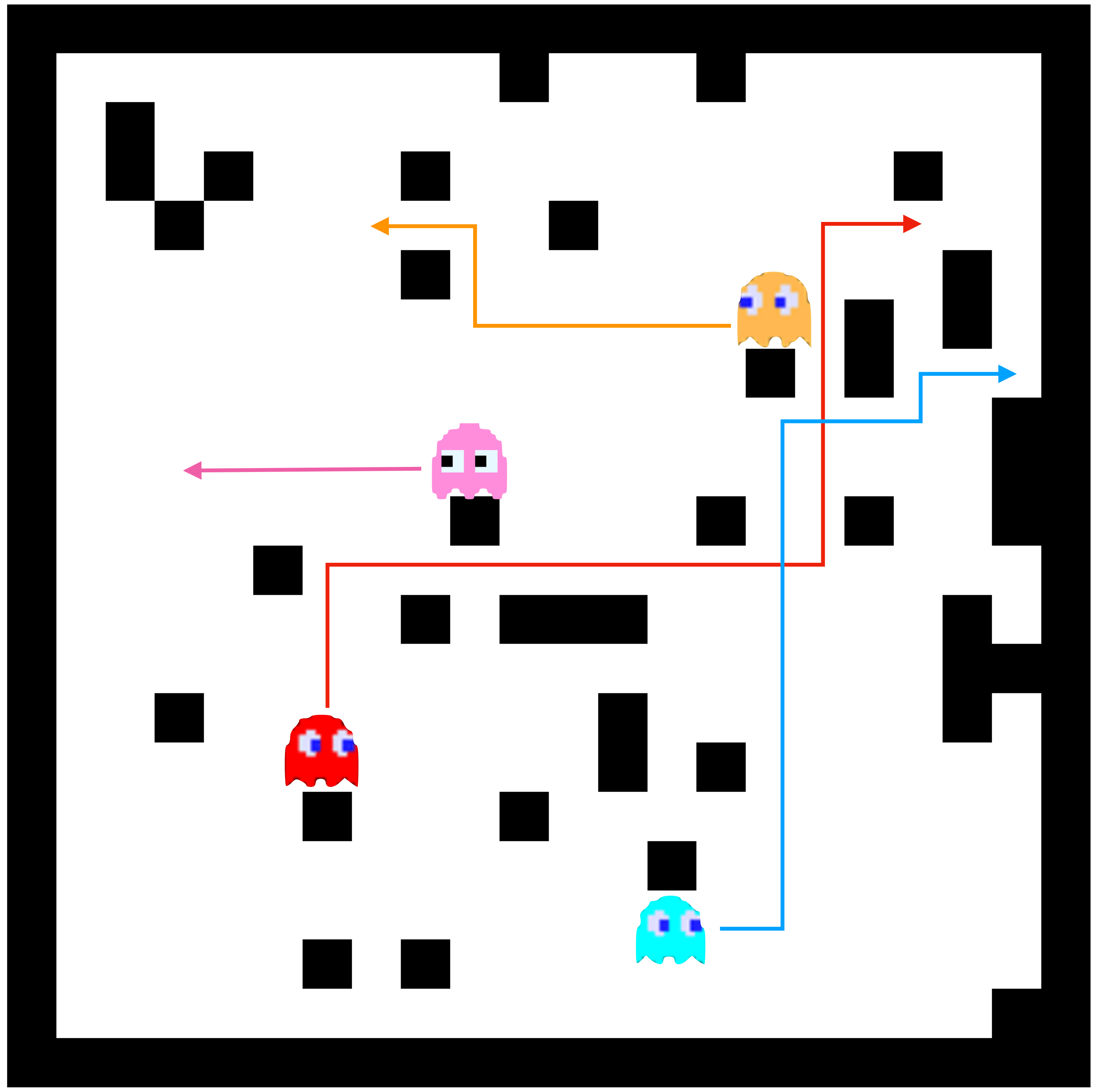
Multi-agent Pathfinding

- Consider an infinite time horizon with discrete **time steps**
- Consider a grid called the **map**
  - The map is divided into cells
  - Cells can be passable or obstacles



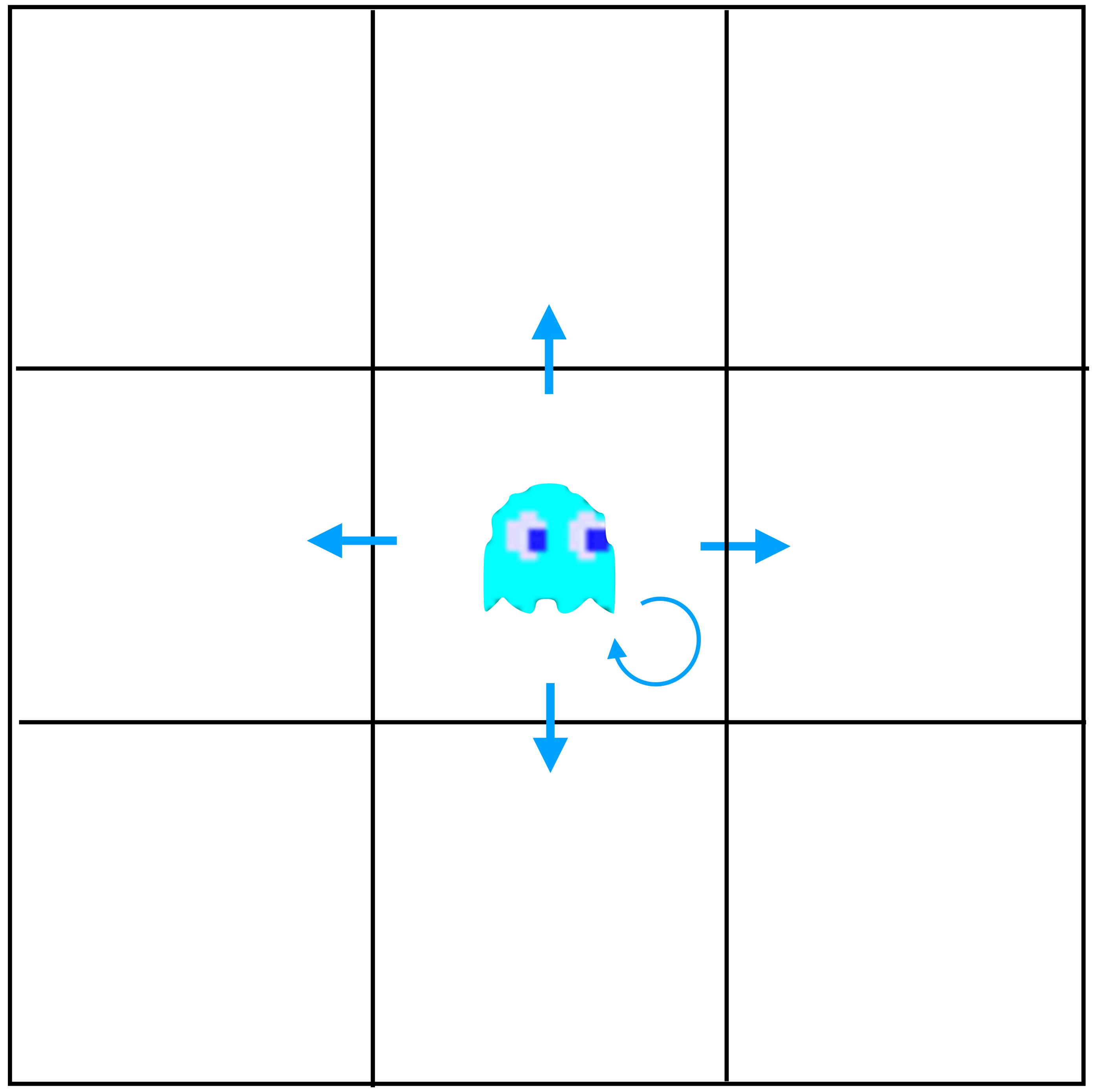
Multi-agent Pathfinding

- Consider a set of **agents**
- Every agent needs to move from its **start cell** to its **end cell**



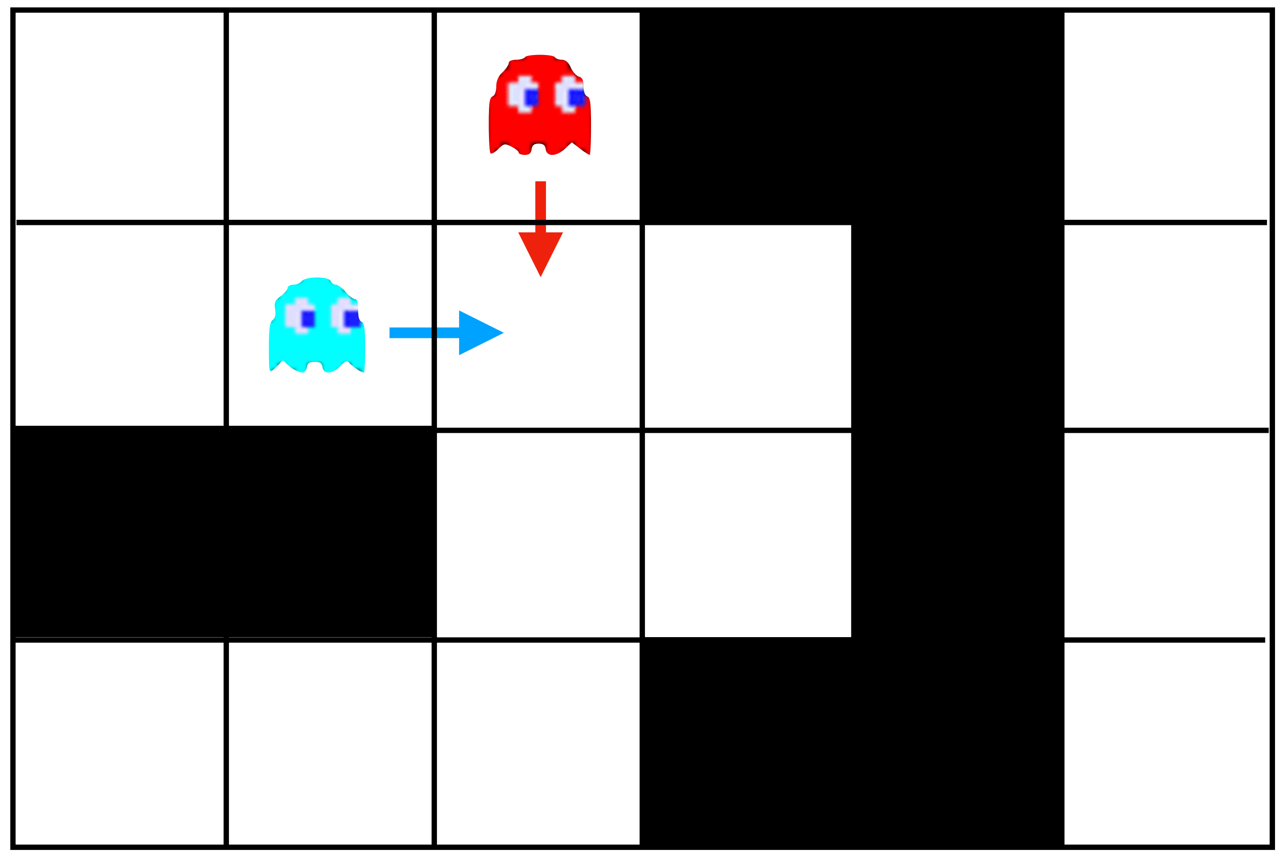
Multi-agent Pathfinding

- At any time step, an agent can **move north, south, east** or **west**, or **wait** at the same cell



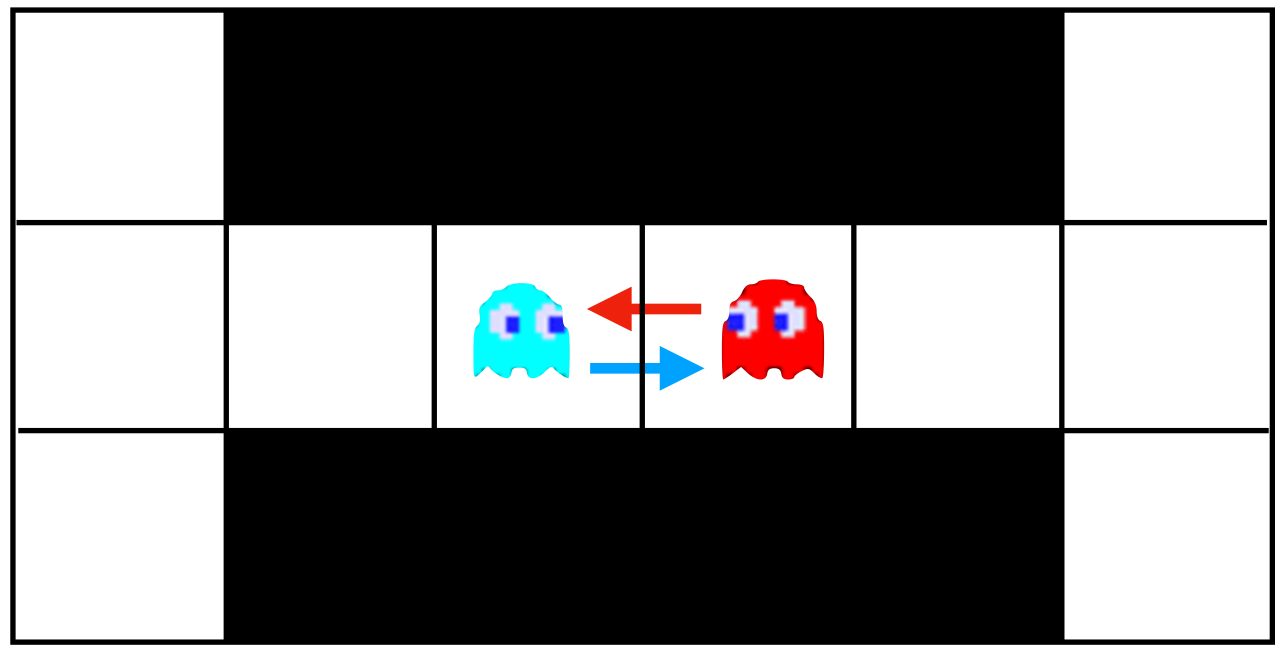
Multi-agent Pathfinding

- **Vertex collision:** two or more agents cannot occupy a cell at any given time



Multi-agent Pathfinding

- **Swap (edge) collision:** two or more agents cannot cross in opposite directions at any given time



Multi-agent Pathfinding

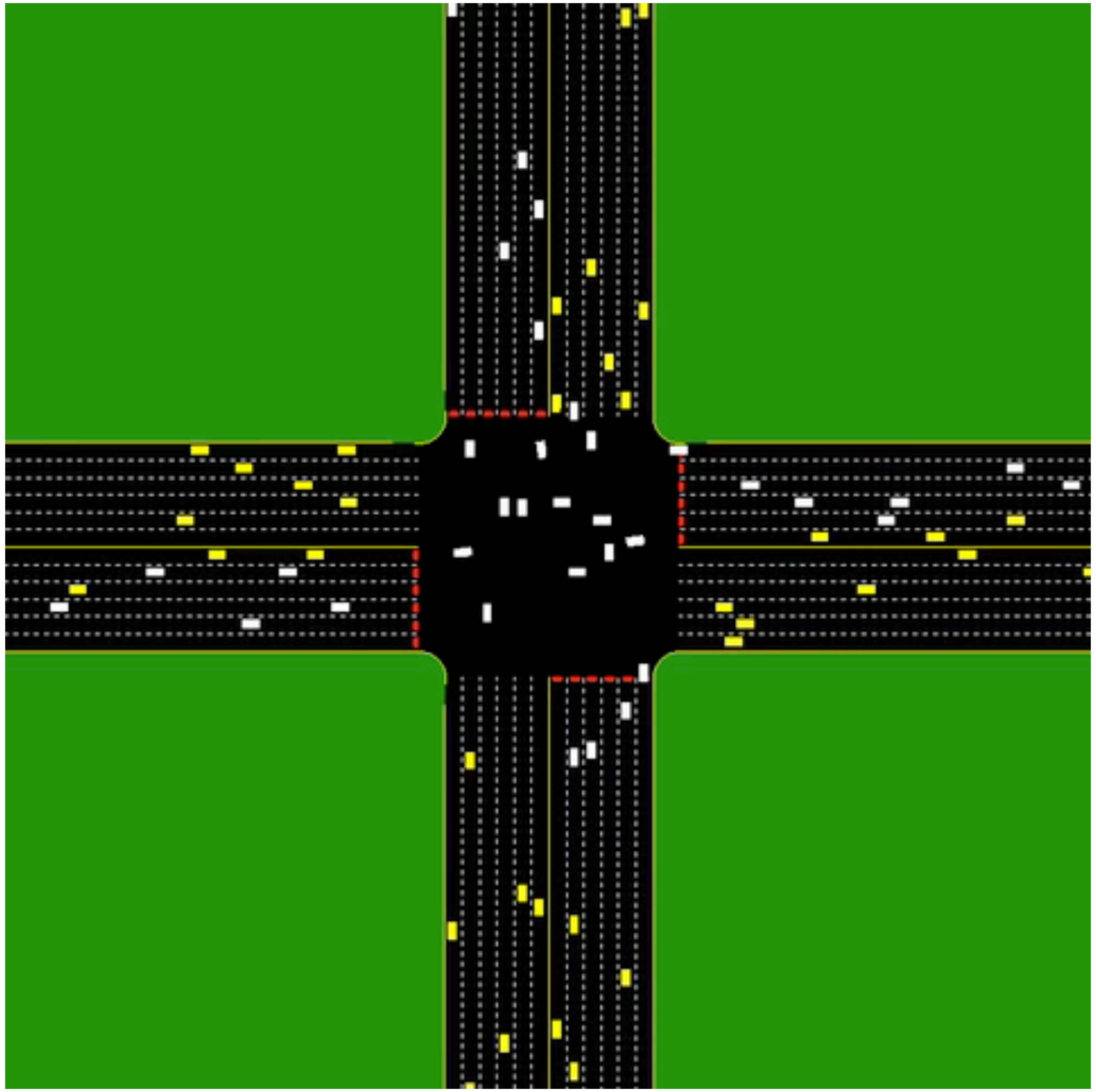
- Minimise sum of individual costs (SIC) (sum of path lengths)
    - alternate makespan (maximum path length) is much easier
- Assumptions:
    - Centralized Solver
    - Offline Task
    - Optimal Solution

Why should we be interested in MAPF

The future of road travel

- From Peter Stone's website

http://www.cs.utexas.edu/~pstone/

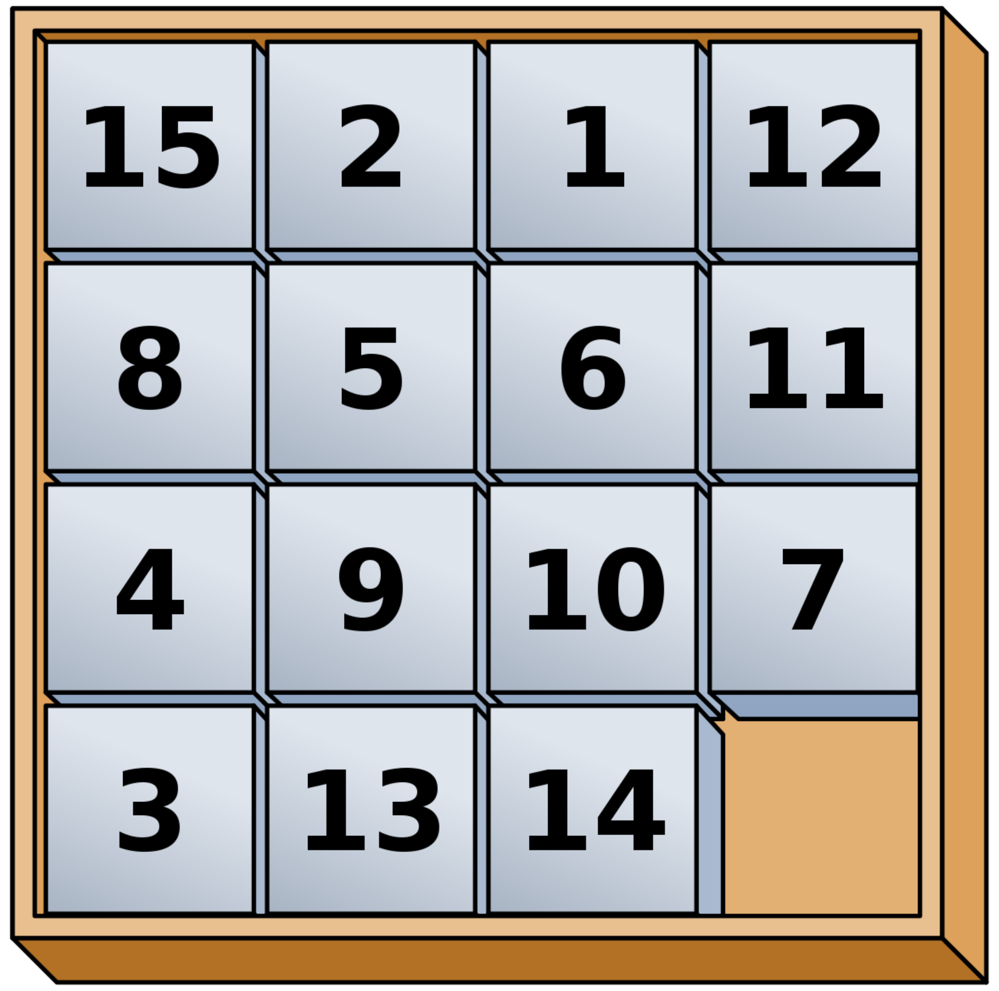


The present of road travel

- Uncontrolled intersection in Ethiopia
- https://youtu.be/SQ3rxwVYs5c

Motivation

- Robotics
- Video games
- Transportation applications
- Warehouse management
- Product assembly




KIVA PICKING

Multi-agent Pathfinding

- The quintessential multi-agent movement coordination problem
  - Real world problems are harder but always have this underlying difficulty
- MAPF is NP-hard
- Who needs optimality?
  - Problems that arise for the optimal version
    - Also hit the bounded sub-optimal case
  - Unbounded sub-optimal solutions can be very bad!

MAPF Solving Methods

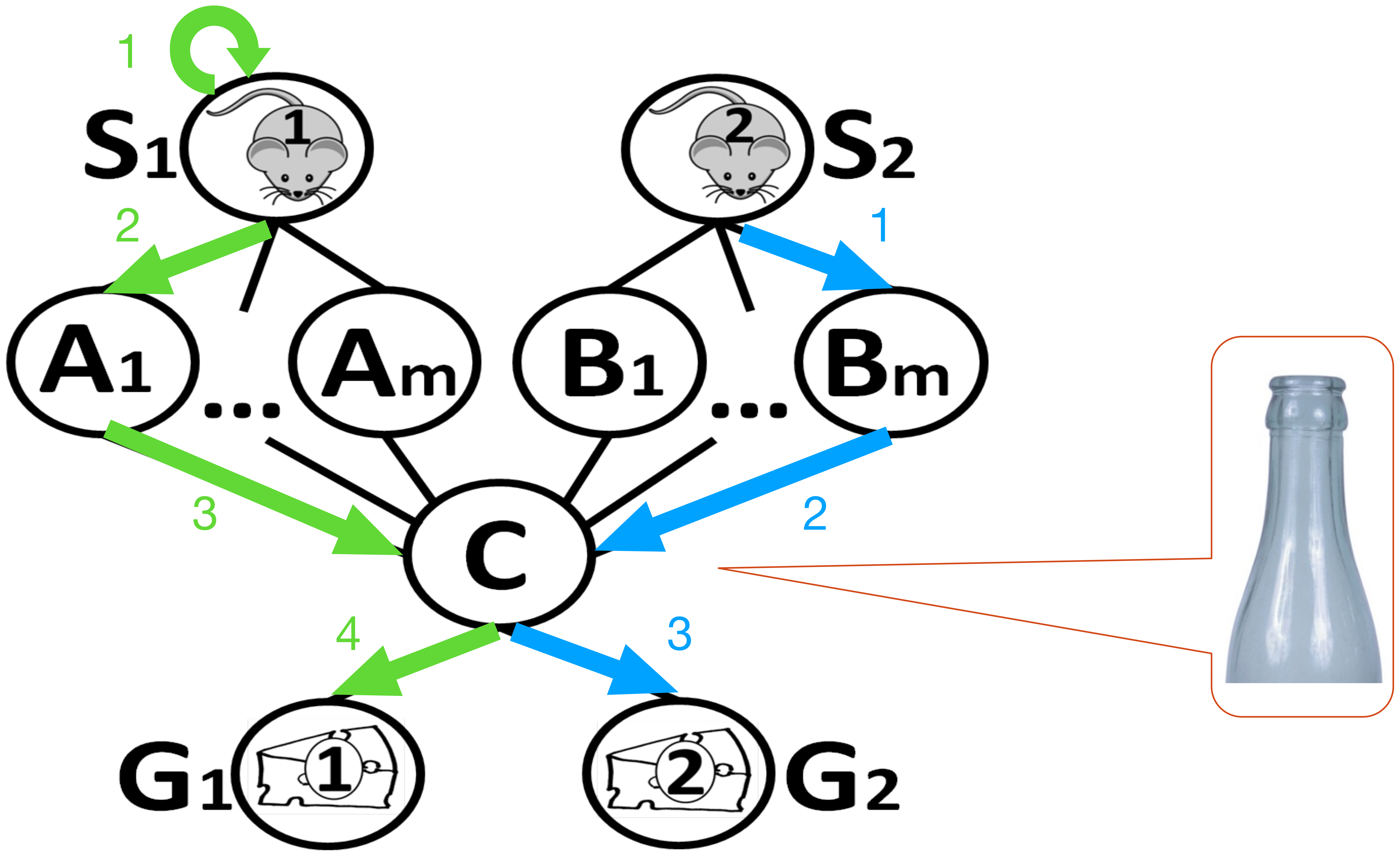
MAPF Solving Approaches

- A* (and variations e.g. M*)
- Conflict Based Search (and variations)
- Encoding/Reduction
  - SAT, ASP, CP, MIP
- Suboptimal methods (we wont talk about)
  - ECBS
  - Push and Swap
  - Pebble rotation coordination (complete)

A* Approach

- **State space:** Permutations of ***n*** agents into ***V*** locations=O(V^n)
- **Operators:** Locations of all agent in the next time step
- **Heuristic function:** Sum of Individual Costs **(SIC)**

Even for this tiny example
m^2 state space

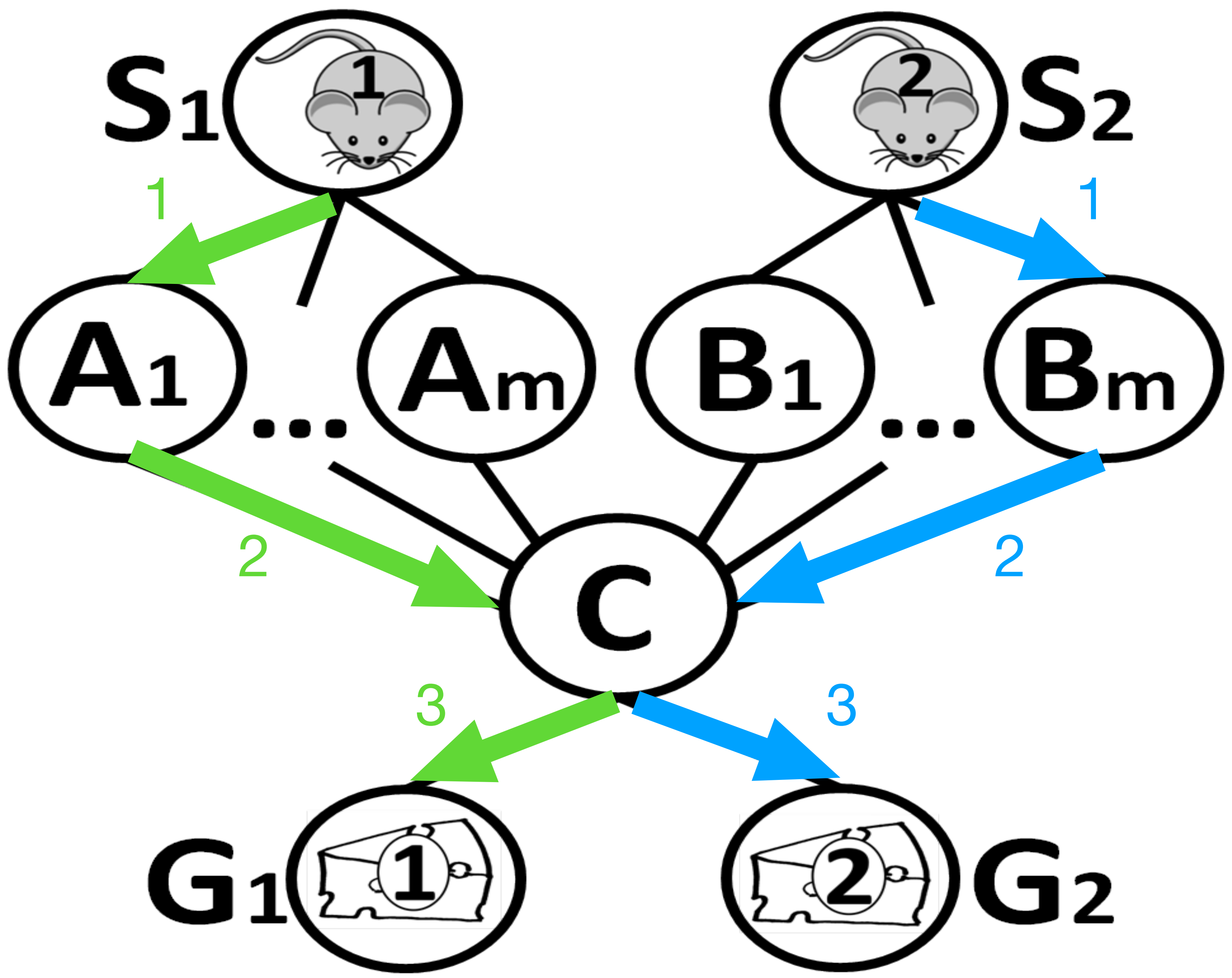


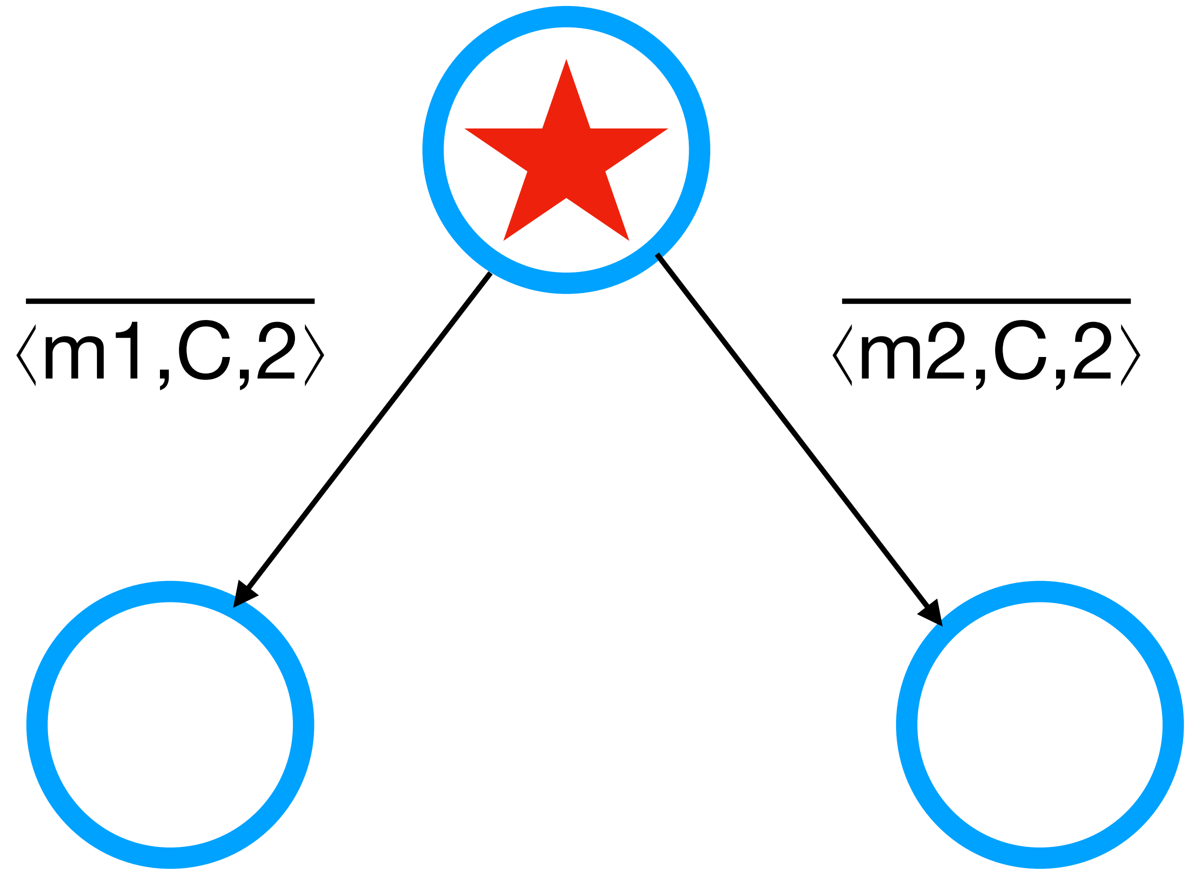
Conflict Based Search

- Plan each agent individually
- **repeat**
  - **if** the plans for pair of agents *A*, *B* conflict (use resource *X* at same time *t*)
    - split the problem into two
      - left child: *A* cannot use *X* at time *t*
      - right child: *B* cannot use *X* at time *t*
    - pick a subproblem to work on
    - replan for the affected agent (enforcing the constraints)
  - **else return** plans

CBS Example

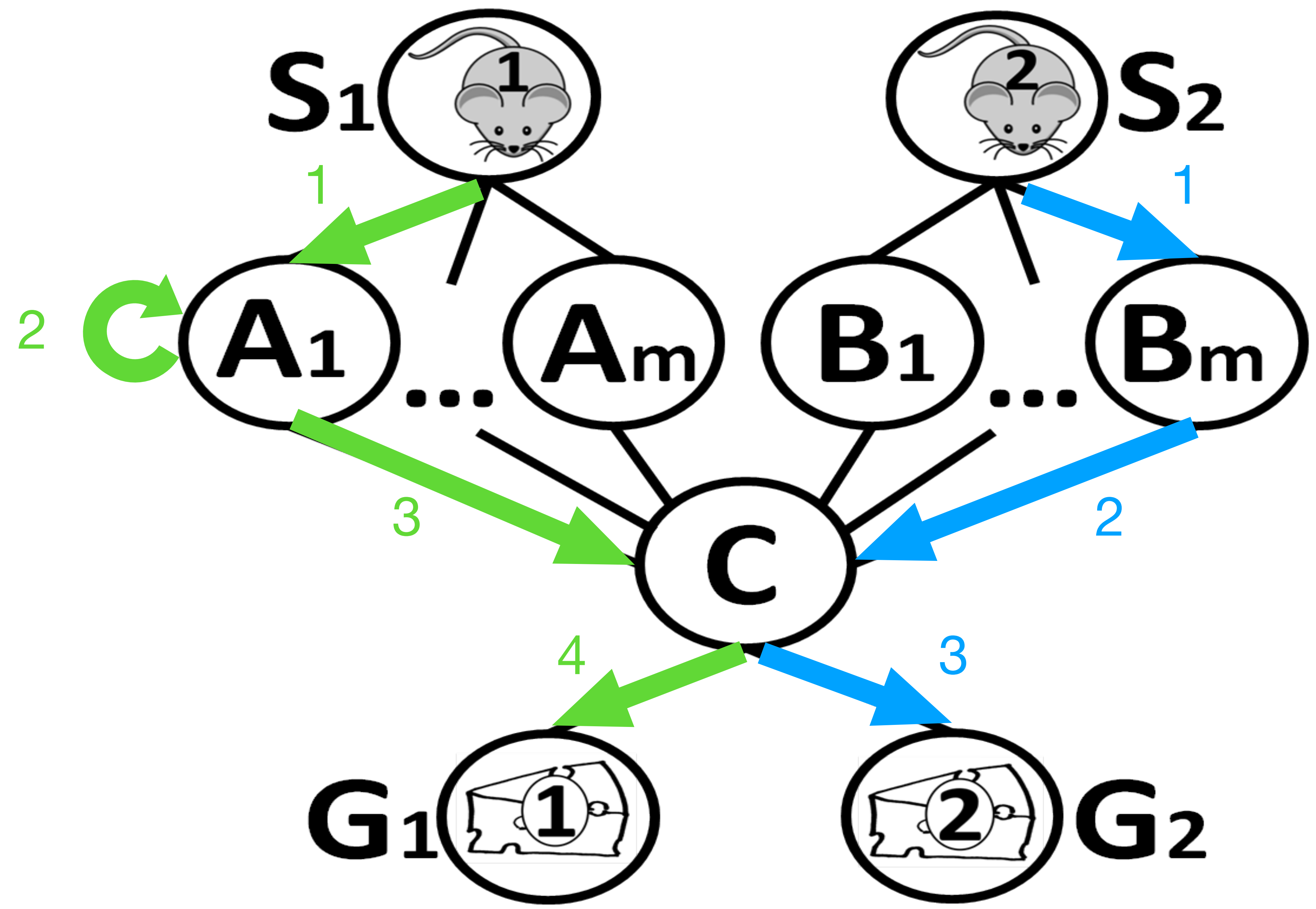
- Initial Plans
- Conflict at location C
- Two subproblems: mouse {1,2} cannot use C at time 2

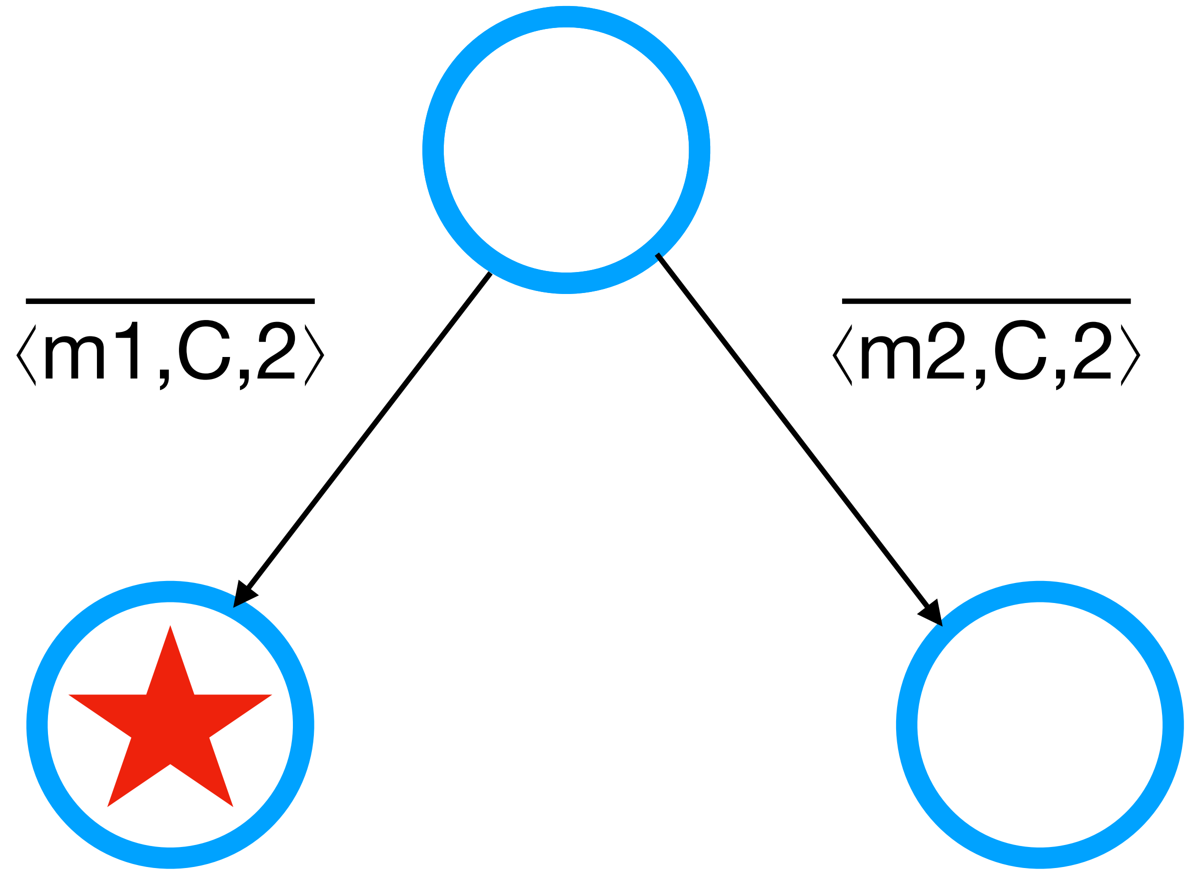




CBS Example

- Select first subproblem
- Replan mouse 1: avoiding C at time 2
- Solution!





Encodings/Reductions

- MIP solution
  - [makespan] multi-commodity flow model, small graphs, ≤ 50 agents (Yu and LaValle, 2013)
- SAT solution
  - [SIC]: on small maps may eventually beat CBS (Surynek et al, 2016)
- CP solution
  - [makespan] Variables choose a path for each agent, dynamic variable domain (J Wang et al, 2019)
- SMT solution
  - [SIC]: Lazy construction of SAT model, better than CBS on tight maps, small number of agents (Surynek, 2019)
- ASP solution
  - [SIC] a tighter model, uses unsatisfiable core based optimization, small graphs ≤ 16 agents (Gomez et al, 2020)

Encodings/Reductions

- Essential Problem for Naive encodings
  - $x_{apt.}$ agent *a* is at position *p* at time *t*
  - $O(nVT)$ *n* is number of agents, *V* is number of vertices, *T* is time bound
- Too many variables, even for smallish cases
  - $2 \times (2m+5) \times 4$ at least for our trivial example
- Tight *T* is hard to determine, $O(V^3)$ for sum of costs

Constraint Programming for MAPF
Lazy-CBS

CP for MAPF

- CP is an integrative solving approach
  - we can hide subproblems inside global constraints
- We can essentially reimplement CBS inside a CP solver
  - What the advantage?
- Lazy-CBS: a CP based approach like CBS for solving MAPF

Lazy CBS

Abstract CP MODEL

```
% MASTER PROBLEM
enum AGENT;    % set of agents
enum LOCATION; % set of locations
int: max_time; % maximum time allowed
set of int: TIME = 1..max_time;
% start and end locations
array[AGENT] of LOCATION: start;
array[AGENT] of LOCATION: end;
% cost of each agents path
array[AGENT] of var 0..infinity: c; % cost of each agents path
% which agent is permitted at location l at time t (edges omitted)
array[LOCATION, TIME] of var AGENT: permitted;
constraint forall(a in AGENT)
               (cost_of_path(a,start[a],end[a],permitted,c[a]));
solve minimize sum(c);
```

CP Path Propagator

- A specialized propagator to update path costs!
- `cost_of_path(a,start[a],end[a],permitted,c)`
  - update the lower bound on `c`
  - compute the shortest path *p* from `start[a]` to `end[a]` that satisfies the permitted constraints. e.g. `permitted[`p`[`t`],`t`] = a` for all *t* in path *p*
  - `c` $\geq$ *length(p)*
- Should update `permitted` to ensure no resource conflicts (ignore for now)
- EXACTLY the same algorithm as the CBS individual agent path finder

Cheating

- Wait a minute!

- Size of the model is still O(nVT)
  - `array[LOCATION, TIME] of var AGENT: permitted;`

- Solution:
  - lazily create the `permitted` variables as required
- Bonus
  - `max_time` no longer required

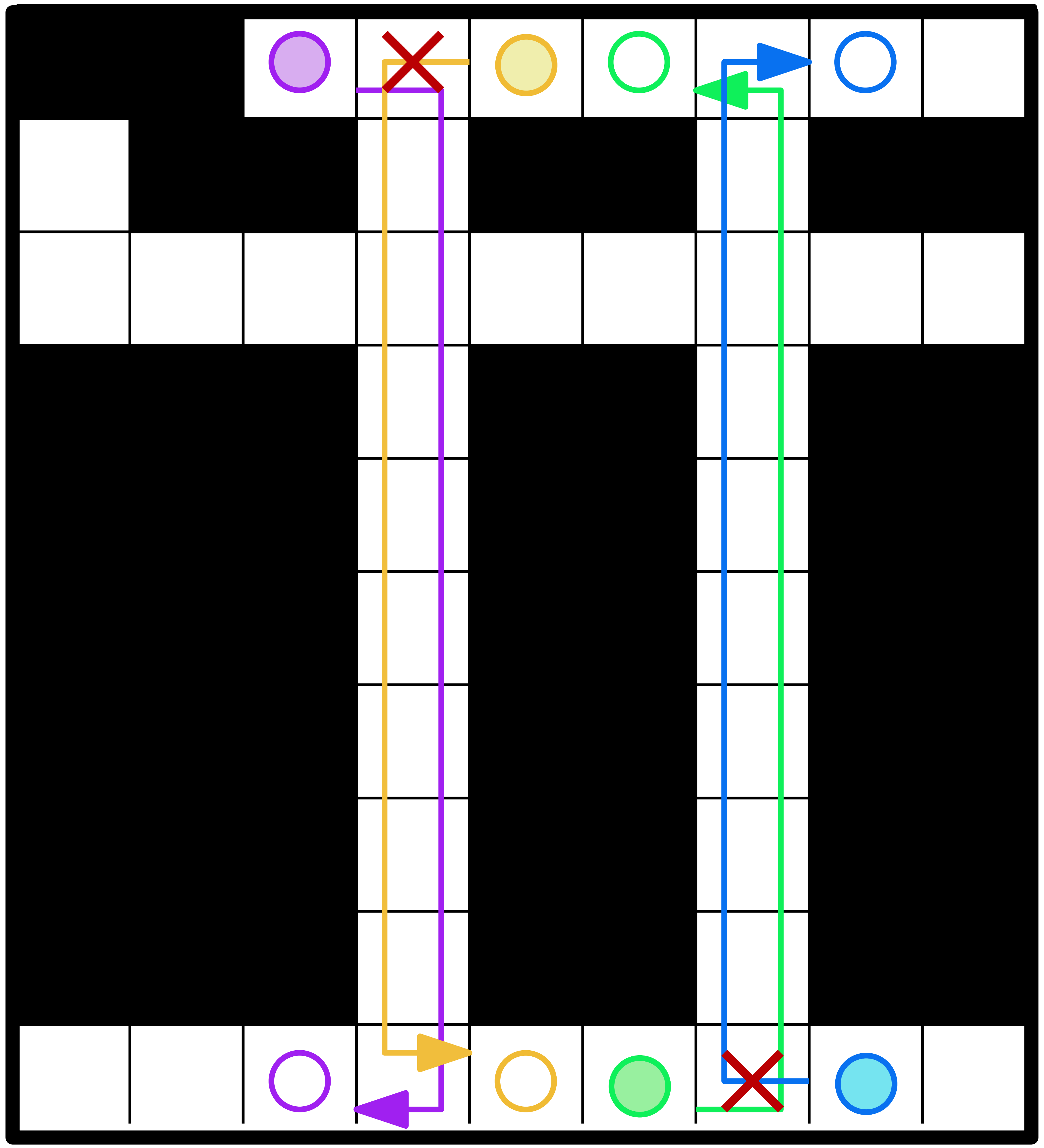
- A permitted variable is only required when two agents use the same location at the same time
- EXACTLY the same as the CBS strategy for adding constraints

WHY? (Reimplement CBS)

- Search is not the same!
  - CBS (best first search)
  - Lazy-CBS (depth first search)
- Learning is possible
  - Avoiding repeated work

CBS Weaknesses

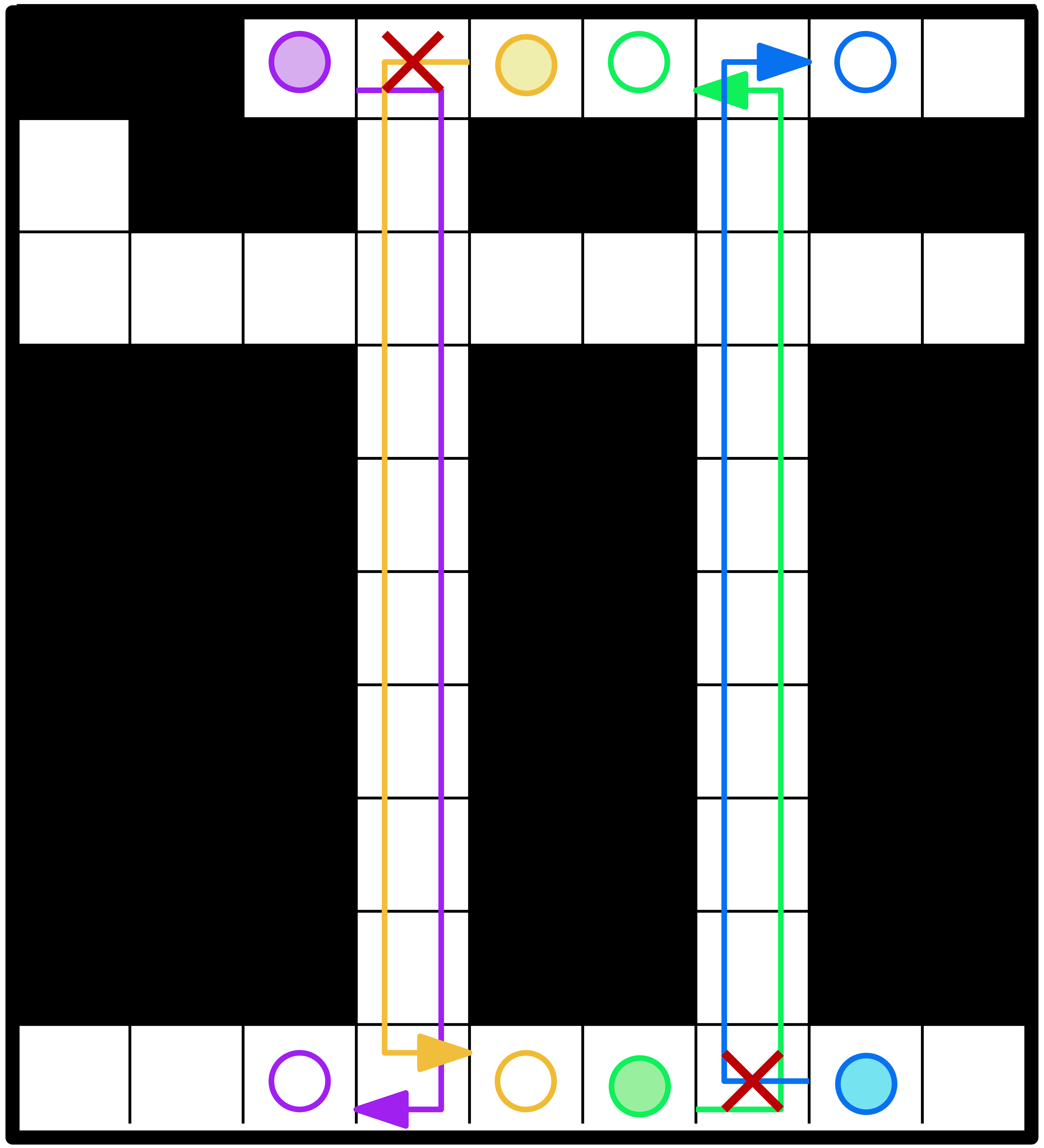
- Two independent conflicts
  - purple/yellow and green/blue



CBS Weaknesses

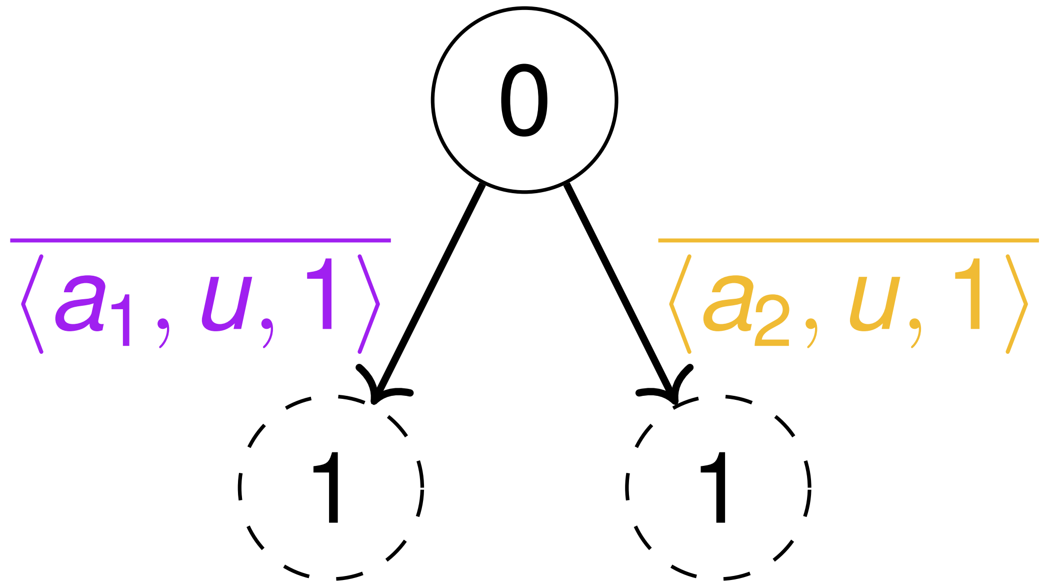
- Two independent conflicts
  - purple/yellow and green/blue

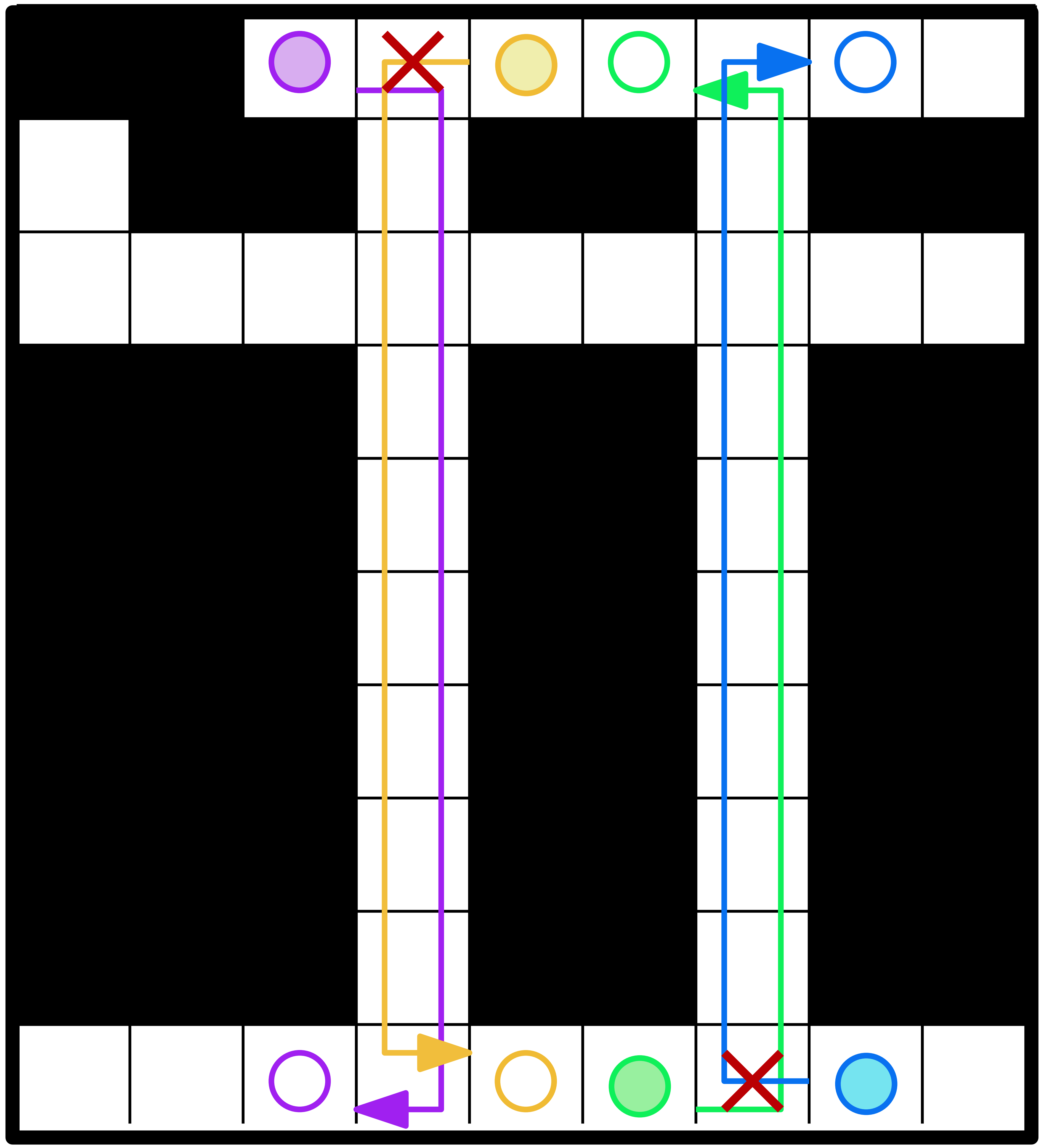
0



CBS Weaknesses

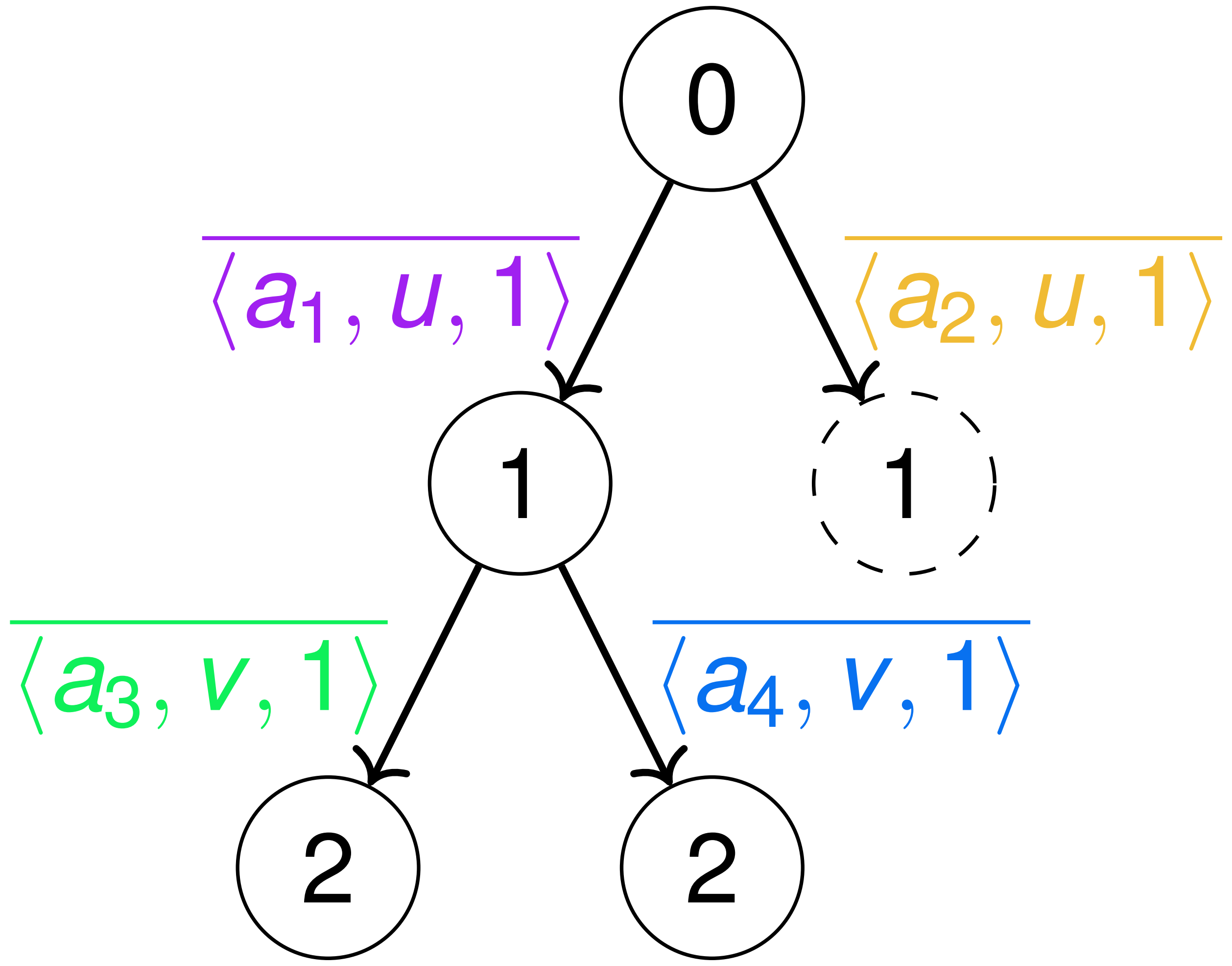
- Two independent conflicts
  - purple/yellow and green/blue





CBS Weaknesses

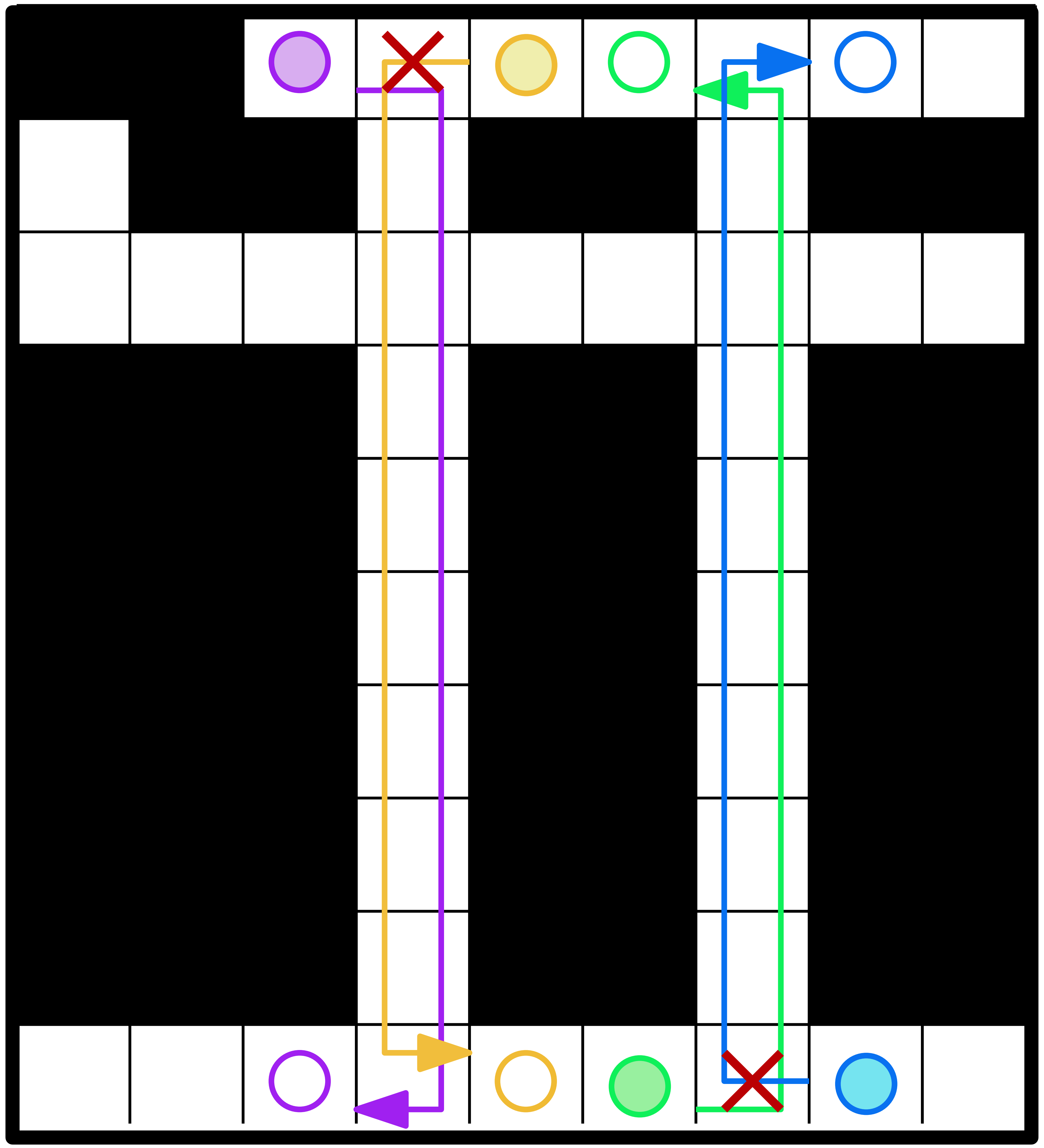
- Two independent conflicts
  - purple/yellow and green/blue



Unexpanded node looks promising

Still contains a_3,a_4 conflict

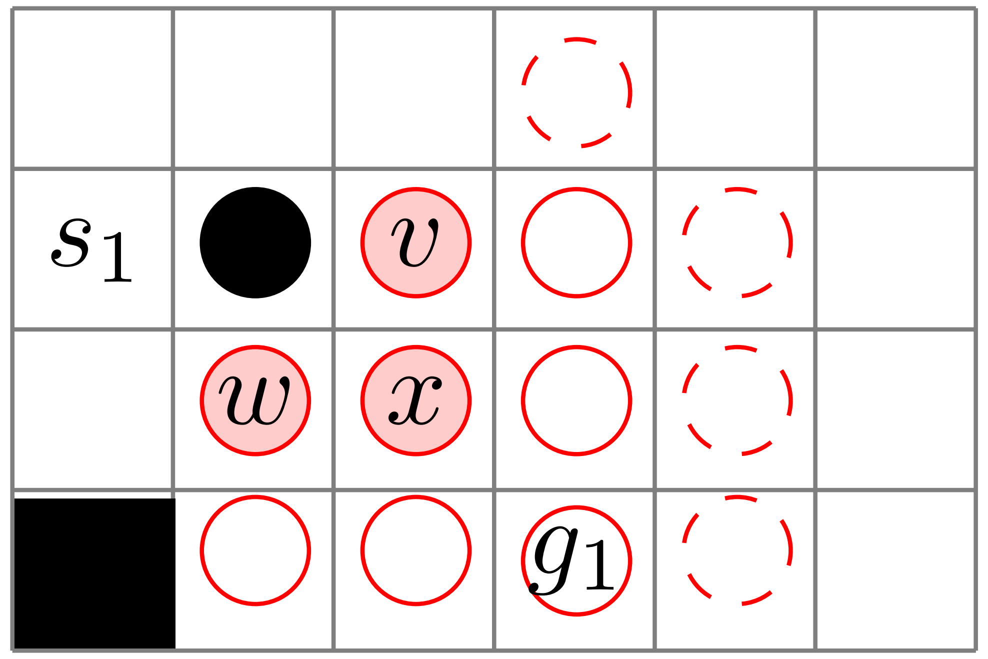
CBS has to resolve the same problem in multiple nodes



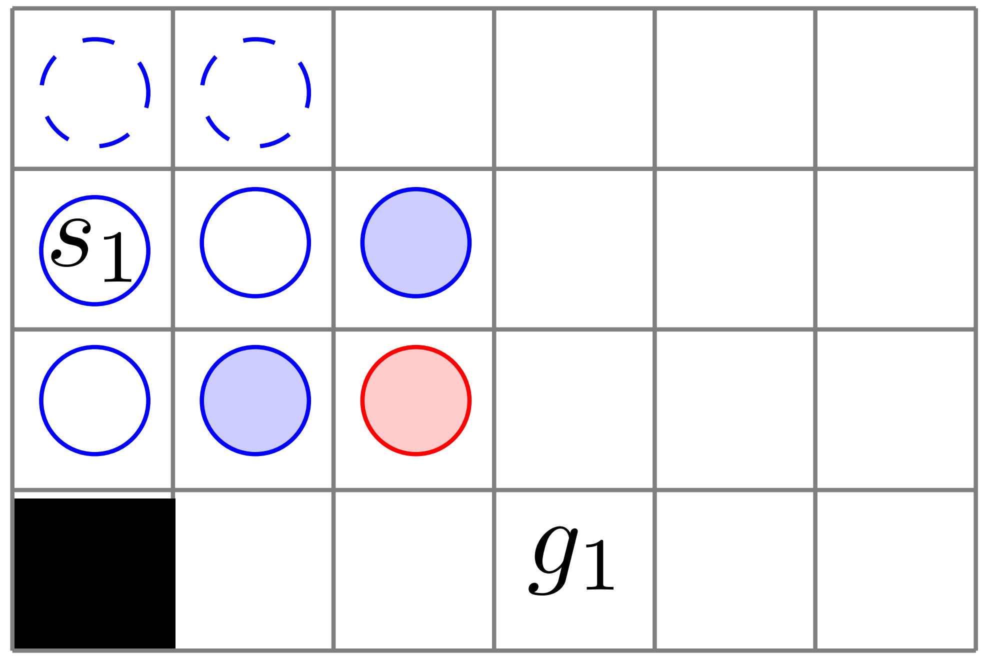
Lazy CBS Search

- CP solvers are astoundingly bad at optimizing sum objectives
  - e.g. Sum of path costs $\sum_{a \in \text{agents}} c_a$
- Core-guided optimization
  - Assume the best possible solution: each path cost c is the shortest
  - Find a contradiction (core)
  - Rewrite the objective with this knowledge
  - Assume the best possible solution under the rewriting
  - **repeat** until solution found
- Core guided optimization requires explanation of failure

Why is a path is too long



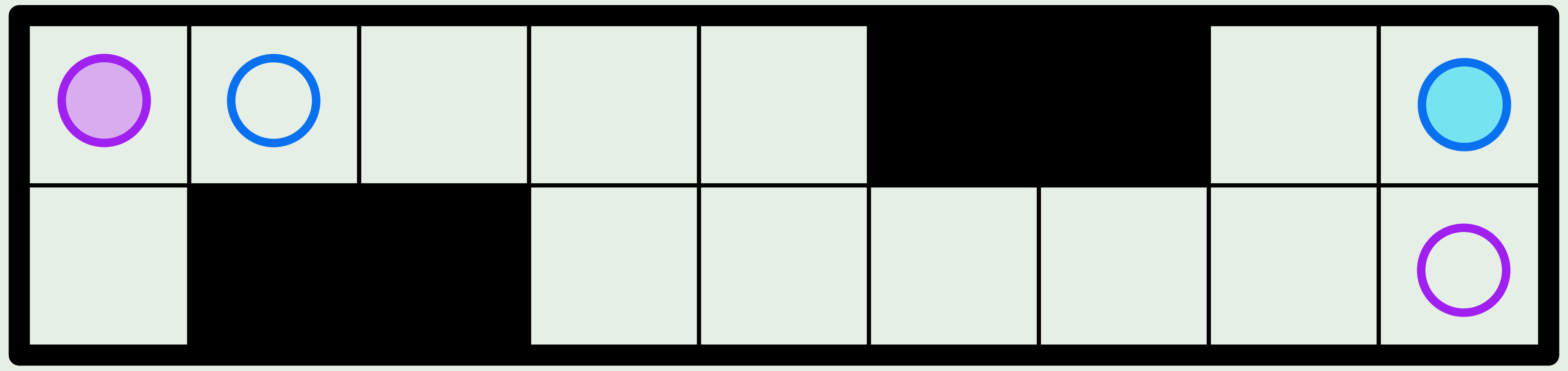
(a) MARK-FORBIDDEN



(b) COLLECT-SUFFICIENT

- Bound $c_1 \leq 5$. Explain why the path from s1 to g1 is more than 5
- Search backwards from goal at time 5
- Mark as forbidden blocked resources that are reached (pink)
- Search from start at time 0
- Skip over unmarked blocked resources (black) , i.e. treat them as available
- Collect reached marked blocked resources R (blue)
- Explanation is $R \rightarrow c_1 > 5$

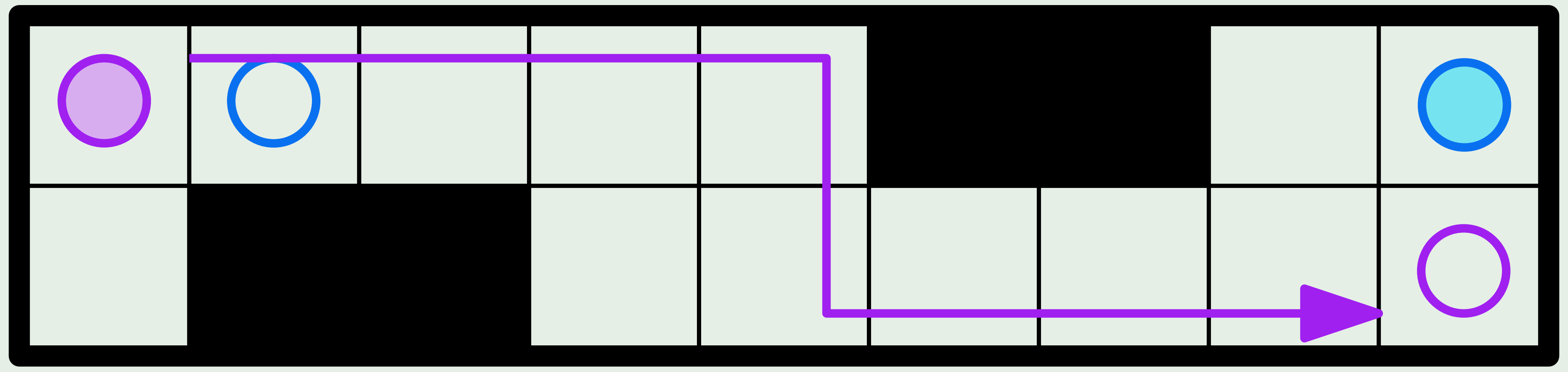
Lazy CBS in Action



c_1 : $[0, 9]$
c_2 : $[0, 9]$

- Initial Problem
- Each Agent constrained to have path cost no greater than shortest path

Lazy CBS in Action

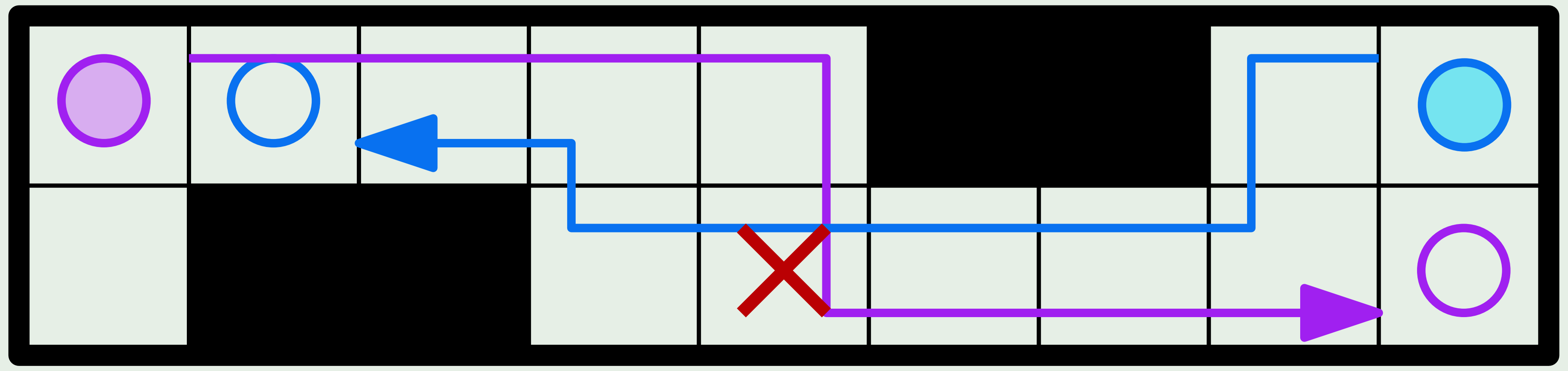


$c_1 : [9, 9]$

$c_2 : [0, 9]$

- Path propagator for agent 1 discovers lower bound on path length
- Generates example path

Lazy CBS in Action

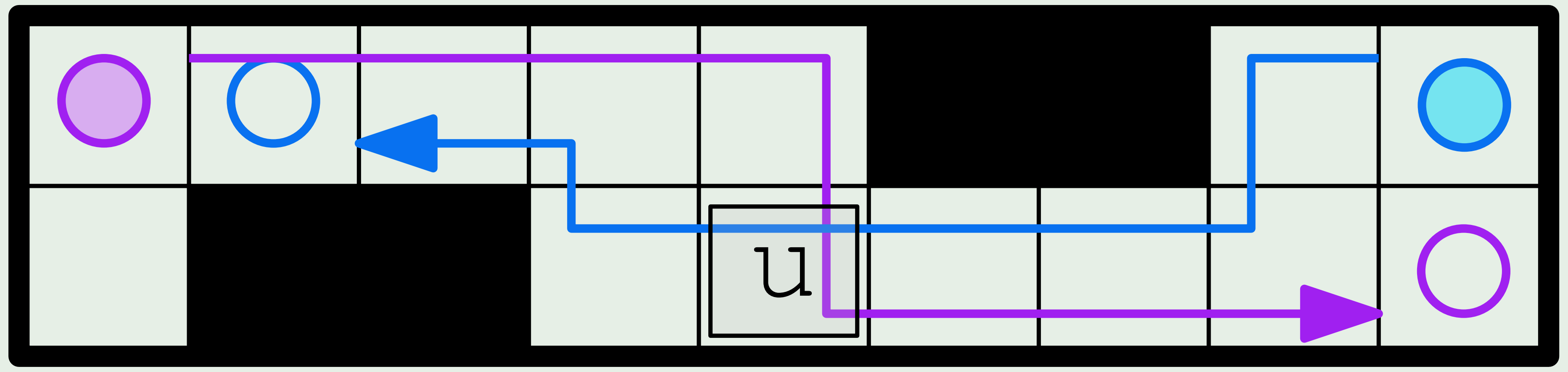


c_1 : $[9, 9]$

c_2 : $[9, 9]$

- Similarly for agent 2
- Conflict discovered

Lazy CBS in Action



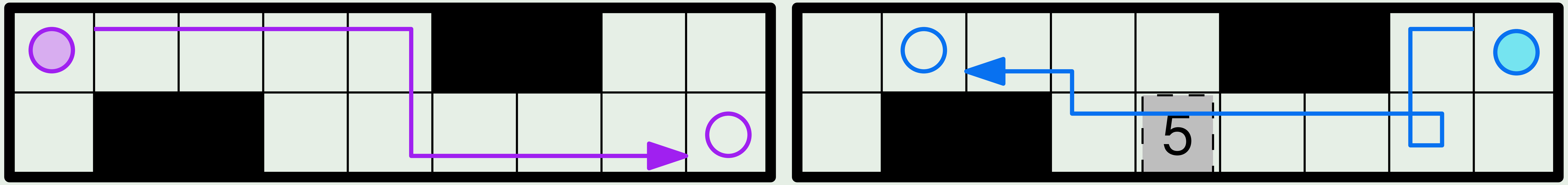
c_1 : $[9, 9]$

c_2 : $[9, 9]$

$\texttt{permitted}[\texttt{u}, 5] \in \{1, 2\}$

- Introduce `permitted` variable for overloaded resource

Lazy CBS in Action



$c_1 : [9, 9]$

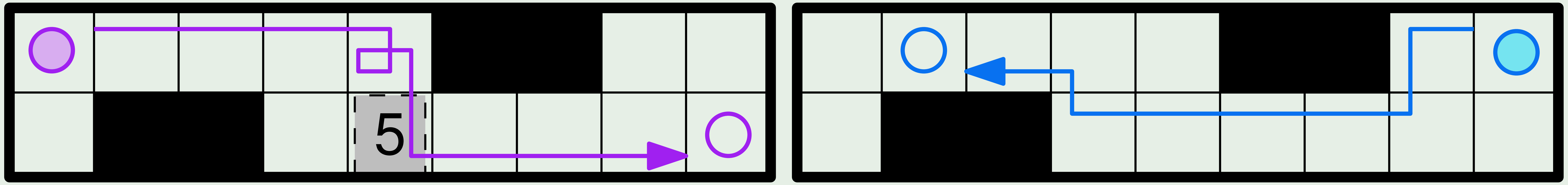
$c_2 : [\color{red}{10}, 9]$

$\texttt{permitted}[\texttt{u}, 5] = 1$

$$\langle \texttt{permitted}[u, 5] \neq 2 \rangle \rightarrow \langle c_2 \geq 10 \rangle$$

- On left branch, discover conflict, and derive explanation

Lazy CBS in Action



$c_1 : [\textcolor{red}{10}, 9]$

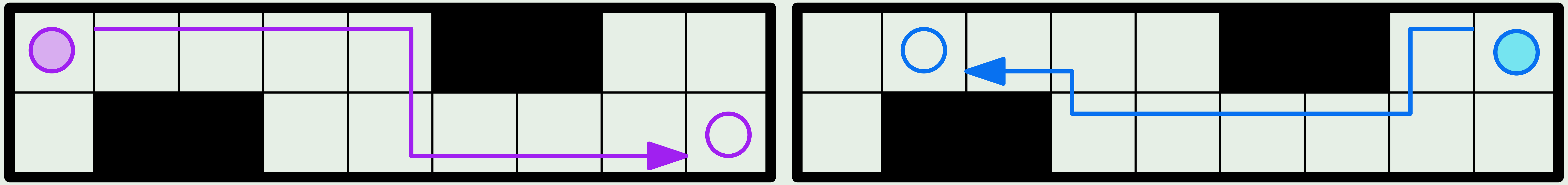
$c_2 : [9, 9]$

$\texttt{permitted}[\texttt{u}, 5] = 2$

$$\langle \texttt{permitted}[u, 5] \neq 2 \rangle \rightarrow \langle c_2 \geq 10 \rangle$$
$$\langle \texttt{permitted}[u, 5] \neq 1 \rangle \rightarrow \langle c_1 \geq 10 \rangle$$

- Right branch, discover conflict, add explanation

Lazy CBS in Action



$c_1 : [10, 9]$
$c_2 : [9, 9]$

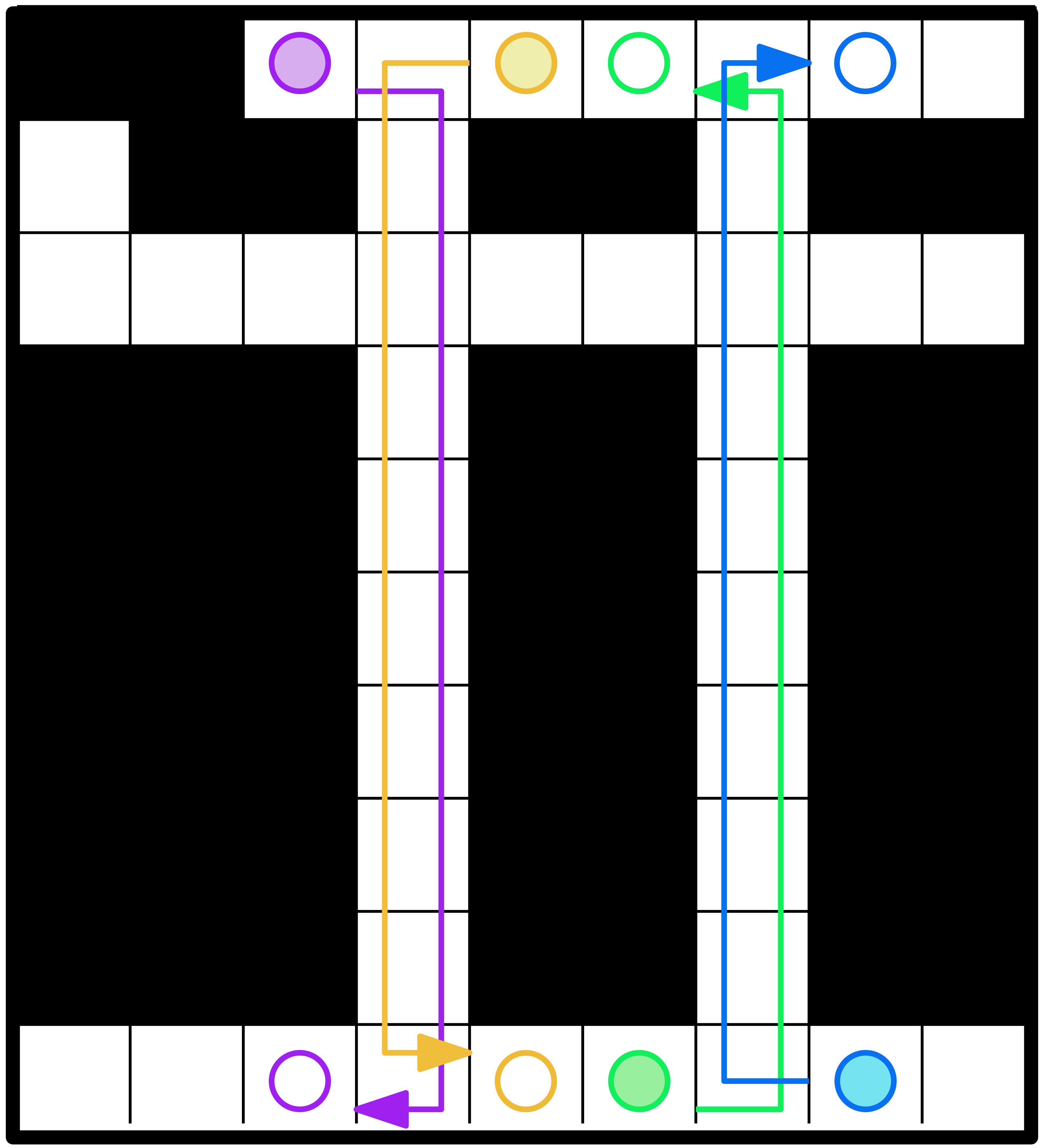
$\texttt{permitted}[u, 5] = 2$

$\langle \texttt{permitted}[u, 5] \neq 2 \rangle \rightarrow \langle c_2 \geq 10 \rangle$
$\langle \texttt{permitted}[u, 5] \neq 1 \rangle \rightarrow \langle c_1 \geq 10 \rangle$
$\langle c_1 \geq 10 \rangle \vee \langle c_2 \geq 10 \rangle$

- Learned nogood is independent of other agents

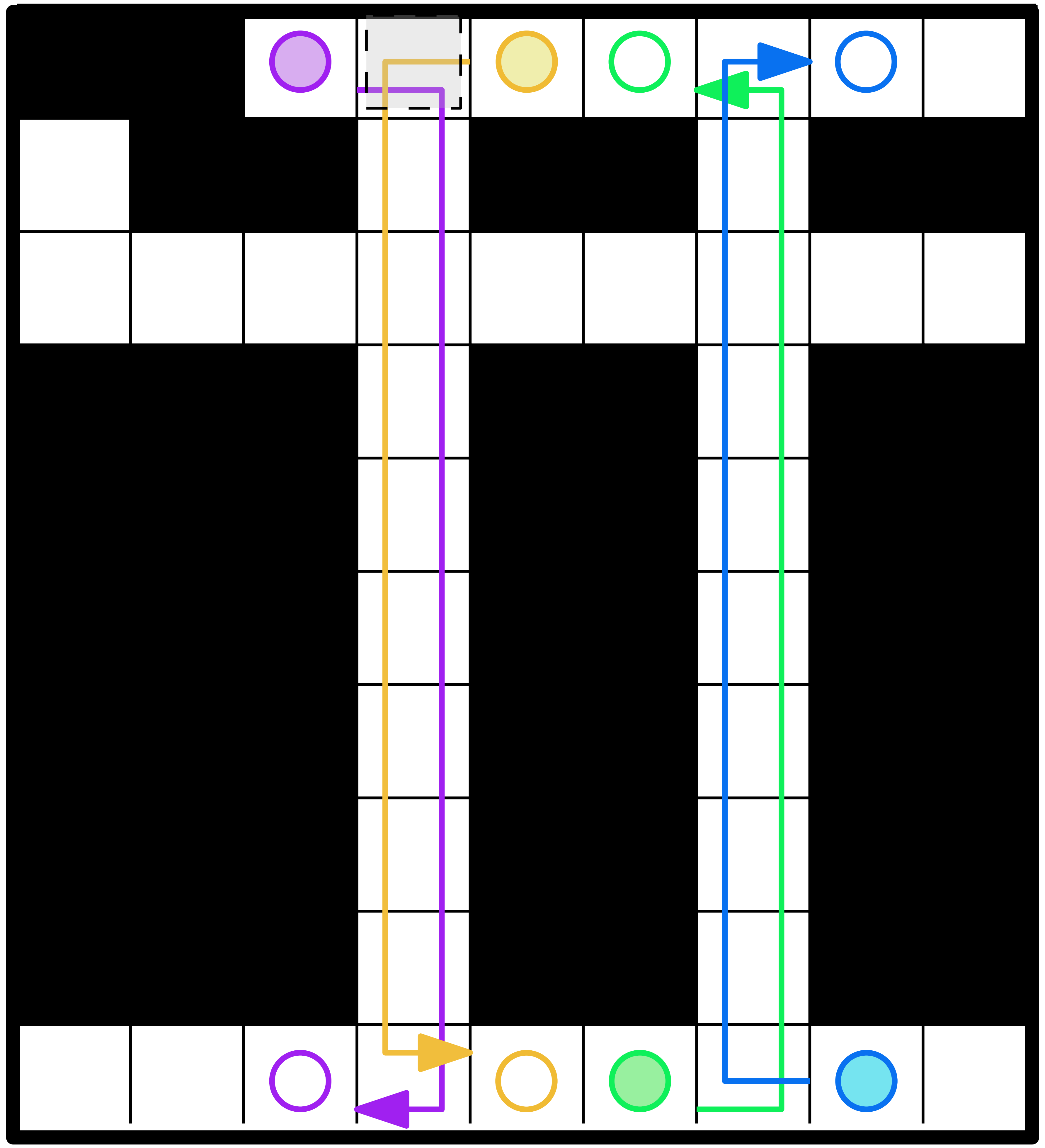
Lazy CBS in Action

- Lower Bound: 44
- Assumptions
  - $c_1 \leq 11$
  - $c_2 \leq 11$
  - $c_3 \leq 11$
  - $c_4 \leq 11$



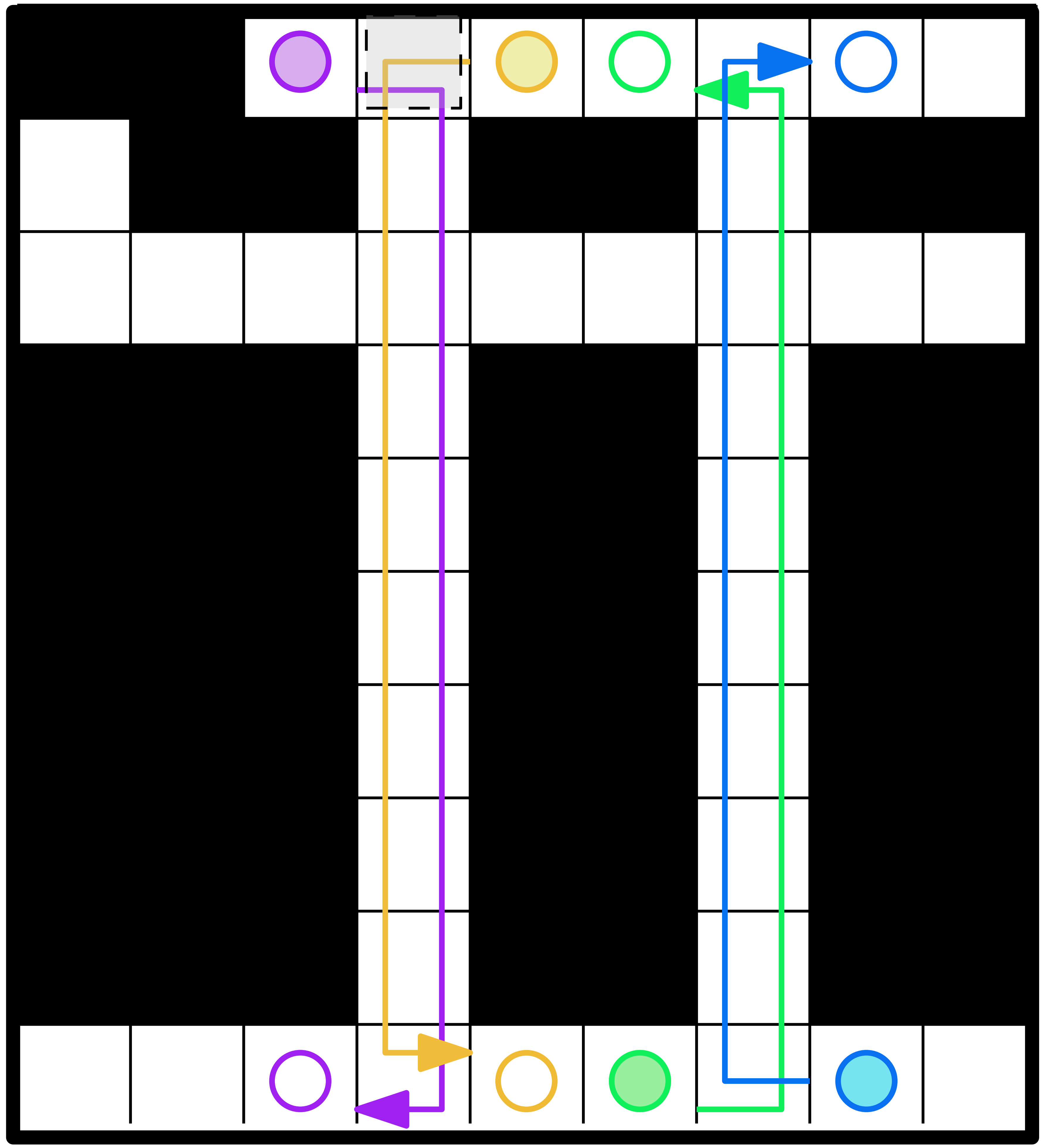
Lazy CBS in Action

- Lower Bound: 44
- Assumptions
    - $c_1 \leq 11$
    - $c_2 \leq 11$
    - $c_3 \leq 11$
    - $c_4 \leq 11$
- Nogood $\langle\, c_1 \geq 12 \,\rangle \;\vee\; \langle\, c_2 \geq 12 \,\rangle$



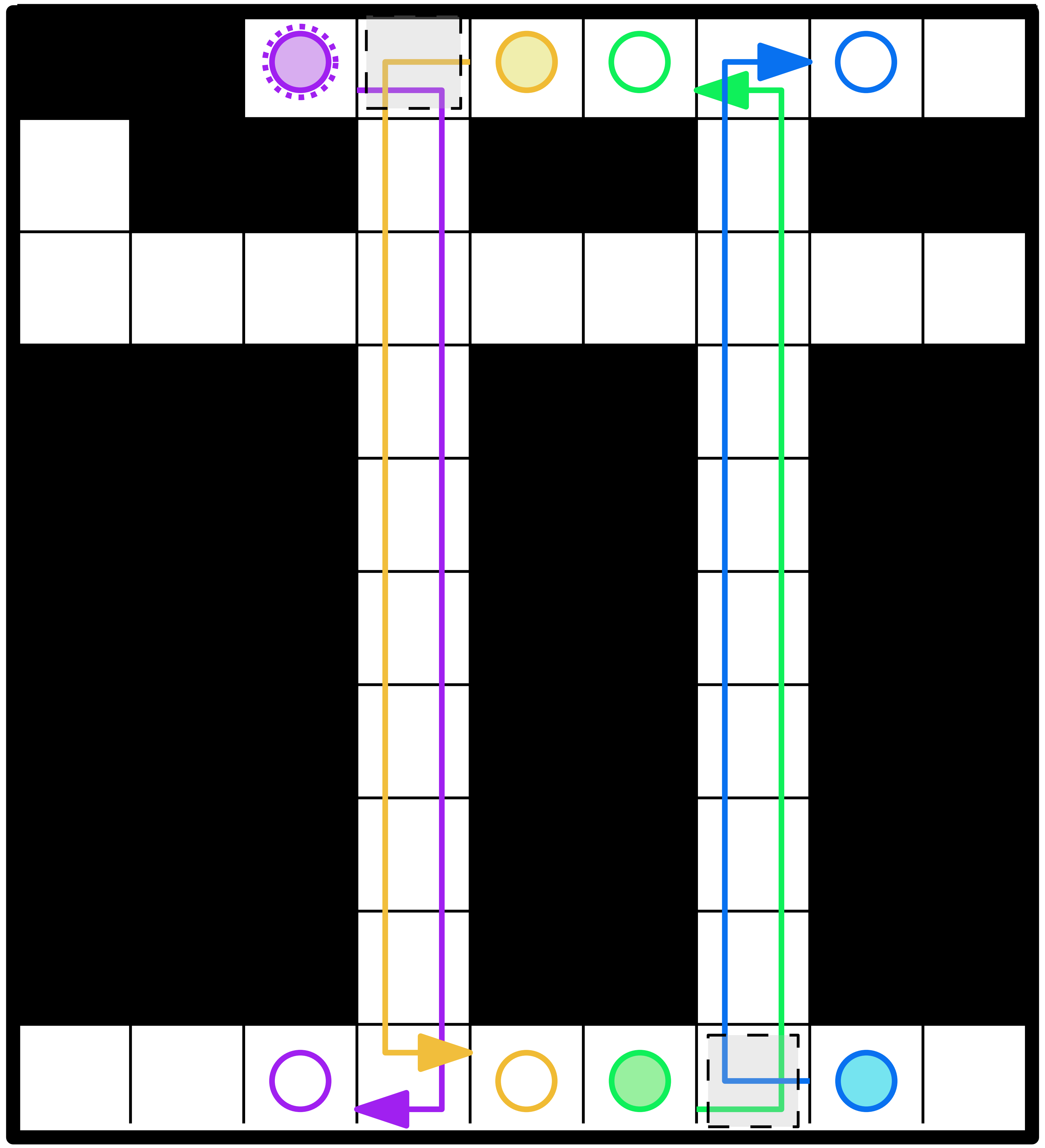
Lazy CBS in Action

- Lower Bound: 45
- Assumptions
  - $c_1 \leq 12$
  - $c_2 \leq 12$
  - $c_3 \leq 11$
  - $c_4 \leq 11$
  - $z_1 \leq 1$
- $z_1 = \langle c_1 \geq 12 \rangle + \langle c_2 \geq 12 \rangle$



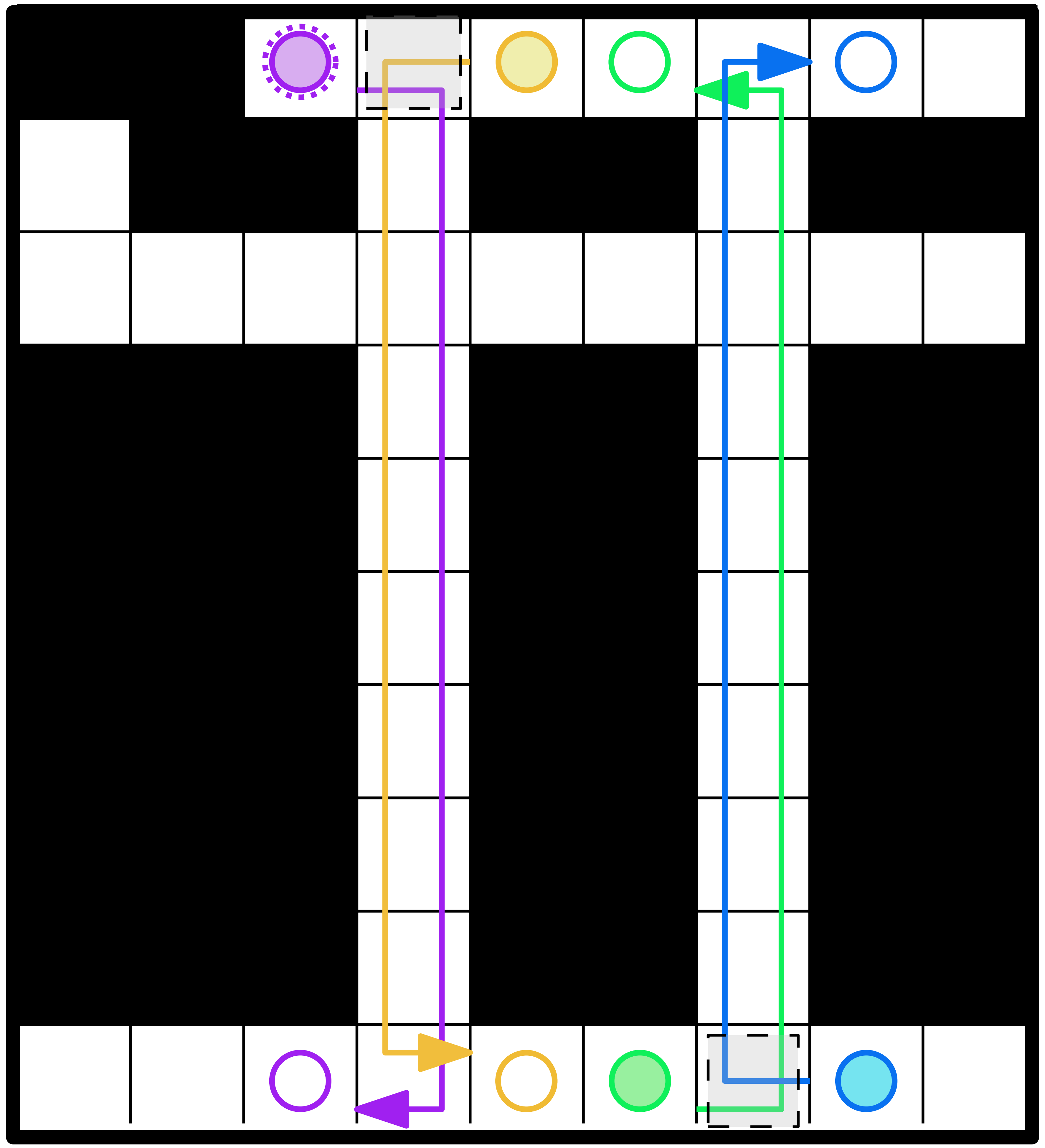
Lazy CBS in Action

- Lower Bound: 45
- Assumptions
  - $c_1 \leq 12$
  - $c_2 \leq 12$
  - $c_3 \leq 11$
  - $c_4 \leq 11$
  - $z_1 \leq 1$
- $\langle c_3 \geq 12 \rangle \vee \langle c_4 \geq 12 \rangle$



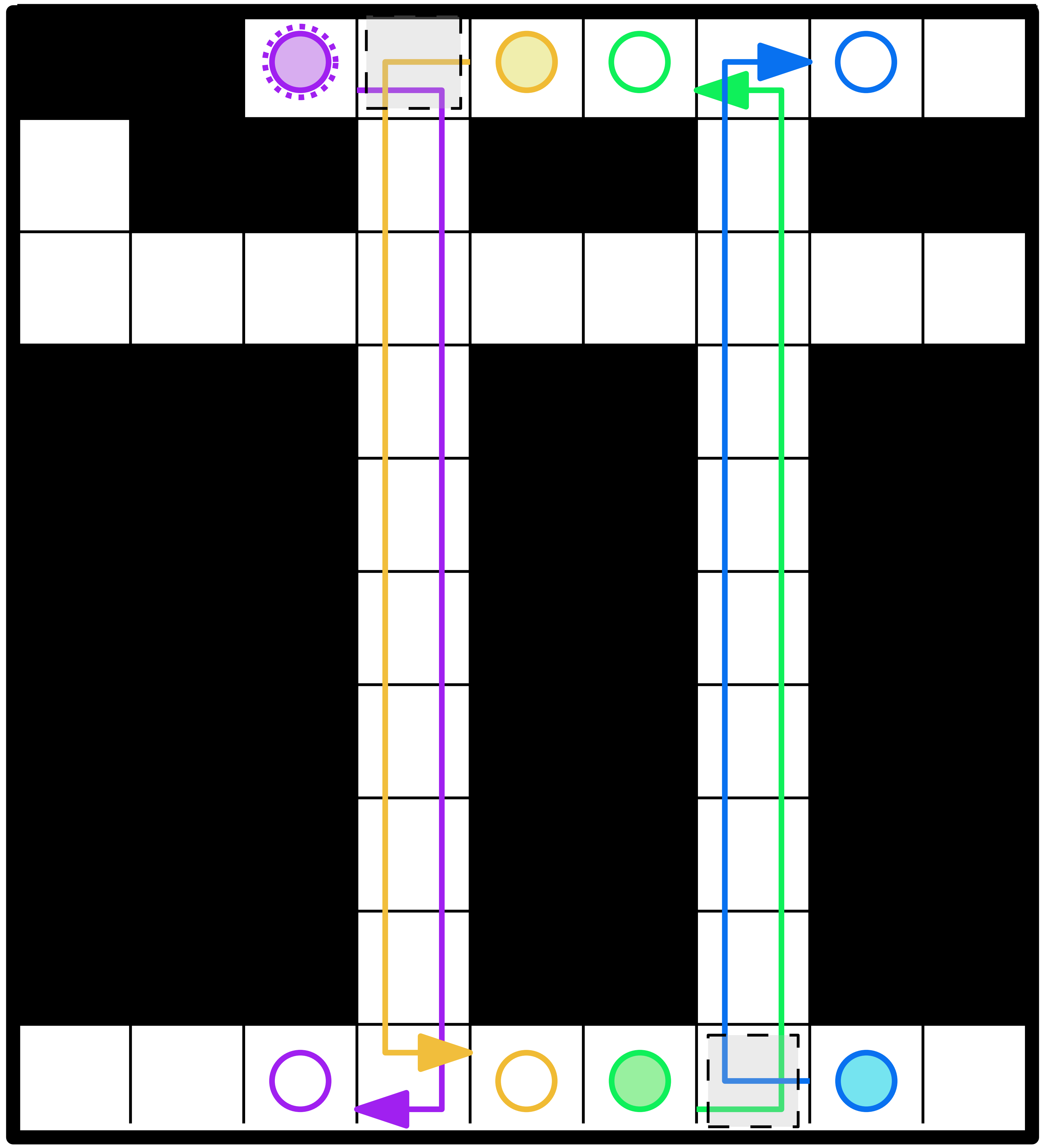
Lazy CBS in Action

- Lower Bound: 46
- Assumptions
  - $c_1 \leq 12$
  - $c_2 \leq 12$
  - $c_3 \leq 12$
  - $c_4 \leq 12$
  - $z_1 \leq 1$
  - $z_2 \leq 1$
- $z_2 = \langle c_3 \geq 12 \rangle + \langle c_4 \geq 12 \rangle$



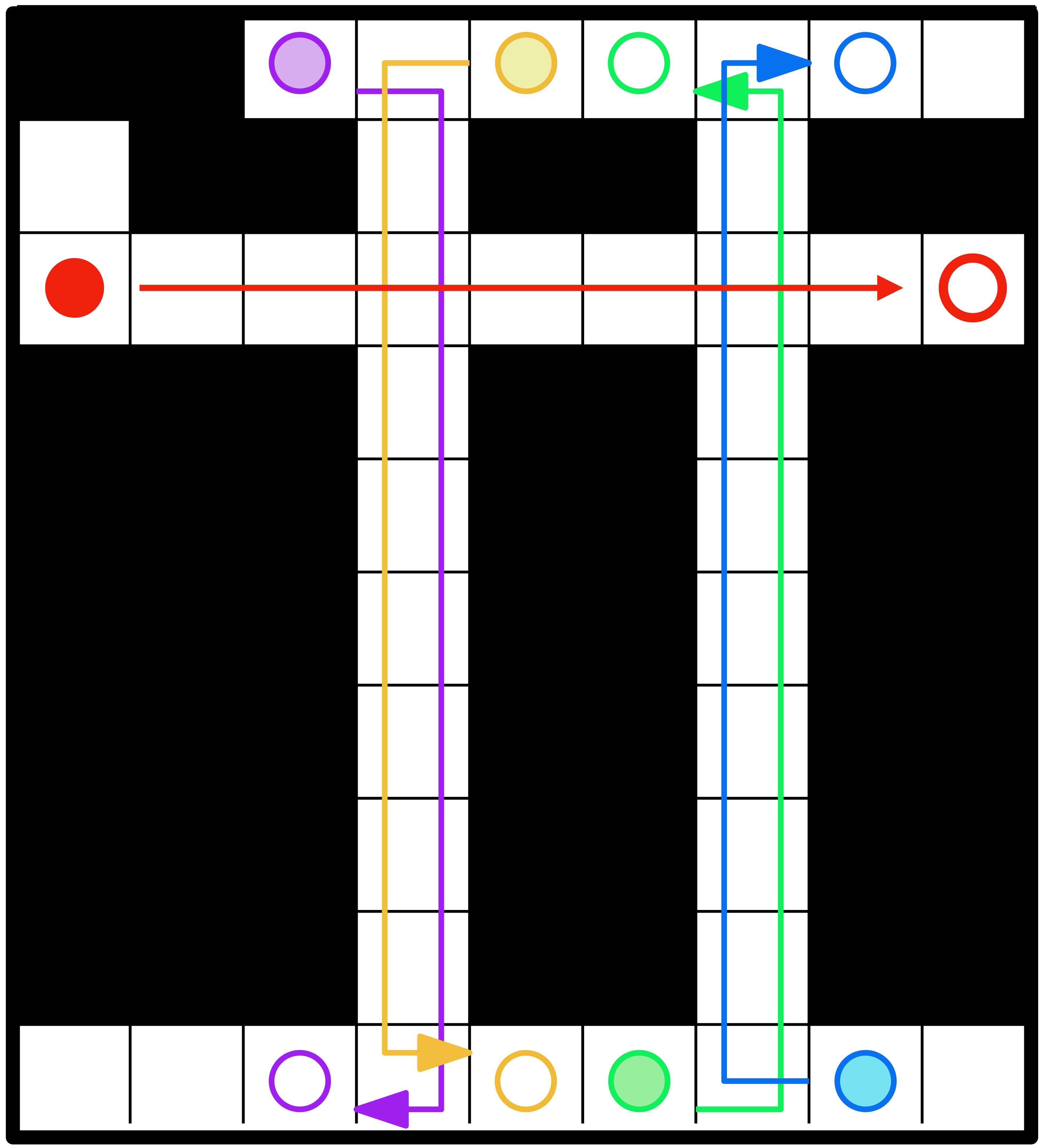
Lazy CBS in Action

- Lower Bound: 46
- Assumptions
  - $c_1 \leq 12$
  - $c_2 \leq 12$
  - $c_3 \leq 12$
  - $c_4 \leq 12$
  - $z_1 \leq 1$
  - $z_2 \leq 1$
- Solution Found: Optimal



Independence Detection

- Independence Detection
  - split agents into sets that never interact
  - solve MAPF problems separately
- Problem
  - as number of agents grow
  - nothing is independent

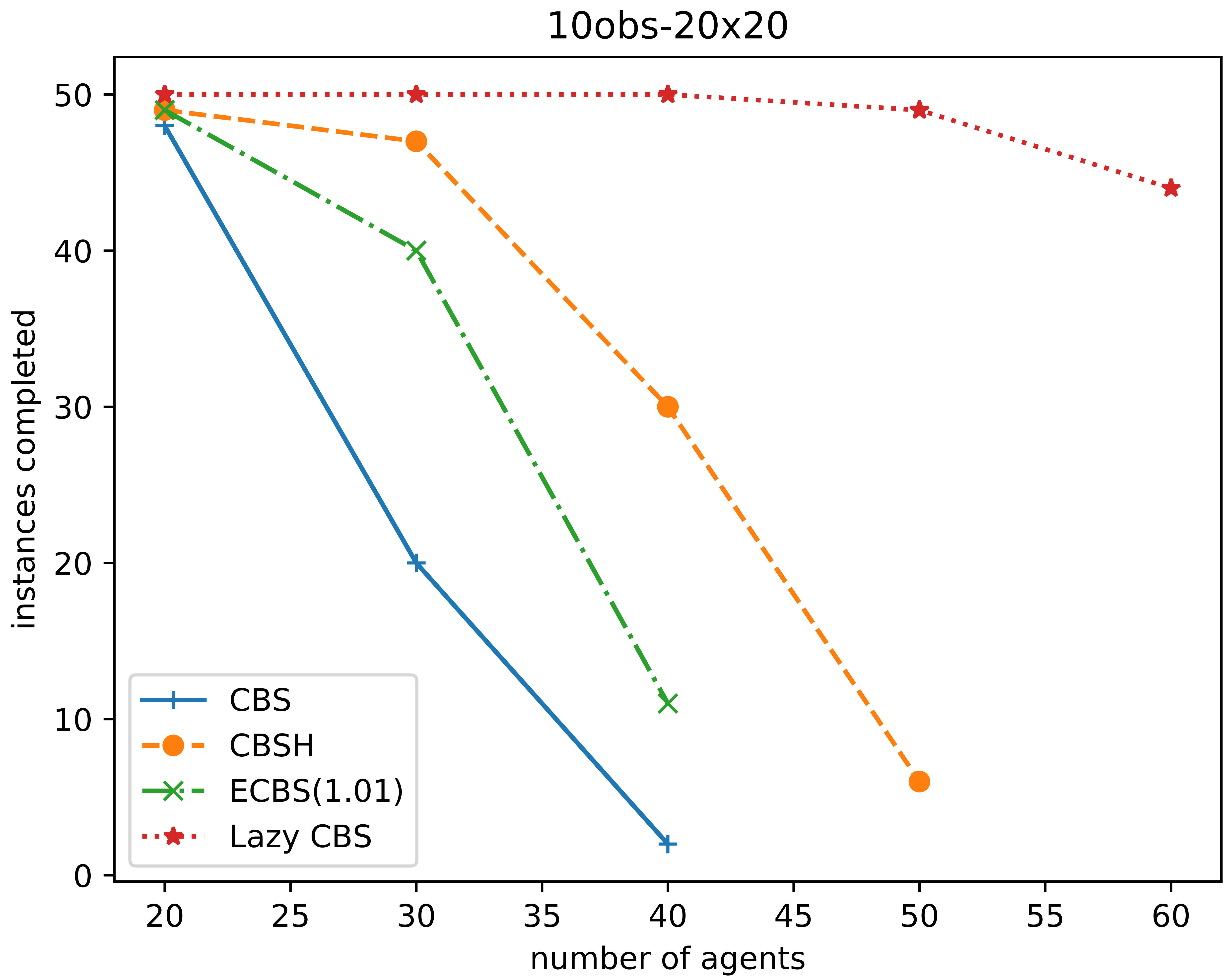


Comparison to CBS

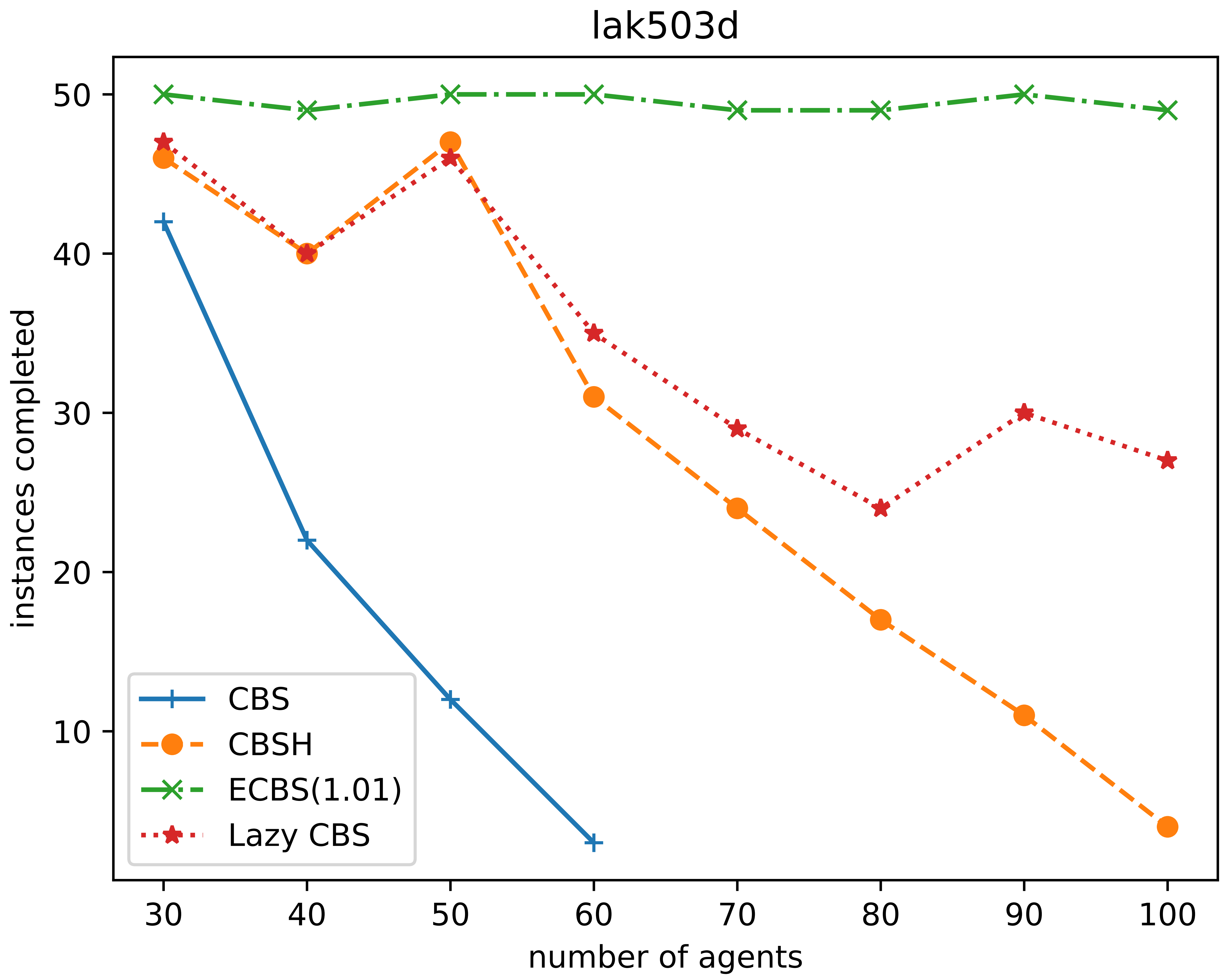
- Similar to Iterative Deepening CBS (IJCAI 2020)
  - Each solve is essentially exploring a whole CBS tree
  - Branches on `permitted[l,t]` are analogous to CBS branches
  - COST more high-level nodes explored
- Lazy introduction of `permitted[l,t]`
  - In a leaf node (all existing `permitted[l,t]` fixed)
  - New conflict between two agents at location `l` time `t`
  - Add a new `permitted[l,t]` variable, branch on it
- Path propagators = low level search in CBS
- NEW: nogoods are universal
  - prevent repeated work across subtrees
  - prevent repeated work in resolves

Lazy CBS Experiments

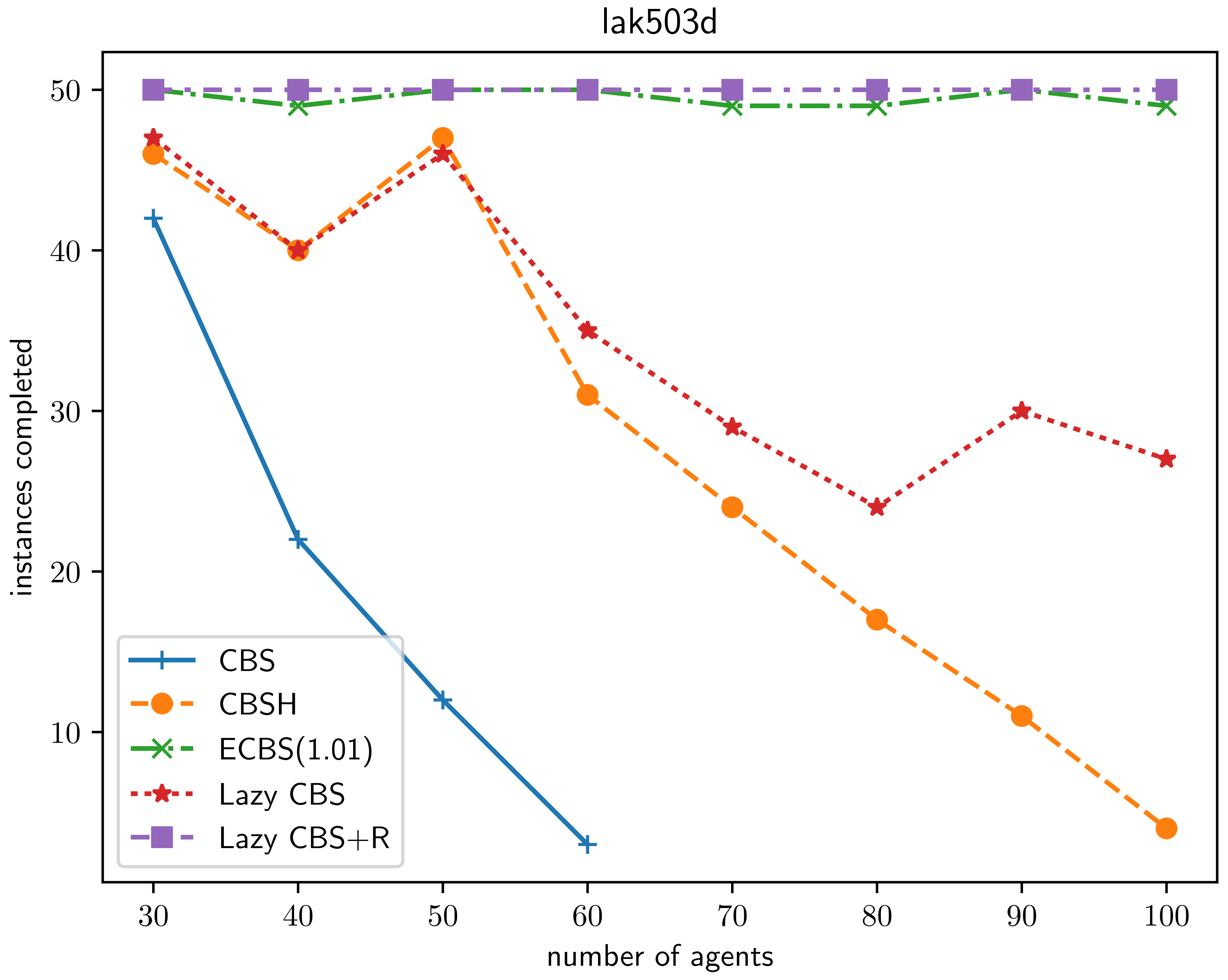
Lazy-CBS Experiments



Lazy-CBS Experiments



Lazy-CBS Experiments



Mixed Integer Programming Approach to MAPF
BCP

Branch and Cut and Price

- A very complicated way of solving a Mixed Integer Program
- Lazily constructing the linear model during solving
- Guaranteed to find optimal solutions

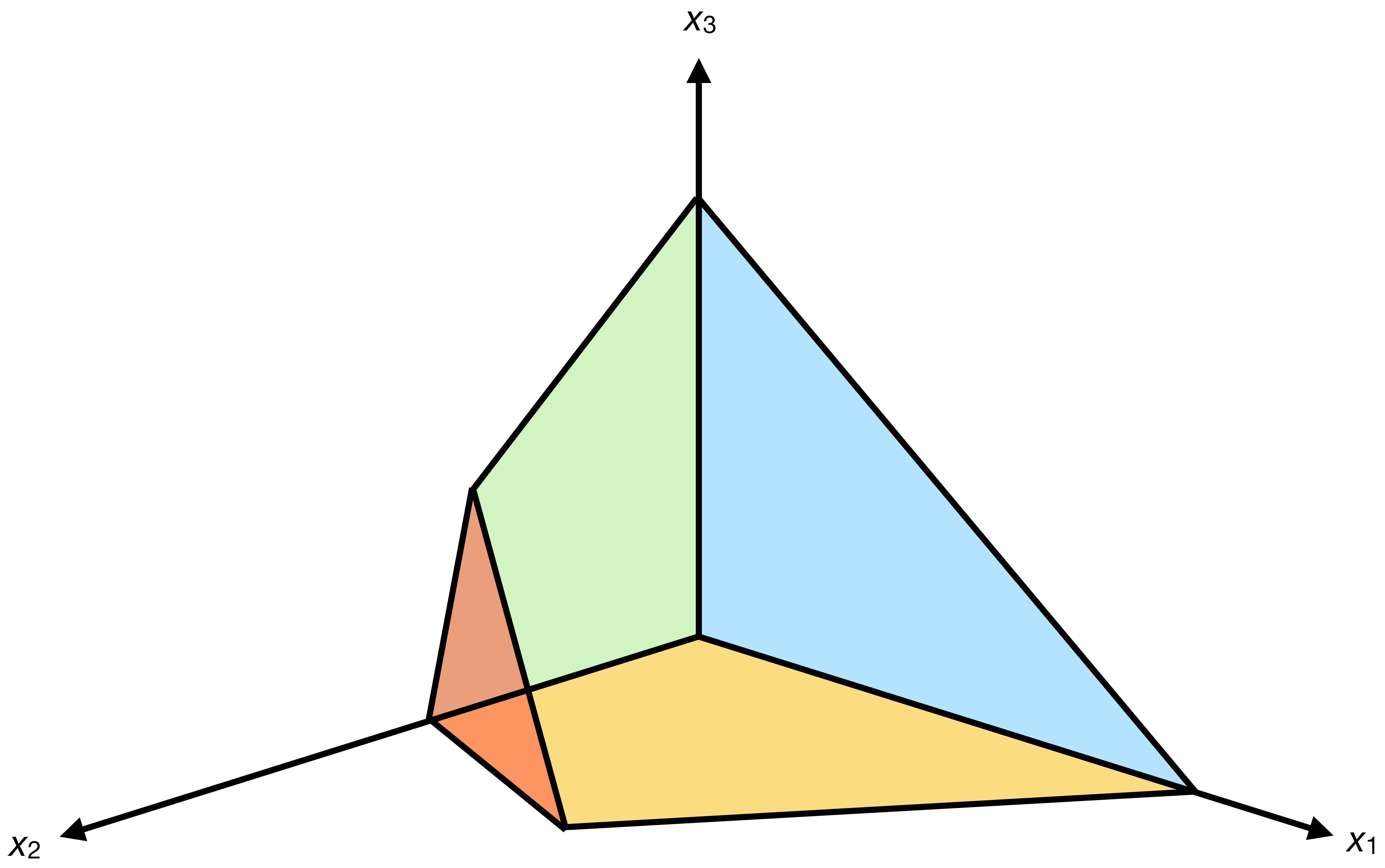
- Allows us to avoid the problem of too big an encoding.

Branch and Cut and Price
(roughly)

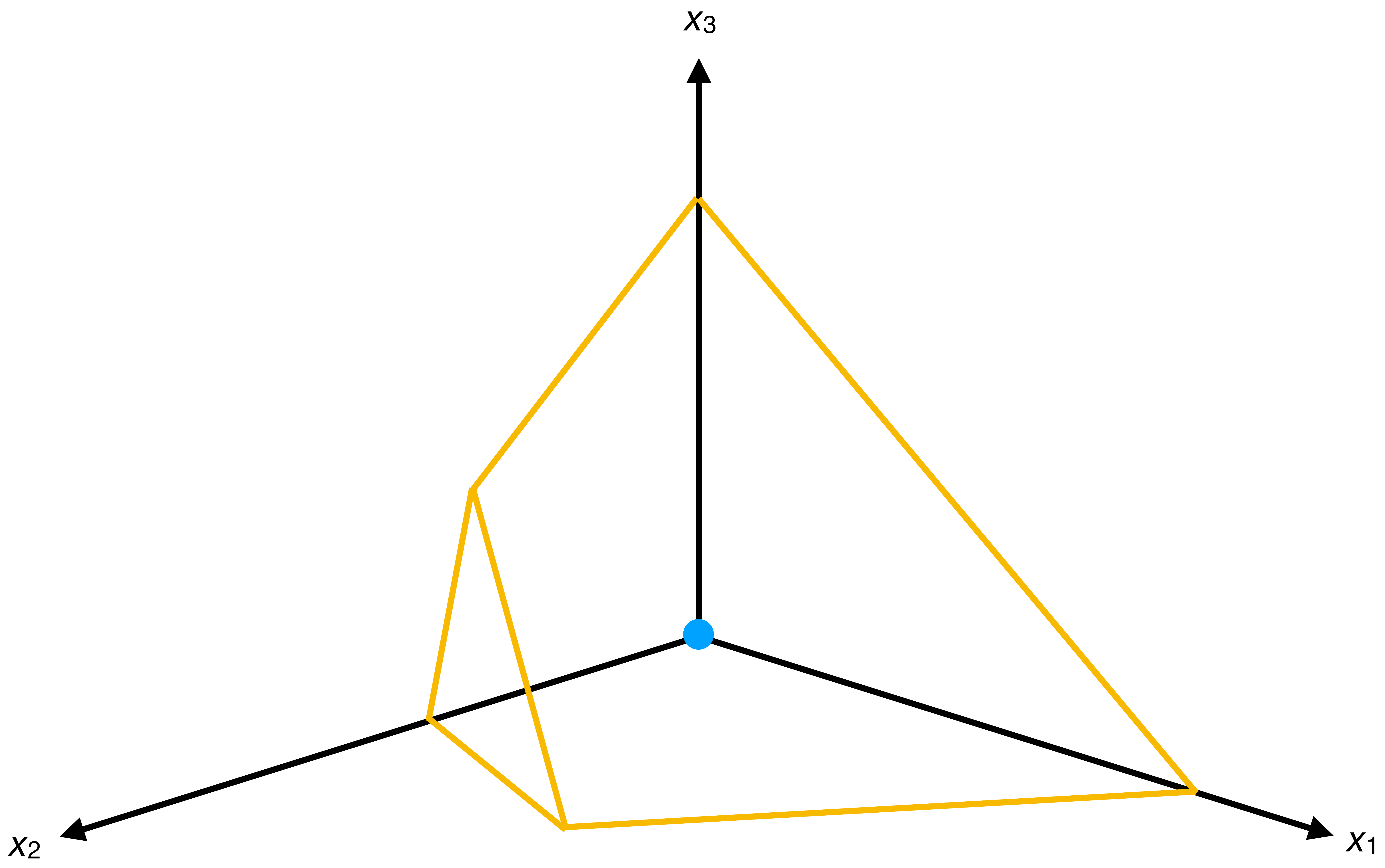
BCP (roughly)

- Linear Program
  - a set of linear constraints with a linear objective
  - solved by the SIMPLEX algorithm (for our purposes)
- Integer Program
  - (some) variables must take integer variables
  - solved by Branch and Bound
- Branch and Cut and Price
  - Generalisation of Branch-and-Bound
  - Start with small matrix and progressively add rows and columns

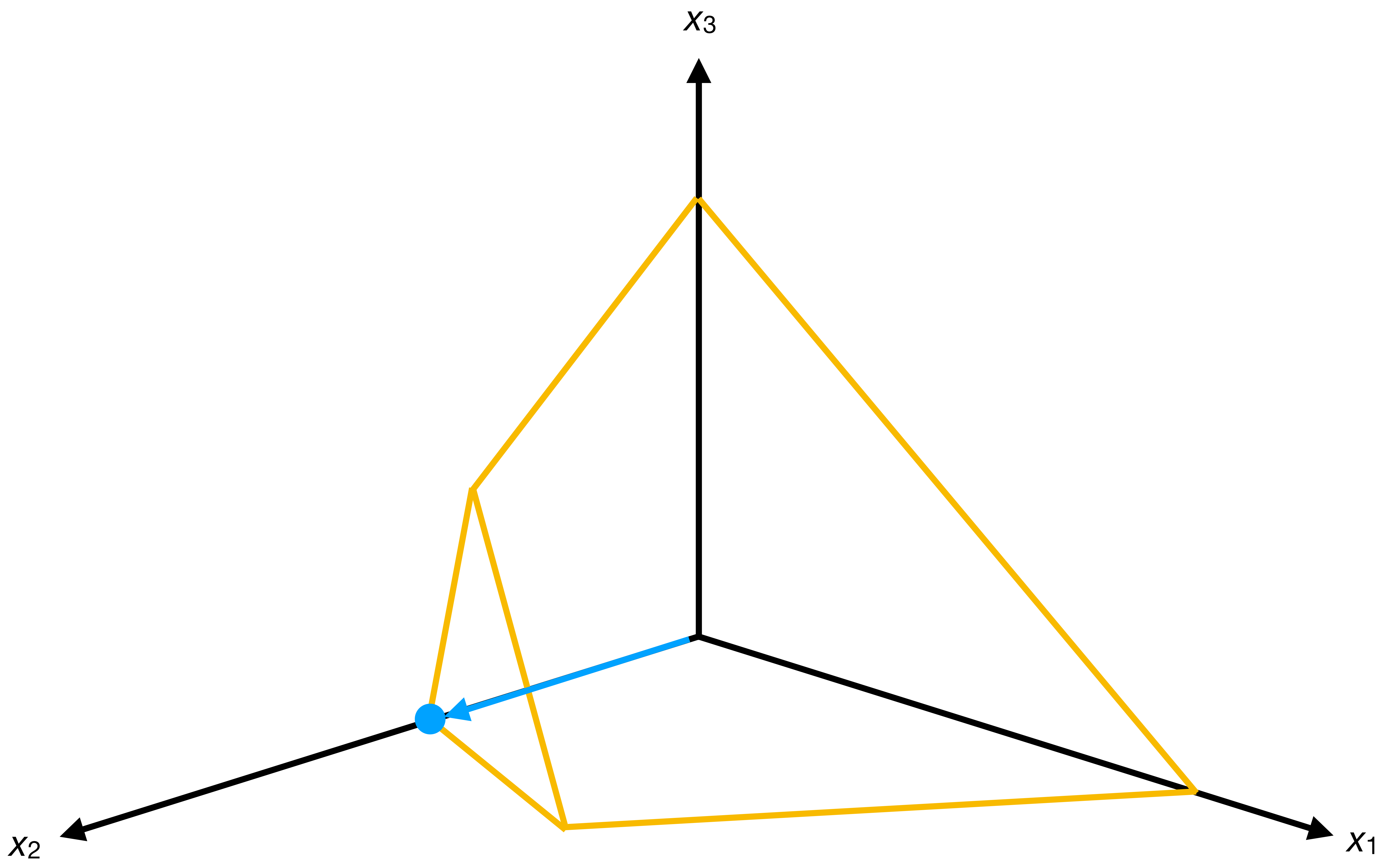
Simple Example



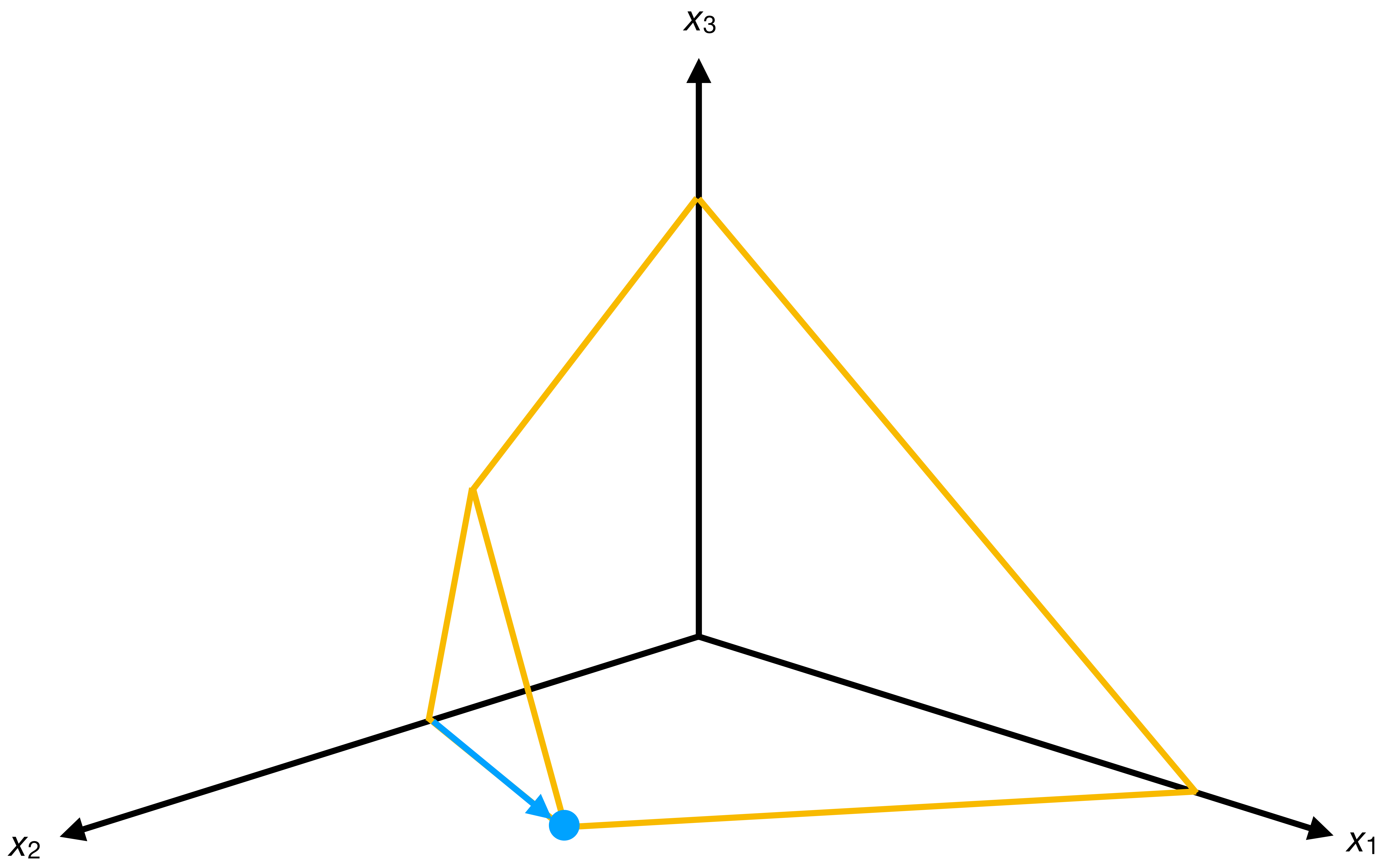
Linear Programming



Linear Programming

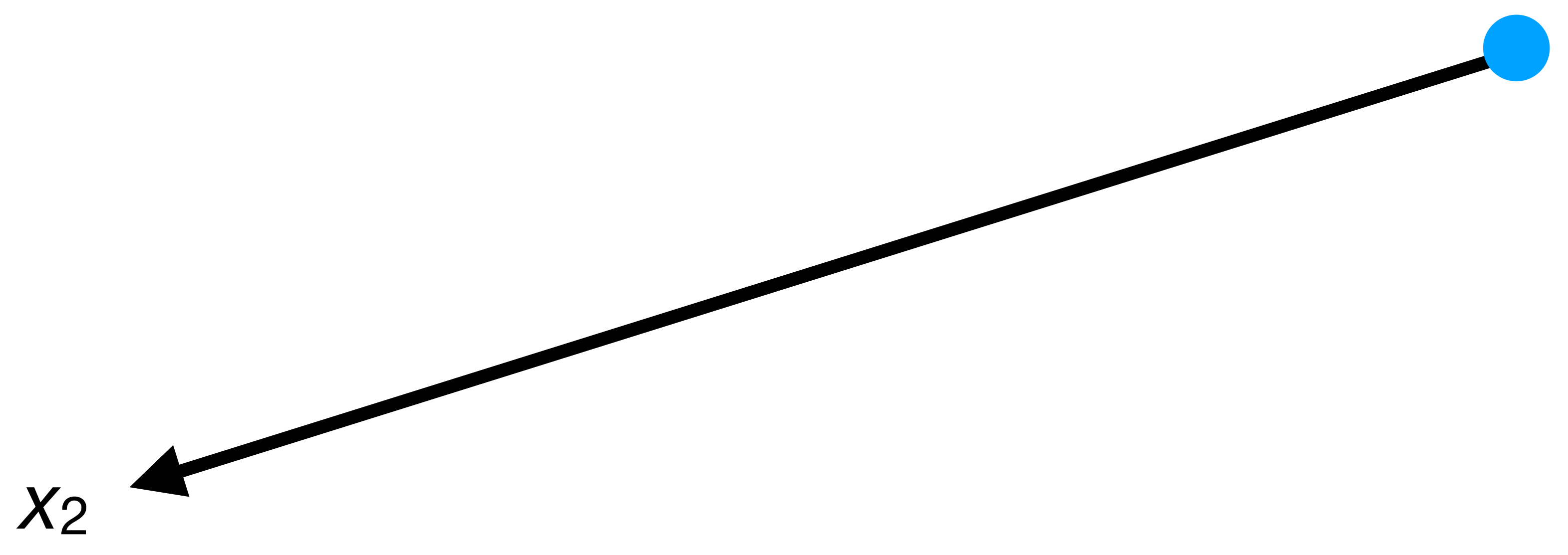


Linear Programming

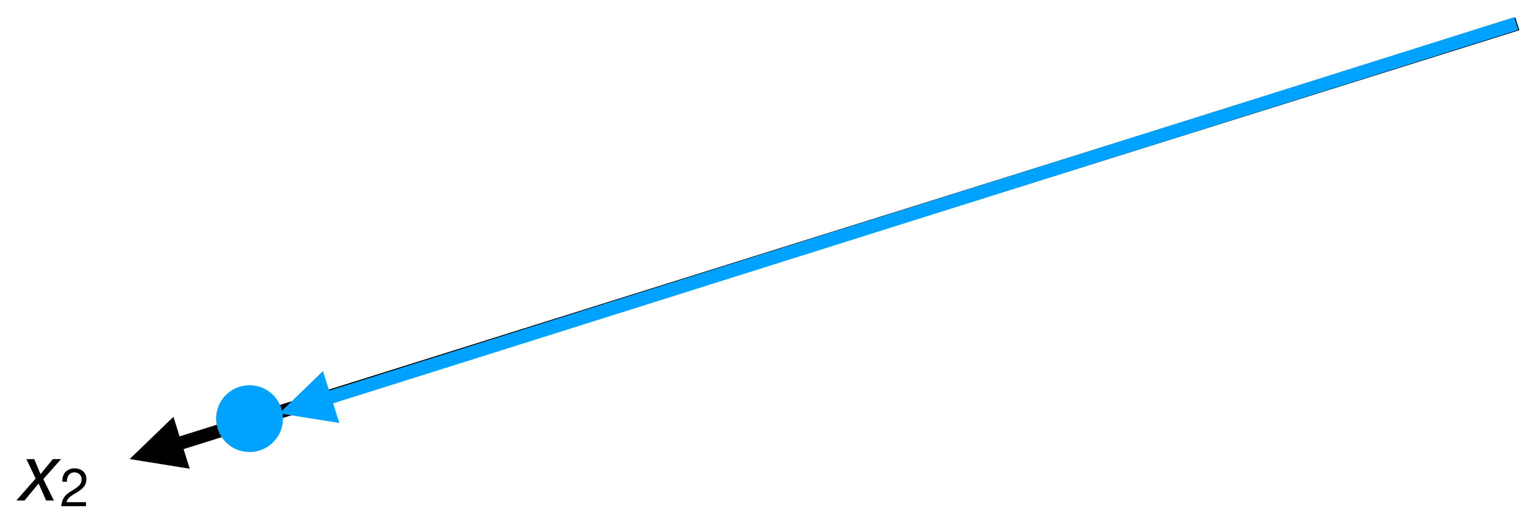


Branch-and-cut-and-price

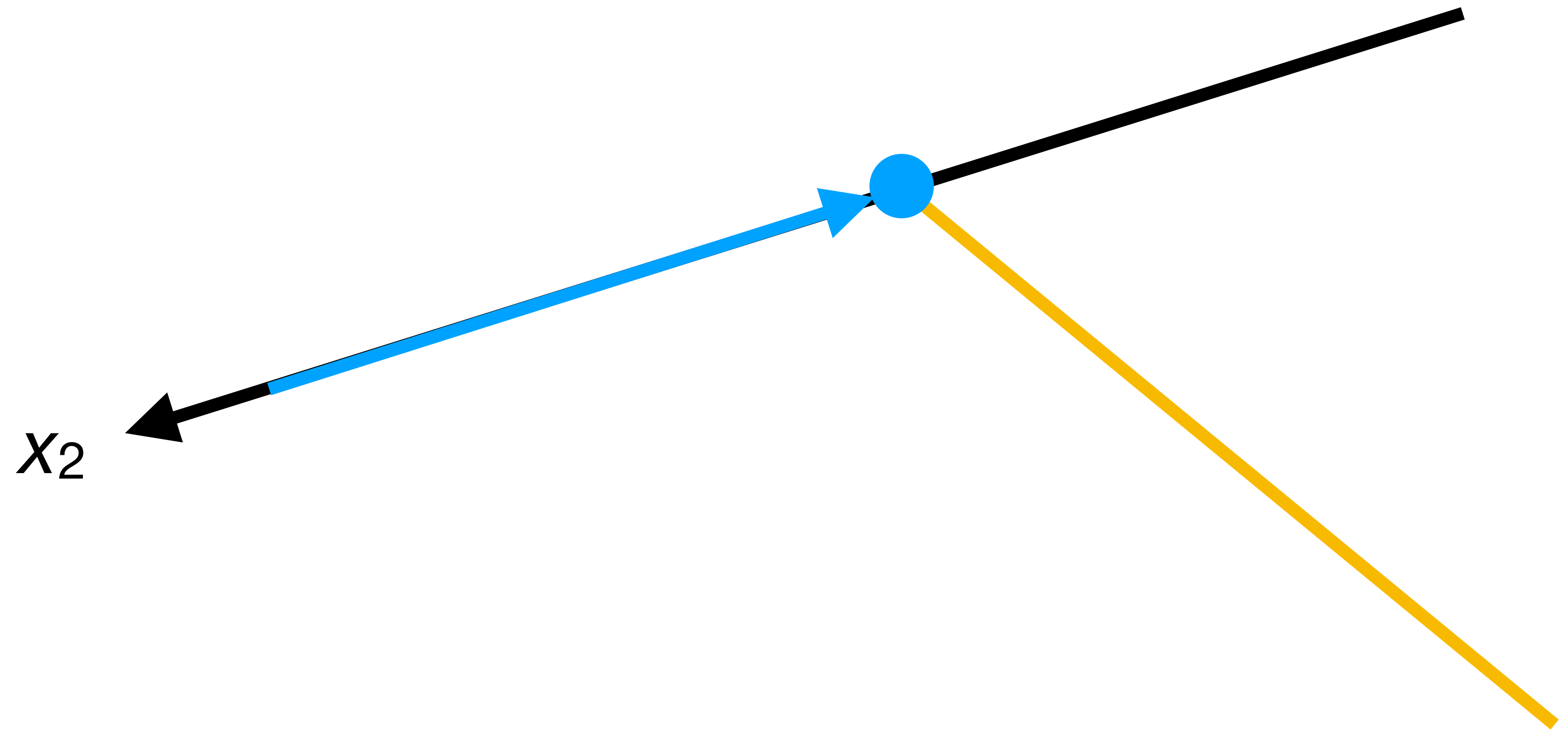
Branch-and-cut-and-price



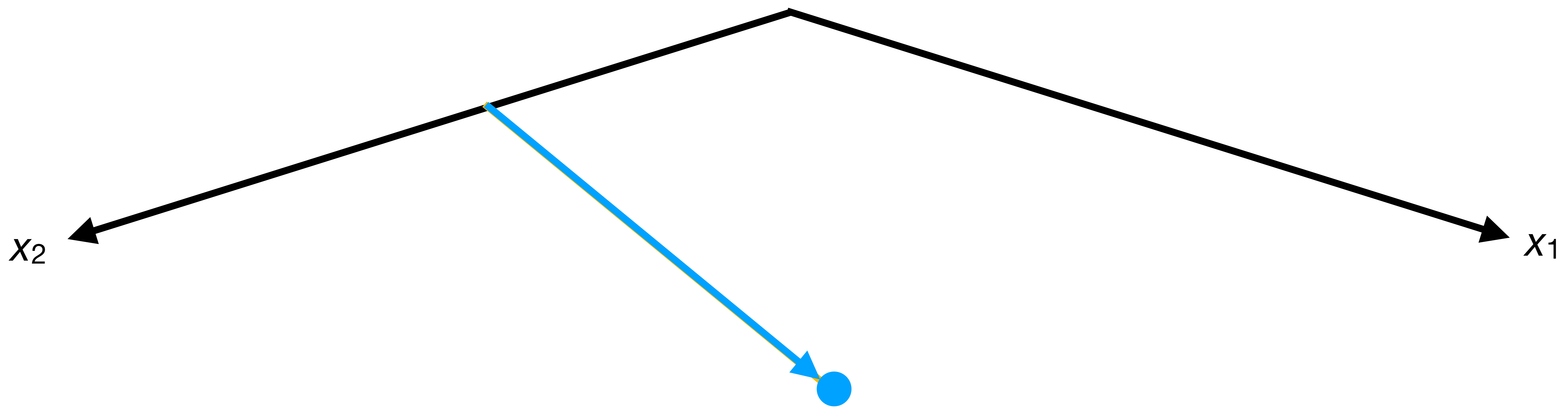
Branch-and-cut-and-price



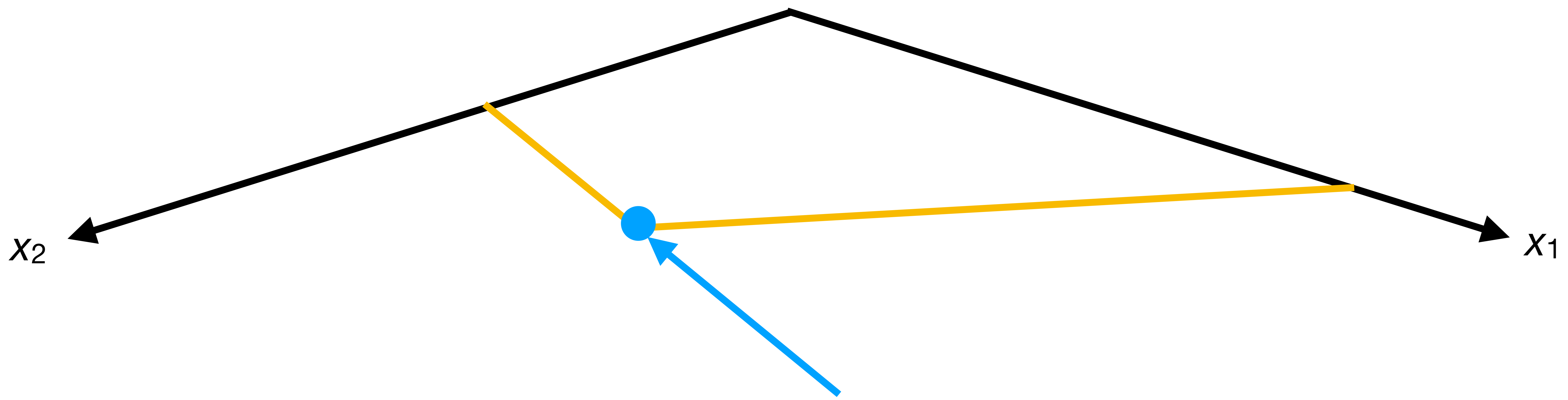
Branch-and-cut-and-price



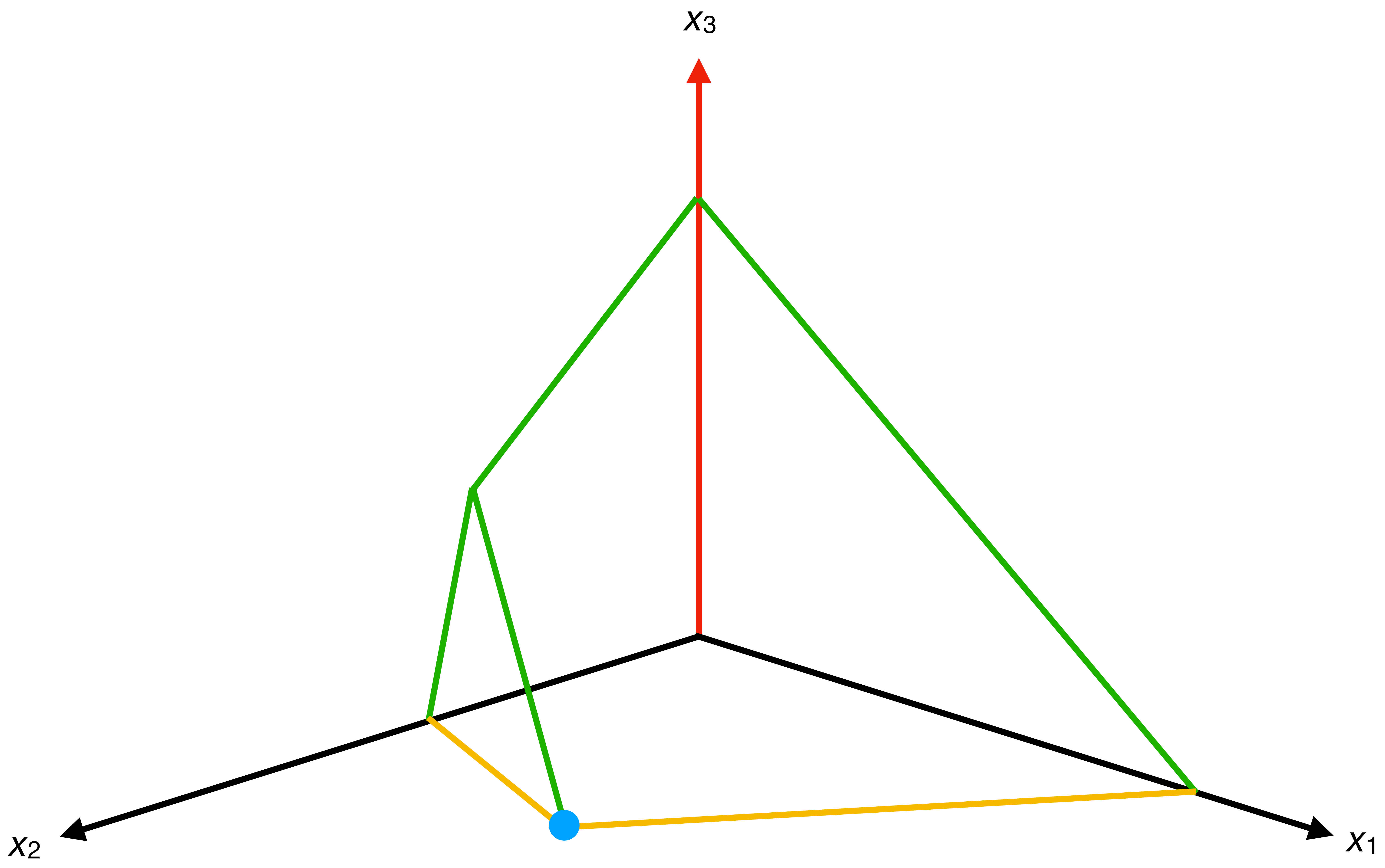
Branch-and-cut-and-price



Branch-and-cut-and-price



Branch-and-cut-and-price



BCP Algorithm for MAPF

BCP Algorithm for MAPF

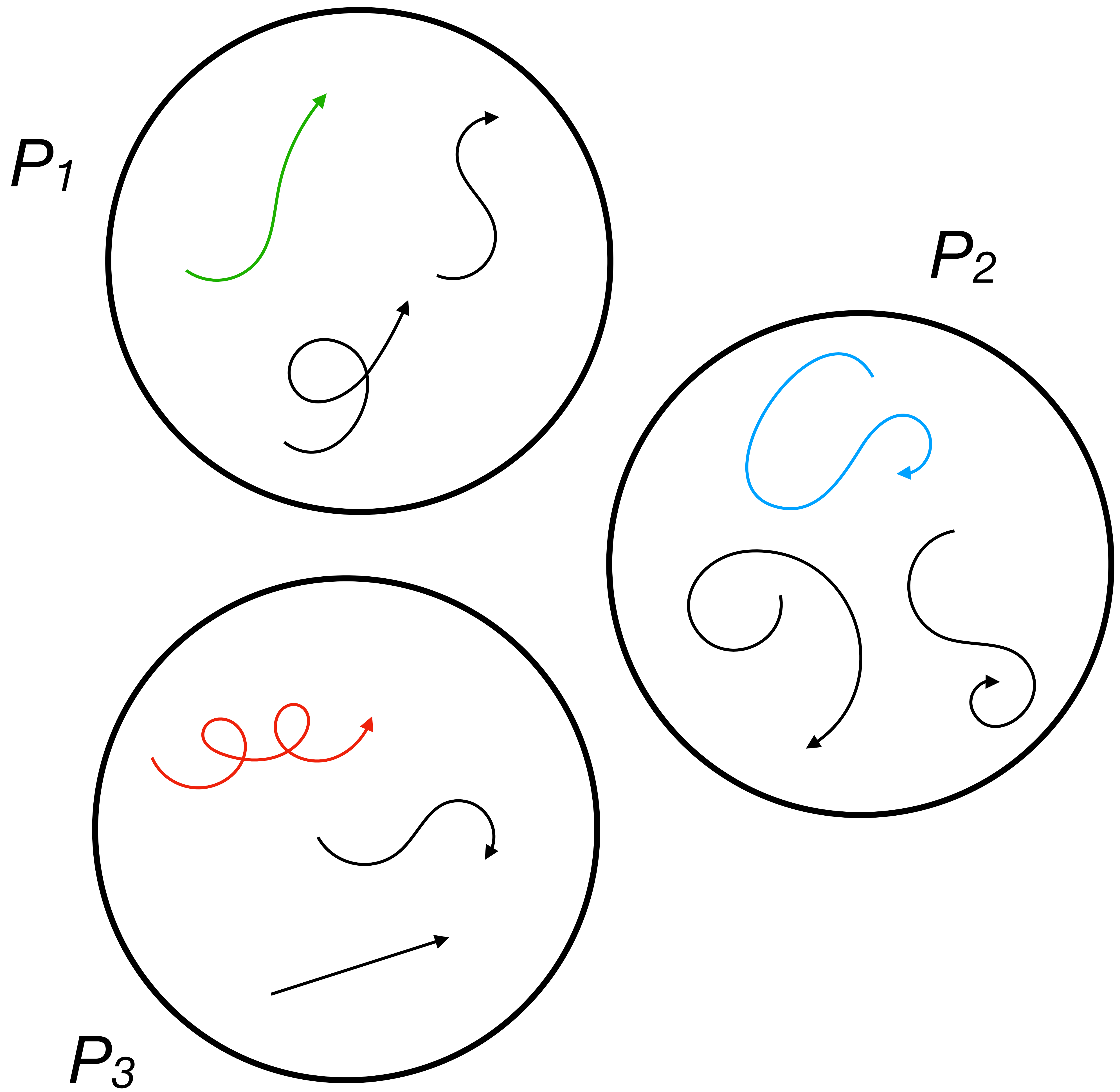
- **Master problem:** select good paths for the agents using LP relaxation
- **Pricing problem:** find better paths for each agent using A*
- **Separation problems:** check for collisions in the selected paths using complete enumeration

Problem Graph

- Define a time-expanded graph:
  - A **vertex** $v = (c,t)$ is a cell-time pair
  - An **edge** $e = (v_1,v_2) = ((c_1,t),(c_2,t+1))$ is a pair of vertices such that:
    - Cell in second vertex is adjacent cell or same cell (wait)
    - Time in second vertex = time in first vertex + 1
  - A **path** is a sequence of vertices such that adjacent vertices are edges

Master Problem

- For each agent a, assume a set P_a of all possible paths
- MIP **master problem**:
  - Variable $\lambda_p \in \{0,1\}$ represents proportion of selecting path p
  - **Path selection constraints:** for each agent a, select one path from P_a
  - **Vertex collision constraints:** each vertex used by at most one agent
  - **Edge collision constraints:** each edge and its reverse used by at most one agent



Master Problem

Cost of path p

Variable representing proportion of selecting path p

Minimise total cost

$$\min \sum_{a \in A} \sum_{p \in P_a} c_p \lambda_p$$

subject to

Every agent must use (at least) one path

$$\sum_{p \in P_a} \lambda_p \geq 1 \qquad \forall a \in A$$

Constant = 1 if path p uses vertex v

Vertex v used at most once across all selected paths

$$\sum_{a \in A} \sum_{p \in P_a} x_v^p \lambda_p \leq 1 \qquad \forall v \in V$$

Constant = 1 if path p uses edge e = (i,j)

Edge e and its reverse used at most once across all selected paths

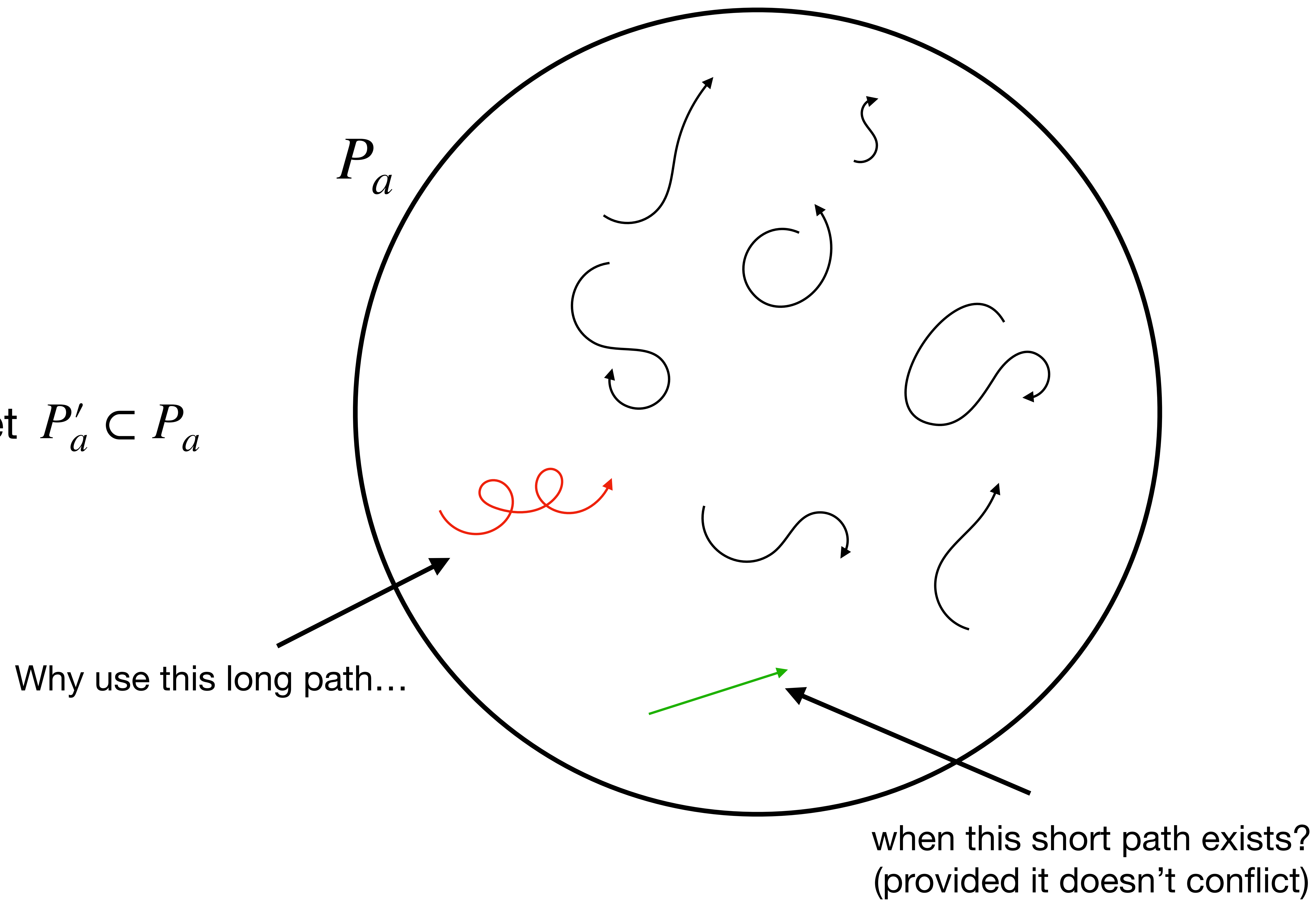
$$\sum_{a \in A} \sum_{p \in P_a} (y_e^p + y_{e'}^p) \lambda_p \leq 1 \qquad \forall e \in E$$

Constant = 1 if path p uses edge e' = (j,i)

$$\lambda_p \geq 0 \qquad \forall a \in A, p \in P_a$$

Master Problem

- Enough to work with a finite subset $P'_a \subset P_a$



Restricted Master Problem

Minimise total cost

$$\min \sum_{a\in A} \sum_{p\in P'_a} c_p \lambda_p$$

subject to

Every agent must use (at least) one path

$$\sum_{p\in P'_a} \lambda_p \geq 1 \qquad \forall a \in A$$

$$\lambda_p \geq 0 \qquad \forall a \in A, p \in P'_a$$

Separating Vertex Conflicts

- Project to network flow formulation: $x_v = \sum_{a \in A} \sum_{p \in P_a} x_v^p \lambda_p$
- Create constraint if $x_v > 1$

Separating Edge Conflicts

- Project to network flow formulation: $y_e = \sum_{a \in A} \sum_{p \in P_a} y_e^p \lambda_p$
- Create constraint if $y_e + y_{e'} > 1$

Pricing Problem

- Use existing A* code with modified objective function:

$$\min \bar{c}_p := c_p - \alpha_a - \sum_{v \in V} \beta_v x_v^p - \sum_{e \in E} \gamma_e (y_e^p + y_{e'}^p)$$

Variable = 1 if path
p uses edge *e'* = (*j*,*i*)

Reduced cost
of path *p*

Value of dual
variable for set
cover constraint

Variable = 1 if path
p uses edge *e* = (*i*,*j*)

Variable = 1 if path
p uses vertex *v*

Original cost
of path *p*

Value of dual variable
for vertex collision
constraint

Value of dual
variable for edge
collision constraint

- Path *p* may improve the master problem LP solution if $\bar{c}_p < 0$ (necessary but not sufficient)

Branching on Vertex

- Solutions have fractional value for λ_p
- Require branching to ensure they are integer {0,1}
- Select a vertex *v* that is fractionally used by an agent *a*
- Agent *a* must:
  - visit *v* in one child. (set $\lambda_p = 0$ for paths *p* where it does not visit)
  - not visit *v* in other child. (set $\lambda_p = 0$ for paths *p* where it does visit)

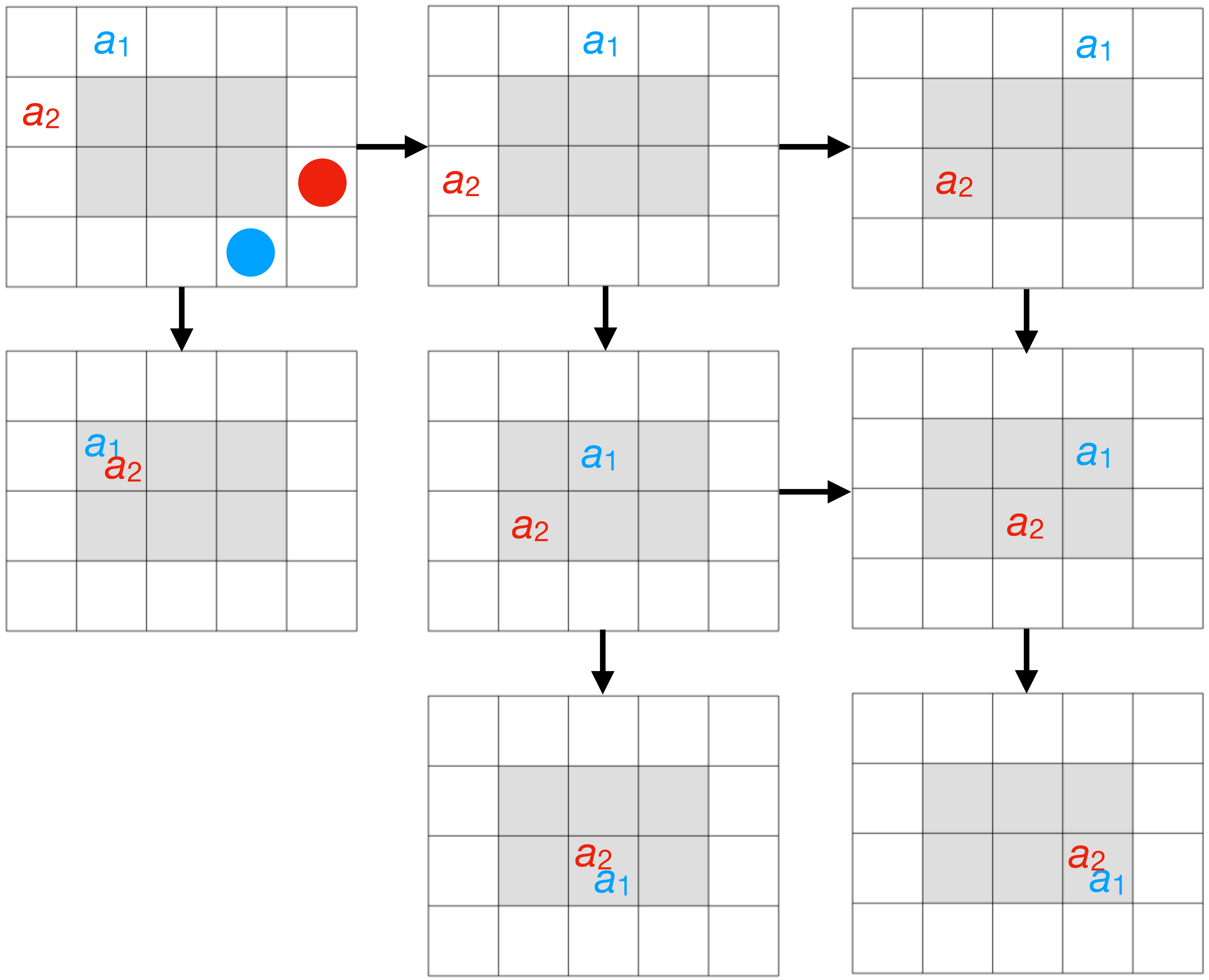
BCP

- BCP iteratively calls three subproblems:
  - **Restricted Master problem:** select good paths using LP relaxation
    - Column (variable) represents the proportion of a path selected
    - Row (constraint) represents resolving a conflict
  - **Pricing problem:** find better paths for each agent using A*
  - **Separation problems:** check for collisions in the selected paths using complete enumeration
- Master problem selects **fractional proportion** of a path
  - Resolve fractionality by branching on agent-vertex: agent must and must not use vertex (disjoint branching)

BCP

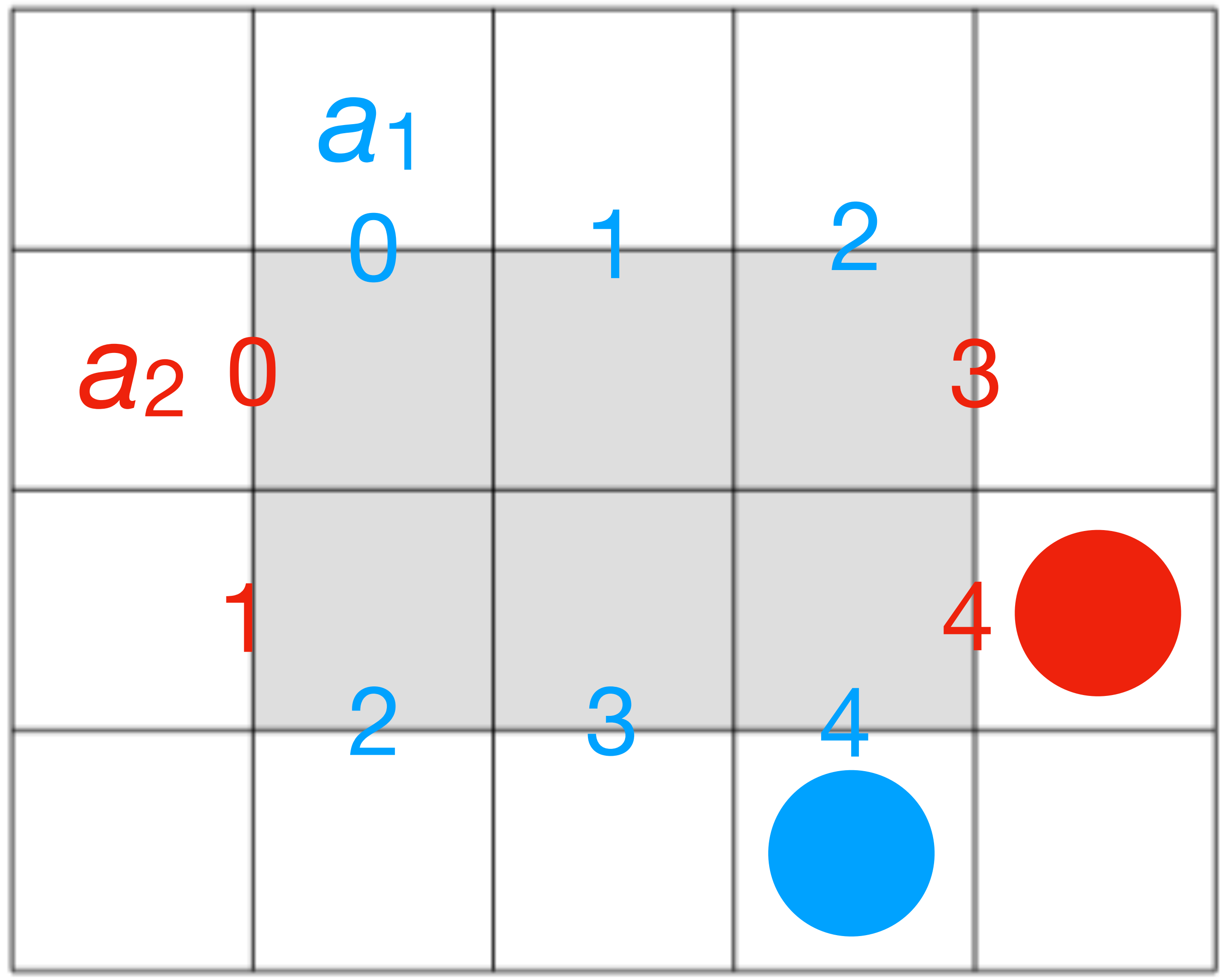
- Implemented in BCP-B algorithm – basic branch-and-cut-and-price
- Now make some improvements:
  - 2 classes of redundant constraints
  - Hierarchical branching rule

Rectangle Symmetry



[Li *et al.*, 2019] J. Li, D. Harabor, P. J. Stuckey, H. Ma, and S. Koenig. Symmetry-breaking constraints for grid-based multi-agent path finding. In *AAAI*, 2019. (to appear).

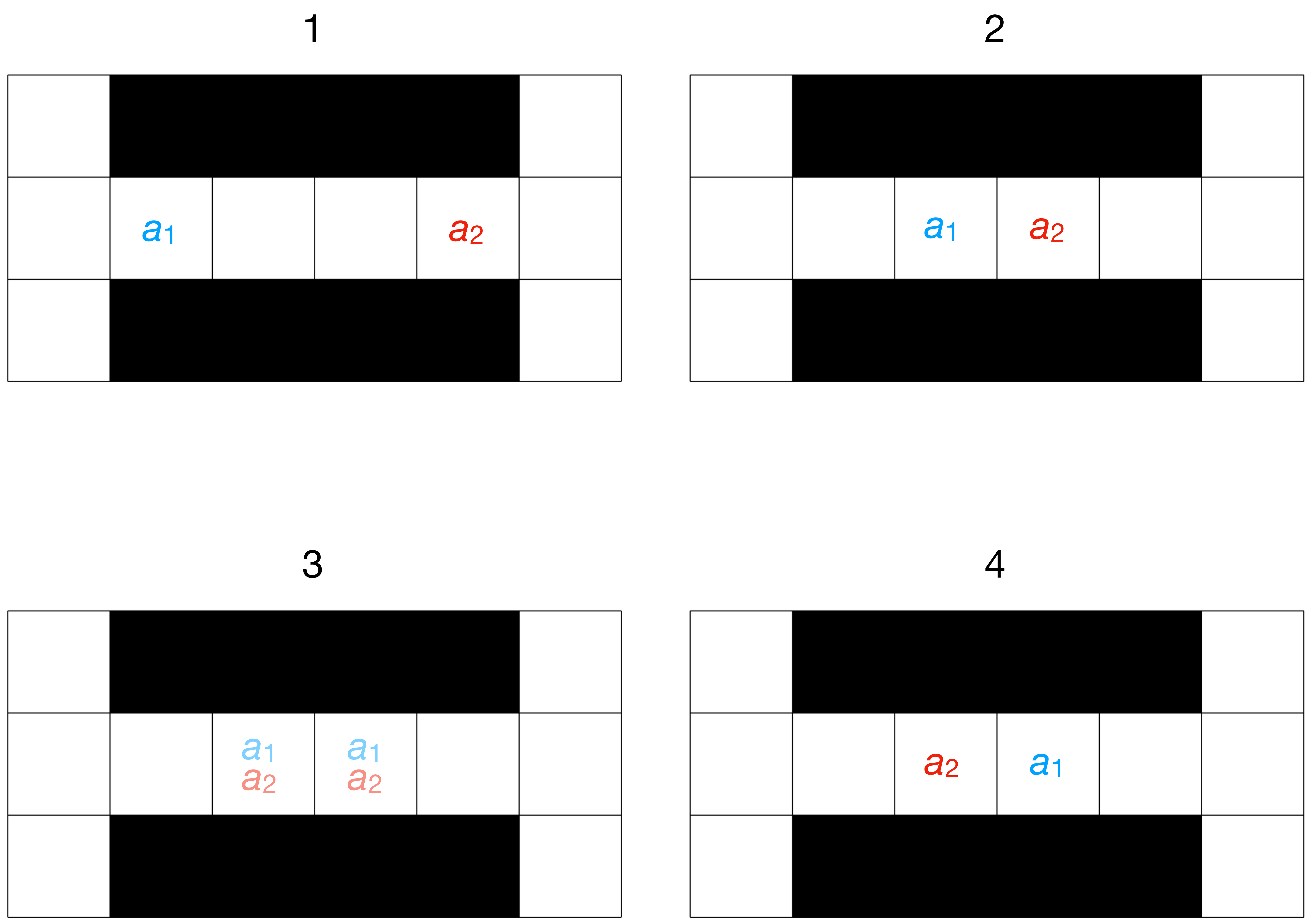
Rectangle Conflicts



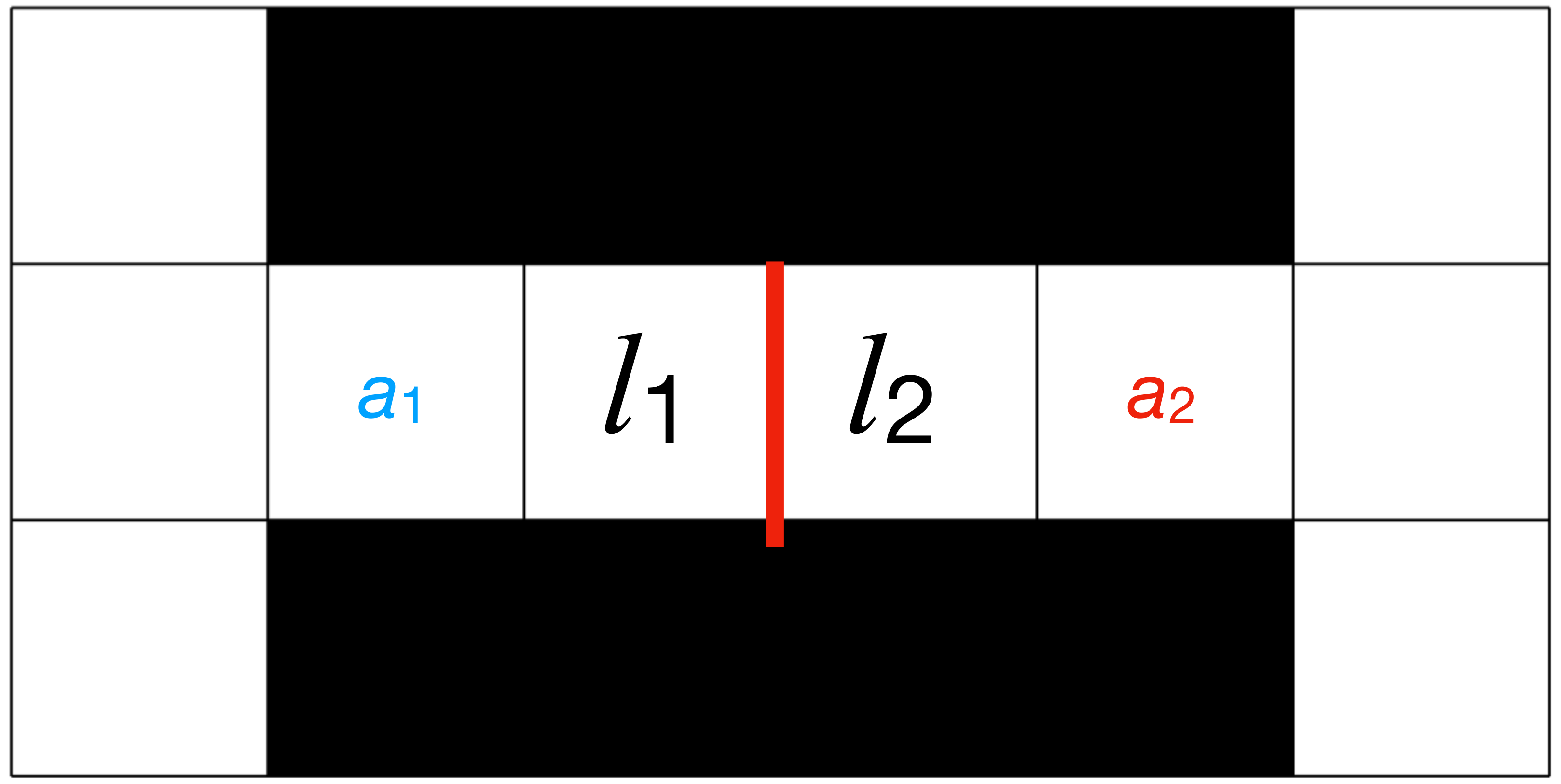
$$\sum_{p \in P_{a_1}} \sum_{e \in E_1} y_e^p \lambda_p + \sum_{p \in P_{a_2}} \sum_{e \in E_2} y_e^p \lambda_p \leq 3 \qquad \forall (a_1, E_1, a_2, E_2)$$

[Li *et al.*, 2019] J. Li, D. Harabor, P. J. Stuckey, H. Ma, and S. Koenig. Symmetry-breaking constraints for grid-based multi-agent path finding. In *AAAI*, 2019. (to appear).

Fractional Corridor Conflicts



Corridor Conflicts



$$\sum_{p\in P_{a_1}} y^p_{((l_1,t),(l_2,t+1))}\lambda_p + \sum_{p\in P_{a_1}} y^p_{((l_1,t+1),(l_2,t+2))}\lambda_p +$$
$$\sum_{p\in P_{a_2}} y^p_{((l_2,t),(l_1,t+1))}\lambda_p + \sum_{p\in P_{a_2}} y^p_{((l_2,t+1),(l_1,t+2))}\lambda_p \leq 1$$
$$\forall (a_1, a_2, l_1, l_2, t)$$

Branching on Path Length

Agent 1 Selected Paths

(c_1,c_2,c_3,c_4,c_5) – cost 5, proportion 0.5

$(c_1,c_2,c_3,c_3,c_4,c_5)$ – cost 6, proportion 0.5

Agent 1 cost = $0.5 \times 5 + 0.5 \times 6 = 5.5$

Agent 1 path length ≤ 5

Agent 1 Selected Paths

(c_1,c_2,c_3,c_4,c_5) – cost 5, proportion 1

~~$(c_1,c_2,c_3,c_3,c_4,c_5)$ – cost 6, proportion 0~~

Agent 1 cost ≤ 5

Agent 1 path length ≥ 6

Agent 1 Selected Paths

~~(c_1,c_2,c_3,c_4,c_5) – cost 5, proportion 0.0~~

$(c_1,c_2,c_3,c_3,c_4,c_5)$ – cost 6, proportion 1

Agent 1 cost ≥ 6

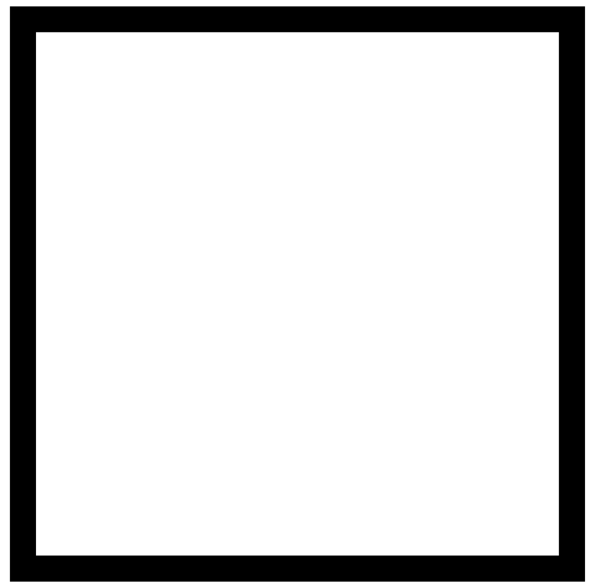
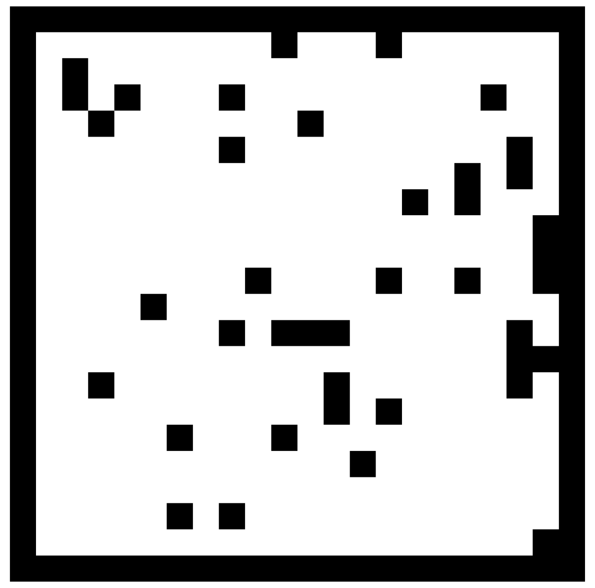
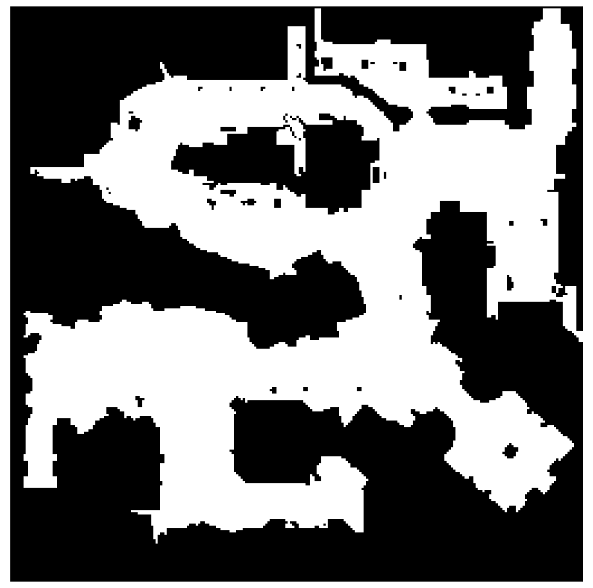
BCP

- BCP-B
  - basic branch and cut and price
  - vertex and edge separation
  - pricing problem solved by A*
  - branching on agent uses vertex (location,time) or does not
- BCB
  - add rectangle conflict cuts
  - add corridor conflict cuts
  - first branch on path length of agents

Experiments

Set-up

- Test 4 algorithms on Intel Xeon E5-2660 V3 CPU at 2.6 GHz with 5 min time limit:
  - BCP
  - BCP-B
  - CBSH
  - CBSH-RM – state-of-the-art as of AAAI19 (Jan 2019)
- 1350 standard benchmarks on 4 maps:


Results

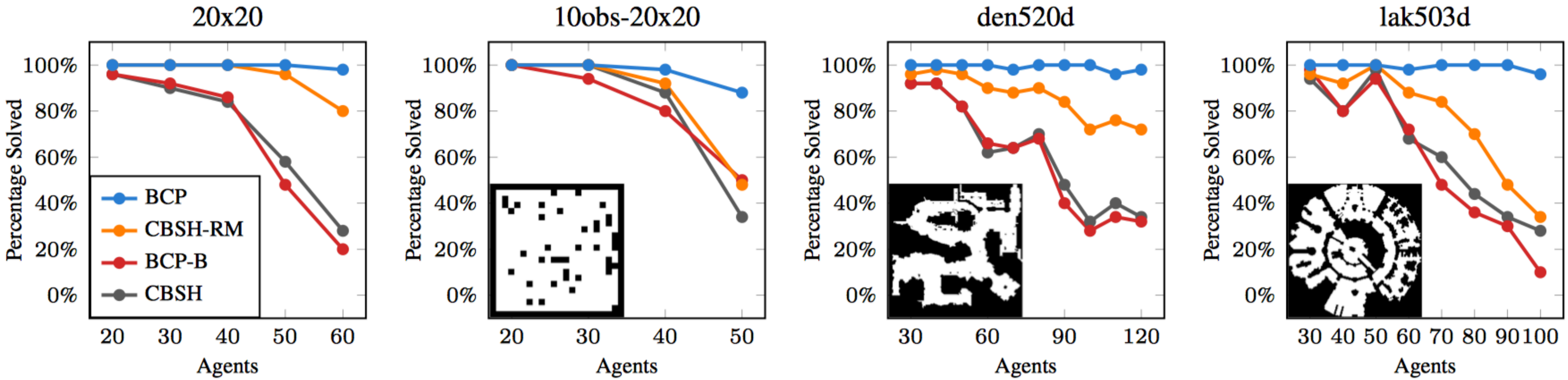
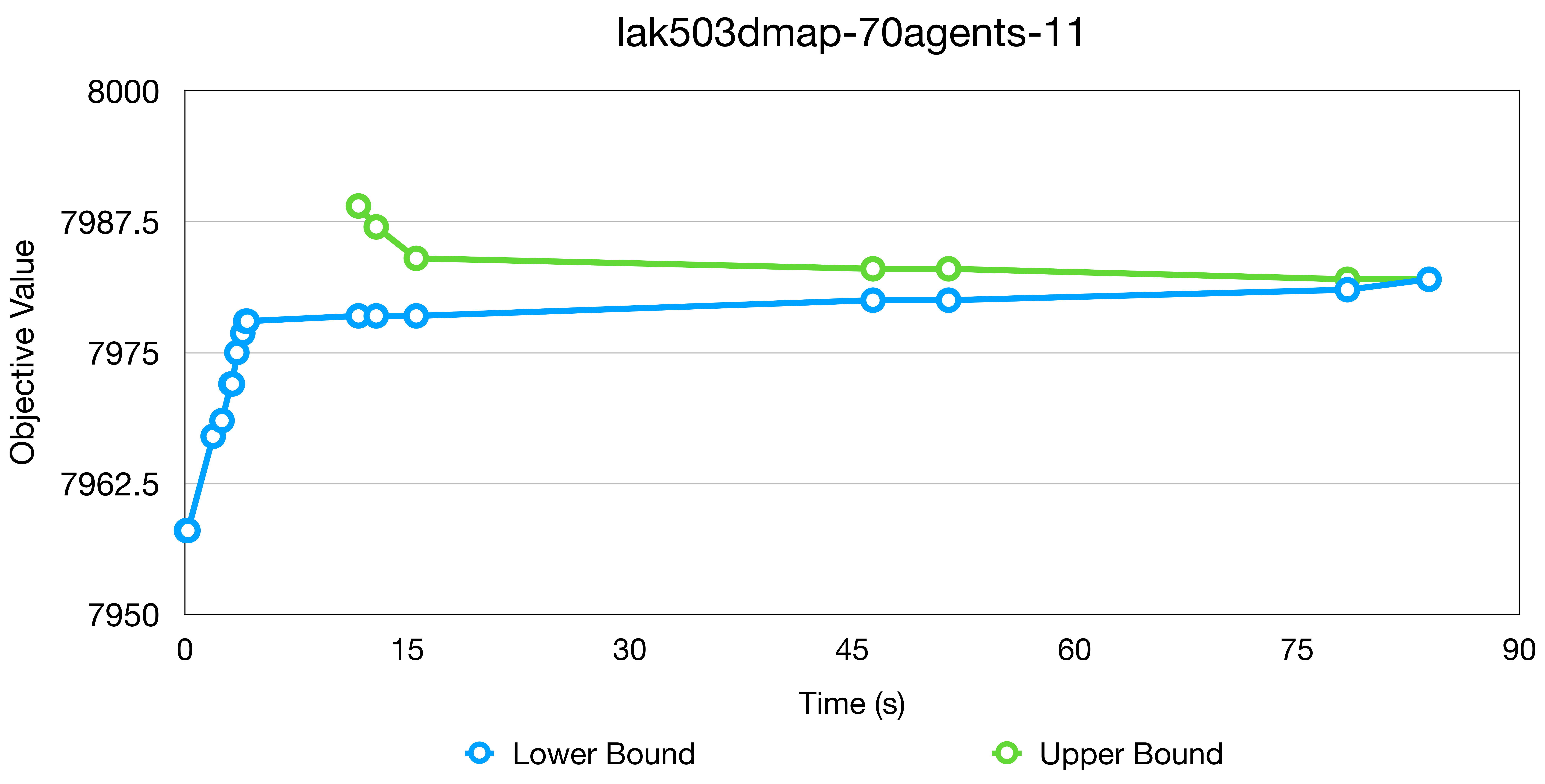


Figure 3: Success rate for each map and number of agents. Higher is better.

- Close all but 15 instances
- Improved to 6 instances in new unpublished work

Objective Bound



Comparison to CBS

Pros

- Reasons across all conflicts and agents simultaneously
- Immediately recalls all paths generated anywhere in high-level search tree
- Extendable with other types of conflicts
- Exploits advances in MIP
  - Primal heuristics!
- Anytime
- Tight lower bound even if no feasible solution (upper bound) is found

Cons

- Difficult to understand
- Difficult to implement
  - 2000 lines of code for A*
  - 12000 lines of code for separators and glue to A*
  - ??? million lines of code for MIP solver (SCIP)
  - ??? million lines of code for LP solver (CPLEX)

Conclusions

Conclusions

- Straight CP solution to MAPF doesnt scale
    - Lazy CBS
        - Benders decomposition (relaxing permitted)
        - Lazy variable addition
        - Core-guided search
- Straight MIP solution to MAPF doesnt scale
    - BCP
        - Dantzig-Wolfe decomposition
        - Specialized Column generator (low level path finding with arbitrary costs)
        - Specialised Cuts (rectangle, corridor, ...)

Discrete Optimization (DO)

- If your problem in NP-hard consider DO techniques
- Straight out of the box may not work
  - But there are lots of DO decomposition approaches
    - Benders, Dantzig-Wolfe, and Lagrangian Decomposition (MIP)
    - Logic-Based Benders, Subproblem encapsulation (CP)

MAPF Arms Race

- Methods from path planning (PP) and discrete optimization (DO) crossover
  - PP to DO:
    - rectangle symmetries
    - low-level search
  - DO to PP
    - corridor symmetries
    - disjoint splitting
    - "depth-first" search

"Better than A*"

- Core-guided search versus A*
  - Use Core-guided search as a plug in replacement for A*
  - Features required
    - Combinatorial state space
    - Poor heuristics
    - "repeated subproblems"

The End