Tidal currents in the northwest African upwelling region

J. M. Huthnance* and P. G. Baines†

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Abstract—Tidal currents are analysed from 27 instruments on five moorings having 28 days in common during “Auftrieb ’75—Upwelling ’75”. The diurnal currents are largely incoherent, probably owing to proximity of the local inertial frequency. The barotropic semi-diurnal currents are modelled by a Kelvin wave propagating northwards with an estimated M₂ tidal energy flux of \(2 \times 10^{11}\) W across 23°N. The baroclinic semi-diurnal currents are much less coherent and are strongest at the moorings in 500-m water depth on the steep continental slope, where they are dominated by intermittent near-surface and bottom-intensified motion. These bottom-intensified currents are mostly aligned with the shelf and can only sometimes be modelled as a combination of Rhines’ (1970, Geophysical Fluid Dynamics, 1, 273–302) bottom-trapped waves. The bottom currents are also much larger than the 0 (2 cm s⁻¹) estimate for internal tides generated by the onshore component of the barotropic current according to conventional models. We suggest that the strong currents are caused by the stronger longshore component of the barotropic current interacting with longshore topographic irregularities in the shelf and slope. Variability in the baroclinic tide is mainly attributed to changing stratification in the upper 150 m. The M₄ harmonic occurs intermittently at 500-m depth on the near-critical continental slope.

INTRODUCTION

The “Auftrieb ’75—Upwelling ’75” experiment took place off northwest Africa from late January to early March 1975 (Brockmann, Hughes and Tomczak, 1977). Numerous CTD casts and five successful current meter moorings along three cross-shelf sections A, B, and C (Fig. 1) enable study of water movements over periods of days or weeks (Tomczak and Hughes, 1980) and of tidal currents in three dimensions.

The tidal currents are interesting owing to (a) their amplitude, which is comparable with that of the longer-period motions; (b) proximity of the diurnal and inertial frequencies; (c) the contribution of northward energy fluxes to the large North Atlantic semi-diurnal tides; and (d) the presence of significant internal tides, already studied in this area by Horn and Meincke (1976) and Gordon (1979). Since the length of continental shelf and slope from 21° to 26°N was chosen because of its minimal longshore topographic variations, the observations provide a favourable opportunity for testing present conceptual models of tidal currents; these models almost invariably assume uniform conditions alongshore.

The plan of the paper is as follows. After a discussion of the data, including spectral analysis that leads us to specialize on the semi-diurnal tides, we consider a Kelvin-wave model for the barotropic currents. The model assists extrapolation oceanwards from the immediate vicinity of the observations, so that an estimate can be made of the total northwards tidal energy flux across 23°N. We then consider the large baroclinic currents at

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B2 and C2 on the steep continental slope in terms of presently known models for (a) free waves propagating over a shelf or slope and (b) internal tides generated over the shelf and slope by the onshore barotropic component of the tidal current. Purely empirical analysis into orthogonal modes is also performed, providing a measure of the efficacy of the conceptual models. In the final sections we discuss various aspects of the interpretation of the data, including the first harmonic ($M_4$), and the conclusions are summarized.

**DATA**

Table 1 shows the positions of the 27 current meters (from five moorings) yielding data for a common period of 28 days. The time series were reduced by separate diurnal and semi-diurnal band-pass filters to 336 2-hourly readings commencing at 1200, 3 February 1975. With regard to the diurnal currents, three aspects of the observations indicate that they contain extra inertial contributions, viz. (a) the diurnal currents were found to have a coherence of only 15% relative to the local surface tide deduced from tide tables (Admiralty, 1979), whereas for the semi-diurnal currents the comparable figure was 53%; (b) the r.m.s. diurnal current speeds were more than half those of the semi-diurnal currents, greatly exceeding expectations based on diurnal tidal elevations, which are only 10% of the semi-diurnal elevations; and (c) the sum of section B current spectra (Fig. 2) shows the diurnal tides barely rising above the broadband inertial motion. These aspects would degrade any analysis for the diurnal tides, which has not been pursued further. Horn and Meincke (1976) found a similar merger of diurnal currents into a broad inertial peak, even though their measurements at 19° to 20°N were further from 'critical latitudes' (e.g., 27.6°N, where the inertial frequency matches the $O_1$ diurnal tidal constituent). We note the possibilities at critical latitudes for internal tide generation by topography (Hendershott,
1973) or by coupling with the barotropic tide via the horizontal components of the Coriolis force (Miles, 1974). The resulting baroclinic currents may be comparable with barotropic diurnal currents (which will not be substantially affected), and incoherent away from the immediate area of generation.

By contrast, the semi-diurnal tides are prominent in Fig. 2, and we follow Horn and Meincke (1976) and Gordon (1979) in concentrating on them. Figure 3 represents the horizontal currents at each meter by a tidal current ellipse, the mean over 28 days = 4 weeks. A measure of variability is

\[ S^2 \equiv \frac{1}{4} \sum_{i=1}^{4} \sum_{j=1}^{2} \frac{|v_{ij} - \bar{v}_j|^2}{\sum_{j=1}^{2} |\bar{v}_j|^2}, \]  

(1)

where \( i \) indexes weekly averages and \( j \) the two component directions. The \( v_{ij} \) are complex current amplitudes relative to the barotropic spring–neap cycle modelled below (so that only variability additional to the spring–neap cycle is measured) and an overbar denotes an average over the four weeks. Variability within weeks is usually somewhat less, owing to the narrow semi-diurnal spectral peak (Fig. 2), which is well covered by the semi-diurnal band-pass filter of width 0.4 cycles day\(^{-1}\), approximately. The values of \( S^2 \) (Fig. 3) show the greatest variability to be at the slope moorings B2 and C2 in 500-m water depth. Unlike observations off Oregon (Torgrímson and Hickey, 1979), there is little correlation between \( 1/S^2 \) and onshore–offshore ellipse alignment, the latter being prominent only at B1 as part of the barotropic Kelvin-wave form (see below).
Fig. 3. Mean tidal current ellipses and normalized variability $S^2$ [see (1)]. The onshore component aligned with the section A, B, or C is to the right in each case. Cyclonic polarization unless marked C. Phase leads of ellipse major axis arc with respect to barotropic Kelvin-wave elevation at section B. Scale applies to mean tides ($M_2$). The current is represented as total = barotropic model + baroclinic remainder at each meter on the mooring.
BAROTROPIC MODEL

Model description

We consider the semi-diurnal band-passed data and take the barotropic current at each mooring to be the mean of the 3, 5, or 6 current records from the various depths. The non-zero internal tide contribution to such simple mooring averages is expected to be removed in subsequent averaging; we seek a coherent motion between the five moorings over a period of four weeks, and internal tides are usually incoherent over such large distances and times (Wunsch, 1975). Such mooring averages are also supported by analysis into empirical orthogonal modes (e.g., Kundu, Allen and Smith, 1975), which shows that most of the coherent energy is retained: the first mode of the mooring-averaged currents includes 90% of the energy contained in the first mode based on all the individual currents. The mooring averages include 64.7% of the total data variance.

The model for the barotropic current comprises (a) a direct tide and (b) a Kelvin wave. Respectively, (a) and (b) are a particular integral and a complementary solution of the linearized inviscid shallow-water (long-wave) equations forced by the tidal potential. The amplitude and phase of the free Kelvin wave are chosen to give the best least-squares fit to the data. This procedure follows Munk, Snodgrass and Wimbush (1970), except that here no Poincaré waves are included, because their pattern of currents is almost indistinguishable from a Kelvin-wave pattern over the limited extent of the observations.

In calculating the direct tide and the Kelvin-wave form for the mean frequency $M_2$, a straight shelf was assumed, but separate calculations were made for the actual offshore depth profiles along each of the three sections A, B, and C, using the numerical method described by Caldwell, Cutchin and Longuet-Higgins (1972). The boundary conditions for both (a) and (b) are: no flow through the coast and decay far offshore. The arbitrary Kelvin-wave amplitudes were linked between the sections A, B, and C to conserve the longshore energy flux, and the phase differences corresponded to the calculated propagation speed or wavelength (7700, 6750, and 7260 km at A, B, and C). Hence, the direct tide, together with any overall amplitude and phase of the calculated Kelvin wave, satisfies all the conditions for barotropic motion against a straight coast and shelf profile. Baroclinic motion generated by interaction between the barotropic current and topography (e.g., Baines, 1973) is considered later.

$S_2$ (only) is added in the same proportion as in the equilibrium tide, so that the model's time dependence is simplified to include only the mean semi-diurnal frequency $M_2$ modulated by a spring-neap cycle. A lag $T$ (the 'age' of the tide) in the Kelvin-wave modulation is also allowed and chosen for the best least-squares fit to the data. By using the Kelvin-wave structure to link all the records in the least-squares fitting process for $T$, we may estimate the age despite its inaccessibility from individual current records, each having incoherent contributions giving almost random ages. This would also be achieved by jointly analysing the barotropic currents into empirical orthogonal functions.

Results: currents

The model currents, after fitting over the whole 28 days, are shown in Fig. 3 and account for 60.4% of the barotropic current variance. The direct tide is at most 0.3 cm s$^{-1}$ for $M_2$, so that most of the current (and surface elevation) is contributed by the Kelvin wave. Confidence limits of 95% determined by the $F$-test for linear regression are approximately ±12% in amplitude and ±7° in phase, given the model.
The given spring–neap cycle appears to be followed. Separate fits for each of the 14 2-day segments of record account for 63% of the barotropic current variance, an insignificant improvement over the 28-day fit assuming the cycle. An age $T$ of $36 \pm 15$ h (95% confidence limits) is found, consistent with observed values averaging about 40 h for tidal elevations in the area.

The model currents match the observations in alignment, amplitude, and phase quite well overall. The figure 60.4% may be compared with the maximum 77.3% of the barotropic current variance attributable to any one spatial form and time dependence (the first empirical mode). [Both figures are sample-based, incorporating artificial coherence, and overestimate any predictive skill of the respective models (Davis, 1976).] However, there are local differences between the Kelvin-wave model and the first empirical mode representing observations. The model currents are mis-aligned (10° to 50°) at A3 and B1, early at B1, and too small (by 20 to 40%) and late at B3 (Fig. 3). Two likely causes for these differences are non-parallel shelf topography and coherent baroclinic contributions to the mooring-averaged current. Moorings A3, B2, B3, and C2 all show coherent variations of the currents with depth (Fig. 3); sampling only the upper levels at A3 and B3 probably causes the mooring average to exceed the barotropic current by a baroclinic contribution. Less probably, the particularly large currents at the deep mooring B3 might result from substantial (but uncharted!) topography; convex features may enhance currents (e.g., Huthnance, 1974). The observed alignment of the barotropic current at B1 cannot be simulated by a model with parallel depth contours; it is probably a result of local topography, of limited extent [0 (10 km) say] so that B2 currents are not similarly deflected.

The currents at B2 and C2 confirm those at Lupine (water depth 400 m, Fig. 1; Gordon, 1979).

Surface elevation

The model's surface elevation is 1.08 m, phase lag 335° for $M_2$ at the coast inshore of section B, and is due almost entirely to the Kelvin wave rather than the direct tide. Much additional information regarding the surface elevation is available for comparison. Many co-tidal charts have been derived both empirically (e.g., Dietrich, 1944) and numerically from first principles (Accad and Peckeris, 1978). Both methods show a combination of (a) an almost plane progressive wave arriving from the South Atlantic and (b) a Kelvin-like wave propagating cyclonically along the North Atlantic coasts around an amphidrome near 50°N, 39°W. Figure 4 for $M_2$ has been sketched using published harmonic constants in the Admiralty Tide Tables (1979), Cartwright, Zetler and Hamon (1979), 29-day analyses of four equatorial Atlantic records obtained by I.O.S. in November to December 1978, and a year's analysis of a Brazilian record on Fernando de Noronha (D. E. Cartwright, personal communication). $S_2$ is similar to $M_2$ with an amplitude factor $= 0.4$ and a phase lag $= 40°$.

Surface elevations in the model are about 20° earlier than observed, and are larger with a longer wavelength (each $\times 1.8$ approximately) than the local estimates 0.6 m, 4000 km from Fig. 4. These amplitude and wavelength factors are consistent, cancelling in their effect on the currents by the longshore momentum equation

$$\frac{\partial v}{\partial t} = -g \frac{\partial \zeta}{\partial y}$$
Fig. 4. Sketch of $M_2$ tidal amplitudes (----) in centimeters and phases (-----).

wherever the onshore velocity component is small relative to $\sigma v/f$, i.e., everywhere except B1 ($f$ is the Coriolis parameter, $\sigma$ the $M_2$ frequency). However, the factor being 1.8 and not 1 does suggest the presence of a Poincaré wave in addition to the Kelvin wave modelled. Accepting in particular the observed 4000-km wavelength alongshore, the dispersion relation (e.g., Munk et al., 1970) in the deep water further offshore (supposed uniform at 2500 m) gives an offshore decay scale of 750 km, considerably less than the figure 2300 km for the Kelvin wave alone. We emphasize that these estimates depend on tidal elevation measurements over a region much more extensive than the current meter sections A to C.

**Energy flux**

Although the current measurements are confined to the coast, we can extrapolate offshore by means of the model to estimate the longshore energy flux

$$\rho g \int h\langle (\zeta - \zeta_e) \nu \rangle \, ds$$

(Cartwright, Edden, Spencer and Vassie, 1980; $\zeta_e$ is the equilibrium tide). Allowing for the reduced elevations and offshore scale discussed above, the flux across 23°N is estimated as $2 \times 10^{11}$ W. The major uncertainty is the offshore decay scale; 750 km is probably a lower limit and if increased the flux estimate increases nearly proportionally.

For comparison, Gordon (1979) found $7 \times 10^4$ W m$^{-1}$ along the 400-m contour at Lupine. Scaled up to $4 \times 10^5$ W m$^{-1}$ in the deeper water (2500 m) offshore, where the longshore current is similar, and extending over the energy flux decay length $\frac{1}{2}(750$ km) offshore, a similar total flux is found.
At 38°N, Cartwright et al. (1980) found a northward flux of \(2.46 \times 10^{11}\) W estimated from observations, and R. A. Flather (personal communication) found a value \(5.5 \times 10^{11}\) W from a numerical model of northeast Atlantic tides (with observed elevations as open-boundary input). The discrepancy between these estimates has to be taken as a measure of their uncertainty. Between 23° and 38°N, there are no obvious energy sinks, but some work,

\[-\rho g \int \langle \zeta e \partial \zeta / \partial t \rangle \, dA = 10^{11}\ W,\]

is done against tidal forces. The balance is completed by a substantial influx of energy from the west between 23° and 38°N, as suggested by the amphidromic system of North Atlantic co-tidal charts; the influx between 23° and 38°N equals

\[
\begin{cases} 
\text{work against tidal forces} \\
+ \text{northward flux across 38°N} \\
- \text{northward flux across 23°N}
\end{cases}
= \begin{cases} 
(1 + 5.5 - 2) \times 10^{11} \ W = 4.5 \times 10^{11} \ W \text{ (maximum)} \\
(1 + 2.5 - 3) \times 10^{11} \ W = 0.5 \times 10^{11} \ W \text{ (minimum)}.
\end{cases}
\]

The observed phase delay in the B1 currents suggests a small shoreward energy flux component \(\rho gh \langle (\zeta - \zeta_e)u \rangle\) of about \(4 \pm 2\) kW m\(^{-1}\), or a total of \(2 \times 10^9\) W if maintained over the whole shelf between sections A and C. These figures are negligible compared either with the longshore flux or with global tidal energy dissipation \(0(3 \times 10^{12}\) W) (e.g., Accad and Pekeris, 1978).

**ANALYSIS OF BAROCLINIC MOTION**

*Distribution*

Both the variability estimates and the mean tidal current ellipses in Fig. 3 suggest that the currents at A3, B1, and B3 are nearly barotropic. Their mooring averages account for, respectively, 74, 94, and 82% of the semi-diurnal variance, contrasting with only 34% at B2 and C2. All these barotropic percentages are larger than, but distributed like, those observed in 19° to 20°N by HORN and MEINCKE (1976), who found a baroclinic energy peak at 1000-m water depth. Our finding of greater baroclinic energy at the slope moorings B2 and C2 is also to be expected in general terms from models of internal tide generation at continental slopes (Baines, 1982).

B2 and C2 also contribute 68% of all the recorded temperature variance with only nine out of 26 records. Only 4% of all the temperature variance was attributable to up-slope–down-slope motion of the barotropic current. Nevertheless, both the currents and the temperature variations according to the barotropic model were subtracted from the original (semi-diurnal-filtered) time series. The ‘baroclinic residual’, described hereafter, remains.

Fig. 5. ‘Baroclinic residual’ currents at (a) B2 and (b) C2. Abscissa is days from 0000, 3 February *1975. Vertical scale is \(-30\) to \(+30\) cm s\(^{-1}\) on all plots. u, onshore; v, longshore north.
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(b)
The currents at B2 and C2 (Fig. 5) are stronger near the sea surface and sea floor, as was found by Horn and Meincke (1976). Root-mean-square values are 9.0, 4.5, 5.7, 7.0, and 11.4 cm s$^{-1}$ on average between B2 and C2, working from the top current meter downwards. Separate surface and bottom-intensified motions are suggested.

**Empirical modes**

The currents ($u$, onshore; $v$, longshore) and temperature ($T$) series at B2 or C2 can be jointly analysed into empirical orthogonal functions:

$$\text{baroclinic residuals} = \sum_{n=1}^{N} b_n \phi_n(t),$$  \hspace{1cm} (2)

where the leading 'organized' contributions $b_1 \phi_1$, $b_2 \phi_2$, ... in turn optimize (for least-square error) the approximation for $N = 1, 2, \ldots, NM$ (e.g., Kundu et al., 1975). The $b_n$ are $NM$-vectors, where $NM$ is the total number of series, 15 at B2 ($u$, $v$, $T$ at 5 meters) and 14 at C2. Temperature is measured in different units from velocity and might accidentally dominate (or not influence at all) the fitting process according to the choice of scale; we give temperature equal weight by scaling to average the same variance numerically as the mean variance of all the $u$ and $v$ series at the station. Two modes (i.e., $N = 2$) are minimal for representing phase differences between $u$, $v$, and $T$, and include a majority of the variance at B2 and C2. If we further impose $\phi_2(t) = d\phi_1(t)/dt$, i.e.,

$$\text{baroclinic residual} = b_1 \phi_1(t) + b_2 d\phi_1(t)/dt,$$  \hspace{1cm} (3)

then $b_1$ and $b_2$ are more readily interpreted as in-phase and quadrature contributions to baroclinic motion oscillating as $\phi_1(t)$.

At B2, the first two (of 15) empirical modes account for 63.8% of the variance; the constrained form (3) accounts for 56.3% and is illustrated in Fig. 6. Figure 7 illustrates the form (3) at C2, where the respective figures for variance accounted for are 68.5 and 55.5%. At both moorings, the temperature is approximately in phase throughout the depth, with a mid-depth minimum, whereas the velocity has one phase reversal and a near-bottom maximum. This structure suggests the lowest baroclinic mode.

Comparing the longshore and onshore pressure gradients calculated from the currents via the inviscid momentum equations

$$\partial p/\partial y = -\rho (\partial v/\partial t + fu)$$
$$\partial p/\partial x = -\rho (\partial u/\partial t - fu),$$  \hspace{1cm} (4)

a larger gradient (or shorter length scale) alongshore than onshore is suggested where $v$ exceeds $u$ (because $\sigma > f$), i.e., near the bottom; at all levels $u$ and $v$ are comparable, so that $\partial p/\partial x$ and $\partial p/\partial y$ are comparable (Figs 6, 7). As a topographically imposed onshore length scale of 10 km is expected, this suggests that a similarly short scale prevails alongshore. With viscosity included the observed $v/u$ ratio is still incompatible with $\partial/\partial y \equiv 0$. At the bottom current meters (only), 10 m above the sea floor, viscous boundary layer effects may increase the longshore component $v$ relative to $u$, but not to the extent observed at B2 if $\partial/\partial y \equiv 0$. In the thicker 'critical' boundary layer considered by Gordon (1980) (assuming $\partial/\partial y \equiv 0$, $v$ and $u$ may both be increased but remain in the relation $|v| = |fu/\sigma| < |u|$.

**Internal tide generation**

All present models assume a straight continental shelf so that internal tides are forced by
the onshore–offshore barotropic component of the current. Then the internal tide can be determined, if the stratification, barotropic tide, and shelf profile are all known. We use the models of Baines (1973, 1981) for 'flat' topography $\alpha < c$ and 'steep' topography $\alpha > c$, respectively, where $\alpha$ is the sea-floor slope and

$$c \equiv (\sigma^2 - f^2)^{1/2}/(N^2 - \sigma^2)^{1/2}$$

is the characteristic (or ray) slope; $N$, $f$, and $\sigma$ are the Brunt-Väisälä, inertial, and tidal frequencies.
For section B, Fig. 8 shows vertical profiles of $\sigma_\theta$ from CTD casts within 5 km of B2 taken at various times while the currents were recorded, and the assumed stratification for modelling. The slope at B2 is 'steep': $0.057 \approx \alpha > c \approx 0.039$; Baines' (1981) model was used. The model slope is uniform and adjoins a level shelf (Fig. 9); internal motion and characteristics are reflected at the underside of a surface mixed layer of depth 40 m.

For section C, Fig. 10 shows $\sigma_\theta$ from CTD casts within 10 km of C2 early in the cruise, and the assumed stratification for modelling. The continental slope is 'steep' between 100 and 220-m depth approximately, where we use Baines' (1981) model, and possibly also below 1050 m. We neglect the latter as a source of internal tides at C2 in view of its distance (35 km) and the absence of CTD data at these depths. Between 220 and 1050-m depth the slope is 'flat' (Fig. 11), for example at C2 $0.04 \approx \alpha < c \approx 0.042$ and $\alpha$ is less elsewhere; here we use Baines' (1973) model. Calculations were made with and without a 30-m surface mixed layer.

Fig. 8. $\sigma_\theta$ near B2. Numbers indicate date in days after 3 February 1975 (when current meter records began). --- $\sigma_\theta$ for $M_2$ characteristics.

Fig. 9. Depth profile, model profile ----, current meter positions +, and $M_2$ characteristics --- near B2.
The Kelvin-wave model gives the onshore–offshore barotropic current component \( U \), say) that forces the internal tidal currents, which may be large near (i.e., within 10 m) the two characteristics originating at the shelf break (Figs 9, 11), \( 0(U) \) between the two characteristics, and small elsewhere. Neither at B2 nor C2 are any current meters close enough to a characteristic for the internal tidal currents to exceed about 2 and 3 cm \( s^{-1} \), respectively [i.e., \( 0(U) \)]. However, the currents at C2 are considerably modified by the surface mixed layer (Fig. 12); the ‘beam’ between characteristics from the shelf break is displaced if reflected at 30 m rather than at the surface.

These calculated internal tides are weak compared with the observed currents, particularly at the bottom meters at B2 and C2, and satisfy \( |v| = |u|f/\sigma \), contrary to observation.
Fig. 12. Calculated onshore $M_2$ baroclinic current at C2. Components —— and ——— are, respectively, in phase with and lag by 90° the onshore barotropic current (1.6 cm s$^{-1}$ at C2). (a) No mixed layer. (b) 30-m mixed layer.

**Bottom-trapped waves**

We subtract the calculated internal tide from the 'baroclinic residual' and regard the remainder as unforced motion to be modelled by free baroclinic waves. (We argue below that this motion is probably due to the large longshore barotropic motion interacting with unknown longshore topographic variations.)

The complete set of free baroclinic waves above the inertial frequency in a continental shelf context awaits systematic study. In the present case, an energy balance for the baroclinic motion of the rough form input rate = radiational loss = energy density $\times$ 'radiation coefficient' suggests a small 'radiation coefficient', as the energy density is large
compared with the forced internal tide, whilst the (unidentified) energy input may reasonably be supposed to be no more than drives the forced internal tide. Hence, we seek fairly well-trapped waves, which suggests short length scales; unless a wave is confined above the slope, a long wavelength alongshore implies substantial radiational losses above the inertial frequency (Hurhance, 1978). Bottom-trapped waves (Rhines, 1970) satisfy the trapping requirement (and indeed are the limiting waveform for short longshore wavelengths at sub-inertial frequencies). The solutions for waves progressing up or down a wedge (Wunsch, 1969) also show bottom intensification but radiate energy rapidly. No other ‘prototype’ solutions appear to be known at present.

Hence, we test whether bottom-trapped waves can model the observed ‘baroclinic residual’. Such waves are possible at the \( M_2 \) frequency \( \sigma \) if \( \sigma < N \alpha/(1+\alpha^2)^{1/2} \approx N \alpha \) (the bottom slope \( \alpha \) is small). At \( B_2 \), \( \sigma < N \alpha \), whilst \( \sigma \approx N \alpha \) at \( C_2 \). Following Rhines (1970), let \( \theta = \pm \cos^{-1} \sigma/N \alpha \) and take rotated cartesian co-ordinates \((x', y', z')\) with \( z' \) normal to the sloping sea floor and \((x', y')\) rotated in the plane of the sea floor through \( \theta \) from (onshore, longshore). Then the model currents are

\[
(u', v', w') = (V, 0, 0) \exp (i k y' - K z') \exp (-i \sigma t),
\]

i.e., rectilinear, parallel to the sea floor, and aligned to make \( \sigma = N \alpha \cos \theta \) the natural buoyancy frequency. The phase velocity (perpendicular to the current) keeps deeper water on the left (northern hemisphere) and has a component into deeper (shallower) water if \( \theta > 0 (\theta < 0) \). In (5), \( K \) is determined by \( k \) and \( l/K = -N \alpha / f \sin \theta \); however, \( k \) is not observable at individual moorings (\( B_2 \) or \( C_2 \)) so that \( K \) may be chosen with \( V \) to fit the observations’ vertical structure. In practice a combination of one up-slope \((\theta < 0)\) and one down-slope \((\theta > 0)\) propagating wave was also considered at \( B_2 \). Fitting was by least squares to the \( M_2 \) complex amplitudes of the ‘baroclinic residual’ current, for each individual week of the 4-week record. Only the bottom three current meters were included. The small energy near 160 m has little effect on a fit by bottom-trapped waves and is excluded as being too near the surface for the bottom-trapped wave form to apply. The meters near 70 m are also excluded on this score and on account of their extra motion to be discussed later. Significance of fits was assessed by the \( F \)-test on residual variance, based on three degrees of freedom for each wave fitted \((K\) and two for complex \( V\)) and 12 degrees of freedom in the data \( (\text{two each for the complex amplitudes of two velocity components at three meters})\).

At \( B_2 \), a significant fit by bottom-trapped waves was found only in the most energetic weeks 2 and 3 (Fig. 5a), when two waves gave a significantly better fit than one. Values of \( V \) and \( K \) for the two waveforms (5) are given in Table 2. The data variance attributable to bottom-trapped waves in weeks 2 and 3 amounts to 30% of the \( B_2 \) total (barotropic and

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**Table 2.** \( B_2 \) bottom-trapped waves. Suffixes 1, 2 denote waves progressing down-slope and up-slope, respectively. \( \theta_1 = 42.4^\circ = -\theta_2 \). \( 1/K_1 = 2.29 \approx 12/K_2 \) in (5). \( \pm 95\% \) confidence limits. Phases relative to model Kelvin-wave elevation

<table>
<thead>
<tr>
<th>Week</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( K_1, \text{ m}^{-1} )</td>
<td>0.0015 ( \pm 50% )</td>
<td>0.0021 ( \pm 100% )</td>
</tr>
<tr>
<td>( K_2, \text{ m}^{-1} )</td>
<td>0.0163 ( \pm 30% )</td>
<td>0.0016 ( \pm 100% )</td>
</tr>
<tr>
<td>( V_1, \text{ cm s}^{-1} )</td>
<td>( -7.73 \pm 4.67 \pm 13% )</td>
<td>( -8.16 \pm 2.69 \pm 30% )</td>
</tr>
<tr>
<td>( V_2, \text{ cm s}^{-1} )</td>
<td>( 8.40 \pm 5.72 \pm 19% )</td>
<td>( 8.67 \pm 1.26 \pm 28% )</td>
</tr>
<tr>
<td>% variance accounted for</td>
<td>98.3</td>
<td>93.9</td>
</tr>
</tbody>
</table>
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baroclinic) over the whole 4 weeks. The associated energy flux (averaged over the 4 weeks) is nearly 500 W m$^{-1}$: 308 W m$^{-1}$ onshore and 177 W m$^{-1}$ offshore (the energy flux is proportional to $|V/K|^2$ and is onshore if phase propagation is down-slope and vice versa). Evidently, the onshore flux is reflected or dissipated before reaching B1, but the current meters at B3 offshore are too high to give firm evidence about fluxes there. The whole flux at B2 represent a loss from the barotropic tide, but is insignificant compared with barotropic energy fluxes. [For comparison with Gordon’s (1979) figure of 300 W m$^{-1}$ at Lupine in 400-m water depth, the total baroclinic energy at B2 propagating seawards at the speed of the first vertical internal mode (assuming a flat bottom) would be a flux of 1500 W m$^{-1}$, an overestimate because (a) the first vertical mode is fastest and (b) the bottom is not flat; e.g., Wunsch’s (1969) solutions propagate energy slower over a slope.]

The vertical velocity $w$ at B2 was inferred from the temperature oscillations. Its phase relative to the ‘baroclinic residual’ onshore current $u$ fluctuated from week to week (Table 3), but the 4-week means show $u$ and $w$ roughly in phase. At mid-depth, $w/u$ was approximately equal to the characteristic slope, as for a plane internal wave propagating on- or offshore. Near the sea floor, $w/u$ approximated the bottom slope $\alpha$. However, these 4-week average results apply only to the individual week 2, when the bottom-trapped waves fitted best. Then, $w$ and $u$ were in phase within 15° at the bottom three meters and $w/u = \alpha$ within 10%, indicating motion parallel to the sea floor.

At C2, the phase of $w$ relative to $u$ fluctuated widely, particularly at the top meter. At 282 and 357 m, $w$ and $u$ were in phase within 11° and $w/u = \alpha = 0.04$ within 20% during week 1 (only), suggesting motion parallel to the sea floor then. Unfortunately, the bottom meter gave no temperature record from which to infer $w$.

The calculated bottom-trapped waves at C2 have $\theta = 0$ because $\sigma = N\alpha$, so that the model currents (5) are aligned onshore-offshore and cannot match the observed predominantly longshore currents. The attempted fit by bottom-trapped waves is never statistically significant.

Discussion of baroclinic models

Available dynamical models (specifically, bottom-trapped waves) give a generally poor representation of the baroclinic motion over the slope at B2 and C2, despite the simple empirical first vertical mode.

The B2, week 3 (only), combination of nearly equal up-slope and down-slope propagating bottom-trapped waves gives net phase and energy propagation mainly along the slope. As bottom-trapped waves at the $M_2$ frequency $\sigma$ require $\sigma < N\alpha$, we speculate that the wave energy is confined over the steepest slope $\alpha$. Such a wave mode is known to be the short-wavelength limit of coastal trapped waves below the inertial frequency.

Table 3. Vertical ($w$)/onshore ($u$) baroclinic currents at B2 (bottom slope 0.057) mean over 4 weeks, and error estimate for the mean

| Depth (m) | $w$ phase lead (°) | $|w|/|u|$ | Characteristic slope |
|-----------|------------------|----------|-------------------|
| 75        | Uncertain        |          |                   |
| 165       | $-38 \pm 32$     | 0.083 ± 0.021 | 0.031 |
| 290       | 28 ± 27          | 0.041 ± 0.014 | 0.039 |
| 365       | 29 ± 29          | 0.061 ± 0.013 | 0.042 |
| 505       | $-12 \pm 21$     | 0.072 ± 0.020 | 0.039 |
(HUTHNANCE, 1978). Its possibility above the inertial frequency merits investigation, despite the inevitability of some radiational energy loss.

In the alternative description in terms of rays, one likely factor contributing to the bottom-intensified currents is simply the superposition of incident and reflected internal waves (or ‘beams’), which are nearly in phase close to the bottom (as in simple modes).

By (4), the greater longshore \((v)\) than onshore \((u)\) currents imply shorter longshore length scales than the onshore-offshore scale, which is at most the topographic scale \(0(10\;\text{km})\). Hence, internal-tide models neglecting longshore variations (as they all do) may be inadequate. Longshore shelf irregularities potentially generate internal tides from the longshore barotropic current component, which is stronger than the onshore component forcing present models.

**DISCUSSION OF INTERNAL TIDE VARIABILITY**

The records from 67 (75) m depth at B2 (C2) appear most energetic in the first \(1\frac{1}{2}\) to 2 weeks, when stratification was strong at this depth (Figs 8, 10). BAINES’ (1981) analysis implies the generation of wave motion on the shallow thermocline; the motion propagates seawards and subsequently leaks downwards on characteristics. Such motion is polarized anticlinonically with \(|v/u| = f/\sigma\), is strongly dependent on the stratification near the shelf break, and is absent when there is no distinct thermocline. A detailed comparison between the model and data at this depth is not feasible, but qualitatively they agree quite well for the first 1 to 2 weeks.

The mixed layer itself extends down to 67 (75) m for part of the time, as indicated by flat-topped temperature records (BROCKMANN et al., 1977). A mixed layer of varying depth reflects characteristics into different paths, affecting the internal tide generated near the shelf break as illustrated by Fig. 12. Near-surface internal-wave reflection may also be affected by a current in the mixed layer (STERN, 1977)—by a factor of two for a 20 cm s\(^{-1}\) current in a 40-m mixed layer at B2, for example.

Stratification changes affect characteristics and \(\sigma_0\) to a similar extent (both are a first integral of \(N^2\)). Figures 8 and 10 indicate that these changes are large in the top 150 m; otherwise modifications to Figs 9 and 11 are slight. Such stratification changes in the generating region over the shelf break, or near the observing point, are sufficient to explain the current variability at the upper level. If the large bottom currents at B2 and C2 are generated near the shelf break (we suggest by the large longshore barotropic currents adjusting to longshore topographic variations), then variability in these bottom-intensified motions may also be attributed to the changing upper-level stratification.

In addition to changes in the surface mixed layer and thermocline (i.e., upper 150 m), there are several other possible sources of internal tide intermittency, or of departure from predictions by linear theory using characteristics. Scattering by microstructure is expected to be small over the short distance from the generation regions to B2 (< 10 km) and C2 (20 km, or about \(\frac{1}{2}\) day at the speed of the lowest internal mode). Interaction with other internal waves is probably also small over only one oscillatory cycle. [WUNSCH (1976) found rapid spatial changes in internal wave energy generated (?) near a seamount, within 10 km for waves of 5-h period. However, the changes were not obviously due to non-linear interactions and did not imply interactions with the tidal currents, to which the internal wave energy was not related.] Mean current shear distorts internal tides (MOOERS, 1975),
but only slightly in our context, because the horizontal shear is much less than \( f \) and the vertical shear causes isopycnal slopes much smaller than the slope of characteristics.

Mean currents \( \langle u \rangle \) can advect the internal tide. Onshore currents are typically 5 cm s\(^{-1}\) or less and probably not generally important, but longshore flows exceed 20 cm s\(^{-1}\) at times. Hence, the internal tide, propagating at only 0 (15 cm s\(^{-1}\)) in its (fastest) first vertical mode form near the shelf edge, may be advected alongshore faster than it can propagate offshore. Then the particular depth profile and characteristics of section B or C (say) no longer apply. Furthermore, if the internal tide wavenumber \( k \) has a component parallel to \( \langle u \rangle \), then the 'intrinsic' frequency \( \sigma - \langle u \cdot k \rangle \) may differ from the \( M_2 \) frequency \( \sigma \), altering the spatial structure of the internal tide. Estimates of \( k \) would require measurements with a longshore spacing of 0 (1 km).

Fluctuating 'mean' currents correspondingly cause intermittency, and also induce changes in the perceived internal tide frequency (of zero crossings, say) in the records. Figure 5 shows only slight variations, usually within the range of periods 12.4 ± ½ h, so that conditions seem to be quasi-steady. Both perceived frequency changes and intermittency (from the various sources mentioned) broaden the tidal peaks in the energy spectrum (Fig. 2). The total effect is rather moderate for semi-diurnal tides.

**BOTTOM BOUNDARY LAYER**

Any boundary layer effects recorded by the lowest meter (10 m above the bottom) at B2 and C2 are masked by other vertical structures. However, at B1, the bottom meter was only 2 m off the sea floor, and Fig. 3 indicates a slight trend towards cyclonic polarization and a phase advance. This trend may be attributed to the boundary layers for the anticyclonic and cyclonically polarized components of the oscillatory tidal current having different thicknesses (Munk et al., 1970). The cyclonic layer is thinner, so that there is a range at the top of the anticyclonic layer where the anticyclonically polarized component has begun to decrease (to zero at the sea floor) but the cyclonic component is still undiminished. [If a uniform eddy viscosity \( v \) is assumed, then the two boundary layer depths are \( \delta_{\pm} = [2v/(\sigma \pm f)]^{\frac{1}{2}} \), where \( \sigma \) is the tidal frequency and \( f \) is the Coriolis parameter; the observations give \( v \leq 10^{-3} \text{ m}^2 \text{ s}^{-1} \) in the bottom 10 m at B1, corresponding to \( \delta_{\pm} \leq 5 \text{ m} \) for \( M_2 \) and \( \delta_{\pm} \leq 11 \text{ m} \) for the diurnal tides (Weatherly, Blumsack and Bird, 1980).] At the end of the cruise, turbid water appeared to occupy the bottom 3 m at B1, beneath clear water (Kullenberg, 1978). This layering may have suppressed bottom turbulence, which is probably generated by the low frequency and semi-diurnal currents (being strongest).

**M\(_4\) HARMONIC**

Onshore-offshore tidal excursions of order 1 km take fluid particles through substantial changes of depth near the shelf edge. Non-linear effects may both influence internal tide generation and generate higher harmonics, notably at the \( M_4 \) frequency (twice \( M_4 \)) near 4 cycles day\(^{-1}\).

\( M_4 \) currents and temperature fluctuations were observed at Lupine (Fig. 1), especially when the baroclinic tide was strongest, but the currents were consistently related to the semi-diurnal tide only near the bottom (Gordon, 1979). The bottom currents were strongest, surface currents were nearly as large, and temperature fluctuations were greatest at mid-depth.
At B2, $M_4$ appears in the onshore-offshore velocity component at the bottom (Fig. 13) when the total semi-diurnal currents are strongest, and in the bottom temperature record in the appropriate phase relation; low temperatures lag the onshore current. In contrast with Lupine, however, there is a minimum in $M_4$ temperature fluctuations at mid-depth, and the near-surface currents contain only 15% of the energy in the bottom currents.

$M_4$ is small at C2 despite even stronger semi-diurnal currents at the end of the record than at B2, and despite being quite close to Lupine. (The C2 and Lupine records are from different times, however.) C2 differs in having an expected minimum period $2\pi/N\alpha$ for Rhines' (1970) bottom-trapped waves of 14 h, compared with 7 h both at Lupine ($N = 1.8$ cycles $h^{-1}$, bottom slope $\alpha = 0.076$) and on the steepest slope just 4 km inshore from B2. Equivalently, the $M_4$ characteristic slope is only just greater than $\alpha$ at B2 and Lupine but twice as steep at C2. Wunsch's (1969) progressive solutions in a wedge therefore suggest that any $M_4$ currents at B2 and Lupine should be concentrated near the bottom.

We conclude that the $M_4$ harmonic is associated with large onshore-offshore semi-diurnal currents and a near-characteristic bottom slope for $M_4$. The latter condition depends on the stratification, making for even more variable $M_4$ generation than implied by changing semi-diurnal currents.

Fig. 13. Hourly onshore ($u$) and longshore ($v$) current and temperature ($T$) at the lowest B2 and C2 meters.
The diurnal currents were largely incoherent with the tidal elevations and have been mostly attributed to a broad inertial peak in the energy spectrum.

The spatial and temporal coherence of the barotropic semi-diurnal tide permits its evaluation from the five current meter moorings. In particular, analysing the moorings simultaneously gives a good estimate of the age of the tide (36 ± 15 h) and confirms the spring-neap cycle. The mooring-averaged currents are generally consistent with a Kelvin-wave model. Extrapolation offshore, using the Kelvin-wave model with allowance for the observed tidal elevations, permits an estimate of the northward tidal energy flux at 23°N of 2 to 3 × 10⁻¹¹ W.

Significant baroclinic currents occur at the two moorings B2 and C2 on the continental slope at 500 m. These currents are intermittent, stronger near the surface with u (onshore) > v (longshore) and, at different times, stronger near the bottom with v > u. The near-surface motion may be attributed to generation near the shelf break on a shallow thermocline, as the motion and thermocline co-exist. The currents in the deeper water are stronger than expected from two-dimensional theory and are probably due to the large longshore barotropic currents interacting with unknown longshore topographic variations. For all of these motions we attribute the intermittency to the varying density stratification in the upper 150 m.

The majority of the baroclinic variance is represented empirically by a first vertical mode form. However, dynamical models for this form over a significant sea-floor slope are lacking. At B2, but not C2, Rhines’ (1970) bottom-trapped waves model the near-bottom motion when its amplitude is large; at both sites large amplitudes near the bottom may be partly due to the superposition of incident and reflected waves.

The M₄ harmonic has (relatively) large amplitude currents at B2 (and Lupine) where the ray slope approximately equals the bottom slope, but not at C2 where the ray slope is twice the bottom slope.

A better understanding of the currents appears to call for (a) models of internal tide generation through the longshore barotropic currents adjusting to an irregular shelf and (b) a more systematic and extensive knowledge of internal wave forms over a continental shelf and slope.

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