Computing the Margin of Victory in Preferential Parliamentary elections

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Abstract. We show how to use automated computation of election margins to assess the number of votes that would need to change in order to alter a parliamentary outcome for single-member preferential electorates. In the context of increasing automation of Australian electoral processes, and accusations of deliberate interference in elections in Europe and the USA, this work forms the basis of a rigorous statistical audit of the parliamentary election outcome. Our example is the New South Wales Legislative Council election of 2015, but the same process could be used for any similar parliament for which data was available, such as the Australian House of Representatives.

1 Introduction

The party that wins a majority of seats in a parliamentary election may not be the party that wins a majority of votes. This has been examined extensively in the United States [7, 8]. In Australian parliamentary elections, even the notion of a "popular majority" is poorly defined because Australian voters rank their candidates in order of preference. But similar results occur: sometimes in practice the Parliamentary winner is not the popular majority winner and there are even some systematic biases [2]. Nevertheless it is often assumed by the public and the media that a party that wins a comfortable overall margin will comfortably win the parliamentary election. Of course, this is not necessarily true.

In this paper we focus on computing the Parliamentary election margin: the minimal number of votes that need to be changed, in a particular election outcome, to switch the Parliamentary winner. This may be much less than the margin between the popular votes of the two major parties.

There are two ways that an Australian parliamentary election may be closer than it seems. First, there may be many seats held by a very small margin. Second, even within one seat, the margin may be smaller than it appears. Australia's preferential voting system proceeds by iteratively eliminating candidates until only two remain, then selecting the one with a larger tally of votes. A naive observer might think that the margin of victory is the number of votes that need to be switched to reverse the winner in this last step (*i.e.* half the difference in the final tallies)—we call this the *last-round margin*. The true margin may be much smaller, however, as changing an early elimination step may cascade into a completely different elimination order. Computing the correct margin for preferential voting is, in general, a computationally difficult problem, but an efficient solution has been demonstrated [1].

In earlier work, Blom *et al.* [1] present an algorithm for computing the margin of victory in Instant Runoff Voting (IRV) elections (also commonly referred to as Alternative Vote elections). In an Australian state or federal Parliamentary election, an IRV election is held in each of a number of districts, electing a single candidate to a seat in the lower house. The party (or coalition of parties) that holds the majority of seats in the lower house, wins the election. Recall that Australian voters rank candidates in order of preference (for example, the ranking [a, c, b] expresses a first preference for candidate a, a second for c, and a third for b). A change to a vote replaces its ranking over candidates with an alternate ranking (for example, replacing ranking [a, c, b] with [c, b, a]). In this paper, we are interested in computing the smallest number of votes (of those cast) that need to be changed to ensure that a different party (or coalition of parties) wins the majority of seats, or that no party (or specific coalition of parties) wins a majority of seats (leading to a hung parliament). Computing the Parliamentary election margin requires a slight modification to the algorithm of [1], in that we must compute the margin of victory *with respect to a specific set of alternate winners* in each seat.

For example, to determine how many votes we would need to change to ensure that Labor wins a majority of seats (in place of the Liberal/National coalition), we would look at manipulations in which seats won by non-Labor candidates are consequently awarded to the Labor candidate.[1] A process of sorting the seats in increasing order of margin, and adding the margins in the necessary number of seats yields the desired Parliamentary election margin.

As a case study, we use data from the 2015 NSW state election to compute the margin by which the Liberal/National coalition won. The popular margin was high—the Liberal/National coalition won 46% of formal first-preference votes compared with 34% for the Labor parties and 10% for The Greens.[2] The coalition won 54 seats compared to Labor's 34. We find, however, that the number of votes necessary to switch the parliamentary outcome is less than 0.1%.

In prior work on US IRV elections, Blom *et al.* [1] found that the true margin is almost always the last-round margin, though exceptions did occur. This is also true of the NSW 2015 election where, for example, the Lismore seat has a last-round margin of 1173, but the true margin of victory is only 209 votes.

The source code used to compute our results is located at:

`https://github.com/michelleblom/margin-irv`

These techniques could be easily applied to any parliamentary outcome for which complete vote data was available. This analysis could become standard procedure for any parliamentary election with automated ballot scanning.

[1] The Liberal, National, and Labor parties are three Australian political parties.
[2] This is from http://pastvtr.elections.nsw.gov.au/SGE2015/la/state/formal/index.htm

1.1 Notation

Below we give common three letter codes used to refer to parties in the 2015 New South Wales (NSW) state election.

LAB	Australian Labor Party
CLP	Country Labor Party
LIB	Liberal Party of Australia
NAT	National Party of Australia
GRN	Australian Greens
IND	Independent (belonging to no party)

1.2 Summary of Results

Of the 4.56 million votes cast in the 2015 New South Wales state election, we have determined that it would have taken:

- 22,746 vote changes for the Labor/Country Labor party to gain the 13 additional seats they need to win government (with 47 seats),
- 16,349 vote changes for a Labor/Greens coalition to gain the 10 additional seats they need to win government, and
- 10,398 vote changes to lose the Liberal/National coalition 8 seats and hence produce a hung parliament.

1.3 Auditing and Accuracy Testing in Elections

The margin computation tools presented in this paper can be used, whenever data[3] is available, to check automatically whether a known problem in an election was large enough to change the outcome. Similarly, when a known number of votes were received over an insecure or unscrutinisable channel, this could be used to decide whether that might have been enough to alter the outcome.

Conversely, it could be used to generate evidence that the election outcome is right.

These calculations could be used as the basis for a rigorous risk-limiting audit to confirm (or overturn) the announced election outcome. Risk limiting audits [4] take an iterative random sample of the paper ballots to check how well they reflect the announced outcome. An audit has *risk-limit* α if a mistaken outcome is guaranteed to be detected with a probability of at least $1 - \alpha$. Either the audit concludes with a certain confidence that the outcome is right, or it finds so many errors that a full manual recount is warranted. The audit process is parameterised by the margin of victory in the election. Kroll *et al.* [3] have devised audits for parliamentary outcomes but, like most US research, they focus on simple first-past-the-post elections in which the margin is obvious.

This is particularly important now that the Australian Parliament's Joint Standing Committee on Electoral Matters has recommended automated scanning of the ballot

[3] An electronic record of the preferences expressed in each paper ballot, after scanning and digitisation

Initially, all candidates remain standing (are not eliminated)
While there is *more than one* candidate standing
  For every candidate c standing
    Tally (count) the votes in which c is the highest-ranked candidate of those standing
  Eliminate the candidate with the smallest tally
The winner is the one candidate not eliminated

Fig. 1: The IRV counting algorithm: the candidate with the smallest tally is repeatedly eliminated, with the ballots in their tally redistributed to remaining candidates according to their next preference.

papers [6]. The overall Parliamentary margin could be quickly calculated using our methods. Rigorous risk-limiting audits could then be performed for each electorate, immediately after the election, in order to provide evidence that the overall election outcome was correct.

In a time where outside influencing of elections is a constant source of news, and where more and more elections systems involve electronic systems, either for voting or counting votes, it is critical that we have mechanisms in place to generate evidence of accurate election results, and indeed to check what degree of manipulation must have taken place for the election result to have been altered.

2 Background

The lower houses of parliaments in the Australian federal and state elections are the result of a number of independent Instant Runoff Voting (IRV) elections for a set of single-member electorates (seats). Each seat has a number of candidates, and each vote consists of an ordered list of the candidates for that seat.[4]

The tallying of votes in an IRV election proceeds by a series of rounds in which the candidate with the lowest number of votes is eliminated (see Figure 1) with the last remaining candidate declared the winner. All votes in an eliminated candidate's tally are distributed to the next most-preferred (remaining) candidate in their ranking.

Let $\mathcal{C}$ be the set of candidates in an IRV election $\mathcal{B}$. We refer to sequences of candidates π in list notation (e.g., $\pi = [c_1, c_2, c_3, c_4]$), and use such sequences to represent both votes and elimination orders. We will often treat a sequence as the set of elements it contains. An election $\mathcal{B}$ is defined as a multiset[5] of votes, each vote $b \in \mathcal{B}$ a sequence of candidates in $\mathcal{C}$, with no duplicates, listed in order of preference (most preferred to least preferred). Let $\mathit{first}(\pi)$ denote the first candidate appearing in sequence π (e.g., $\mathit{first}([c_2, c_3]) = c_2$). In each round of vote counting, there are a current set of eliminated candidates $\mathcal{E}$ and a current set of candidates still standing $\mathcal{S} = \mathcal{C} \setminus \mathcal{E}$. The winner c_w of the election is the last standing candidate.

[4] Most Australian elections require all preferences to be filled in, but some allow partial lists or several equal-last candidates. Our analysis extends to all these cases.

[5] A multiset allows for the inclusion of duplicate items.

Each candidate $c \in \mathcal{C}$ has a *tally* of votes. Votes are added to this tally upon the elimination of a candidate $c' \in \mathcal{C} \setminus \{c\}$, and are redistributed from this tally upon the elimination of c.

Definition 1. ***Tally*** $\mathbf{t}_{\mathcal{S}}(\mathbf{c})$ *Given candidates $\mathcal{S} \subseteq \mathcal{C}$ are still standing in an election $\mathcal{B}$, the tally for candidate $c \in \mathcal{C}$, denoted $t_{\mathcal{S}}(c)$, is defined as the number of votes $b \in \mathcal{B}$ for which c is the most-preferred candidate of those remaining. Let $p_{\mathcal{S}}(b)$ denote the sequence of candidates mentioned in b that are also in $\mathcal{S}$.*

$$t_{\mathcal{S}}(c) = \ | \, [b \mid b \in \mathcal{B}, c = \mathit{first}(p_{\mathcal{S}}(b))] \, | \tag{1}$$

Definition 2. ***Margin of Victory (MOV)*** *The MOV in an election with candidates $\mathcal{C}$ and winner $c_w \in \mathcal{C}$, is the smallest number of votes whose ranking must be modified (by an adversary) so that a candidate $c' \in \mathcal{C} \setminus \{c_w\}$ is elected.*

Often the last round margin (LRM) is used as a proxy for the margin of victory.

Definition 3. ***Last Round Margin (**LRM**)*** *The LRM of an election, in which two candidates $\mathcal{S} = \{c, c'\}$ remain with $t_{\mathcal{S}}(c)$ and $t_{\mathcal{S}}(c')$ votes in their tallies, is equal to half the difference between the tallies of c and c' rounded up.*

$$LRM = \lceil \frac{|t_{\mathcal{S}}(c) - t_{\mathcal{S}}(c')|}{2} \rceil \tag{2}$$

In this paper, we are interested in a more restricted version of margin of victory, which is the margin of victory over a subset of the non-winning candidates.

Definition 4. ***Margin of Victory over Candidates*** $\mathcal{A}$ ***(MOVC)*** *The MOVC in an election with candidates $\mathcal{C}$ and winner $c_w \in \mathcal{C}$ over the alternate candidates $\mathcal{A} \subseteq \mathcal{C} \setminus \{c_w\}$, is the smallest number of votes whose ranking must be modified (by an adversary) so that a candidate $c' \in \mathcal{A}$ is elected.*

While the MOV calculates the number of votes required to be changed to alter the winner, the MOVC calculates the number of votes required to be changed to alter the winner *to one of a set* $\mathcal{A}$. We will require this finer information in order to calculate the smallest number of votes for a different party or coalition to win the election.

Example 1. Consider an election between candidates a, b, and c with the election profile shown in Table 1. The initial tallies of a, b, and c are 55, 41, and 40 votes, respectively, hence c is eliminated. Candidates a and b consequently have tallies of 80 and 41 votes, giving a the victory with a last round margin of 20 votes. Consider changing 1 of the [b, c] votes to a [c] vote. Then the initial tallies are $\{a : 55, b : 40, c : 41\}$ and b is eliminated. Candidates a and c consequently have tallies of 55 and 81 votes, and c is the winner of the election.

Clearly the MOV is 1 vote. The MOVC for $\{b\}$ is 10, which is achieved by changing 5 votes from [a] to [b, c] and 5 from [a] to [c], giving first round tallies of $\{a : 45, b : 46, c : 45\}$. An adversary can choose to eliminate a leaving b and c with tallies of 46 and 45 votes, and b winning the election. □

Ranking	Count
$[a]$	55
$[c, a]$	25
$[b, c]$	41
$[c]$	15

(a)

Candidate	Round 1	Round 2
a	55	80
b	41	41
c	40	—

(b)

Table 1: IRV example, with (a) the number of votes cast with each listed ranking over candidates a, b, c, and (b) tallies after each round of vote counting.

2.1 Computing margins for an IRV election

Blom *et al.* [1] present a branch-and-bound algorithm (denoted *margin-irv*) for efficiently computing the margin of victory in an IRV election. This algorithm improves upon an existing method by Magrino *et al.* [5].

Given an IRV election with winning candidate c_w, *margin-irv* traverses a tree defining all possible *alternate* orders of candidate elimination (that result in a winning candidate other than c_w). As the algorithm explores these alternate elimination sequences, it solves a mixed integer program (MIP) to determine the minimum number of vote manipulations required to realise each elimination order. The ultimate goal is to find an elimination sequence, in which an alternate winner is elected, that requires the smallest number of vote changes to realise. Searching through the entire space of alternate elimination sequences would be too combinatorially complex, however, and so *margin-irv* incorporates rules for pruning sections of this tree from consideration. The result is an efficient algorithm for computing electoral margins.

A description of both the *margin-irv* algorithm, and the original branch-and-bound method of Magrino *et al.* [5], can be found in Blom *et al.* [1]. We summarise *margin-irv* in this section, and outline how it can be altered to compute a margin over a set of candidates $\mathcal{A}$ (the MOVC). Appendix B provides the full *margin-irv* algorithm for computing the MOVC for a single seat.

Given an IRV election with candidates $\mathcal{C}$ and winner $c_w \in \mathcal{C}$, the *margin-irv* algorithm starts by adding $|\mathcal{C}| - 1$ partial elimination sequences to the search tree, one for each of alternate winner $c'_w \in \mathcal{C} \setminus \{c_w\}$. These partial sequences form a frontier F. Each of these sequences contains a single candidate – the alternate winner in question. Following the basic structure of a branch-and-bound algorithm, we compute, for each partial sequence $\pi \in F$, a lower bound on the number of vote changes required to realise a elimination sequence that *ends* in π. These lower bounds are used to guide construction of the search tree, and are computed by both solving a MIP, and applying several rules for lower bound computation. The partial sequence π with the smallest lower bound is selected and *expanded*. For each candidate $c \in \mathcal{C}$ that is not already present in π, we create a new sequence with c appended to the front. For example, given a set of candidates c_1, c_2, and c_3, with winning candidate c_3, the partial sequence $\pi = [c_2]$ will be expanded to create two new sequences $[c_1, c_2]$ and $[c_3, c_2]$. We evaluate each new sequence π' created by assigning it a lower bound on the number of votes required to realise any elimination order ending in π'.

While exploring and building elimination sequences, *margin-irv* maintains a running *upper bound* on the value of the true margin. This upper bound is initialised to the last round margin of the election. When a sequence π containing all candidates is constructed, our MIP computes the exact number of vote manipulations required to realise it. If this number is lower than our current upper bound, the upper bound is revised, and all orders on our frontier with a lower bound greater than or equal to it are pruned from consideration (removed from our frontier). This process continues until our frontier is empty (we have considered or pruned all possible alternate elimination sequences). The value of the running upper bound is the true margin of victory of the election.

Its easy to extend the *margin-irv* algorithm to also calculate MOVC for a set of alternate winners $\mathcal{A}$. In the first step of the algorithm, rather than adding a node for each alternate winner in $\mathcal{C} \setminus \{c_w\}$ we add a node only for each of the alternate candidates in $\mathcal{A}$. The remainder of the algorithm is unchanged. With this modification, *margin-irv* will only explore alternate election outcomes that result in one of the candidates in $\mathcal{A}$ winning the election.

3 Calculating the Number of Votes to Change a Parliamentary Election Outcome

Given a set S of seats in a parliament, a winning coalition P is a set of parties such that the number of seats won by that coalition is at least some defined threshold T. Usually $T = \lceil \frac{|S|+1}{2} \rceil$, requiring the coalition to win more than half the seats. The NSW Legislative Assembly has 93 seats, and so 47 are required to win government.

We can use this threshold to calculate the number of vote changes required to change a parliamentary election result as follows. Assume the coalition won $W \geq T$ seats. We calculate the MOVC for each seat s won by the coalition P for the set of alternate candidates in that election *not* in coalition P. We then sort the MOVC values, and choose the $W - T + 1$ seats O with the least MOVC values. The sum of the MOVC of these seats O is the number of changes in votes required to remove the victory of the winning coalition P, and hence change the outcome of the election.

Note that if the coalition is a single party $P = \{p\}$, or more generally if no seat has two candidates from the coalition, then the MOVC values required are identical to MOV values. This is the case for the NSW Legislative Election where no seat has both a Liberal (LIB) and National (NAT) candidate. The above procedure examines how we might rob the original winning coalition P of its victory. However, we are interested in computing the number of vote changes required to award victory to a specific party or coalition of parties P' (such as a Labor (LAB)/Greens (GRN) coalition).

We can use a similar approach to calculate the number of vote changes required to change a parliamentary election outcome so that another coalition P' would win instead. Assume P' won $W' < T$ seats. We calculate the MOVC for each seat s not won by coalition P' with the set of alternate candidates $\mathcal{A}$ equal to the set of candidates belonging to parties in P'. We then sort the MOVC values, and choose the $W' - T$ seats O' with the least MOVC values. The sum of the MOVC of these seats O' is the number of changes in votes required to give a parliamentary victory to coalition P'.

Seat	$\|\mathcal{C}\|$	Last-round margin	True margin	Winner
Lismore	6	1173	209	NAT
Balina	7	1267	1130	GRN
Heffron	5	5835	5824	LAB
Maitland	6	5446	4012	CLP
Willoughby	6	10247	10160	LIB

Table 2: The 5 seats in the 2015 NSW Legislative Assembly Parliamentary Election in which the last-round margin did not equal the true margin of victory.

Seat	$\|\mathcal{C}\|$	Last-round margin	True margin	Winner
East Hills	5	189	189	LIB
Lismore	6	1173	209	NAT
Upper Hunter	6	866	866	NAT
Monaro	5	1122	1122	NAT
Coogee	5	1243	1243	LIB
Tweed	5	1291	1291	NAT
Penrith	8	2576	2576	LIB
Holsworthy	6	2902	2902	LIB

Table 3: The 8 seats, won by a LIB or NAT candidate, with the lowest MOV.

Again if the coalition P' was always the alternate winner in the calculation of the MOV, then the MOVC and MOV calculations will coincide, and indeed if P' is a strong existing coalition it is likely that it is the alternate winner in most seats with the lowest MOVC.

4 Results

The NSW Legislative Assembly Parliamentary Election of 2015 was contested by major parties: Liberal (LIB), National (NAT), Green (GRN), Labor (LAB) and Country Labor (CLP); as well as a number of minor parties and independents (IND). We found 5 seats in which the true margin was not the last-round margin. These seats are listed in Table 2, alongside the number of candidates up for election in each electorate ($|\mathcal{C}|$), the last-round margin for the seat, the true margin of victory for the seat, and the party whose candidate won the seat.

The LIB/NAT coalition won 54 seats to have a winning majority. In order to lose this majority, they must lose $54 - 47 + 1 = 8$ seats. Since no seat ran both a LIB and a NAT candidate, we can use the MOV values to calculate the number of votes required to lose 8 seats. The 8 LIB/NAT seats with the lowest MOV are listed in Table 3. For Lismore, the MOV differs substantially from the last-round margin. Hence, the total number of votes required for the LIB/NAT coalition to lose their majority is 10,398 (the sum of the ‘True margin’ values in the 4^{th} column of Table 3).

For a LAB and CLP coalition to win the election we need them win to $47 - 34 = 13$ more seats. The 13 seats with the lowest MOVC for a change to LAB/CLP are listed in Table 4.

Seat	$\|\mathcal{C}\|$	Last-round margin	True margin	Winner	MOVC
East Hills	5	189	189	LIB	189
Lismore	6	1173	209	NAT	209
Upper Hunter	6	866	866	NAT	866
Monaro	5	1122	1122	NAT	1122
Balina	7	1267	1130	GRN	1130
Coogee	5	1243	1243	LIB	1243
Tweed	5	1291	1291	NAT	1291
Balmain	7	1731	1731	GRN	1731
Penrith	8	2576	2576	LIB	2576
Holsworthy	6	2902	2902	LIB	2902
Goulburn	6	2945	2945	LIB	2945
Oatley	5	3006	3006	LIB	3006
Newtown	7	3536	3536	GRN	3536

Table 4: The 13 seats with the lowest MOVC for a change in winner to LAB/CLP.

The total number of votes required to give an LAB/CLP victory is hence 22,746. In this case we can see, since the LAB/CLP is a strong alternate coalition, that all the MOVC calculations agree with the MOV calculations. Note that this is not true for all seats. For example in the NSW data Sydney is the first seat where the MOVC (= 5583) for the LAB and CLP coalition is different from the MOV (=2864). This is because the runner-up was an Independent. Note that if we used MOV instead of MOVC we would incorrectly treat Sydney as one of the seats to change, and incorrectly calculate the number of votes required for an LAB/CLP coalition to win.

The full results for all seats are in Appendix A. The total numbers for changing the parliamentary outcome are computed by simply adding together the smallest margins for the necessary number of seats.

5 Conclusion

We have shown an efficient method of automated margin computation that can be used to identify the minimum number of vote changes (or errors) necessary to alter a parliamentary election outcome using single-member preferential voting. Our example was the NSW Legislative Assembly election of 2015, but the same tools and techniques could be immediately applied to any other parliament constructed in the same way for which full voting data was available, such as the Australian House of Representatives or other state lower houses.

Accurate electoral margins can form the basis of rigorous statistical auditing of paper ballot records to check the official election result. This would be valuable in any scenario, but is particularly important when an electronic (and hence unobservable) process such as automated ballot scanning is part of the count. Since these are exactly the scenarios that tend to produce detailed vote data, this work provides the basis for a count that is automated and fast (because of automated ballot scanning) and also transparent and verifiably accurate, because of rigorous auditing given an accurately computed election margin.

A Full list of margins for the NSW 2015 state election

Table 5 records the last-round and true victory margins for each seat in the 2015 NSW lower house elections. In most seats, the last-round margin – the difference between the two last candidates in the elimination order – is the true margin. Exceptions to this rule are marked with an asterisk. The 8 Liberal/National coalition seats with the smallest margins are shown in bold. The total of the margins of these 8 seats gives the smallest number of vote changes required to produce a hung parliament, 10,398.

Table 5: LRM and MOV for each seat in the 2015 NSW lower house election.

Seat	$\|\mathcal{C}\|$	LRM	MOV	Winner	Seat	$\|\mathcal{C}\|$	LRM	MOV	Winner
Gosford	6	102	102	LAB	The Entrance	5	171	171	LAB
East Hills	5	189	**189**	**LIB**	***Lismore**	6	1173	**209**	**NAT**
Strathfield	5	770	770	LAB	Granville	6	837	837	LAB
Upper Hunter	6	866	**866**	**NAT**	**Monaro**	5	1122	**1122**	**NAT**
*Balina	7	1267	1130	GRN	**Coogee**	5	1243	**1243**	**LIB**
Tweed	5	1291	**1291**	**NAT**	Prospect	5	1458	1458	LAB
Balmain	7	1731	1731	GRN	Rockdale	6	2004	2004	LAB
Port Stephens	5	2088	2088	CLP	Auburn	6	2265	2265	LAB
Penrith	8	2576	**2576**	**LIB**	Kogarah	6	2782	2782	LAB
Sydney	8	2864	2864	IND	**Holsworthy**	6	2902	**2902**	**LIB**
Goulburn	6	2945	2945	LIB	Oatley	5	3006	3006	LIB
Campbelltown	5	3096	3096	LAB	Newcastle	7	3132	3132	LAB
Wollongong	7	3367	3367	LAB	Macquarie Fields	7	3519	3519	LAB
Newtown	7	3536	3536	GRN	Heathcote	6	3560	3560	LIB
Blue Mountains	6	3614	3614	LAB	Myall Lakes	6	3627	3627	NAT
Bega	5	3663	3663	LIB	Wyong	7	3720	3720	LAB
Londonderry	5	3736	3736	LAB	Seven Hills	7	3774	3774	LIB
Summer Hill	7	3854	3854	LAB	Kiama	5	3856	3856	LIB
*Maitland	6	5446	4012	CLP	Terrigal	5	4053	4053	LIB
South Coast	5	4054	4054	LIB	Clarence	8	4069	4069	NAT
Lake Macquarie	7	4253	4253	IND	Mulgoa	5	4336	4336	LIB
Oxley	5	4591	4591	NAT	Tamworth	7	4643	4643	NAT
Maroubra	5	4717	4717	LAB	Swansea	8	4974	4974	LAB
Ryde	5	5153	5153	LIB	Barwon	6	5229	5229	NAT
Riverstone	5	5324	5324	LIB	Wagga Wagga	6	5475	5475	LIB

Continued

Seat	$\|\mathcal{C}\|$	LRM	MOV	Winner	Seat	$\|\mathcal{C}\|$	LRM	MOV	Winner
Parramatta	7	5509	5509	LIB	Charlestown	7	5532	5532	LAB
Bankstown	6	5542	5542	LAB	Blacktown	5	5565	5565	LAB
Coffs Harbour	5	5824	5824	NAT	*Heffron	5	5835	5824	LAB
Albury	5	5840	5840	LIB	Miranda	6	5881	5881	LIB
Mount Druitt	5	6343	6343	LAB	Canterbury	5	6610	6610	LAB
Fairfield	5	6998	6998	LAB	Epping	6	7156	7156	LIB
Bathurst	5	7267	7267	NAT	Hawkesbury	8	7311	7311	LIB
Wollondilly	6	7401	7401	LIB	Shellharbour	7	7519	7519	LAB
Cabramatta	5	7613	7613	LAB	Lane Cove	6	7740	7740	LIB
Drummoyne	6	8099	8099	LIB	Keira	5	8164	8164	LAB
Camden	5	8217	8217	LIB	Lakemba	5	8235	8235	LAB
Liverpool	5	8495	8495	LAB	North Shore	7	8517	8517	NAT
Murray	8	8574	8574	NAT	Hornsby	6	8577	8577	LIB
Dubbo	7	8680	8680	NAT	Port Macquarie	5	8715	8715	NAT
Cessnock	5	9187	9187	CLP	Cootamundra	5	9247	9247	NAT
Wallsend	5	9418	9418	LAB	Cronulla	5	9674	9674	LIB
Vaucluse	5	9783	9783	LIB	Baulkham Hills	5	10023	10023	LIB
Orange	5	10048	10048	NAT	Ku-ring-gai	5	10061	10061	LIB
*Willoughby	6	10247	10160	LIB	Wakehurst	6	10770	10770	LIB
Manly	5	10806	10806	LIB	Pittwater	5	11430	11430	LIB
Northern Tablelands	6	11969	11969	LIB	Davidson	5	12960	12960	LIB
Castle Hill	5	13160	13160	LIB					

Table 6 lists the number of vote changes (denoted Δ) necessary to elect an LAB or CLP candidate. This is at least the true margin (from the previous table), but may be strictly more, for example if an independent candidate was the runner-up. The rows inside the double lines are the 10 seats with the smallest changes necessary to give the labor parties 47 seats. The combined total number of votes needed to produce this is the sum of those rows: 22746.

Table 6: LRM, MOV, and the number of vote changes (Δ) required to elect an LAB or CLP candidate for each seat in the 2015 NSW lower house election.

Seat	$\|\mathcal{C}\|$	LRM	MOV	Winner	Δ	Seat	$\|\mathcal{C}\|$	LRM	MOV	Winner	Δ
Auburn	6	2265	2265	LAB	0	Bankstown	6	5542	5542	LAB	0
Blacktown	5	5565	5565	LAB	0	Blue Mountains	6	3614	3614	LAB	0
Cabramatta	5	7613	7613	LAB	0	Campbelltown	5	3096	3096	LAB	0
Canterbury	5	6610	6610	LAB	0	Cessnock	5	9187	9187	CLP	0
Charlestown	7	5532	5532	LAB	0	Fairfield	5	6998	6998	LAB	0
Gosford	6	102	102	LAB	0	Granville	6	837	837	LAB	0
Heffron	5	5835	5824	LAB	0	Keira	5	8164	8164	LAB	0
Kogarah	6	2782	2782	LAB	0	Lakemba	5	8235	8235	LAB	0
Liverpool	5	8495	8495	LAB	0	L-derry	5	3736	3736	LAB	0
Macq. Fields	7	3519	3519	LAB	0	Maitland	6	5446	4012	CLP	0
Maroubra	5	4717	4717	LAB	0	Mt. Druitt	5	6343	6343	LAB	0

Continued

Seat	$\|\mathcal{C}\|$	LRM	MOV	Winner	Δ	Seat	$\|\mathcal{C}\|$	LRM	MOV	Winner	Δ
Newcastle	7	3132	3132	LAB	0	P. Stephens	5	2088	2088	CLP	0
Prospect	5	1458	1458	LAB	0	Rockdale	6	2004	2004	LAB	0
Shellharbour	7	7519	7519	LAB	0	Strathfield	5	770	770	LAB	0
Summer Hill	7	3854	3854	LAB	0	Swansea	8	4974	4974	LAB	0
The Entrance	5	171	171	LAB	0	Wallsend	5	9418	9418	LAB	0
Wollongong	7	3367	3367	LAB	0	Wyong	7	3720	3720	LAB	0
East Hills	5	189	189	LIB	189	Lismore	6	1173	209	NAT	209
U. Hunter	6	866	866	NAT	866	Monaro	5	1122	1122	NAT	1122
Balina	7	1267	1130	GRN	1130	Coogee	5	1243	1243	LIB	1243
Tweed	5	1291	1291	NAT	1291	Balmain	7	1731	1731	GRN	1731
Penrith	8	2576	2576	LIB	2576	Holsworthy	6	2902	2902	LIB	2902
Goulburn	6	2945	2945	LIB	2945	Oatley	5	3006	3006	LIB	3006
Newtown	7	3536	3536	GRN	3536						
Heathcote	6	3560	3560	LIB	3560	M. Lakes	6	3627	3627	NAT	3627
Bega	5	3663	3663	LIB	3663	Seven Hills	7	3774	3774	LIB	3774
Kiama	5	3856	3856	LIB	3856	Terrigal	5	4053	4053	LIB	4053
South Coast	5	4054	4054	LIB	4054	Clarence	8	4069	4069	NAT	4069
Lake Macq.	7	4253	4253	IND	4253	Mulgoa	5	4336	4336	LIB	4336
Oxley	5	4591	4591	NAT	4591	Ryde	5	5153	5153	LIB	5153
Barwon	6	5229	5229	NAT	5229	Riverstone	5	5324	5324	LIB	5324
W-Wagga	6	5475	5475	LIB	5475	Parramatta	7	5509	5509	LIB	5509
Sydney	8	2864	2864	IND	5583	C. Harbour	5	5824	5824	NAT	5824
Albury	5	5840	5840	LIB	5840	Miranda	6	5881	5881	LIB	5881
Epping	6	7156	7156	LIB	7156	Bathurst	5	7267	7267	NAT	7267
Hawkesbury	8	7311	7311	LIB	7311	W-dilly	6	7401	7401	LIB	7401
Lane Cove	6	7740	7740	LIB	7740	D-moyne	6	8099	8099	LIB	8099
Camden	5	8217	8217	LIB	8217	Hornsby	6	8577	8577	LIB	8577
Dubbo	7	8680	8680	NAT	8680	Port Macq.	5	8715	8715	NAT	8715
North Shore	7	8517	8517	NAT	8798	C-mundra	5	9247	9247	NAT	9247
Murray	8	8574	8574	NAT	9483	Cronulla	5	9674	9674	LIB	9674
B. Hills	5	10023	10023	LIB	10023	Orange	5	10048	10048	NAT	10048
Ku-ring-gai	5	10061	10061	LIB	10061	Willoughby	6	10247	10160	LIB	10160
Vaucluse	5	9783	9783	LIB	10581	Wakehurst	6	10770	10770	LIB	10770
Tamworth	7	4643	4643	NAT	11283	N. T-lands	6	11969	11969	LIB	11969
Manly	5	10806	10806	LIB	12106	Pittwater	5	11430	11430	LIB	12181
Davidson	5	12960	12960	LIB	13065	Castle Hill	5	13160	13160	LIB	13160

Table 7 records the margins for a Labor-Green coalition. In this case the total number of vote changes required to produce this outcome is 16349.

Table 7: LRM, MOV, and the number of vote changes (Δ) required to elect a LAB, CLP, or GRN for each seat in the 2015 NSW lower house election.

Seat	$\|\mathcal{C}\|$	LRM	MOV	Winner	Δ	Seat	$\|\mathcal{C}\|$	LRM	MOV	Winner	Δ
Auburn	6	2265	2265	LAB	0	Balina	7	1267	1130	GRN	0

Continued

Seat	$\|\mathcal{C}\|$	LRM	MOV	Winner	Δ	Seat	$\|\mathcal{C}\|$	LRM	MOV	Winner	Δ
Balmain	7	1731	1731	GRN	0	Bankstown	6	5542	5542	LAB	0
Blacktown	5	5565	5565	LAB	0	B. Mountains	6	3614	3614	LAB	0
Cabramatta	5	7613	7613	LAB	0	C-belltown	5	3096	3096	LAB	0
Canterbury	5	6610	6610	LAB	0	Cessnock	5	9187	9187	CLP	0
Charlestown	7	5532	5532	LAB	0	Fairfield	5	6998	6998	LAB	0
Gosford	6	102	102	LAB	0	Granville	6	837	837	LAB	0
Heffron	5	5835	5824	LAB	0	Keira	5	8164	8164	LAB	0
Kogarah	6	2782	2782	LAB	0	Lakemba	5	8235	8235	LAB	0
Liverpool	5	8495	8495	LAB	0	L-derry	5	3736	3736	LAB	0
M. Fields	7	3519	3519	LAB	0	Maitland	6	5446	4012	CLP	0
Maroubra	5	4717	4717	LAB	0	Mt. Druitt	5	6343	6343	LAB	0
Newcastle	7	3132	3132	LAB	0	Newtown	7	3536	3536	GRN	0
P. Stephens	5	2088	2088	CLP	0	Prospect	5	1458	1458	LAB	0
Rockdale	6	2004	2004	LAB	0	S-harbour	7	7519	7519	LAB	0
Strathfield	5	770	770	LAB	0	S. Hill	7	3854	3854	LAB	0
Swansea	8	4974	4974	LAB	0	The Entr.	5	171	171	LAB	0
Wallsend	5	9418	9418	LAB	0	Wollongong	7	3367	3367	LAB	0
Wyong	7	3720	3720	LAB	0						
East Hills	5	189	189	LIB	189	Lismore	6	1173	209	NAT	209
U. Hunter	6	866	866	NAT	866	Monaro	5	1122	1122	NAT	1122
Coogee	5	1243	1243	LIB	1243	Tweed	5	1291	1291	NAT	1291
Penrith	8	2576	2576	LIB	2576	Holsworthy	6	2902	2902	LIB	2902
Goulburn	6	2945	2945	LIB	2945	Oatley	5	3006	3006	LIB	3006
Heathcote	6	3560	3560	LIB	3560	M. Lakes	6	3627	3627	NAT	3627
Bega	5	3663	3663	LIB	3663	Seven Hills	7	3774	3774	LIB	3774
Kiama	5	3856	3856	LIB	3856	Terrigal	5	4053	4053	LIB	4053
S. Coast	5	4054	4054	LIB	4054	Clarence	8	4069	4069	NAT	4069
Lake Macq.	7	4253	4253	IND	4253	Mulgoa	5	4336	4336	LIB	4336
Oxley	5	4591	4591	NAT	4591	Ryde	5	5153	5153	LIB	5153
Barwon	6	5229	5229	NAT	5229	Riverstone	5	5324	5324	LIB	5324
W-Wagga	6	5475	5475	LIB	5475	Parramatta	7	5509	5509	LIB	5509
Sydney	8	2864	2864	IND	5583	C. Harbour	5	5824	5824	NAT	5824
Albury	5	5840	5840	LIB	5840	Miranda	6	5881	5881	LIB	5881
Epping	6	7156	7156	LIB	7156	Bathurst	5	7267	7267	NAT	7267
H-bury	8	7311	7311	LIB	7311	W-dilly	6	7401	7401	LIB	7401
Lane Cove	6	7740	7740	LIB	7740	D-moyne	6	8099	8099	LIB	8099
Camden	5	8217	8217	LIB	8217	N. Shore	7	8517	8517	NAT	8517
Hornsby	6	8577	8577	LIB	8577	Dubbo	7	8680	8680	NAT	8680
Port Macq.	5	8715	8715	NAT	8715	C-mundra	5	9247	9247	NAT	9247
Murray	8	8574	8574	NAT	9483	Cronulla	5	9674	9674	LIB	9674
Vaucluse	5	9783	9783	LIB	9783	B. Hills	5	10023	10023	LIB	10023
Orange	5	10048	10048	NAT	10048	Ku-ring-gai	5	10061	10061	LIB	10061
Willoughby	6	10247	10160	LIB	10160	Wakehurst	6	10770	10770	LIB	10770

Continued

margin-irv($\mathcal{C}$, $\mathcal{B}$, c_w, $\mathcal{A}$)
1 $F := \emptyset$
2 $U := LRM_{\mathcal{B}}$
3 **for**($c \in \mathcal{A}$)
4 $\quad \pi' := [c]$
5 $\quad l :=$ LOWERBOUND(π')
6 $\quad$**if**($l < U$)
7 $\qquad F := F \cup \{(l, \pi')\}$
8 **while** $F \neq \emptyset$
9 $\quad (l, \pi') := \arg\min F$
10 $\quad F := F \setminus \{(l, \pi')\}$
11 $\quad U :=$ expand($l, \pi', U, F, \mathcal{C}, \mathcal{B}$)
12 **return** U

expand($l, \pi', U, F, \mathcal{C}, \mathcal{B}$)
13 $l' := \max\{l,$ DISTANCETO(π', $\mathcal{C}$,$\mathcal{B}$)$\}$
14 **if**($l' \geq U$)
15 $\quad$**return** U
16 **for**($c \in \mathcal{C} \setminus \pi'$)
17 $\quad \pi := [c] +\!\!+ \pi'$
18 $\quad$**if**($|\pi| = |\mathcal{C}|$)
19 $\qquad$**return** $\min\{U,$DISTANCETO(π,$\mathcal{C}$,$\mathcal{B}$)$\}$
20 $\quad l'' = \max\{l',$ LOWERBOUND(π)$\}$
21 $\quad$**if**($l'' < U$)
22 $\qquad F := F \cup \{(l'', \pi)\}$
23 **return** U

Fig. 2: MOVC computation for an IRV election $\mathcal{B}$ with candidates $\mathcal{C}$, winner $c_w \in \mathcal{C}$, and alternate winner set $\mathcal{A}$.

Seat	$\|\mathcal{C}\|$	LRM	MOV	Winner	Δ	Seat	$\|\mathcal{C}\|$	LRM	MOV	Winner	Δ
Manly	5	10806	10806	LIB	10806	Tamworth	7	4643	4643	NAT	11283
Pittwater	5	11430	11430	LIB	11430	N. T-lands	6	11969	11969	LIB	11969
Davidson	5	12960	12960	LIB	12960	Castle Hill	5	13160	13160	LIB	13160

B Modified *margin-irv*: Computing the MOVC

The *margin-irv* algorithm for computing the MOVC for an IRV election $\mathcal{B}$ given a set of alternate winners $\mathcal{A}$ is shown in Figure 2. An initial upper bound on the MOVC is initialised to the last round margin ($LRM_{\mathcal{B}}$) in Step 2. For each candidate in $\mathcal{A}$, we add a partial elimination order to our frontier F. Each order π' is assigned a lower bound (computed as described by Blom *et al.* [1]) on the degree of manipulation required to realise an elimination sequence *ending* in π' – only orders with an estimated lower bound (l) that *is less than* the current MOVC upper bound (U) are added to the frontier (Steps 6 and 7). Steps 8 to 12 repeatedly select the partial order π' in F with the smallest associated lower bound for expansion. To expand an order π', we create a new order for each candidate c *not* already present in π', appending c to the start of the sequence (Step 17). If the created sequence π contains all candidates, it is a leaf node, and we evaluate the exact number of vote changes required to realise the sequence with a mixed integer linear program (MIP) denoted DISTANCETO.

Section B.1 provides the formulation of the DISTANCETO MIP, replicated from Blom *et al.* [1]. Otherwise, we compute a lower bound on the on the degree of manipulation required to realise an elimination sequence *ending* in π (l'') and add π to our frontier if this lower bound is less than our current upper bound on the MOVC (Steps 21 to 22). The algorithm terminates once there are no further partial orders to be expanded in our frontier, returning the current MOVC upper bound (U) as the computed MOVC.

B.1 The DISTANCETO MIP

The following MIP formulation, originally presented in the work of Magrino *et al.* [5], has been replicated as it appears in Blom *et al.* [1]. Let $\mathbf{R}$ denote the set of possible (partial and total) rankings R of candidates $\mathcal{C}$ that could appear on a vote, N_R the number of votes cast in the election with ranking $R \in \mathbf{R}$, and N the total number of votes cast. For each $R \in \mathbf{R}$, we define variables:

q_R integer number of votes to be changed into R;
m_R integer number of votes with ranking R in the unmodified election to be changed into something other than R; and
y_R number of votes in the modified election with ranking R.

Given a partial or complete order π, the DISTANCETO MIP is:

$$\min \sum_{R \in \mathbf{R}} q_R$$

$$N_R + q_R - m_R = y_R \qquad \forall R \in \mathbf{R} \quad (3)$$

$$\sum_{R \in \mathbf{R}} q_R = \sum_{R \in \mathbf{R}} m_R \quad (4)$$

$$\sum_{R \in \mathcal{R}_{i,i}} y_R \leq \sum_{R \in \mathcal{R}_{j,i}} y_R \qquad \forall c_i, c_j \in \pi \ . \ i < j \quad (5)$$

$$n \geq y_R \geq 0, \ \ N_R \geq m_R \geq 0, \ \ q_R \geq 0 \qquad \forall R \in \mathbf{R} \quad (6)$$

Constraint (3) states that the number of votes with ranking $R \in \mathbf{R}$ in the new election is equal to the sum of those with this ranking in the unmodified election and those whose ranking has *changed to* R, minus the number of votes whose ranking has been *changed from* R. Constraint (5) defines a set of *special elimination constraints* which force the candidates in π to be eliminated in the stated order. $\mathcal{R}_{j,i}$ denotes the subset of rankings in $\mathbf{R}$ ($\mathcal{R}_{j,i} \subset \mathbf{R}$) in which c_j is the most preferred candidate still standing (i.e., that will count toward c_j's tally) at the start of round i (in which candidate c_i is eliminated). Constraint (4) ensures that the total number of votes cast in the election does not change as a result of the manipulation.

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