Obstacle drag in stratified flow

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This paper describes an experimental study of the drag of two- and three-dimensional bluff obstacles of various cross-stream shapes when towed through a fluid having a stable, linear density gradient with Brunt-Vaisala frequency, N. Drag measurements were made directly using a force balance, and effects of obstacle blockage (h/D, where h and D are the obstacle height and the fluid depth, respectively) and Reynolds number were effectively eliminated. It is shown that even in cases where the downstream lee waves and propagating columnar waves are of large amplitude, the variation of drag with the parameter K (=ND/πU) is qualitatively close to that implied by linear theories, with drag minima existing at integral values of K. Under certain conditions large, steady, periodic variations in drag occur. Simultaneous drag measurements and video recordings of the wakes show that this unsteadiness is linked directly with time-variations in the lee and columnar wave amplitudes. It is argued that there are, therefore, situations where the inviscid flow is always unsteady even for large times; the consequent implications for atmospheric motions are discussed.

1. Introduction

There is an extensive literature on the now classical problem of stratified flow over obstacles. Many of the early theoretical works, most of which were essentially linear theories of various kinds, have described the relationship between the obstacle drag and the amplitude of internal waves generated by the obstacle (see, for example, Drazin & Moore 1967; Miles 1968; Trustrum 1971). The ‘wave’ drag has components linked both to stationary lee waves and to ‘columnar’ waves which can propagate upstream and downstream of the obstacle. Despite the importance of drag in the meteorological context, there seems to have been less emphasis on this link in more recent work, with attention being concentrated largely on the nature and form of the wave field. For example, in one of the few
recent attempts to construct a nonlinear theory describing the flow over a two-
dimensional barrier when the parameter $K$ is close to an integer (the ‘near-
resonant’ condition), Grimshaw & Smyth (1986) do not mention drag at all. ($K$ is
defined in the usual way as $ND/\pi U$, where $D$ is the channel depth and $U$ is the
upstream flow velocity, and note that $K$ is the ratio of the propagation speed of
the lowest wave mode to the undisturbed flow velocity.) This is not intended as a
criticism of such work; indeed, it would appear that the wave drag-wave
amplitude links are most clearly formulated in works which are, arguably, rather
less mathematically rigorous (see, for example, Trustrum 1971; Janowitz 1981).

What is more surprising, perhaps, is that so few attempts have been made to
measure drag forces experimentally. Accurate determination of the drag of an
obstacle under stratified flow conditions would, in principle, be of considerable
help in determining the nature of the flow. The longstanding discussions
surrounding the question of upstream influence and its implications for towing
tank experiments, for example, would have been helped if even qualitative drag
data had been available. As far as the authors are aware, the only published
experimental results on obstacle drag in stratified flow are those obtained by
Davis (1969, immortalized in the text by Turner, 1973) and the data of Lofquist
& Purcell (1984). (See Note added in proof.) Only Davis made any attempt to
compare his measurements with the wave drag deduced theoretically from the lee
wave amplitude measurements. Now a close examination of Davis’s experiment
raises certain difficulties. The obstacle used for the drag measurements – a thin
vertical barrier, or ‘fence’ – was suspended beneath the surface (and not
symmetrically between the surface and the base of the tank). Vortex shedding
would therefore almost certainly have occurred, although strongly affected no
doubt by the stratification. Reynolds numbers, based on barrier height and tow
speed, were less than about 5000, so that substantial Reynolds number effects
could also be anticipated. These two features alone, neither of which are discussed
by Davis, make an accurate interpretation of the drag measurements difficult. It
is also worth pointing out that the drag force never exceeded about 14 g, which
made it difficult to measure accurately. Furthermore, Davis noted that in many
cases the lee wave structure was far from steady. This made estimation of the lee-
wave generated drag uncertain but, more importantly, it implies an unsteady drag
force.

Despite these difficulties, some of Davis’s conclusions have been broadly
confirmed by the work of Lofquist & Purcell (1984), who made very careful
measurements of the drag of spheres. (Again, the Reynolds numbers were quite
low but the authors successfully separated the effects of stratification from the
effects of Reynolds number.) First, both found that drag can be substantially
increased or reduced as a result of stratification. Secondly, at large values of $K$ (in
excess of 3, say), Davis found no organized wave structure and the very large drag
coefficients predicted by Drazin & Moore (1967) and Miles (1968) did not seem to
occur in either set of experiments. This was attributed by Davis to the breakdown
of the waves to turbulence, and the consequent inapplicability of Long’s model, on
which the theories were based. Miles did, in fact, anticipate this conclusion by
pointing out that, even if the predicted results are legitimate steady-state
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solutions, they probably represent unstable motions. Davis also concluded that the drag associated with lee-wave generation was not (for a thin barrier) the dominant effect of stratification.

Now, over recent years substantial evidence has accumulated that, for two-dimensional obstacles at least, columnar wave modes of significant amplitudes can be generated and propagate upstream once $K > 1$. Small-parameter ‘weakly nonlinear’ theories (McIntyre (1972) for small obstacle height, and Baines & Grimshaw (1979) for small stratification) indicate only second-order upstream disturbances. However, experiments have clearly identified first-order disturbances (Wei et al. 1975; Baines 1977, 1979; Castro & Snyder 1988) and these can occur even in effectively infinite depth cases (Baines & Hoinka 1985; Pierrehumbert & Wyman 1985). The generation mechanism is almost certainly a nonlinear one and there have as yet been no theories which explain the process adequately. None the less, it is evident that the columnar modes, as well as the lee-wave modes, must be associated with additional obstacle drag. It is interesting that the Oseen type (linear) theories can give reasonable quantitative agreement with experiment, at least as far as the columnar wave amplitudes are concerned (Castro & Snyder 1988), although this requires specification of the obstacle drag coefficient and has only been tested at large values of $K (O(10))$.

Our initial objectives in undertaking direct drag measurements were four-fold. Firstly, we wanted to compare measured drag coefficients with those required by the Oseen theories to give agreement with measured wave amplitudes. Secondly, it was of interest to compare the variations in drag with $K$ (and $F_b = U/N_h$) with those implied by the linear theories and by Long’s model. All these theories (and the time-dependent ones based, alternatively, on the Oseen assumption) predict minimum wave drag at integral values of $K$, with maxima occurring at intermediate values. Bluff obstacles cannot have zero drag, of course, because of the inevitable pressure drag associated with flow separation and a turbulent wake, but if the wave drag component were significant, one might expect variations which qualitatively follow the variations given by inviscid theories. Thirdly, we wanted to assess the influence of stratification on obstacle drag in the regime where no waves are generated ($K < 1$). Finally, we anticipated that direct drag measurements would provide both a more immediate and a clearer indication of whether the flow ever reached a steady state during the course of the tow. The intention in all of this was to concentrate on surface-mounted obstacles, for which classical (Karman) vortex shedding could not occur; this seems most relevant to atmospheric flows. Direct drag measurements have not previously been reported for such cases. It was also felt important to minimize the possible complicating effects of Reynolds number by comparing results obtained with and without stratification at the same Reynolds number.

Our initial measurements have been partially discussed already (Castro & Snyder 1987, hereafter referred to as CS), where it was shown that under some circumstances large, periodic oscillations in drag could occur. This was an unexpected finding; it was not caused by wave reflections from the tank end-walls and there seemed to be no obvious explanation for the phenomenon. In this paper we first summarize the more important results of this initial work (in §3) and
Figure 8. Photographs taken when the drag was (a) maximum and (b) minimum (times of 25 and 46 s, respectively, on figure 6).
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FIGURE 9. Raw drag trace for two-dimensional fence. $K = 1.4$, $F_h = 1.05$.

FIGURE 10. Variation in (normalized) drag with $K$. $\Delta$, two-dimensional fence (various $F_h$); $\triangle$, two-dimensional fence, $F_h = 1.0$; $\blacktriangle$, Witch of Agnesi (various $F_h$).

decreasing $F_h$ corresponds to increasing blockage ($h/D$) and figure 11 includes an $h/D$ scale. Evidently the unsteadiness only occurs over a limited range of $F_h$ and this range will presumably depend, to some extent at least, on the particular value of $K$. Note that the fence results for $K > 2$ in figure 10 were all obtained at $F_h = 1.0$; oscillations might well occur at different $F_h$, but we have no data to confirm that. Also included in figure 11 is wave-amplitude data obtained from the
corresponding video recording. As before, the results represent the amplitude as deduced from a single dye streamer. For \( F_h < 1.25 \), the high drag state corresponded to the occurrence of wave breaking in the lee, and wave amplitude measurements were only possible during those times when the drag was near its minimum. At the very low end of the \( F_h \) range, where oscillations again did not occur, wave breaking occurred steadily throughout the tow.

These periodic oscillations in wake structure and obstacle drag might be explained, first, by reference to the flow variations with \( K \) in cases where oscillations do not occur. As discussed in §3.3, both drag and wave amplitude have a maximum in between integral values of \( K \). Recall that \( K = ND/\pi U \). After starting a tow the lee and upstream propagating columnar waves will gradually increase in strength. (Note, incidentally, that a number of specially designed tests demonstrated that the final flow was essentially independent of how the tow was started.) The upstream waves will cause an effective reduction in the upstream velocity ahead of the obstacle, with a corresponding increase in the effective value of \( K \). In cases for which this effective \( K \) is higher than the value corresponding to the maximum wave amplitude condition (i.e. \( 1.5 < K < 2.0 \)) this will lead in turn to a reduction in the wave amplitude (because \( dA/dK \) is negative), which will then cause a reduction in the effective value of \( K \) and a consequent increase in the wave amplitude. There seems no reason why this cycle of increasing and decreasing effective \( K \) (with the consequent decreasing and increasing wave amplitude and

![Figure 11](image_url)

**Figure 11.** Variation of (a) drag and (b) lee-wave amplitude with \( F_h \) at fixed \( K = 1.7 \). ~ denotes wave breaking. \( F_h \) and \( h/D \) scales apply to both (a) and (b).
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drag) could not repeat continuously. The inherent instability arises because $dA/dK$ is negative. For $1.0 < K < 1.5$, $dA/dk$ is positive and no such instability exists. Note that this explanation does not essentially depend on nonlinear processes near the obstacle. The requirement is only that $dA/dK$ has a turning point, which many steady state linear theories yield. Whether oscillations actually do occur in cases when, in principle, they can, will depend on the amplitude of the columnar wave. If this is sufficiently small, viscous effects can presumably damp out the small oscillations that would otherwise occur.

As a second and alternative explanation for the flow and drag variations, it is possible that the nonlinear processes in the region of the obstacle are, in fact, crucial. Apart from Long's model, two dynamical models have been advanced to describe the nonlinear flow of stratified fluid of finite depth over topography. Grimshaw & Smyth (1986) considered the self-interaction of the critical mode (when, in the present context, $K$ is near an integer) and showed that the mode amplitude satisfies a forced Korteweg-de-Vries equation. In the case of uniform stratification, however, the nonlinear interaction term vanishes so the theory is not applicable directly to the present case. Baines & Guest (1986) described a model in which it is assumed that the flow reaches a steady state. The model requires that the flow become critical over the obstacle to control the nonlinear character and, for this to occur with uniform stratification, the Long's model solution must become unstable. However, in the present experiments drag variations are observed for cases in which the Long's model solution is stable (no vertical streamlines anywhere), so that this model is also inapplicable here.

Recent theoretical studies, to be reported elsewhere, indicate that a possible nonlinear explanation is along the following lines. For strictly hydrostatic flow, the linear solution for the perturbation streamfunction when the flow is initiated from rest contains a steady part (which becomes Long's model for finite $h$) plus two time-dependent terms for each mode (Baines & Guest 1988). One of these propagates against the stream and the other propagates with it. The steady-state part and the upstream propagating part become large as $K$ approaches an integer. As the wave propagates slowly through the steady state part of the solution, however, its propagation speed may be affected by the changed flow; it may even change direction, so that the leading part of the wave may propagate upstream but the trailing part may never escape from the region of the obstacle. A perturbation expansion in disturbance amplitude to obtain the effects on wave speed does give a form containing terms which modify the usual linear wave speed. Now for non-hydrostatic flow with shorter obstacles, the equation for the amplitude of the upstream propagating mode is expected to contain a wave dispersion term, giving an equation of the Korteweg-de-Vries or a similar form. By analogy with the work of Grimshaw & Smyth (1986) such a system may be expected to give periodic solutions in some parameter ranges and it is these which may correspond to the drag variations observed in our experiments.

Whatever the explanation for the observed drag variations, this distinction between cases in which towing-tank experiments on two-dimensional obstacles may or may not be steady, quite independently of the effects of columnar-wave reflections, appears not to have been recognized previously. A number of authors
have noted unsteady behaviour, including Davis (1969), but the possibility of columnar mode reflections, particularly in cases where the tank was relatively short, tends to obscure matters. However, there have been previous results which, with hindsight, would appear to confirm the present conclusions. A recent example is provided by the work of Boyer & Tao (1987a), who studied stratified flow over triangular and cosine-shaped obstacles. Careful inspection of their data indicates that the flows which were most clearly unsteady were those for which $1.5 < K < 2.0$ and $2.5 < K < 3.0$. They also found that, for certain parameters ($K = 1.3$, $F_h = 0.7$), the flow was relatively insensitive to the starting conditions, while for others ($K = 1.6$, $F_h = 0.76$), the final flow depended critically on the translational history of the obstacle (Boyer & Tao (1987b), although presumably such dependence would disappear if the tank were sufficiently long). This is precisely what would be expected on the basis of the present work.

During preparation of the final version of this paper, the recent work of Hanazaki (1989) came to our notice. He has undertaken a numerical study of stratified flow past a square plate, and found similar periodic variations in the drag, which were explained as arising from ‘successive release of the upstream columnar disturbance (or the ‘eddy’) of the first internal wave mode’. In Hanazaki’s work no distinction was made between cases for which $K$ was less than or greater than 1.5, although the numerical results clearly indicate much larger fluctuations for $K > 1.5$, in line with the present work. Other authors have also talked in terms of ‘detaching upstream eddies’. We prefer to think simply in terms of periodic variations in the strength of the columnar mode, arising from changes in the effective value of $K$. For $K < 1.5$, such variations will (usually) eventually die away, so that no further apparent ‘perturbation eddies’ will ‘detach’. Hanazaki’s computations did not continue long enough for a steady state to be reached in those cases. It is also worth noting that these computations, for $Re = 50$, demonstrate variations in drag with $K$ which are very similar to the present experimental results. Indeed, $C_d/C_{do}$ at $K = 1$ (0.71) and $K = 1.5$ (1.11) are remarkably close to our results for the narrow three-dimensional fence (figure 2c), which – taking the water surface as a symmetry plane – has the same aspect ratio. This is a further confirmation that the wave-induced motions and pressure field dominate the influences of turbulence.

It remains to discuss the mechanisms which determine the timescale of the periodic oscillations in drag. Figure 12 shows the period, $T$, of these oscillations, normalized by towing speed and water depth, as a function of (a) $K$ and (b) $F_h$ for the various obstacles used. In every case the period falls with increasing $K$, starting from a maximum at the value of $K$ for which oscillations first begin (1.5 and 2.5). According to our first possible explanation for the drag variations, fluctuations in the apparent value of $K$ drive the instability and these will only occur once the wave amplitudes have grown sufficiently. Then the period will depend essentially on the time required to develop a significant lee (and columnar) wave amplitude. If this were a function mainly of the lee-wave wavelength ($2\pi U/N$), then $TU/D$ would be inversely proportional to $K$. The data certainly behave, qualitatively, in that way, as do the results of Hanazaki’s (1989) computations, referred to above. However, $TU/D$ would then also be independent
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![Figure 12. Period of unsteadiness, as a function of (a) K or (b) F$.]

(a)
(b)

$\frac{T_{U/D}}{K}$

0
1.0
1.5
2.0
2.5
3.0

0.0
0.3
0.2
0.15
0.1

$h/D$

Figure 12. Period of unsteadiness, as a function of (a) $K$ or (b) $F_h$. ▲, Witch of Agnesi, $F_h = 1.0-1.2$; ◆, $F_h = 1.7-1.9$; ▲, $F_h = 0.7-0.9$. □, wide three-dimensional fence, $F_h = 0.8-1.0$; ■, two-dimensional fence, $K = 1.7$.

Note that the timescale is much larger than any scale based on towing speed and some typical longitudinal length scale characterizing the flow. The lee-wave wavelength, $\lambda$, is in many cases the longest such length scale, and $T$ can be more than an order-of-magnitude larger than $\lambda/U$. If the towing tank is too short (or more strictly, tank length/water depth is too small) then the oscillation period may be of the same order as, or even less than, the tow time. This was, in fact, the case for some of the earlier full-depth tows discussed by CS; it may also have been true for other cases in the literature reported as unsteady.

5. Conclusions

The major conclusions of the work can be summarized as follows.

1. For bodies of any shape but sufficiently bluff to promote separation in the lee, stratification first acts so as to reduce the obstacle drag. This is largely a consequence of the damping of vertical motions and the reduction of vertical mixing and shear layer entrainment, and occurs in both laminar and turbulent flows.

2. Once the stratification has become strong enough to allow strong stationary
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Lee waves and propagating columnar waves ($K > 1$), the subsequent variation of drag with increasing stratification is qualitatively similar to that predicted by linear theories, whether these are based on Long's model or on the time-dependent, Oseen equations. The drag follows the wave amplitudes in having minima at integral values of $K$ with maxima in between. Changes in the drag associated with separation and the turbulent wake are small and, while such drag is certainly not negligible, the total drag is dominated by the effects of the wave motions. Since the large-amplitude columnar waves are probably a direct result of nonlinear effects, it would seem that even a full nonlinear theory would give a drag variation qualitatively similar to that implied by linear theories.

3. For two-dimensional obstacles in conditions such that $K$ is in the upper half of the range between two integral values, strong oscillations in the wave-field and, consequently the obstacle drag, can occur. This phenomenon has nothing to do with the arrival of wave reflections from the tank end-walls. The relative magnitude of the oscillation depends both on obstacle shape and on $F_h$, via the influence of these parameters on the wave amplitudes. Three-dimensional obstacles can also generate these oscillations, provided they are sufficiently long in the spanwise direction.

Since large-amplitude columnar modes can occur even in infinite depth situations (Pierrehumbert & Wyman 1985; Baines & Hoinka 1985) the instability could, in principle, arise in the atmosphere. The authors are not aware of any atmospheric observations which indicate this behaviour, but in view of its inherently long timescale it may be a rather rare occurrence.

All the data reported here and in previous papers on obstacle drag measurements are in the context of relatively bluff obstacles which lead to separation in the lee. However, there seems no reason to suppose that such oscillatory motions could not be generated with obstacles of rather lower slope. Separation is not a requirement for the generation of strong columnar modes and it is the latter that drive the instability through their influence on the apparent value of $K$.

4. At extreme levels of stratification (very low obstacle Froude number, $U/Nh$) the wake flow and the drag are more likely to be steady even if lee-wave breaking occurs. Lee-wave drag is then relatively small, but the presence of a spectrum of rapidly propagating columnar waves leads, in towing tank experiments, to multiple reflections from end walls which imply, in the limit of zero $F_h$, an asymptotically large drag coefficient. In this respect our drag measurements at $K > 9$ agree qualitatively with the theory of Foster & Saffman (1970). They do not agree with the linear, time-dependent theory of Janowitz (1981), in that the measured $C_d$ values are much higher than required by the theory to match the columnar wave amplitudes measured in our earlier experiments (Castro & Snyder 1988). Since the generation of first-order amplitude waves is a strongly nonlinear phenomenon, this is not, perhaps, surprising.

5. As a final comment, it is worth pointing out that in cases where the obstacle either spans the towing tank completely or is sufficiently wide to lead to significant upstream motions (under appropriate circumstances), wave reflections can drastically reduce the effective towing time. For a case where the distance between the starting position of the obstacle and the upstream end wall is $L$ and $K > 1$, the
reflection of the fastest moving mode arrives back at the obstacle after a time
given by $Ut/D = [2/(1 + K)] L/D$. Only if the wave amplitude is small can the tow
continue (until $Ut/d = L/D$ at the very most) without serious distortion of the flow
field.

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Note added in proof (23 February 1990). We recently became aware of the work of Mason (Geophys. Astrophys. Fluid Dyn. 8, 137–154 (1977)), who measured body forces on spheres in rotating stratified fluid.