Forcing of the Antarctic Circumpolar Wave by El Niño-Southern Oscillation teleconnections

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Abstract. We address several issues regarding the Antarctic Circumpolar Wave (ACW). These include the extent to which the ACW is related to El Niño-Southern Oscillation (ENSO) cycles and the need to reconcile differences between the observed zonal wavenumber 2 pattern and the wavenumber 3 pattern found in coupled ocean-atmosphere models. In the present study we revisit the sea surface temperature (SST) signal of the observed ACW. We find that the associated SST variations may be decomposed into several empirical orthogonal functions (EOFs) that are linearly independent. The first four EOFs describe 84% of the total variance on timescales of 3-7 years, and superposition of SST signals of the four EOFs reproduces SST signals of the observed ACW. Given that EOF analysis can only describe standing oscillations, we then employ complex EOF (CEOF) analysis suitable for studying propagating waves. The first two CEOFs (denoted CEOFSST1 and 2) account for 70% of the total variance, and the SST variability reconstructed from them again reproduces the observed ACW. The real part of CEOFSST1 (44% of the total variance) is similar to the first real EOF (EOFST1). Both EOFST1 and CEOFSST1 may be identified with ENSO and have a zonal wavenumber 2 zonal pattern in the southern middle-to-high latitudes. CEOFSST1 is the major constituent of the ACW in the period of 1981-1997. The associated atmospheric circulation displays a Pacific-South America (PSA) pattern. Statistical analyses demonstrate that the PSA pattern teleconnected to ENSO is the major mechanism that forces this principal constituent of the ACW. CEOFSST2 (26% of the total variance) has a zonal wavenumber 3 pattern in the same latitude band, similar to that simulated by several coupled models. We suggest that if ENSO is not adequately resolved in coupled models but structures with zonal wavenumber 3 pattern are, then the modeled ACW would appear to have a zonal wavenumber 3 pattern.

1. Introduction

The recently discovered interactive phenomenon in the southern middle-to-high latitudes, the Antarctic Circumpolar Wave (ACW), is a near 4 year climate signal, with phase-locked eastward propagating oceanic and atmospheric signals, taking ~8 years to encircle the Southern Ocean with a global zonal wavenumber 2 pattern [White and Peterson, 1996, hereinafter referred to as WP; Jacobs and Mitchell, 1996]. The discovery has inspired a great deal of research interest. White et al. [1998] constructed an idealized coupled ocean-atmosphere model of the Southern Ocean region that captured many features of the observed ACW, and Peterson and White [1998] have suggested that El Niño-Southern Oscillation (ENSO) can influence the ACW by advective oceanic teleconnections from the western subtropical South Pacific. From these studies and others [e.g., Baines and Cai 2000] it seems possible that the ACW may be maintained by coupling between the extratropical ocean and atmosphere. The impact of the ACW on climate variability of southern continents has also attracted attention. White and Cherry [1999] have focused on the impact upon rainfall and temperature variability over New Zealand, and White [2000] has examined the influence on Australia.

ACW-like phenomena have been found in coupled ocean-atmosphere models [Christoph et al., 1998, hereinafter referred to as CBR; Motoi et al., 1998; Cai et al., 1999, hereinafter referred to as CBG]. In the study by Motoi et al. [1998] a zonal wavenumber 2 pattern dominates, as in the observations, and the variability is linked to ENSO. The CBR and CBG studies show that the eastward movement appears in sea surface temperature (SST) signals but not in atmospheric anomaly fields. In those fields the signal is dominated by standing oscillations, and there is no phase-locking between oceanic and atmospheric anomalies. Although local signals are generated every 4-5 years as in the observa-
tions, the SST signal takes 12-16 years to complete a circuit around the pole, and there is a predominant zonal wavenumber 3 pattern. CBR examined the possibility of a teleconnection between ACW and ENSO events in their models and found that ENSO does play a role, but it is not essential. As for the difference in the modeled and observed wavenumber pattern, they reported that in their model in some decades, the wavenumber 2 pattern dominates. In the Commonwealth Scientific and Industrial Research Organisation (CSIRO) model (CBG) it seems that wavenumber 3 is the preferred pattern, and there is no evidence of a wavenumber 2 pattern in the 60 year period examined. Reconciliation between the observed and modeled ACW and between ACWs in different models is therefore needed.

In the present study we revisit the observed SST signal of the ACW. We find that the SST signal may be decomposed into several empirical orthogonal functions (EOFs) that are linearly independent. Some of these EOFs are familiar ones already extensively studied. The first EOF is ENSO’s manifestation in the Southern Ocean, which, as will be shown, has a wavenumber 2 signal pattern in the southern middle-to-high latitudes. The second EOF has a discernible wavenumber 3 pattern in the same southern latitude band. Given that EOF analysis can only pick up standing oscillatory modes, we also employ complex EOF (CEOF) analysis. We show that the sum of SST variations reconstructed from the first two CEOFs reproduces most of the SST signal of the observed ACW. Like the first EOF mode, the first CEOF is ENSO’s manifestation in the Southern Ocean, and it has a zonal wavenumber 2 pattern. We demonstrate that this CEOF is forced by the atmospheric Pacific-South America (PSA) pattern teleconnected to ENSO. The second CEOF has a wavenumber 3 pattern similar to the second EOF. In the period of 1981-1997 the wavenumber 2 signal dominates the wavenumber 3 signal. As stated above, many coupled models (e.g., CBR and CBG) produce ACW-like variability with a predominant wavenumber 3 pattern, rather than a wavenumber 2 pattern. We suggest that this difference between the observed and modeled wavenumber patterns may be due to the fact that ENSO is not adequately resolved in these models.

2. Data and Method

The SST data used in the present study are from the new version (version 3) of gridded historical Global Sea Ice and SST (GISST3) fields [Rayner et al., 1996]. This is an updated version of the quality-controlled U. K. Meteorological Office Historical SST data set [Parker et al., 1994, 1995]. Our analysis differs from that of WP and Peterson and White [1998], who, instead, use the data set of Reynolds and Smith [1994] and express their results in terms of a single extended EOF.

We focus on the 17 year period of 1981-1997. We did not extend our analysis to the period prior to 1981 because there were few observations over the southern middle-to-high latitudes. Reliable data coverage over these latitudes only became available in the early 1980s from satellite observations that were incorporated in GISST3. A climatological mean over the 17 years is formed for each month. These means are subtracted, by month, from the time series of monthly averages to remove, as much as possible, the seasonal cycle in the series. The remaining quantity is hereinafter referred to as the SST “signal.” The discovery of the ACW, in terms of SST, was based on data covering 1982-1995 (WP). This period lies within that chosen here, providing an opportunity to benchmark against WP’s results. To focus on the ACW time scale (4-5 years), the monthly signals are band-passed with a 3-7 year (Butterworth) admittance window, as by WP; this procedure suppresses seasonal and possible biennial signals and removes the long-term trend. EOF analysis and CEOF analysis are then applied to the filtered signals over the Southern Hemisphere. Both EOFs and CEOFs are obtained from covariance matrix of the gridded data, and the gridded data are not area weighted. We exclude the Northern Hemisphere in our analysis so as to provide a better focus on the region of interest, the southern middle-to-high latitudes. This also ensures that the analysis is not affected by dominant modes of the Northern Hemisphere such as the North Pacific mode [Deser and Blackmon, 1993] and North Atlantic Oscillation [van Loon and Rogers, 1979; Wallace and Gutzler, 1981]. To explore the nature of these modes, we employ 500 hPa geopotential height fields covering the period of 1981-1997, from reanalysis conducted by the U.S. National Centers for Environmental Prediction/National Centers for Atmospheric Research NCEP/NCAR (see Hines et al. [2000] for potential shorting comings), and mean sea level pressure (MSLP) fields covering the period from 1981 to 1994, from the Hadley Centre [Bassinett and Parker, 1997]. The atmospheric circulation patterns of these fields and their association with the EOFs of SST are explored.

3. Results

3.1. Standard EOFs of SSTs

The spatial patterns of the first four EOFs for the whole Southern Hemisphere are shown in Figure 1 and the corresponding time series are shown in Figure 2. They account for 47, 15, 13 and 9% of the total variance, respectively. We describe these EOFs because they may be regarded as a succinct description of the data and seem to be the ingredients of the observed ACW. The first EOF (Figure 1a), denoted EOF1, may be indentified with ENSO. The largest weights are in the equatorial Pacific, where there is opposing polarity between east and west. The temporal part (Figure 2a) for this EOF is consistent with its ENSO-like spatial pattern, showing large positive amplitudes in 1983,
1987, and 1997 and weak positive amplitudes from 1991 to 1994. In the 45°-65°S latitude band a clear zonal wavenumber 2 pattern appears, and these signals seem to be connected with those in the low latitudes. In particular, the positively weighted region between South Africa and Antarctica is connected with weights of the same polarity in the low-latitude Indian Ocean, while negative weights in the midlatitude South Pacific and south of Australia are connected with negative weights in the west Pacific warm pool region. This coexistence of midlatitude South Pacific signals with signals of opposing polarity in the eastern equatorial Pacific mirrors that reported in the North Pacific in many studies [e.g., Deser and Blackmon, 1995; Zhang et al., 1996].

In EOFST2 (Figure 1b) the equatorial regions of the Atlantic, Pacific, and Indian Oceans are mostly in phase, with maximum weights occurring near 20°S in the eastern Indian Ocean immediately west of Australia. At higher latitudes, in the band of 45°-65°S, a wavenumber 3 pattern is seen, again with maximum weights in the longitudes of the Indian Ocean.

In the third and fourth EOFs the principal features of interest are found outside the tropics. Like EOFST1, EOFST3 (Figure 1c) also displays a wavenumber 2 pattern in SST signals at high latitudes. The maximum weights appear near the Drake Passage and south of Australia, and the signal centers are not colocated with those of EOFST1. EOFST4 (Figure 1d) shows an identifiable wavenumber 3 pattern in the latitude band 45°-65°S as in EOFST2 (though not as clear as wavenumber 2), but, again, the centers of the signals of these two EOFs are not colocated. The fifth and sixth EOFs contain 6 and 5% of the total variance, respectively, but their features are not strong, and they are omitted from this discussion.

The robustness of this EOF decomposition has been tested by repeating the process but excluding the region north of 30°S. The structure of the first four EOFs in SST in this reduced domain is almost identical to those for the full hemisphere, and the fractions of variance explained by each are now 33, 20, 16, and 12%, respectively. The time series for EOFST1 is also similar to that for this EOF for the full hemisphere. Hence the high-latitude structure of all four EOFs is not induced by the inclusion of low-latitude phenomena, and they appear to be robust entities.

### 3.2. Composition of the ACW

Figure 3a shows a time-longitude plot of filtered SST signals meridionally averaged over the 45°-65°S latitude band. The eastward propagation is clearly seen, especially between 1985 and 1994, and the dominant pattern is wavenumber 2. However, since 1995 the wavenumber...
period considered, despite the inclusion of the wavenumber 3 pattern (EOFs SSTs 2 and 4) the dominant pattern is wavenumber 2. This suggests that the signals associated with wavenumber 3 are weak for the period considered, although a wavenumber 3 is conspicuous at times, for example, during 1993. The dominance of wavenumber 2 over wavenumber 3 is reflected in time series shown in Figure 2; for example, the amplitude for EOF1 (wavenumber 2) is much larger than that for EOF2 (wavenumber 3).

We therefore constructed meridionally averaged ACW signals by using signals due to EOFs SST1 and EOFs SST3 together and EOFs SST2 and EOFs SST4 together. The signals due to EOFs SST1 alone display a standing oscillation. When the signals due to EOFs SST3 are added to those due to EOFs SST1, major features seen in Figure 3a are reproduced, and the direction of the propagation becomes definitely eastward. The sum of EOF-

Figure 2. (a) Time series of temporal coefficient of EOF1 (solid curve) and EOF3 (dot-dashed curve). (b) The same as Figure 2a but for EOF2 and EOF4.

pattern and the eastward propagation are less clear. We shall demonstrate that most of these signals that form the ACW can be reconstructed from the above four EOFs.

The signal of an EOF $k$ at each location and at a time $t$ (in month), $T_k(i,j,t)$, is obtained by

$$T_k(i,j,t) = A_k(i,j) \times f_k(t).$$

Here $f_k(t)$ is the temporal coefficient of the EOF and $A_k(i,j)$ is the spatial pattern. For each month, $T_k(i,j,t)$ is meridionally averaged over 45$^\circ$-65$^\circ$S to form a time-longitude field.

The sum of meridionally mean SST signals due to the four EOFs discussed above is shown in Figure 3b. Indeed, most ACW signals can be reconstructed by the signals attributable to these four EOFs. For the pe-

Figure 3. (a) Time-longitude diagram of filtered (retaining 3-7 years) SST signals (°C) meridionally averaged over 45$^\circ$-65$^\circ$S. (b) The same as Figure 3a but for signals that are linearly attributable to the sum of EOFs SST1, EOFs SST2, EOFs SST3, and EOFs SST4.
SST2 and EOF SST4 also displays clear wavenumber 3 pattern and shows a tendency to more eastward movement than does EOF SST2 alone, although this effect is much less marked than that obtained from the sum of EOF SST1 and EOF SST3.

In terms of spatial patterns of SST signals the EOFs constituting the ACW may be seen as two separate groups: EOF SSTs 1 and 3 that have zonal wavenumber 2 and EOF SSTs 2 and 4 that have wavenumber 3, in the latitudes of the Southern Ocean. Time-lagged cross correlations (figure not shown) between the time series of EOF SST1 and EOF SST3 and between those of EOF SST2 and EOF SST4 show that EOF SST1 and EOF SST3, and EOF SST2 and EOF SST4, indeed, correlate at a lag, with EOF SST1 leading EOF SST3 and EOF SST2 leading EOF SST4 by about 10 months. However, the amount of this lag is such that EOF SSTs 1 and 3 (or EOF SSTs 2 and 4) are not exactly 90° out of phase.

3.3. Complex EOFs of SSTs

To illustrate better the propagation feature, we employ CEOF analysis to the filtered SST signals. The programme was kindly provided by T. Walker and W. White of Scripps Institution of Oceanography. Within the routine, time sequence data at each spatial location is Hilbert-transformed to obtain a complex time sequence following the transformation method described by Barnett [1983], the real part being the input data and the imaginary part representing signals that are identified to be related with the real part but are 90° out of phase. The complex covariance matrix is then computed, and the eigenvalues, complex eigenvectors, and complex principal components are then found by singular value decomposition of this matrix.

In CEOF analysis the standing oscillations of a mode represented by the real parts of both the pattern and the temporal coefficient are often similar to a mode of the standard EOF analysis. The additional information of the imaginary part provides both temporal and spatial phase relationships between the standing oscillation and its subsequent features. Together, they provide information on evolution and propagation of the mode.

Two cases are conducted here. In one case the domain is the same as in the standard EOF analysis, i.e., the whole Southern Hemisphere; in the other case the domain is southward of 20°S. The CEOFs from the two cases are similar. We shall focus on the latter case as it illustrates that while the first CEOF may be identified with ENSO, the high-latitude structure is not induced by the inclusion of the tropics in the analysis.

The spatial patterns of the real and imaginary part of the first CEOF are displayed in Figure 4, and their time series are displayed in Figure 5. The first and second CEOFs (denoted CEOFSST1 and 2) account for 44 and

![Figure 4](image_url)  
*Figure 4. Patterns of weights of (a) real part of CEOFSST1, (b) imaginary part of CEOFSST1, (c) real part of CEOFSST2, and (d) imaginary part of CEOFSST2, of filtered SST retaining timescales of 3-7 years. CEOFSST1 and CEOFSST2 account for 44 and 26%, respectively, of the total variance on these timescales.*
Here \( Re \) and \( Im \) denote real and imaginary parts, \( f_k(t)^* \) is the complex conjugate of the temporal coefficient of \( f_k(t) \), and \( A_k(i,j) \) is the corresponding complex spatial pattern. Both \( A_k(i,j) \) and \( f_k(t) \) contain real and imaginary parts. For each month, \( T_k(i,j,t) \) is meridionally averaged over \( 45^\circ-65^\circ S \) to form a time-longitude field.

The meridionally mean SST signals due to EOFSSST1, EOFSSST2, and their sum are shown in Figure 6. EOFSSST1 (Figure 6a) produces major features seen in Figure 3a, including the definite eastward direction of propagation and the dominant wavenumber 2 pattern. The signals associated with EOFSSST2 (Figure 6b) show a tendency to eastward propagation and have a predominant wavenumber 3 pattern. Comparing Figures 6a and 6b, one sees that in the period 1981-1992 the wavenumber 2 signals are larger than the wavenumber 3 signals. However, from 1993 onward the wavenumber 3 signals appear to be stronger than the wavenumber 2 signals. The sum of the SST signals due to EOFSSST1 and 2 is shown in Figure 6c. Indeed, most ACW signals seen in Figure 3a can be reconstructed by the signals of these two EOFs.

**3.4. Associated Atmospheric Flow**

Time series for the atmospheric 500 hPa geopotential heights at each Southern Hemisphere grid location have been regressed onto each of the two SST EOFs, and the results are shown in Figure 7. The anomaly pattern in 500 hPa geopotential height associated with the real part of EOFSSST1 (Figure 7a) is familiar. This is the so-called PSA pattern, which is the Southern Hemisphere’s equivalent to the Northern Hemisphere’s Pacific-North American (PNA) pattern [Wallace and Gutzler, 1981; Deser and Blackmon, 1995; Zhang et al., 1996]. The PSA pattern is consistent with the Rossby wave response to equatorial anomalous heating [Hoskins and Karoly, 1981] and was first found by Karoly [1989] in a composite analysis of winter anomalies over several ENSO events. It emerges as one of the leading circulation patterns in the Southern Hemisphere, ranging from daily [Mo and Ghil 1987] to intraseasonal [Mo and Higgins, 1998], interannual [Kidson, 1988] and interdecadal timescales [Garreau and Battisti, 1999]. Hence the 500 hPa regression pattern for the real part of EOFSSST1 exists as an independent dynamical entity. In fact, it has been identified as a near-neutral mode of the zonal mean atmospheric flow [Frederiksen and Webster, 1988; Frederiksen and Frederiksen, 1996]. Their studies suggest that PSA is a preferred mode that can be generated by internal dynamics.

The 500 hPa height regression pattern on the imaginary part of EOFSSST1 (Figure 7b) shows a similar PSA pattern, with two significant differences. The first of these is the development and enhancement of the negative regression maximum around \( 60^\circ E, 55^\circ S \). The second is the eastward movement of the center of positive regression maximum to a position about \( 45^\circ \) east of the corresponding maximum in Figure 7a. Since the

![Figure 5.](image-url)
imaginary part is $90^\circ$ out of phase (in time) with the real part, the pattern in Figure 7b corresponds to the atmospheric circulation pattern $90^\circ$ out of phase with that of Figure 7a.

The anomaly patterns in 500 hPa geopotential height regressed on CEOFSST2 (Figures 7c and 7d) display a discernible wavenumber 3 pattern in the latitudes of the Southern Ocean, the real part having a uniform phase over the Antarctic continent. This stationary atmospheric wavenumber 3 pattern is another dominant mode operating on a wide range of timescales, ranging from day-to-day [e.g., Wallace and Hsu, 1983; Mo, 1986; Mo and Ghil, 1987; Kidson, 1988] to intraseasonal [e.g., Mo and White, 1985; Karoly et al., 1989; Mo and Higgins, 1998] and interannual [Mo and van Loon, 1984; Karoly et al., 1989]. This atmospheric pattern, including the uniform phase over Antarctica (especially the pattern associated with the real part), appears to be the dominant mode in the Southern Hemisphere low-frequency atmospheric circulation and may be, in part, associated with the mode that is called “zonal symmetric mode” [Kidson, 1988] and has recently been named as the “Antarctic Oscillation” [Thompson and Wallace, 2000] because of its similarity to the “Arctic Oscillation” in the Northern Hemisphere. That the patterns of Figure 7 are so familiar is a clear indication that they are significant and not just a random chance occurrence. This is further demonstrated below.
4. Teleconnections

Our results above suggest that to a large extent the observed ACW reported by WP is a manifestation of ENSO in the Southern Ocean. The regression pattern upon CEOFSST1 of the 500 hPa geopotential heights suggests that the atmospheric PSA mode is the major forcing component of the ACW. We proceed to demonstrate the strength of this connection, focusing on the CEOF analysis. Statistical uncertainties in the results have been estimated using procedures outlined in Appendix A. These uncertainties have been expressed in terms of error bars based on the standard errors of the correlation coefficients and eigenvalues of the CEOFs. The effective number of degrees of freedom \( (N^*) \) of these various quantities has been calculated and employed in these estimates.

Figures 4a and 5a clearly show the well-known ENSO events. The variance explained by CEOFSST1 is relatively high and is well separated from variance explained by other CEOFs. However, since filtering is applied, the effective number of degrees of freedom in the time series of CEOFSST1.r is small (according to (A7), \( N^* \) is about 6). We decided to repeat our CEOF analysis on raw (i.e., unfiltered) monthly SST signals, which admit variability on timescales 2 months. The domain is from 20°S southward. Our approach is to establish the statistical significance of the teleconnections when total variability on all timescales is considered and demonstrate that such significance carries over to variability on timescales of 3-7 years. For clarity we shall refer to CEOFs from this case as CEOFrSST1 and 2 ("r" indicating raw data) and those in section 3 as CEOFSST1 and 2.

4.1. Correlation Between Southern Oscillation Index and CEOF1 of Raw SSTs

CEOFSST1 accounts for 8.9% of the total variance. Its pattern (real part) has a spatial pattern similar to CEOFSST1 shown in Figure 4a, and is not shown here (the pattern correlation coefficient is 0.96). The time series of the real part of CEOFrSST1 is displayed in Figure 8a (thick solid curve). The interannual fluctuations displayed in CEOFSST1 (solid curve, Figure 5a) are reflected in the time series of CEOFrSST1, and the correlation between these two time series is 0.91. These high correlations indicate that CEOFSST1 and CEOFSST1 both belong to the same mode, which is identifiable with ENSO.

The subsequent three CEOFs of raw SSTs account for 6.42, 5.87, and 5.10% of the total variance. We have examined the statistical significance of these unfiltered CEOFs using the procedure summarized in Appendix
Figure 8. (a) Time series of temporal coefficient of the real part of CEOFrSST1 from CEOF analysis on raw SST monthly signals (thick solid curve), SOI (dot-dashed curve, rescaled for plotting), and temporal coefficient of the real part of CEOFr500H2, referred to in the text as the PSA mode (thin solid curve, rescaled for plotting). The PSA mode is the second mode from CEOF analysis on 500 hPa raw geopotential height anomalies. (b) Time-lagged cross correlations between the three time series. A positive lag means that the first time series leads the second time series.

A, based on that of North et al. [1982]. The effective number of degrees of freedom $N^*$ for the terms $S_{ij}$ in the associated covariance matrix had a mean value of 68, with most values in the range 60-80. Taking this mean value for $N^*$, the uncertainties in these eigenvalues as given by (A17) are $(8.9\pm1.5), (6.42\pm1.1), (5.87 \pm 1.0)$, and $5.0(\pm0.87)\%$. By this criterion the first CEOF is marginally separated from the rest, and the others are not separated. However, numerical computations of the uncertainties using (A15) give much smaller values of uncertainty for CEOFrSST1 and 2. We infer that CEOFrSST1 at least is significant and argue that this carries over to the filtered form CEOFSST1, which has very similar spatial structure. We therefore focus on CEOFrSST1 and its atmospheric teleconnections.

The real part of CEOFrSST1 is in an antiphase relationship with the Southern Oscillation Index (SOI), which is also plotted in Figure 8a (dot-dashed curve, rescaled for plotting). Lag correlation between the two time series is shown in Figure 8b (thick solid curve).
A positive lag indicates that SOI is leading. At zero lag the correlation coefficient is 0.47±0.12. With the number of degrees of effective degrees of freedom $N^* = 71$ this is unquestionably significant at a 99% level. Thus CEOFSST1 is significantly correlated with ENSO variability, strongly supporting our proposition that CEOF1 of raw SSTs represents ENSO.

At a lag of 2 months, i.e., when SOI leads by 2 months, the correlation coefficient reaches a maximum of 0.58. Thus the atmospheric MSLP signals lead those of SSTs by 2 months. On the timescales of 3-7 years the lead time may be different, and we shall discuss this later.

4.2. Correlation Between SOI and PSA

We now test the significance of the connections between the SOI and the 500 hPa geopotential height fields. To this end, CEOF analysis is applied to the unfiltered monthly 500 hPa geopotential height anomalies in the domain of the whole Southern Hemisphere. These anomalies are obtained as follows. A climatological mean over the 16 years is formed for each month, and then monthly anomalies are constructed with reference to the climatology. The first CEOF (CEOFr500H1) is the so-called Antarctic Oscillation, and the second (CEOFr500H2) is the PSA pattern; they account for 39.0, and 11.5% of the total variance, respectively. The uncertainties in these eigenvalues were calculated using the method in Appendix A, as for the SSTs in section 4.1. The mean effective number of degrees of freedom in the covariances is $N^* = 38$, so that by (A17) these eigenvalues are (39.0±8.9) and (11.5±5.2)%. The third and the fourth CEOFs both have wavenumber 3 patterns, and they account for (7.7±1.7) and (5.8±2.1)% of the total variances, respectively. By (A17) therefore the first and second CEOFs are well separated, and the second and third are marginally so. We shall focus on CEOF500H2, abbreviated by PSA. The time series of the real part of the PSA mode is also plotted in Figure 8a. The patterns of both the real and imaginary parts are plotted in Figures 9a and 9b. These patterns resemble the regression patterns associated with CEOF1 of filtered SSTs (Figures 7a and 7b).

Previous studies have found that the PSA is a preferred pattern of atmospheric internal variability in the absence of any tropical forcing [Frederiksen and Webster, 1988; Frederiksen and Frederiksen, 1996]. Because of its correlation with the SOI, the identified PSA pattern must contain components generated by tropical convective sources as well those induced locally. It is therefore necessary to test whether the ENSO-
induced component is significant. Lag correlation between SOI and PSA is plotted in Figure 10b. At zero lag the coefficient is -0.40 (±0.03). The high correlation supports a strong teleconnection between the tropics and the middle-to-high latitude atmospheric circulation variability. Further, the correlation coefficient is largest at zero lag, indicating simultaneous variations of SOI and PSA, which with \( N^* = 81 \) is significant at the 99% level. We have shown that SOI leads CEOFrSST1 by 2 months; the simultaneous coherence of SOI and PSA therefore means that PSA also leads CEOFrSST1 by 2 months.

4.3. Correlation Between the PSA Mode and CEOF1 of Raw SSTs

Figure 10b also plots the correlation coefficient between time series of the real part of CEOFrSST1 and the PSA mode at various lags. At zero lag the coefficient is 0.30. This is lower than that between the CEOFrSST1 and SOI and may be, in part, due to the fact that the PSA mode contains a locally induced component. Nevertheless, this is still significant at the 99% significance level given the large number (\( N^* = 125 \)) of effective degrees of freedom. As expected, the PSA mode leads CEOFrSST1 by 2 months, giving a maximum correlation coefficient of 0.41 at a lag of 2 months. There is virtually no correlation between CEOFrSST1 and other CEOFs of 500 hPa geopotential height fields, suggesting that other modes of geopotential height variations do not contribute to CEOFrSST1.

4.4. Statistical Significance on Timescales 3-7 Years

We have demonstrated that without separating into specific frequency bands the connections among CEOF1 of SSTs, SOI and PSA are significant. This significance provides a necessary but not a sufficient condition for statistical significance on a specific range of timescales. In the following, we demonstrate that such statistical significance carries over to the timescales of 3-7 years.

First, it is well known that SOI fluctuates mainly on the timescales of 3-7 years. The time series of PSA and CEOFrSST1 also display significant fluctuations on this timescale range. Second, when the variability on timescales longer than 3 years is removed from the three time series of Figure 8a (retaining variability on timescales shorter than 3 years), the correlation coefficients (at zero lag) between the remaining variability drop considerably. The coefficients between SOI and CEOFrSST1, between PSA and SOI, and between PSA and CEOFrSST1 decrease to 0.24, -0.22, and 0.15, from 0.47, -0.40, and 0.3, respectively. The dramatic decrease of correlation suggests that it is the variability on timescales longer than 3 years that contributes to the high statistical significance of the teleconnections among SST, SOI, and PSA on all timescales established in the previous sections since there is little variability on timescales longer than 7 years. Third, the correlation map (Figure 10a) between the time series of the real part of CEOFrSST1 (solid curve, Figure 5a) and the raw grid point 500 hPa geopotential height fields shows that the maximum correlations are in the tropics.

We now compute the correlation coefficient between time series of the CEOFSST1 (solid curve, Figure 5a) and an interannual PSA index to assess directly the teleconnections on 3-7 year timescales. The time series of the interannual PSA index retaining variability on timescales of 3-7 years is obtained by band-pass filtering the time series of PSA mode shown in the thin solid curve of Figure 8a. The effective number of degrees of freedom \( N^* \) of the two time series is 6.9. The correlation coefficient is 0.683, marginally greater than the value of 0.670 required for the 95% significance level for the degrees of freedom of 6.9. On the basis of this result and those we have described above in this section we conclude that the teleconnections on 3-7 year timescales are significant.

As a further test of the importance of ENSO in the forcing of the ACW, we removed the ENSO signal from the data and examined whether the ACW remains. There are at least two ways of forming “non-ENSO” monthly SST signals. One is to remove signals linearly attributable to SOI by regressing SST signals onto SOI and subtracting them. The other is to remove signals reconstructed from a CEOF identified with ENSO. ENSO signals from linear regression on SOI could be different from those derived from CEOF analysis because the latter takes account of both simultaneous and lagged responses while the former includes only the simultaneous signal. We chose the latter because the response to ENSO will have a lag at locations remote from the tropics. The CEOF of ENSO was obtained by applying CEOF analysis to raw monthly SST signals in a global domain. The signals at each month and at each gridpoint were reconstructed and then subtracted from the records. The remaining SST signals were then filtered as before to retain variability on timescales of 3-7 years, and CEOF analysis was carried out on these records.

The first CEOF of these filtered non-ENSO global SSTs (denoted CEOFnESST1 and 2) accounts for 40%, and the second CEOF accounts for 28% of the total remaining variance. The real part of CEOFnESST1 displays maximum loadings in the vicinity of Drake Passage (Figure 11a). Interestingly, this CEOF also shows a maximum correlation in the tropics with the 500 hPa geopotential height fields (Figure 11b), suggesting that it is mainly driven by tropical non-ENSO convective sources. CEOFnESST2 again shows a wavenumber 3 pattern in the middle-to-high latitudes (figure not shown). Meridionally averaged SST signals over 45°-66°S due toCEO1 and due to the sum of the first four CEOFnESSTs (90% of the total variances) are shown in Figures 11c and 11d, respectively. There is no continuous global encircling propagation. Without ENSO, im-
important "connections" that constitute the global route in Figure 3a disappear in Figure 11d. This result further highlights the importance of ENSO to the ACW.

4.5. Phase Relationship Between Atmospheric Flow and CEOFSST1

The ENSO component of the PSA pattern can be further illustrated by decomposing the time series of the real part of the PSA mode, shown in Figure 8a. One objective method for decomposition is the so-called empirical mode decomposition (EMD) method recently developed by Huang et al. [1998]. This analysis decomposes a time sequence into various frequency bands, and these bands are identified automatically by the analysis. Thus this method has the advantage over a filter of not having to specify a frequency band. Figure 12a shows a time series of interannual variation of the PSA mode identified by EMD, with the noisy higher-frequency components (termed intrinsic mode functions by Huang et al.) removed. Signals of recent ENSO events are apparent. Lag correlation between this time series and that of the real part of CEOFSST1 is displayed in Figure 12b. We see that the PSA mode leads CEOF1 of filtered SSTs by about 5 months.

To relate the atmospheric flow to SST, it is appropriate to consider the variations of MSLP. For this pur-
Figure 12. (a) Variability on interannual timescales of the PSA mode. The time series is obtained by applying an EMD method recently developed by Huang et al. [1998]. (b) Time-lagged cross correlations between the time series shown in Figure 12a and that of the real part of CEOFSST1 of filtered SSTs (solid curve, Figure 5a). A positive lag means that PSA leads SSTs.

Pose we apply CEOF analysis to MSLP anomalies constructed from the data set of Basnett and Parker [1997] for the period of 1981-1994. Prior to the analysis a monthly climatology is formed by averaging the full 14 year monthly fields, and for each month of each year the anomalies are constructed by subtracting the monthly climatology from the initial records. The anomalies are then filtered retaining variation on timescales of 3-7 years. To focus on the southern middle-to-high latitudes, the domain studied covers an area from 30° to 65°S. The first CEOF (termed CEOFMSLP1) accounts for 40% of the total variance. The spatial patterns at various phases of a half cycle are shown in Figure 13 (left). The temporal component of the real part of this CEOF is plotted in Figure 14a (solid curve). The pattern at zero phase and its temporal part together describe a standing oscillation, which is essentially in phase with the SOI. To relate this to CEOFSST1, the corresponding pattern and temporal sequence of the latter are also plotted in Figure 13 (right) and Figure 14a (dot-dashed curve).

Several features emerge from Figures 13 and 14a. First, CEOFMSLP1 contains propagation, and eastward propagation in the Atlantic and Indian sectors is clearly seen. Second, there exists remarkable coherence between the time series, albeit with a lag and with MSLP leading SSTs by about 5 months (Figure 14b). This coherence exists, although the two time series are generated from independent CEOF analyses. Again, this supports our hypothesis of forcing of the ACW by ENSO. Finally, the relationship between spatial patterns of SST and MSLP are such that the atmospheric pattern acts to reinforce the SST pattern, with northerly (southerly) winds located over warm (cold) regions. To this effect, major centers of MSLP anomalies, negative or positive, are located to the east of the signal centers of SSTs. This reinforcing process also appears in CEOFSST2. We shall discuss this process further in section 5.

The feature that CEOFSST1 lags the PSA by 5-6 months is interesting and is consistent with findings of previous studies. Kiladis and Mo, [1999] compiled 500 hPa height anomalies for each season for the warm ENSO events from 1958 onward. They found that the dominant atmospheric circulation pattern for the June-August and September-November seasons is a PSA pattern [Kiladis and Mo, 1999, Figure 8.4], and the pattern is more or less stagnant during these two seasons. This feature implies that the PSA pattern tends to persist for about 6 months. (In contrast, in their analysis the patterns in December-January and March-May seasons are not PSA like and are similar to that of the Antarctic Oscillation, with uniform phase over the Antarctic and discernible wavenumber 3 structure in the middle-to-high latitudes. For an oscillatory cycle of 4-5 years, a time lag of 5-6 months corresponds to a phase lag of about 45°. Such a phase relationship between the atmospheric variability and that of the ocean is also evident in the analysis of Baines and Cai [2000].

5. Discussion

5.1. Wavenumber 2 ACW Component and PSA

We have shown that in terms of the spatial patterns of SST signals the CEOFs constituting the ACW form two separate entities: CEOFSST1 that has zonal wavenumber 2 and CEOFSST2 that has wavenumber 3. We have demonstrated that CEOFSST1 is the predominant component for the ACW in the period from 1981 to 1997, which is significantly correlated with ENSO variabil-
ity, and that this component is mainly forced by the atmospheric PSA pattern teleconnected to ENSO. A scale analysis shows that physically this is quite plausible. The 500 hPa height differences of 50 m give atmospheric flow perturbations of 2-3 m s\(^{-1}\), and the associated north/south atmospheric advection can give air-sea temperature differences of 5°C. If these flows are maintained for several months, conventional expressions for heat flux [e.g., Baines and Cai, 2000] give SST perturbations of the order of 1 K, consistent with the observed SST signals of the ACW.

That middle-to-high latitude SST variability is atmospherically teleconnected with tropical variability has been proposed by many previous studies focusing on the situation in the North Pacific. There it was found that indices of the associated winter season variabili-
associated with the highly energetic atmospheric motions in midlatitudes, which also have PNA as their preferred pattern. Deser and Blackmon [1995] found, in an EOF analysis of cold season (November to March) SST variability over the tropical and North Pacific domain (20°-60°N) for the period of 1950-1992, that the “North Pacific” mode (second after ENSO) is linearly independent of ENSO and that the associated atmospheric anomaly also shows a PNA pattern in the 500 hPa height anomalies. These features were confirmed by Zhang et al. [1996]. The results from these latter two studies advocate the possibility that the North Pacific mode is one of its own, independent of ENSO, despite the fact that the associated pattern in 500 hPa geopotential height anomaly resembles that associated with ENSO. An ultimate conclusion regarding the relative importance of the ENSO forcing and local dynamic processes in the cause of the high-latitude SST variability has yet to be drawn.

Wang and Weisberg [1998] proposed another mechanism in which tropical ocean warming induced by the Walker Circulation causes the tropical air there to rise and flow toward the subtropics, where it sinks. When the sinking air approaches the sea surface, it splits and flows both equatorward and poleward, reinforcing the Hadley circulation and strengthening tropical easterlies and extratropical westerlies, the latter producing extratropical cooling. Thus the Walker Circulation and Hadley Circulation result in tropical warming and extratropical cooling. This mechanism can account for the cool signal at midlatitudes but not for the associated warm signal at South Pacific high-latitudes in CEOFSST1 of our analysis.

It is likely that the internally induced PSA contributes to the development of the wavenumber 2 SST variability on a broad range of timescales but on the timescales of 3-7 years any such contribution appears to be much smaller than that induced by the PSA component teleconnected to ENSO. As shown in Figure 10a, maximum correlation between this wavenumber 2 component of the ACW and atmospheric circulation exists in the tropical latitudes rather than in the middle-to-high latitudes. This implies that the source regions are in the tropics. These source regions do not show significant amplitude in the 500 hPa height field (Figure 7) because the atmospheric response to equatorial heating generally does not produce much vorticity in the heating region [Simmons, 1982; Sardeshmukh and Hoskins, 1988]. Instead, the process that generates PSA-ENSO teleconnections is divergence caused by tropical convection, which produces Rossby wave trains propagating southward into high latitudes. The timescale of this process is of the order of days to weeks for both the tropical convective events and propagation of the Rossby waves. The resulting patterns in Figures 1a and 10a are seen as the cumulative effect of a large number of events of this type over a period of about 5-6 months.

As a manifestation of this ENSO teleconnection, a warm SST signal is seen near the Ross Sea (Figures 1a
and 4a), to the west of the positive signal of the PSA pattern. Positive SST signals appear here when the SST in the eastern equatorial Pacific is anomalously high, i.e., during an El Niño event. This feature is consistent with results in a steadily expanding literature on the relationship between ENSO and the atmospheric circulation over the Antarctic. These studies show that there is strong modulation of the atmospheric circulation that seems to concentrate in the vicinity of the Ross Sea. Using observational data in the Ross Sea over the period of 1982-1994, Ledley and Huang [1997] showed that there is a statistically significant positive correlation between SST signals in the Ross Sea and Niño3 SST signals, with ENSO signals leading Ross Sea signals by about 3 months, and as the Ross Sea SST warms, sea ice concentration there decreases. Such connections are supported by Yuan and Martinson [2000] and Simmonds and Jacka [1995]. Using monthly data of surface pressure and temperature in Antarctica between 1957 and 1984, Smith and Stearns [1993] found a sharp change in the sign of pressure and temperature signals between the year before and the year after the minimum in the SOI and suggested that such changes are related to the 500 hPa height fields. These findings are consistent with our PSA teleconnection hypothesis.

Numerous studies have also examined the relationship between SOI and rainfall over the Antarctic. Cullather and Bromwich [1996] and Bromwich et al. [2000] showed that in the period between 1980 and 1990, precipitation in the vicinity of the Ross Sea (180°-120°W) is positively correlated with SOI. After 1990 the relationship becomes anticorrelated; Marshall [2000] found that this anticorrelation also exists in the region of Thurston Island (72°S, 99°W) in the period after 1990. Marshall suggests that this may be, in part, due to increased intensity of cyclones in the region during an ENSO. The change of the relationship in time and space may indicate propagation of the rainfall signal associated with the ACW, but further studies are needed.

5.2. Zonal Wavenumber 3 ACW Component and Other Atmospheric Modes

We turn now to CEOF2 of filtered SSTs (CEOF-SST2). Because of the short length of observational record, we are unable to establish the statistical significance of this CEOF, but its features and its connections imply that it is important. We have shown that the atmospheric pattern associated with the real part of this CEOF in Figure 7c displays a discernible wavenumber 3 pattern with a uniform phase over Antarctica. The latter feature indicates that this CEOF is in part associated with the Antarctic Oscillation. A quarter period later, the wavenumber 3 pattern (Figure 7d) becomes clear, and the connection to the Antarctic Oscillation weakens. The relationship between these atmospheric patterns and those of CEOFSST2 (Figures 4c and 4d) implies that it acts to reinforce the SST pattern, with northerly (southerly) winds located over warm (cold) regions. Correlation between the real part of the time series of CEOFSST2 and grid point 500 hPa height fields shows that maximum correlations occur over the Antarctic (Figure 10b).

In many coupled ocean-atmosphere models, SST variability associated with atmospheric standing oscillations is advected by the modeled Antarctic Circumpolar Current (ACC), producing an ACW-like variability. The variability displays a dominant wavenumber 3 pattern (e.g., CBR and CBG). The lack of a wavenumber 2 pattern may be due to the inability of these models to produce ENSO realistically. This is certainly the case for CBG’s study; in their model the amplitude of ENSO, as measured by the NINO3 SST signal, is about one third of the observed, and the signal pattern in the Southern Ocean, is predominantly wavenumber 3.

CBG also found in their model that after generation the SST signals do not persist for an entire zonal circle without reinforcement. They demonstrated that consistent with the wavenumber 3 pattern in their model, a signal is reinforced 6 times over within a global circle. The dynamics involves a feedback between propagating SST signals and MSLP anomalies. This feedback process generates meridional wind stress anomalies, which in turn produce latent and sensible heat flux anomalies, in a manner that reinforces SST signals. A linear model of the essentials of this interaction [Baines and Cai, 2000] that includes all essential momentum and heat flux exchanges between atmosphere and ocean shows that it can lead to growing disturbances, but the interaction is not strong, and the growth rates are small. However, it is possible that this interaction may be strengthened by the inclusion of the nonlinear effects of atmospheric eddies and associated storm tracks [Palmer and Sun, 1985; Lau, 1997].

In practice, the ocean will experience the superimposed effect of all modes in the atmosphere and vice versa. In the case of the ocean, where warm and cold signals mainly represent northward or southward displacements of the mean isotherm pattern, the motions may well be linear. In the atmosphere, where the circulation patterns include the effects of eddies and storm tracks, they may well be nonlinear, but the fact that the atmospheric anomaly patterns of Figure 7 are of large scale, and appear to conform to dynamical patterns, suggests that these nonlinearities may cause quantitative rather than qualitative changes overall.

We expect the overall SST pattern to be advected eastward by the ACC. Previous studies have found that the regularity of the ACW may be determined by the speed of the ACC and the global zonal wavenumber pattern. The observed ACC speed is about 8-9 cm s⁻¹, as opposed to about 5-6 cm s⁻¹ in the coupled models of CBR and CBG. This difference in ACC speed is offset by the difference in the zonal wavenumber patterns between the observed wavenumber 2 and the modeled wavenumber 3. In CBG’s study it is this offsetting that
leads to the similarity between the observed and the modeled local ACW oscillation period.

6. Summary

Since the discovery of the ACW, there has been increasing interest in exploring its nature and the associated dynamics. The present study suggests that the observed ACW may be seen as the sum of two linearly independent complex EOFs: CEOFSST1 with a zonal wavenumber 2 pattern and CEOFSST2 with a wavenumber 3 pattern. We find that during the 1981-1997 period, CEOFSST1 is the dominant component of the ACW, and CEOFSST2 is relatively less important, leading to the dominant wavenumber 2 pattern of the observed ACW. We demonstrate that this wavenumber 2 component of the ACW is forced by the PSA pattern teleconnected to ENSO, a well-known dynamical entity.

The picture that emerges of the ACW is the following. During phases of ENSO, and particularly El Niño, convective events in the equatorial Pacific continually generate the PSA atmospheric pattern over a period of 6 months. The associated wind stresses and heat fluxes cause SST changes of the order of 1°C over large areas of the southern South Pacific. These regions are then advected eastward by the ACC. They cause associated perturbations in the atmospheric flow that help to maintain them. In other words, the principal component of the ACW is initiated by atmospheric teleconnections from ENSO in the South Pacific, propagated by the ACC, and maintained by the air-sea interactions.

In some coupled ocean-atmospheric models (e.g., CBG) ENSO is not adequately resolved, but a mode with a wavenumber 3 signal pattern in SST and ocean heat content is found in the Southern Ocean. When these wavenumber 3 signals are advected by the ACC, an ACW-like phenomenon is produced. Our results indicate that the lack of wavenumber 2 pattern may be due to the fact that ENSO is not adequately simulated. There is also the possibility that in some decades the wavenumber 3 pattern may become predominant when the intensity of ENSO decreases. Although there is recent further evidence of wavenumber 3 pattern in sea ice anomalies [Comiso, 2000], for the period considered the major features of SST signals of the ACW are mainly described by the signals associated with CEOFSST1.

Further studies are needed to improve our understanding of the process that maintains SST signals as they propagate eastward. It is now well established that air-sea interactions can provide a positive feedback between atmospheric flow anomalies and signals in SST and corresponding ocean heat content [Palmer and Sun, 1985]. This positive feedback occurs in linear models, but it is too weak to initiate the process [Baines and Cai, 2000]. However, it is possible that it could be strengthened by atmospheric nonlinearities involving eddies and storm tracks over the Southern Ocean.

Appendix A: Statistical Significance of CEOFs Based on Monthly and Filtered Time Series

The conventional theory of statistical significance and estimation is highly developed and extensive. However, its application to specific data sets depends on the following assumptions. (1) The data are jointly statistically stationary over the length of the record. (2) The variables being considered are all jointly normally distributed, being part of respective populations that have underlying means, variances, and covariances. (3) The data sets used are unbiased samples of these populations.

In an analysis of climate data, there may be problems with each of these assumptions. Long-period variations such as the Interdecadal Pacific Oscillation may affect data sets of length < 20 years or so, and the normality of variables on these timescales is uncertain. The representativeness of samples is perhaps the least questionable of these assumptions, but this is also sensitive to processes on timescales longer than the length of the record. Given these uncertainties, a statistical analysis of uncertainties of climate data sets is probably only useful as a guide, and though it provides answers, these should not be expected to constitute firm conclusions about significance. Nevertheless, we pursue it here.

The relationship between the time series of any two observed variables \( x(t) \) and \( y(t) \) (such as the SST at two particular grid points) is measured by the covariance between them and, as computed from the data, is denoted \( s_{xy} \). However, this computed covariance is just an estimate from the sample of data available of the underlying covariance \( \mu_{xy} \) of the assumed population of SSTs at these two grid points. It is this latter quantity that we are trying to determine. How good is the estimate? A measure of this is given by the variance \( s_{xy}^2 \) (or standard error \( s_{xy} \)) of \( v_{xy} \), defined by

\[
s_{xy}^2 = E(v_{xy} - \mu_{xy})^2 = E(v_{xy})^2 - \mu_{xy}^2, \quad (A1)
\]

where \( E \) denotes the expected value, and \( \mu_{xy} = E(v_{xy}) \). We do not know \( \mu_{xy} \) but must estimate it from \( v_{xy} \), so that

\[
s_{xy}^2 = E(v_{xy})^2 - v_{xy}^2. \quad (A2)
\]

If \( x \) and \( y \) are complex time series,

\[
v_{xy} = \text{cov}(x, y) = \frac{1}{N} \sum_{i=1}^{N} [x(t_i) - \bar{x}][y(t_i) - \bar{y}]^*, \quad (A3)
\]

\[
\bar{x} = \frac{1}{N} \sum_{i=1}^{N} x(t_i),
\]

\[
s_{xy}^2 = \frac{1}{N^2} E(\sum_{i=1}^{N} \sum_{j=1}^{N} [x(t_i) - \bar{x}][y(t_i) - \bar{y}]^* \times [x(t_j) - \bar{x}][y(t_j) - \bar{y}]^*) - E([v_{xy}])^2, \quad (A4)
\]
where an asterisk denotes a complex conjugate. Covariances such as \( v_{xy} \) may be complex, but variances and \( s_{xy}^2 \) are real. We assume that \( x \) and \( y \) are jointly normally distributed, with means \( \mu_x \) and \( \mu_y \), variances \( \sigma_x^2 \) and \( \sigma_y^2 \), and (complex) correlation coefficient \( r_{xy} \). Then (A4) reduces to [Papoulis, 1965; Davis, 1976]

\[
\begin{align*}
    s_{xy}^2 &= \frac{\sigma_x^2 \sigma_y^2}{N} \left[ 1 + |r_{xy}|^2 \right] + \sum_{j=1}^{N-1} \left[ \text{Re}(r_{xz}(j)r_{yy}(j)) + |r_{xy}(j)|^2 \right],
\end{align*}
\]

(A5)

where \( N \) is assumed to be odd for convenience, \( \text{Re} \) denotes real part, and \( r_{xz}(j) \) denotes the time-lagged correlation coefficient for \( x \) with lag \( j \Delta t \), where \( \Delta t \) is the time interval between samples. Quantities on the right-hand side of (A5) must be estimated from their sample values, taking \( \sigma_x^2 = v_{xx}, \sigma_y^2 = v_{yy}, r_{xy} = v_{xy}(v_{xx}v_{yy})^{1/2} \), etc. If \( \Delta t \) was significantly large, so that the values of \( x(t_i), y(t_i) \) were all independent from each other, the lag correlation terms would all vanish. We would have \( N \) independent sample pairs, or “degrees of freedom,” so that in this case,

\[
\begin{align*}
    s_{xy}^2 &= \frac{\sigma_x^2 \sigma_y^2}{N} \left[ 1 + |r_{xy}|^2 \right].
\end{align*}
\]

(A6)

Equation (A5) may then be used to define an effective number of degrees of freedom \( N^* \) in (A6) [Davis, 1976]

\[
\begin{align*}
    N^* &= \frac{N(1 + |r_{xy}|^2)}{\{1 + |r_{xy}|^2 + \sum_{j=1}^{N-1} \left[ \text{Re}(r_{xz}(j)r_{yy}(j)) + |r_{xy}(j)|^2 \right]\}},
\end{align*}
\]

(A7)

Taking \( v_{xy} +/ - s_{xy} \) gives an estimate of \( \mu_{xy} \), with error bars the width of the standard error.

To test the significance of CEOFs, we follow the procedure described by North et al. [1982] and consider an array of \( m \) grid points at each of which there is an observed series of values of a variable \( x \) at \( N \) times. These times are simultaneous and evenly spaced with interval \( \Delta t \). Hence we have an array of data \( x_j(t_k), j = 1, \ldots, m \) and \( k = 1, \ldots, N \). From these we may obtain CEOF by taking the Hilbert transform \( h_j(t_k) \) of the time series at each grid point and then defining the complex time series

\[
    X_j(t_k) = x_j(t_k) + ih_j(t_k).
\]

(A8)

We then define the \( m \times m \) matrix of complex covariances \( S_{ij} = \text{cov}(X_i, X_j) \) by (A3). The spatial structure of the CEOFs is then given by the eigenvalue equation

\[
\begin{align*}
    \sum_{j=1}^{m} S_{ij} f_{\alpha}(j) &= l_{\alpha} f_{\alpha}(i), \quad \alpha, i = 1, \ldots, m,
\end{align*}
\]

(A9)

where \( S_{ij} \) is hermitian, so that the eigenvalues \( l_{\alpha} \) are all real and positive and the eigenvectors \( f_{\alpha}(i) \) are complex. These form a linearly independent set, with \( \alpha = 1, \ldots, m \).

In accordance with the assumptions mentioned above the background population of which the \( x_i \) represent a sample are assumed to be statistically stationary and have underlying values of the covariances at the \( m \) grid locations that may be represented by \( v_{ij} \). From these exact (but unknown) covariances the exact eigenvalues \( \lambda_{\alpha} \) and eigenvectors \( f_{\alpha}(i) \) are given by

\[
\begin{align*}
    \sum_{j=1}^{m} v_{ij} f_{\alpha}(j) &= \lambda_{\alpha} f_{\alpha}(i), \quad \alpha, i = 1, \ldots, m.
\end{align*}
\]

(A10)

One may expect that the deviations of the sample covariances \( S_{ij} \) from the actual covariances \( v_{ij} \) will be of the order of the standard errors of the \( S_{ij} \), so that we may write

\[
\begin{align*}
    S_{ij} &= v_{ij} + \varepsilon V_{ij},
\end{align*}
\]

(A11)

where \( \varepsilon \) is a small parameter of order \( (2/N^*)^{1/2} \) and \( V_{ij} \) is of the order of

\[
\begin{align*}
    V_{ij} &= \left[ \text{v} \left( v_{ij} v_{ij}^* \right) / 2 \right]^{1/2},
\end{align*}
\]

(A12)

where \( N^* \) is given by (A7) with \( X_i \) and \( X_j \) in the place of \( x \) and \( y \). In practice, we take a mean value for \( N^* \) over all pairs \( X_i, X_j \). For values of the \( v_{ij} \) we use the sample values \( S_{ij} \). One may then use a standard perturbation expansion of the form

\[
\begin{align*}
    f_{\alpha}(i) &= \phi_{\alpha}(i) + \varepsilon f_{\alpha}(1) + \ldots, \quad (A13) \\
    l_{\alpha} &= \lambda_{\alpha} + \varepsilon l_{\alpha}^{(1)} + \ldots, \quad (A14)
\end{align*}
\]

to obtain the first-order deviations due to the perturbations in the \( v_{ij} \). These give [North et al., 1982]

\[
\begin{align*}
    l_{\alpha}^{(1)} &= \sum_{i=1}^{m} \sum_{j=1}^{m} \phi_{\alpha}(i) V_{ij} \phi_{\alpha}(j), \quad (A15a) \\
    \lambda_{\alpha}^{(1)} &= \sum_{i=1}^{m} \sum_{j=1}^{m} \phi_{\alpha}(i) v_{ij} \phi_{\alpha}(j), \quad (A15b) \\
    f_{\alpha}(i) &= \sum_{\beta=1}^{m} a_{\alpha \beta} \phi_{\beta}(i), \quad \beta \neq \alpha, \quad (A16a) \\
    a_{\alpha \beta} &= \sum_{i=1}^{m} \sum_{j=1}^{m} \phi_{\beta}(i) V_{ij} \phi_{\alpha}(j), \quad (A16b)
\end{align*}
\]

Since \( V_{ij} \) is of the same order as \( v_{ij} \) (and \( V_{ii} \approx v_{ii} \)), from (A15) we have \( l_{\alpha}^{(1)} = O(\lambda_{\alpha}) \), so that the magnitude of the uncertainty in the eigenvalue \( \delta \lambda_{\alpha} \) is approximately

\[
\begin{align*}
    \delta \lambda_{\alpha} = \varepsilon l_{\alpha}^{(1)} \approx \left( \frac{2}{N^*} \right)^{1/2} \lambda_{\alpha}.
\end{align*}
\]

(A17)

If one eigenvalue is much larger than the others, the form of (A15) indicates that (A17) may be an overestimate. In these cases, sample computed values for \( l_{\alpha}^{(1)} \) using (A15) generally give much smaller values than those from (A17). Clearly, this criterion should be taken as a guide, and these uncertainties are interpreted here as error bars.
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