A Decomposition-Based Heuristic for Collaborative Scheduling in a Network of Open-Pit Mines: Computational Results

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This document presents material supplementary to our companion paper *A Decomposition-Based Heuristic for Collaborative Scheduling in a Network of Open-Pit Mines.* In this companion paper, we consider the short-term production scheduling problem for a network of multiple open-pit mines and ports. Ore produced at each mine is transported by rail to a set of ports and blended into signature products for shipping. Consistency in the grade and quality of production over time is critical for customer satisfaction, while the maximal production of blended products is required to maximise profit. In practice, short-term schedules are formed independently at each mine, tasked with achieving the grade and quality targets outlined in a medium-term plan. However, due to the dynamics of the mining environment, such targets may not be feasible in the short-term. We have presented a decomposition-based heuristic for this short-term scheduling problem in which the grade and quality goals assigned to each mine are collaboratively adapted – ensuring the satisfaction of blending constraints at each port, and exploiting opportunities to maximise production in the network that would otherwise be missed. This document presents an evaluation of our decomposition-based heuristic in a real-world open-pit network, the data for which has been provided by an industry partner.

Notation used in this document is defined in Appendices A and B of our companion paper. All further references to material in our companion paper are prefixed with CP.

We have constructed a test suite with which to evaluate our decomposition-based heuristic, and contrast its performance with alternative solution methods. These tests define an 8-mine, 2-port network, characterised using data provided by an industry partner. This network represents a currently operating system of open-pit mines that produce over 200 million tons of ore annually. In each test case, we provide each mine with: a set of grade blocks available for extraction, listing their grade, quality profile, and tonnage; the mining precedences that exist between blocks; compositions and sizes for each high and low grade stockpile; and a limit on the tons of material extracted in each pit, and hauled mine-wide.

Test cases have been generated using historical block extraction data obtained for each mine. This data lists the set of grade blocks that have been defined by geologists at each mine, over the period of a year, and the dates by which they have been extracted. Each test case has been generated by selecting a date in the year long period covered by the historical block extraction data, and determining the state of each mine (the grade blocks available for extraction) at this time point. The number of grade blocks available for scheduling at each mine, across the test suite, ranges from 34 to 297. Haulage capacities at each mine, minimum production requirements, port throughput capacities, and blend requirements at each port are fixed across all test cases. In each test, each port produces one product of each granularity ($|N_l^\pi| = 1$ for all ports $\pi \in \Pi$ and granularities $l \in \mathcal{L}$).

For the MINLP model of the single week multiple mine planning problem (MMPP) in Section CP-5, Section 1 identifies the gap in quality between model solutions, found by our heuristic, and known lower bounds on the MINLP objective, across our test suite. In

Section 1.1 we solve a series of linear relaxations of our MINLP model. We first replace each bilinear term with its convex envelope (McCormick 1976) to obtain a lower bound on the objective in each test. We consequently generate and solve several piecewise-linear relaxations (Gounaris et al. 2009), of increasing fidelity, of the model. Due to discrepancies between the evaluation of port product composition in these relaxed models, and their actual composition, port products were not correctly blended in the obtained solutions. We use the magnitude of these discrepancies to narrow the bounds describing desired product composition, and resolve the piecewise-linear relaxed models. The composition of port products in the resulting solutions lie within the original bounds.

We additionally consider the ALT heuristic (Audet et al. 2004), for general bilinear programs, as an alternative to our approach (Section 1.3). Lower bounds obtained on the MINLP objective, and the quality of solutions found via the use of piecewise-linear relaxation and ALT, are used to evaluate our decomposition-based heuristic.

All evaluations have been conducted on a 2.40 GHz Intel Xeon CPU with 8 GB RAM.

1. Computational Results

We have used our decomposition-based heuristic to solve each test case generated for our 8-mine, 2-port network. IBM CPLEX 12.5 was used to solve all MIPs.

Table 1 records the results of the decomposition-based heuristic, averaged over 10 seeded runs on each of our benchmark tests, with: $N = 10, 15$, and 20; $\gamma = 0.75$; and priority weightings $W_{l=0} = 0.6$ and $W_{l=1} = 0.4$ assigned to lump and fines production at each mine. Table 2 records the results of our heuristic with $N = 10$, and varying γ. We record, for the best solution found by the heuristic, $\vec{s}_{best}$: the elapsed time to termination (s); revenue achieved via the sale of products formed at each port ($); the total utilisation of trucking resources, and the dry and wet processing plants (stated as a percentage of total haulage capacity across the set of mines); the total percentage (%) of (network-wide) haulage capacity spent on undesirable stockpiling across all mines; the maximum deviation (%) from desired bounds present in port products formed across the 10 seeded runs (deviation in metal grade is listed separately from that in other attributes); and the gap (%) between $Z'_{MMPP}(\vec{s}_{best})$[1] and the best lower bound discovered in Section 1.1. Quantities have been averaged over 10 seeded runs, with the average (μ) and standard deviation (σ) recorded.

Increasing N, the number of schedules formed during the solve of each $\mathcal{O}_m$ in each iteration of the heuristic, and γ, altering the degree to which the standard deviations given to each $\mathcal{O}_m$ as input are increased or decreased (a larger γ results in smaller changes), improves, in general, the quality of solutions found by the heuristic. The heuristic is successful, across all tested combinations of the N and γ parameters, at discovering near optimal solutions to the MMPP – with gaps of less than 2% achieved (in all but one test case) between $Z'_{MMPP}(\vec{s}_{best})$ and its best known lower bound. For $N = 10{,}15{,}20$ and $\gamma = 0.50{,}0.75$, gaps of less than 1% are reported in a majority of test cases. Decreasing γ results in the heuristic performing less iterations, reducing the time it takes to solve, but limiting its opportunities to improve the quality of its current best found solution.

We have evaluated the extent to which our choice of port-to-mine feedback (see Table CP-1) improves the performance of our heuristic by considering two alternative schemes. The first, denoted $R2$, replicates our existing rules but does not increase the standard deviations provided to each mine at any stage. The second, denoted $R3$, replicates $R2$, but

[1] The objective in the MINLP MMPP model, denoted Z'_{MMPP}, is defined in Equation CP-13 of the companion paper.

Table 1 Best solution $\vec{s}_{best}$ found by our heuristic for $N = 10, 15, 20$, and $\gamma = 0.75$, in each test #, recording: elapsed time to completion of solve (s); revenue achieved (\$); the total utilisation of trucks, and processing plants (% of network-wide capacity); the total percentage (%) of (network-wide) haulage capacity spent on undesirable stockpiling; the max deviation (%) from desired bounds (on metal grade, and other attributes) present in port products across 10 seeded runs; and the gap (%) between $Z'_{MMPP}(\vec{s}_{best})$ and the best known lower bound. Quantities have been averaged over 10 seeded runs, with the average (μ) and standard deviation (σ) recorded.

$N = 10$, $\gamma = 0.75$					Utilisation (over all mines) (%)								Deviation (%)		Gap to (%)	
#	Time (s)		Revenue (\$)		Trucking		Dry		Wet		Stockpiling		Metal	Other	MINLP$_{lb}$	
	μ_T	σ_T	μ_R	σ_R	μ_K	σ_K	μ_D	σ_D	μ_W	σ_W	μ_S	σ_S			μ_G	σ_G
1	201	18.56	318297600	226800	99.95	0.01	100	0.95	100	0	2.70	0.05	0	0	0.98	0.01
2	360	44.17	321132600	226800	97.93	0.03	99.23	0.92	100	0	3.22	0.08	0	0	0.09	0.01
3	271	30.93	317957400	226800	98.32	0.06	100	0.92	100	0	4.08	0.10	0	0	1.08	0.01
4	333	46.03	316426500	380355	98.77	0.11	95.90	0.85	100	0	3.55	0.06	0	0	1.56	0.01
5	358	35.70	317277000	0	97.59	0.09	97.95	0.86	100	0	4.95	0.08	0	0	1.29	0
6	310	66.78	319091400	941971	99.50	0.02	99.60	0.89	100	0	5.47	0.09	0	0	0.73	0.03
7	350	63.84	316710000	253570	99.22	0.09	97.44	0.85	100	0	5.50	0.12	0	0	1.47	0.01
8	363	19.68	321246000	0	99.73	0.02	99.24	0.87	100	0	6.05	0.11	0	0	0.06	0
9	159	21.16	316993500	380355	99.84	0.02	100	0.92	100	0	4.05	0.07	0	0	1.38	0.01
10	319	61.08	321132600	226799	99.13	0.02	99.01	0.91	100	0	3.78	0.05	0	0	0.09	0.01
11	363	57.86	318354300	534906	99.84	0.01	100	0.94	100	0	2.89	0.09	0	0	0.96	0.02
12	222	44.49	317617200	1322459	99.73	0.02	97.66	0.88	100	0	3.76	0.06	0	0	1.19	0.04
13	250	38.55	319148100	259832	99.23	0.03	99.99	0.95	100	0	2.76	0.07	0	0	0.71	0.01
14	177	32.18	318581100	442841	99.53	0.03	100	0.98	100	0	1.21	0.05	0	0	0.89	0.01
15	428	73.59	317163600	833316	99.82	0.04	99.73	0.92	100	0	3.90	0.05	0	0	1.33	0.03
16	230	20.84	320962500	457130	99.06	0.06	99.32	0.87	100	0	5.98	0.07	0	0	0.15	0.01
17	220	27.57	321246000	0	99.83	0.02	99.44	0.93	100	0	2.96	0.11	0	0	0.06	0
18	195	18.71	321246000	0	99.71	0.03	97.16	0.89	94.33	1.70	2.56	0.04	0	0	0.06	0
19	227	19.35	321246000	0	99.35	0.05	99.77	0.92	100	0	3.62	0.07	0	0	0.06	0
20	456	63.52	313351200	1315828	97.47	0.06	93.15	0.77	99.96	0.01	5.67	0.04	0	0	2.52	0.04
$N = 15$,$\gamma = 0.75$																
1	282	39.88	318637800	277772	99.43	0.08	100	0.94	100	0	2.87	0.05	0	0	0.87	0.01
2	435	36.12	321246000	0	97.61	0.10	98.97	0.91	100	0	3.41	0.06	0	0	0.06	0
3	368	11.14	318297600	226800	98.22	0.03	100	0.91	100	0	4.52	0.07	0	0	0.98	0.01
4	393	49.36	316256400	941971	98.78	0.08	96.33	0.86	100	0	3.66	0.06	0	0	1.61	0.03
5	535	38.37	317220300	170100	97.91	0.04	98.67	0.87	100	0	5.14	0.06	0	0	1.31	0.01
6	383	65.74	320395500	1022173	99.22	0.04	97.68	0.85	100	0	5.60	0.06	0	0	0.32	0.03
7	462	77.65	317277000	760710	98.60	0.12	96.79	0.85	100	0	4.80	0.09	0	0	1.29	0.02
8	413	42.62	321246000	0	99.72	0.04	99.57	0.87	100	0	6.20	0.11	0	0	0.06	0
9	247	38.01	317390400	424303	99.67	0.04	100	0.92	100	0	3.92	0.05	0	0	1.26	0.01
10	348	43.16	321246000	0	99.01	0.04	98.12	0.89	100	0	3.98	0.10	0	0	0.06	0
11	488	52.84	318581100	363057	99.80	0.02	100	0.95	100	0	2.47	0.06	0	0	0.89	0.01
12	361	60.82	318581100	1015864	99.79	0.00	96.56	0.86	100	0	3.58	0.05	0	0	0.89	0.03
13	324	47.51	319091400	226799	99.25	0.05	100	0.95	100	0	2.75	0.08	0	0	0.73	0.01
14	280	37.78	319091400	424303	99.70	0.05	100	0.97	100	0	1.31	0.05	0	0	0.73	0.01
15	494	90.89	317787300	692111	99.86	0.02	99.72	0.93	100	0	3.32	0.09	0	0	1.14	0.02
16	308	28.44	321132600	340200	98.82	0.05	98.95	0.87	100	0	5.85	0.07	0	0	0.09	0.01
17	260	18.58	321246000	0	99.91	0.01	99.60	0.93	100	0	3.32	0.12	0	0	0.06	0
18	232	11.76	321246000	0	99.71	0.02	97.30	0.90	100	0	2.41	0.06	0	0	0.06	0
19	285	20.98	321246000	0	99.57	0.03	99.41	0.92	100	0	3.71	0.06	0	0	0.06	0
20	522	69.28	313804800	1437059	98.01	0.07	92.71	0.76	100	0	5.77	0.07	0	0	2.37	0.04
$N = 20$,$\gamma = 0.75$																
1	363	66.95	318581100	259832	99.81	0.02	100	0.95	100	0	2.74	0.05	0	0	0.89	0.01
2	523	55.34	321246000	0	97.64	0.05	98.65	0.90	100	0	3.70	0.08	0	0	0.06	0
3	444	33.52	318297600	226800	98.07	0.06	100	0.90	100	0	4.95	0.11	0	0	0.98	0.01
4	458	73.17	316426500	923009	99.14	0.04	96.10	0.85	100	0	3.72	0.05	0	0	1.56	0.03
5	701	108.19	317277000	0	98.08	0.03	99.69	0.90	100	0	4.69	0.04	0	0	1.29	0
6	455	38.72	321189300	170099	99.40	0.03	96.32	0.83	100	0	5.30	0.08	0	0	0.08	0.01
7	581	143.39	317560500	957206	99.23	0.07	96.60	0.84	100	0	5.06	0.07	0	0	1.21	0.03
8	497	49.23	321246000	0	99.88	0.02	99.47	0.86	100	0	6.58	0.09	0	0	0.06	0
9	322	28.94	317560500	380355	99.69	0.04	100	0.93	100	0	3.75	0.06	0	0	1.21	0.01
10	396	63.20	321246000	0	99.07	0.04	97.58	0.88	100	0	3.71	0.05	0	0	0.06	0
11	584	94.76	318751200	453600	99.75	0.05	100	0.95	100	0	2.70	0.05	0	0	0.84	0.01
12	490	63.72	319374900	623700	99.75	0.00	95.19	0.83	100	0	4.02	0.05	0	0	0.64	0.02
13	401	50.52	319318200	277772	99.35	0.05	100	0.95	100	0	2.51	0.06	0	0	0.66	0.01
14	343	35.62	319148100	363057	99.60	0.04	100	0.97	100	0	1.58	0.08	0	0	0.71	0.01
15	596	68.63	317957400	424303	99.83	0.02	99.70	0.93	100	0	3.24	0.04	0	0	1.08	0.01
16	344	30.74	321246000	0	98.86	0.05	98.68	0.87	100	0	5.45	0.05	0	0	0.06	0
17	299	10.67	321246000	0	99.79	0.01	99.67	0.92	100	0	3.65	0.12	0	0	0.06	0
18	269	13.24	321246000	0	99.54	0.03	97.47	0.91	100	0	2.32	0.08	0	0	0.06	0
19	326	11.09	321246000	0	99.49	0.05	99.48	0.92	100	0	3.73	0.06	0	0	0.06	0
20	631	70.57	314158500	633925	98.13	0.07	92.52	0.75	100	0	5.65	0.06	0	0	2.26	0.02

reduces these standard deviations only after two consecutive iterations have failed to yield an improved $\vec{s}_{best}$. For $N = 10$ and $\gamma = 0.75$, we have found that, relative to our existing rules, $R2$ results in similar heuristic solve times, but lower quality solutions, on a majority of tests. $R3$ results in solutions that are slightly higher in quality than those of Table 1,

Table 2 Best solution $\vec{s}_{best}$ found by heuristic for $\gamma = 0.25, 0.50$, and $N = 10$. Columns are defined as in Table 1. Quantities have been averaged over 10 seeded runs, with the average (μ) and standard deviation (σ) recorded.

$N=10,\ \gamma=0.25$					Utilisation (over all mines) (%)								Deviation (%)		Gap to (%)	
#	Time (s)		Revenue ($)		Trucking		Dry		Wet		Stockpiling		Metal	Other	MINLP$_{lb}$	
	μ_T	σ_T	μ_R	σ_R	μ_K	σ_K	μ_D	σ_D	μ_W	σ_W	μ_S	σ_S			μ_G	σ_G
1	98	17.12	318127500	283500	99.80	0.02	100	0.95	100	0	2.56	0.05	0	0	1.03	0.01
2	202	52.71	320225400	1262768	98.14	0.05	99.15	0.91	100	0	3.79	0.05	0	0	0.38	0.04
3	135	25.37	317673900	363057	98.26	0.05	99.79	0.92	100	0	4.08	0.08	0	0	1.17	0.01
4	162	38.17	315235800	1416365	98.93	0.06	95.43	0.84	99.64	0.11	3.70	0.05	0	0	1.93	0.04
5	235	41.38	316880100	510300	97.85	0.04	97.51	0.85	100	0	5.36	0.05	0	0	1.42	0.02
6	167	44.46	319318200	955525	98.77	0.12	98.95	0.88	100	0	5.26	0.07	0	0	0.66	0.03
7	208	34.04	316653300	305338	99.33	0.09	96.80	0.83	100	0	5.72	0.13	0	0	1.49	0.01
8	188	19.79	321019200	277772	99.66	0.03	99.11	0.87	100	0	5.97	0.11	0	0	0.13	0.01
9	79	21.98	316766700	396900	99.84	0.02	100	0.92	100	0	4.18	0.05	0	0	1.45	0.01
10	192	63.47	320679000	439196	99.01	0.10	99.60	0.92	100	0	3.73	0.09	0	0	0.24	0.01
11	160	39.51	317844000	439196	99.73	0.02	100	0.94	100	0	3.04	0.06	0	0	1.12	0.01
12	111	35.63	317220300	996695	99.78	0.01	97.30	0.88	100	0	3.65	0.05	0	0	1.31	0.03
13	122	31.35	318807900	259832	99.01	0.11	99.96	0.94	100	0	2.93	0.08	0	0	0.82	0.01
14	82	20.78	318411000	0	99.48	0.04	100	0.98	100	0	1.24	0.04	0	0	0.94	0
15	252	54.81	316539900	983708	99.94	0.01	99.33	0.91	100	0	4.03	0.06	0	0	1.52	0.03
16	123	12.06	320679000	760710	99.02	0.04	99.46	0.88	100	0	5.92	0.07	0	0	0.24	0.02
17	92	14.90	321246000	0	99.83	0.02	99.44	0.93	100	0	3.00	0.10	0	0	0.06	0
18	92	13.52	321246000	0	99.62	0.03	97.16	0.90	94.33	1.70	2.55	0.03	0	0	0.06	0
19	94	13.68	321246000	0	99.35	0.05	99.77	0.92	100	0	3.62	0.07	0	0	0.06	0
20	267	84.52	290417400	45159371	97.78	0.08	92.34	0.75	99.94	0.01	6.02	0.07	0.02	0	>100	–
$N=10,\ \gamma=0.50$																
1	133	34.53	318240900	259832	99.56	0.04	100	0.95	100	0	2.62	0.06	0	0	0.99	0.01
2	256	29.16	320962500	283500	97.32	0.15	98.93	0.92	100	0	3.17	0.07	0	0	0.15	0.01
3	173	29.34	317900700	305338	98.04	0.06	100	0.91	100	0	4.45	0.10	0	0	1.10	0.01
4	230	55.80	315576000	1216079	98.83	0.11	96.47	0.86	100	0	3.78	0.07	0	0	1.82	0.04
5	231	41.04	316880100	259832	97.56	0.08	97.41	0.85	100	0	5.14	0.06	0	0	1.42	0.01
6	210	69.28	318978000	760710	99.35	0.03	99.85	0.89	100	0	5.31	0.09	0	0	0.76	0.02
7	251	41.48	316710000	253570	98.98	0.09	97.00	0.83	100	0	5.86	0.12	0	0	1.47	0.01
8	224	16.10	321189300	170099	99.68	0.03	98.95	0.86	100	0	6.24	0.10	0	0	0.08	0.01
9	98	16.95	316823400	340200	99.59	0.05	100	0.92	100	0	4.02	0.07	0	0	1.43	0.01
10	230	49.37	320849100	259832	99.22	0.02	99.61	0.91	100	0	4.25	0.06	0	0	0.18	0.01
11	197	27.79	317787300	396900	99.81	0.02	99.99	0.95	100	0	2.66	0.06	0	0	1.14	0.01
12	157	38.21	317560500	957206	99.78	0.00	97.34	0.88	100	0	3.79	0.05	0	0	1.21	0.03
13	151	25.77	318807900	259832	98.79	0.12	100	0.95	100	0	2.69	0.09	0	0	0.82	0.01
14	105	14.99	318467700	170100	99.53	0.03	100	0.98	100	0	1.15	0.04	0	0	0.92	0.01
15	283	37.08	316596600	941971	99.95	0.01	99.52	0.93	100	0	3.30	0.06	0	0	1.51	0.03
16	151	10.48	320735700	737100	98.91	0.05	99.82	0.88	100	0	5.96	0.07	0	0	0.22	0.02
17	124	17.42	321246000	0	99.83	0.02	99.44	0.93	100	0	2.96	0.11	0	0	0.06	0
18	117	14.42	321246000	0	99.71	0.03	97.16	0.89	94.33	1.70	2.60	0.05	0	0	0.06	0
19	124	16.55	321246000	0	99.35	0.05	99.77	0.92	100	0	3.62	0.07	0	0	0.06	0
20	248	76.29	276115500	58427971	97.51	0.09	91.02	0.73	100	0	5.86	0.11	0.04	0	>100	–

on a majority of tests, but increases heuristic solve time by almost 200s on average. For brevity, the full results of this evaluation have been omitted from this paper.

1.1. Generation of lower bounds

We find lower bounds on the value of Z'_{MMPP}, in each test, via the use of linear (McCormick 1976) and piecewise-linear (Gounaris et al. 2009) relaxations of our non-linear model.

We first relax each bilinear term, $v^m_{l,q}\tau^m_l$, in the MINLP of Section CP-5 with its convex envelope (McCormick 1976). Default optimality tolerances could not be reached, in any test case, when the resulting MIP was solved. In each test, a gap of 0.06% was achieved, with respect to a lower bound obtained via an LP relaxation of the MIP (after 12 hours of solving). The average deviation between desired bounds on the percentage of metal in each lump and fines port product, and its actual composition, across the solutions of the relaxed model, was 0.56% and 0.16% (with standard deviations of 0.27 and 0.20). The maximum deviations in metal percentage, across all tests, were 1.14% and 0.81% in the lump and fines products formed across the port system. To generate relaxations of greater fidelity, we linearise each bilinear term, $v^m_{l,q}\tau^m_l$ for $m \in \mathcal{M}$, $l \in \mathcal{L}$, and $q \in \mathcal{Q}$, by partitioning the domain of the τ^m_l variable into $N_\tau = 2, 5, 10$, and 20, intervals. We reformulate each τ^m_l

as shown in Equations (1)–(4).

$$\tau_l^m = D_l^m + \sum_{j=0}^{N_\tau - 1} j\,\Delta\tau_l^m\,\hat{\tau}_{l,j}^m + \Delta\tau_l^m\,\tilde{\tau}_l^m, \quad \Delta\tau_l^m = \frac{U_l^m - D_l^m}{N_\tau} \qquad \forall\, m \in \mathcal{M}, l \in \mathcal{L} \quad (1)$$

$$0 \le \tilde{\tau}_l^m \le 1 \qquad \forall\, m \in \mathcal{M}, l \in \mathcal{L} \quad (2)$$

$$\hat{\tau}_{l,j}^m \in \{0,1\} \qquad \forall\, j = 0 \ldots N_\tau - 1,\ m \in \mathcal{M}, l \in \mathcal{L} \quad (3)$$

$$\sum_{j=0}^{N_\tau - 1} \hat{\tau}_{l,j}^m = 1 \qquad \forall\, m \in \mathcal{M}, l \in \mathcal{L} \quad (4)$$

The binary variable $\hat{\tau}_{l,j}^m$ forms part of an SOS1 constraint (Equation (4)), and is active ($\hat{\tau}_{l,j}^m = 1$) only when variable τ_l^m lies between the value $D_l^m + j\,\Delta\tau_l^m$ and $D_l^m + (j+1)\,\Delta\tau_l^m$, where U_l^m denotes the maximum tons of granularity $l \in \mathcal{L}$ producible by m. The variable $\tilde{\tau}_l^m$ forms part of a slack term, allowing the value of each τ_l^m to lie between the discrete points in its domain characterised by $D_l^m + j\,\Delta\tau_l^m$ for $j = 0 \ldots N_\tau - 1$.

We substitute the expression in Equation (1) for τ_l^m in each of the bilinear terms in our MINLP. The terms $\hat{\tau}_{l,j}^m\, v_{l,q}^m$ and $\tilde{\tau}_l^m v_{l,q}^m$ appearing in Equation (5) are replaced with variables $w_{l,j,q}^m = \hat{\tau}_{l,j}^m\, v_{l,q}^m$ and $\tilde{v}_{l,q}^m = \tilde{\tau}_l^m v_{l,q}^m$, yielding Equation (6). Each $w_{l,j,q}^m$ is constrained as shown in Equations (7)–(10). Variable $\tilde{v}_{l,q}^m$ is constrained as shown in Equations (11)–(14), where $L_{l,q}^m$ and $U_{l,q}^m$ denote lower and upper bounds on the domain of variable $v_{l,q}^m$.

$$v_{l,q}^m \tau_l^m = D_l^m v_{l,q}^m + \sum_{j=0}^{N_\tau - 1} j\,\Delta\tau_l^m\,\hat{\tau}_{l,j}^m\, v_{l,q}^m + \Delta\tau_l^m\,\tilde{\tau}_l^m v_{l,q}^m \qquad \forall\, m \in \mathcal{M}, l \in \mathcal{L}, q \in \mathcal{Q} \quad (5)$$

$$v_{l,q}^m \tau_l^m = D_l^m v_{l,q}^m + \sum_{j=0}^{N_\tau - 1} j\,\Delta\tau_l^m\, w_{l,j,q}^m + \Delta\tau_l^m\,\tilde{v}_{l,q}^m \qquad \forall\, m \in \mathcal{M}, l \in \mathcal{L}, q \in \mathcal{Q} \quad (6)$$

$$w_{l,j,q}^m \le U_{l,q}^m \hat{\tau}_l^m \qquad \forall\, m \in \mathcal{M}, l \in \mathcal{L}, q \in \mathcal{Q} \quad (7)$$

$$w_{l,j,q}^m \ge L_{l,q}^m \hat{\tau}_l^m \qquad \forall\, m \in \mathcal{M}, l \in \mathcal{L}, q \in \mathcal{Q} \quad (8)$$

$$w_{l,j,q}^m \le v_{l,q}^m + L_{l,q}^m (1 - \hat{\tau}_l^m) \qquad \forall\, m \in \mathcal{M}, l \in \mathcal{L}, q \in \mathcal{Q} \quad (9)$$

$$w_{l,j,q}^m \ge v_{l,q}^m - U_{l,q}^m (1 - \hat{\tau}_l^m) \qquad \forall\, m \in \mathcal{M}, l \in \mathcal{L}, q \in \mathcal{Q} \quad (10)$$

$$\tilde{v}_{l,q}^m \le U_{l,q}^m \tilde{\tau}_l^m \qquad \forall\, m \in \mathcal{M}, l \in \mathcal{L}, q \in \mathcal{Q} \quad (11)$$

$$\tilde{v}_{l,q}^m \ge L_{l,q}^m \tilde{\tau}_l^m \qquad \forall\, m \in \mathcal{M}, l \in \mathcal{L}, q \in \mathcal{Q} \quad (12)$$

$$\tilde{v}_{l,q}^m \ge U_{l,q}^m \tilde{\tau}_l^m + v_{l,q}^m - U_{l,q}^m \qquad \forall\, m \in \mathcal{M}, l \in \mathcal{L}, q \in \mathcal{Q} \quad (13)$$

$$\tilde{v}_{l,q}^m \le L_{l,q}^m \tilde{\tau}_l^m + v_{l,q}^m \qquad \forall\, m \in \mathcal{M}, l \in \mathcal{L}, q \in \mathcal{Q} \quad (14)$$

The maximum deviation in metal percentage, from desired bounds, across all port products, was found to be 1.02%, 0.69%, 0.36%, and 0.16%, respectively, in solutions to the models generated with $N_\tau = 2, 5, 10$, and 20. All MIP models generated to approximate the MINLP could not be solved to default optimality tolerances in any of the 20 tests, in a 12 hour period. Lower bounds obtained from the LP relaxation of each of these MIPs (after 12 hours of solving) have been used to assess the quality of solutions found by our heuristic in Tables 1–2.

Table 3 Comparison of piecewise-linear relaxation (PLR) and our heuristic. For the best solution $best$ found by PLR, we record for each test #: elapsed time (s) to completion of solve ('–' denotes that default optimality tolerances were not reached in 12hrs); elapsed time (s) to discovery of $best$; revenue from correctly blended port products (\$); the N_τ value used to generate each solution; utilisation of trucks, and dry/wet processing plants (% of network-wide capacity); percentage of network-wide haulage capacity spent on undesirable stockpiling; and the gap (%) between $Z'_{MMPP}(best)$ and the best known lower bound. Columns 11-16 compare PLR and our heuristic. Given $N=10$, $\gamma=0.25$, and $N=20$, $\gamma=0.75$, we record for $\vec{s}_{best}$ in each test #: the gap between $Z'_{MMPP}(\vec{s}_{best})$ and the best known lower bound (Gap, %); heuristic (elapsed) solve time (Time, s); and the elapsed time (s) taken by PLR (PLR, s) to find an equally good solution ('–' indicates that no such solution was found). Differences in mine productivity across solutions are not evident in gaps rounded to two decimal places. In #1 and 7, the heuristic finds a better solution than PLR, despite both achieving gaps of 1.12 and 1.47, respectively.

									Gap to	$N=10, \gamma=0.25$			$N=20, \gamma=0.75$		
	Solve	Best	Revenue	N_τ	Utilisation (%)				$MINLP_{lb}$	Gap	Time	PLR	Gap	Time	PLR
#	(s)	(s)	(\$)		Trucking	Dry	Wet	Stockpiling	(%)	(%)	(s)	(s)	(%)	(s)	(s)
1	–	42793	317844000	10	98.74	99.28	100	1.49	1.12	1.12	89	–	0.94	340	–
2	–	41042	319545000	20	98.32	100	100	2.24	0.59	1.29	139	38977	0.06	554	–
3	–	30584	317844000	20	98.90	100	100	2.22	1.12	1.47	110	1390	1.12	357	30584
4	–	42696	316143000	20	98.73	96.98	100	3.09	1.65	2.70	166	39621	2.35	352	39621
5	–	40379	316710000	20	98.40	99.24	100	3.44	1.47	1.82	235	39053	1.29	565	–
6	–	42793	319545000	20	99.40	100	100	3.46	0.59	0.94	119	39177	0.24	501	–
7	–	41502	316710000	20	99.52	97.02	100	3.87	1.47	1.65	234	40245	1.47	390	–
8	–	39323	321246000	20	99.45	100	100	2.64	0.06	0.24	198	38879	0.06	448	39323
9	–	41753	316710000	10	100	100	100	2.31	1.47	1.65	50	41528	1.47	265	41753
10	–	41798	321246000	20	99.43	100	100	3.61	0.06	0.59	110	39432	0.06	312	41371
11	–	42680	318411000	20	99.65	100	100	2.64	0.94	1.47	130	39590	1.12	407	41583
12	–	41529	317844000	20	99.34	97.58	100	2.49	1.12	1.65	130	38832	0.94	336	–
13	–	40835	318411000	20	99.28	100	100	2.60	0.94	0.94	90	40835	0.76	383	–
14	–	40679	318978000	20	99.89	100	100	0.77	0.76	0.94	88	1123	0.94	250	1123
15	–	42052	317844000	20	99.90	99.75	100	1.85	1.12	1.82	236	39176	1.29	678	41004
16	–	43075	321246000	20	98.35	99.96	100	5.19	0.06	0.76	100	39046	0.06	343	41359
17	–	3346	321246000	20	100	100	100	0.35	0.06	0.06	88	3346	0.06	295	3346
18	–	2405	319545000	10	100	98	100	1.59	0.59	0.06	101	–	0.06	256	–
19	–	1027	321246000	20	100	100	100	1.03	0.06	0.06	79	679	0.06	328	679
20	–	43089	301428000	20	98.12	87.15	100	6.26	6.22	>100	148	39074	2.71	692	–

1.2. Piecewise-linear relaxations (PLR)

To determine whether piecewise-linear relaxation is capable of finding high quality solutions to the MMPP, in which port products are correctly blended, we re-solve the $N_\tau = 10$, and 20 relaxed models (generated in Section 1.1) with narrowed bounds on each attribute $q \in \mathcal{Q}$. Each set of bounds is narrowed to offset the maximum deviations incurred on the relevant attribute in the solutions to each model. Each model was able to produce solutions in which no deviation existed between port product composition and the original bounds.

Table 3 records for the best solution (*best*) found, in each test: the elapsed time (s) to the completion of solve ('–' denotes that default optimality tolerances were not reached in a 12 hour period); the elapsed time (s) to the discovery of *best*; the total revenue achieved (\$) via the sale of ore products formed across the port system; the value of N_τ which generated the best solution for the test case; the total utilisation of trucking resources, and the dry and wet processing plants (% of network-wide capacity); the total percentage of network-wide haulage capacity spent on undesirable stockpiling; and the gap (%) between the objective value of *best* and the best known lower bound on Z'_{MMPP} for the test case.

We compare the results of the piecewise-linear relaxed (PLR) solver with those obtained by our heuristic, using both the worst and best performing combination of N, and γ, parameters: $N = 10$, $\gamma = 0.25$; and $N = 20$, $\gamma = 0.75$, respectively. As we perform 10 seeded runs of our heuristic on each test, and average the results of those runs in Tables 1 and 2, we use the worst performing run (producing the highest value for Z'_{MMPP}) obtained for each test and $N - \gamma$ parameter combination in our comparison. The final six columns of Table 3 denote: the gap (%) between $Z'_{MMPP}(\vec{s}_{best})$, where $\vec{s}_{best}$ is the solution found by our heuristic for the given $N - \gamma$ combination, and the best known lower bound; the elapsed time (s) at which the heuristic discovered this solution; and the time required by

Table 4 Comparison of ALT and our heuristic. For the best solution $best$ found by ALT in each test #, we record: the elapsed time (s) to the discovery of $best$, and convergence ('–' indicates that convergence did not occur in 12hrs); revenue from correctly blended port products ($); time limit (s) on each MIP solve; utilisation of trucks, and dry/wet processing plants (% of network-wide capacity); percentage of network-wide haulage capacity spent on undesirable stockpiling; and the gap (%) between $Z'_{MMPP}(best)$ and the best known lower bound. Columns 11-16 compare ALT and our heuristic. Given $N = 10,\ \gamma = 0.25$, and $N = 20,\ \gamma = 0.75$, we record for the lowest quality $\vec{s}_{best}$ found across all seeded runs of each test #: the gap between $Z'_{MMPP}(\vec{s}_{best})$ and the best known lower bound (Gap, %); the elapsed time (Time, s) taken by our heuristic to solve; and the elapsed time (ALT, s) taken by ALT to find an equally good solution ('–' indicates that no such solution was found).

					Utilisation (%)				Gap to	$N=10,\gamma=0.25$			$N=20,\gamma=0.75$		
#	Best (s)	Converges (s)	Revenue ($)	MIP_L (s)	Trucking	Dry	Wet	Stockpiling	MINLP_{lb} (%)	Gap (%)	Time (s)	ALT (s)	Gap (%)	Time (s)	ALT (s)
1	36000	–	318978000	500	99.47	99.19	100	0.67	0.76	1.12	193	1000	0.94	340	1000
2	24500	–	321246000	500	98.40	99.98	100	1.86	0.06	0.24	282	1000	0.06	554	24500
3	22000	–	318411000	1000	98.58	100	100	1.06	0.94	1.12	323	12000	1.12	357	12000
4	34000	–	315009000	500	97.29	98.83	100	3.92	2.00	1.65	246	–	2.35	352	6500
5	12000	26000	316710000	500	98.63	99.79	100	1.96	1.47	1.29	379	–	1.29	565	–
6	6000	16000	319545000	500	99.26	98.83	100	2	0.59	0.94	290	1000	0.24	501	–
7	41000	–	316710000	500	99.87	97.89	100	2.63	1.47	1.65	284	41000	1.47	390	41000
8	30000	–	320679000	1000	99.59	97.8	100	2.67	0.24	0.06	390	–	0.06	448	–
9	2000	30000	317844000	500	99.88	100	100	1.19	1.12	1.47	142	1000	1.47	265	1000
10	37000	–	320679000	1000	99.25	100	100	1.35	0.24	0.24	390	37000	0.06	312	–
11	17000	38000	319545000	1000	99.79	100	100	0.86	0.59	1.12	315	3000	1.12	407	3000
12	8000	–	320679000	1000	99.76	100	100	3.19	1.65	1.65	166	8000	0.94	336	–
13	14000	–	318978000	1000	100	100	100	1.72	0.76	0.76	170	14000	0.76	383	14000
14	22000	–	318411000	500	100	100	100	0	0.94	1.12	154	22000	0.94	250	22000
15	2000	12000	315576000	1000	100	99.75	100	2.42	1.82	1.82	311	2000	1.29	678	–
16	6000	–	321246000	1000	98.81	99.89	100	2.78	0.06	0.41	263	6000	0.06	343	6000
17	6000	–	321246000	1000	100	100	100	0.28	0.06	0.06	196	3000	0.06	295	3000
18	6100	11800	320679000	100	100	98.29	100	1.71	0.24	0.06	203	–	0.06	256	–
19	8000	–	320679000	1000	99.25	99.94	100	0.54	0.24	0.06	204	–	0.06	328	–
20	10642	11142	196020000	500	97.41	97.55	100	4.60	> 100	3.63	382	–	2.71	692	–

the PLR solver to find a solution of equivalent quality (a '–' in the PLR column indicates that the PLR solver did not find such a solution in a 12 hour timeframe). In tests 1 and 7, for $N = 10, \gamma = 0.25$ and $N = 20, \gamma = 0.75$, respectively, the gap between the objective of solutions found by the heuristic and the PLR solver, to the best known lower bounds, appears to be the same, at 1.12 and 1.47. The total productivity of the mine network is higher, however, in the heuristic solutions – the scaling that exists between port product deviation, revenue, and productivity, in Z'_{MMPP}, results in productivity changes equating to small differences in gap values, not evident when rounded to two decimal places.

Table 3 shows that, for $N = 20$ and $\gamma = 0.75$, our heuristic discovers solutions equally as good, or better, than the PLR solver, in orders of magnitude less time, on a majority of tests (16/20). For the worst performing parameter combination of $N = 10$ and $\gamma = 0.25$, the PLR solver finds higher quality solutions in a majority of tests (16/20), but requires, in 14 of the 20 tests, orders of magnitude more time to do so. The PLR solver is consequently not a viable alternative – it rarely displays good performance, and requires knowledge of the extent to which bounds on port product composition should be narrowed.

1.3. The ALT Heuristic

The ALT heuristic generates and solves a series of linear programs (LPs), by alternately fixing each set of variables that appear in the bilinear constraints of a general BLP (Audet et al. 2004). We first fix the $v^m_{l,q}$ variable in each bilinear term, $v^m_{l,q}\tau^m_l$, of our MINLP to its instantiation in the solution to the envelope-based relaxation of Section 1.1. We solve the resulting MIP to obtain a set of values for each τ^m_l variable. These values are then used to fix each τ^m_l variable, and solve for a new instantiation of each $v^m_{l,q}$. This process of alternate variable fixing repeats until two successive iterations of the heuristic yield equal (to a tolerance) values for either of the $v^m_{l,q}$ or τ^m_l variable sets. On our set of benchmark tests, the MIP generated from fixing each $v^m_{l,q}$ to its first value could not be solved to default optimality tolerances within a 12 hour period. We have run a variation of the ALT algorithm in which each MIP solve is given a time limit. The best solution found in that time limit is used to obtain new instantiations of the $v^m_{l,q}$ and τ^m_l variable sets. In this setting, convergence to a local optimum is no longer guaranteed, and the MINLP objective value in successive solutions may not monotonically improve. This modified ALT heuristic has been applied to each of our benchmark tests, and the best solution found over all iterations, until convergence or the 12 hour cut-off point is reached, recorded.

We have applied modified ALT with a MIP time limit of 100, 500, and 1000 seconds. We record, in Table 4, for the best solution *best* found in each test: the elapsed time (s) to discovery, and convergence; the MIP solve limit (s) used to generate the best solution for the test; the revenue ($) achieved via the sale of ore products; the utilisation of trucking resources, and the dry and wet processing plants (% of network-wide capacity); the percentage of network-wide haulage capacity spent on undesirable stockpiling; and the gap (%) between $Z'_{MMPP}(best)$ and the best known lower bound on Z'_{MMPP} for the test case. The final six columns of Table 4 denote: the gap (%) between $Z'_{MMPP}(\vec{s}_{best})$, where $\vec{s}_{best}$ is the solution found by our heuristic for the given $N - \gamma$ combination, and the best known lower bound; the elapsed time (s) at which the heuristic discovered this solution; and the time required by the ALT solver to find a solution of equivalent quality (a '–' in the ALT column indicates that ALT did not find such a solution in a 12 hour timefame).

Table 4 shows that, for both $N - \gamma$ combinations, our heuristic discovers solutions equally as good, or better, than ALT, on a majority of tests (15/20 for $N = 20$, $\gamma = 0.25$, and 11/20 for $N = 10$, $\gamma = 0.25$). The performance of ALT, across the tests, is inconsistent, often requiring orders of magnitude more time, than our heuristic, to discover solutions of comparable quality. Moreover, Table 4 shows that ALT was unable to converge in a reasonable timeframe. This lack of convergence arises as a result of the time limit imposed on each MIP solve, preventing it from being solved to optimality.

2. Concluding Remarks

We have evaluated our decomposition-based heuristic on a suite of test cases generated for an 8-mine, 2-port network, using data provided by an industry partner – contrasting its performance with a range of solvers for a MINLP modelling of the problem. The presented decomposition-based heuristic was found to find solutions of higher quality, on a subset of test cases, than a range of alternatives, described in Section CP-5 of our companion paper. Each alternative was afforded 12 hours, for each test case, in which to find a solution. Where the heuristic did not find a solution higher in quality than that found by an alternative, it returned a good quality solution for which the alternative required orders of magnitude more time, relative to the heuristic run time, to match. Overall our decomposition-based heuristic approach provides a highly competitive solution to the short-term multiple port and mine open-pit production scheduling problem.

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