Auditing Hamiltonian Elections

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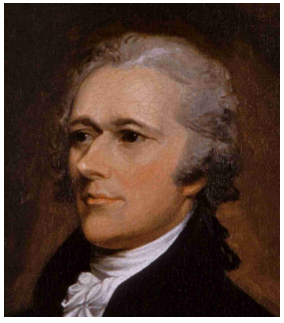
Voting'21

Overview

- Hamiltonian Elections
  - Case study: DNC Presidential Primaries (2020)
- Primer on Risk Limiting Audits
- RLAs for Hamiltonian Elections

Hamiltonian Elections



Use the **largest remainder method** (also known as the Hare-Niemeyer method) to allocate seats to parties. Attributed to Alexander Hamilton (1792).

Case study: 2020 DNC Presidential Primaries

- Using a Plurality or an IRV (ranked-choice) election, determine **viable** candidates;
- Viable candidates need to amass at least 15% of the vote;
- Delegates are awarded to viable candidates using the **largest remainder method**.

Example: Determining Viability

We have 4 candidates and 5 delegates to award.

Candidate	Votes	Proportion
Ann	57,532	76.1%
Bob	15,630	20.6%
Cal	1,600	2.1%
Dee	846	1.1%
Total Votes	**75,608**	**100.0%**

(a)

Candidate	Votes	Proportion
Ann	57,532	78.6%
Bob	15,630	21.4%
Qual Votes	**73,162**	**100.0%**

(b)

Figure: (a) Votes and (b) Qualified Votes in a Hamiltonian election with plurality-based exclusion (viability threshold of 15%).

In IRV-based exclusion, we eliminate candidates and redistribute their votes until all remaining candidates have at least 15% of the vote.

Example: Awarding Delegates

Let p_c denote the proportion of the qualified vote in c's tally.
$p_{ann} = 0.786$
$p_{bob} = 0.214$

We compute the 'delegate quota' for each candidate ($D = 5$):
$q_{ann} = D \times p_{ann} = 3.932$
$q_{bob} = D \times p_{bob} = 1.068$

There are two rounds of delegate allocation:

- We allocate $\lfloor q_c \rfloor$ delegates to candidate c (Ann $\leftarrow$ 3, Bob $\leftarrow$ 1).
- The remaining delegates are awarded to the candidates in decreasing order of their 'remainder'. Ann has the largest remainder (0.932) and gets last delegate (Ann $\leftarrow$ 4, Bob $\leftarrow$ 1).

Risk Limiting Audits

- Developed by Philip Stark from UC Berkeley, initially for first-past-the-post (FPTP) elections.
- The *risk limit* of the audit controls the degree of confidence attained, and influences auditing effort required.

In a nutshell

The probability that the audit fails to detect a wrong outcome is bounded by the risk limit. An RLA with a risk limit of 1%, for example, has at most a 1% chance of failing to detect that a reported election outcome is wrong.

- Randomly sample paper ballots, and maintain running statistics that indicate when the audit can stop.

Auditing with Assertions: Viability

We create a set of *assertions*. Each assertion can be statistically tested in an audit using SHANGRLA[1].

Auditing Viability:

- Viable(c, E, t): majority assertion $p_c \geq t$
- NonViable(c, E, t): majority assertion $\tilde{p}_c \geq 1 - t$
- IRV(c, c', E): pairwise majority assertion $p_c \geq p_{c'}$

To audit the viability of Ann and Bob we have four assertions:

Viable(Ann, $\emptyset$, 0.15); Viable(Bob, $\emptyset$, 0.15); NonViable(Cal, $\emptyset$, 0.15); and NonViable(Dee, $\emptyset$, 0.15).

Estimated effort (# ballots): 1; 17; 46; and 42.

[1] https://github.com/pbstark/SHANGRLA

Auditing with Assertions: Delegates

We create a pairwise-difference assertion for each pair of viable candidates m and n:

$$p_m \geq p_n + \frac{a_m - a_n - 1}{D}, \qquad \forall n, m \in V, n \neq m$$

V is the set of viable candidates, and a_c is the number of delegates awarded to c.

Each assertion says: the 'delegate quota' for n is not 1 more than the quota for m after removing all received delegates but the last.

In our example, we create two assertions to check: $p_{ann} \leq p_{bob} + 4/5$ and $p_{bob} \leq p_{ann} - 2/5$. These require an estimated 5, and 59, ballot samples.

Case Study: 2020 DNC Primaries

Table: Hard (top) and relatively easy (bottom) primaries for which to audit the assigned at-large delegates. The number of at-large delegates D; the delegate quotas for Biden and Sanders; and the difference between the remainder of their quotas (divided by D) is reported.

State	D	Quotas		Rem. Diff. / D	ASN
		Biden	Sanders		
CA	90	50.688	39.312	0.004	3.2×10^6
MO	15	9.524	5.476	0.003	–
NY	61	47.629	13.371	0.004	486,495
SC	12	8.533	3.467	0.006	34,546
ME	5	2.050	1.993	0.19	189
AZ	14	8.010	5.990	0.07	120
OR	13	9.948	3.052	0.07	191

In the paper ...

- More detail on the assertions, how they are generated, and how they are tested by SHANGRLA.
- How auditing effort is estimated given a set of assertions.
- Proof showing that if the delegate assignment to candidates is incorrect, at least one of the pairwise-difference assertions we create will be violated.
- Estimated auditing effort required to audit the 2020 DNC primaries that use either plurality or IRV-based exclusion.

Questions?

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