Rate Equilibria in WLANs with Block ACKs

Suong H. Nguyen*, Ihsan Ayyub Qazi†, Lachlan L. H. Andrew* and Hai L. Vu*

* Centre for Advanced Internet Architectures
Swinburne University of Technology, Australia
Email: {hsnguyen, hvu, landrew}@swin.edu.au

† Department of Computer Science, LUMS, Lahore, Pakistan
Email: ihsan.qazi@lums.edu.pk

Abstract—To achieve high system efficiency with increasing speeds, recent WiFi standards, such as IEEE 802.11e/n, allow burst transmissions with block acknowledgements, provided the initial packet is successfully received. Consequently, a user can sometimes improve its throughput by sending the initial packet at a lower rate than other users. We model such a system as a game. Our results show that the socially optimal strategy is to send the initial packet at a lower rate than the rest of the burst. Such a strategy results in a better Nash Equilibrium than using the same rate for the entire burst. Moreover, we show that using the rate that maximizes the per-packet throughput, as commonly done, can result in performance that is far from the social optimum.

I. INTRODUCTION

IEEE 802.11 wireless standards allow users to adapt to channel conditions by selecting either high, error-prone bit rates, or low, reliable bitrates. Since using a lower rate causes a delay to all stations but benefits only the sending station, nodes' incentives are not aligned and a game ensues: If stations send a single packet after successfully contending for the channel, then their optimal strategy is to transmit at a rate lower than socially optimal [1].

It was also shown in [1] that this mismatch disappears when stations can send multiple packets per contention, as allowed in 802.11e, provided that the duration for which a station can transmit per channel access is independent of its bit rate. However, as explained in Section III, if a channel error corrupts the first packet in a burst then the standards say the remainder of the burst is not transmitted. Consequently, a game remains.

In particular, the initial packet is different from the other packets in a burst since its successful transmission determines the transmission of the rest of the burst. Therefore, sending it at the "myopic" rate that maximizes the per-packet throughput [2], [3] may not necessarily optimize overall throughput. Hence, a station can use a lower rate for the initial packet to increase the probability of a burst transmission, as lower rates are more robust to channel losses. However, this can adversely affect the throughput achieved by other stations. In this paper, we ask, "*Should the first packet be allowed to be sent at a different rate from the rest of the burst?*". Ideally, users should use rates that maximize the system throughput, called the *Social Optimum* (SO). However, a system with selfish users often operates at the *Nash Equilibrium* (NE): a configuration where users have no incentive to change unilaterally [4], [5]. Therefore, mechanisms should be in place to achieve a desirable NE. To this end, we study two rate policies: *uniform* (U) in which users must use the same rate for the entire burst, and *independent* (I) in which users can use different rates for the initial packet and the rest of the burst.

In this paper, we propose a game-theoretic model to investigate the properties of the rate game in which users are allowed to send the initial packet with a different rate from the remaining packets in a burst. Our numerical results show that the social optimum strategy under both policies is to send the first packet at a lower rate than the myopic rate, the commonly used rate in practice [2], [3], and users can obtain significant throughput gains by doing so. The social optimum and NE under I are always better than those under U. Moreover, performance at the NE under I can even exceed the social optimum under U and the converse was not observed. This suggests that the first packet should be allowed to be sent at a different rate from the rest of the burst.

The rest of the paper is organized as follows. First, a brief description of the IEEE 802.11 protocol and game theory is provided in Section II. Then, we present a game theoretic model of the rate game in Section III, followed by Section IV which describes how the Nash equilibrium of the rate game is determined from the proposed model. Numerical results are provided in Section V. Finally, we offer concluding remarks in Section VI.

II. BACKGROUND

Here we will first briefly describe the Distributed Coordination Function (DCF) channel access mechanism with TXOP limit and block acknowledgement defined in the IEEE 802.11 standard [6]. Note that TXOP limit allows multiple packets to be transmitted per channel access. Then, we will provide some background about game theory.

A. 802.11 DCF

The DCF channel access mechanism enables users to contend for the common wireless channel using a carrier sense multiple access mechanism with collision avoidance (CSMA/CA). To reduce collisions, it employs both sensing

of the channel to detect channel activity and truncated binary exponential backoff (BEB) to randomize the start times of packet transmissions. When a packet arrives to an idle source, the source senses the channel for a period DIFS. If it is idle during this whole time, the packet is transmitted immediately. Otherwise, the source waits until the channel is continuously idle for DIFS, and then starts a backoff process. A backoff counter is initialized to a random integer uniformly distributed between 0 and (CW-1), where CW is the current contention window. For each new transmission, CW is initialized to CWmin and doubles after each unsuccessful transmission until it reaches CWmax, after which it remains constant until the packet is either successfully received or a retry limit is exceeded. The backoff counter is decreased by one at every idle slot time and frozen during periods of channel activity. When the backoff counter reaches zero, the source is allowed to transmit for a TXOP limit period of time, which may allow one or more packets to be transmitted. When a burst of multiple packets is sent per TXOP limit, we consider the scheme where an acknowledgment (ACK) is sent back from the receiver after a Short Inter-Frame Space (SIFS) for the first packet and the subsequent packets will be acknowledged in a block ACK frame after a Block ACK Request from the source. If an ACK is not received, the source increases CW as described above, and attempts again until the retry limit is reached. After receiving an ACK for a single-packet burst or a block ACK for a multiple-packet burst, the source performs a "post-backoff" process with contention window CWmin before being allowed to restart the above procedure. This prevents back-to-back packet transmission.

B. Game theory

Game theory is a collection of analytical tools designed to help us understand the phenomena that we observe when decision-makers interact [4]. More specifically, it provides a mathematical basis for the analysis of interactive decision-making processes.

A non-cooperative game usually consists of the following three components [4], [7], [8], [9]:

- a finite set of players,
- a set of possible actions for each player, and
- a payoff function for each player.

Players: These are the decision makers in the modeled scenario. In the wireless scenario, players are usually the nodes of the network.

Actions: An action is the "move" (or decision) a player makes. In the wireless scenario, actions can be transmission rates, backoff time or transmit power level [4].

Payoffs: For each strategy profile, each player receives a payoff, which represents the value of the outcome to the user. In particular, a payoff is a number assigned to each possible outcome through a utility function. A higher payoff represents a more desirable outcome [4]. In the wireless scenario, energy saving and throughput are some examples of players' payoff.

Nash Equilibrium: One of the goals of game theory is to predict what will happen when a game is played. The most common prediction of what will happen is called an "equilibrium". The most well-known equilibrium concept in game theory is the "Nash equilibrium" [4]. Recall that a Nash equilibrium is an action profile at which no player has any incentive for unilateral deviation.

Price of Anarchy (POA): This is defined as the ratio of throughput at the social optimum to that at the worst-case Nash Equilibrium.

III. RATE GAME MODEL

In this section, we first describe the structure of the game and then present the users' utility functions.

It will be shown in Section III-A that users need only decide the bit rate of the first packet in each burst, since, given the rate at which the first packet in a burst is sent, the optimal rate for the remaining packets is determined.

We define a rate game $\mathcal{G}$ as a triple (Ψ, A_i, S_i) where $\Psi = \{1, 2, \ldots, N\}$ is the set of users, the action set $A_i \subseteq [R_{\min}, R_{\max}]$ is the set of rates at which user i can send its first packet, and the payoff $S_i(R_{f,i}, R_{f,-i})$ is the throughput user i achieves when it uses rate $R_{f,i}$ and others use rates denoted by a vector $R_{f,-i}$, as calculated in the remaining part of this section. Note that the bitrate a user selects will determine the packet error rate, which is usually an increasing function of the bitrate[1] [3], [10], [11], [12].

A. Payoff function

As mentioned above, the payoff in the rate game is the throughput a user achieves. As in [13], in the context of 802.11e WLANs, the throughput of user u, S_u, is

$$S_u = \frac{b_u E[D_u]}{E[Y]} \qquad (1)$$

where b_u is the probability of a burst transmission in a given slot, $E[D_u]$ is the average number of bits delivered in a burst, and $E[Y]$ is the average slot duration. These are given by (4), (5) and (9) in the following model of 802.11.

B. 802.11 model

To describe the model, we first summarize notation and assumptions. Then, we present the fixed point equations central to this model. Finally, we calculate the components of S_u as shown in (1).

We assume stations are saturated, symmetric and use block ACKs [14]. (Note that it is straightforward to modify the model to take into account asymmetric stations. We speculate in this case that the qualitative properties will carry over but the quantitative ones will be different. However, the investigation of this is out of scope of this paper.)

We assume control packets such as ACKs are not lost due to channel errors[2], and a fractional number of packets

---

[1]In 802.11a/b/g, the packet error rate increases monotonically with the bitrate [10], [11]. However, this is not the case in WLAN standards, such as 802.11n and 802.11ac, that provide MIMO functionality and support spatial multiplexing and spatial diversity modes. While packet error rate increases monotonically with the bitrate within each mode, it does not *across* these modes [12].

[2]Note that ACKs are generally sent at a lower bitrate than data packets and thus the latter has a lower chance of getting lost due to channel error [15].

can be transmitted in a burst so that stations use the entire transmission opportunity (TXOP) duration, T_b, similar to [14].

A station i tries to obtain a TXOP at the beginning of an arbitrary slot with probability τ_i [15]. With this probability, it sends a packet at rate R_f bps, and awaits an ACK.

If an ACK is received, the station sends B_u packets for the remaining time in TXOP at a rate R_r bps: under U, $R_r = R_f$, whereas under I, R_r is equal to the "myopic" rate that maximizes the per-packet throughput, given by $R_M = \arg\max_R \frac{1 - P_e(R)}{L_u/R + K + Z}$ where L_u is the packet size, K is the physical layer overhead, G is an inter-frame gap, and $P_e(R)$ is the probability that a packet sent at rate R bps gets corrupted due to a channel error.

Stations do not back off when packets are lost after the initial ACK is received. This is modeled in (2), which shows that the collision probability of a user i depends on the packet error rate of only the first packet.

Stations can send the initial packet at different rates. For the first packet, station i uses a bit rate $R_{f,i}$ bps and the remaining $N-1$ stations use bit rates denoted by a vector $R_{f,-i}$. The backoff process is modeled as in [13]. Since the first packet can be lost due to a collision or an error, the transmission failure probabilities F_i and F_{-i} for the ith and each of the other $N-1$ stations, respectively, are given by

$$F_i = 1 - \prod_{j \neq i}(1 - \tau_j)\Big(1 - P_e(R_{f,i})\Big) \tag{2}$$

From [13], the attempt probability τ_u is

$$\tau_u = \frac{2}{1 + CW + F_u CW \sum_{j=0}^{m-1}(2F_u)^j} \tag{3}$$

where CW is the minimum contention window size, m is the maximum number of time a station doubles its contention window after a collision, and $u \in \{1, \ldots, N\}$.

Solving the fixed point of (2) and (3) gives τ_u, F_i, and F_{-i}. All terms in (1) can now be determined. The probability user u transmits a full burst is

$$b_u = \tau_u(1 - F_u). \tag{4}$$

The average number of bits sent in a burst is

$$E[D_u] = L_u(1 + B_u(1 - P_e(R_r))). \tag{5}$$

where B_u solves $T_b = B_u(L_r/R_r + K + Z) + L_{f,u}/R_{f,u} + T_{ack} + T_{bar} + T_{ba} + \text{DIFS} + 3\,\text{SIFS}$, and T_{ack}, T_{bar}, T_{ba}, DIFS and SIFS are the transmission time of an ACK frame, a Block ACK Request frame, a Block ACK frame, Distributed Inter-Frame Space, and Short Inter-Frame Space, respectively.

Substituting (4) and (5) into (1) gives

$$S_i = \frac{\tau_i(1 - F_i)\Big(1 + (H_2 - L_f/R_{f,i})H_3\Big)}{E[Y]} \tag{6}$$

where H_2 and H_3 are given by

$$H_2 = T_b - (T_{ack} + T_{bar} + T_{ba} + \text{DIFS} + 3\,\text{SIFS}) \tag{7}$$

$$H_3 = \frac{(1 - P_e(R_r))}{L_r/R_r + K + Z} \tag{8}$$

To find the average slot duration $E[Y]$, let σ be the duration of an idle slot, which occurs with probability $P_{idle} = \prod_{k=1}^{N}(1 - \tau_k)$.

Without loss of generality, players are indexed in non-increasing order of the first packet's duration. That is, $T_i \geq T_j$ for $i < j$.

Define

$$\begin{aligned} c_i = \tau_i\Bigg(&\prod_{j \neq i}(1 - \tau_j)P_e(R_{f,i}) \\ &+ \Big(1 - \prod_{k=i+1}^{N}(1 - \tau_k)\Big)\prod_{j=1}^{i-1}(1 - \tau_j)\Bigg) \end{aligned}$$

to be the probability of a collision/corruption involving only station i and/or stations $j > i$. This has duration

$$T_{c,i} = L_{f,i}/R_{f,i} + K + \text{EIFS}$$

Note that $\sigma < T_{c,i} < T_b$. Then

$$E[Y] = P_{idle}\sigma + T_b\Big(\sum_{i=1}^{N} b_i\Big) + \sum_{i=1}^{N} c_i T_{c,i}. \tag{9}$$

Note that the above model is related to the one presented in [14], but the latter does not account for scenarios in which stations can use different bitrates for the first packet and experience different packet error rates.

In summary, the proposed model allows the payoff S_i of a user i to be obtained given a rate vector $(R_{f,i}, R_{f,-i})$. In the next section, we will present how the Nash equilibrium of the rate game can be determined.

IV. NASH EQUILIBRIA SEARCH

In general, Nash equilibria of the rate game can be found by discretizing the rate space and then performing an exhaustive search on the discrete space to find the best response of every user at different strategy profiles of other users. Then, the Nash equilibria are the strategy profile in which the strategy of every user is its best response.

However, this method is computationally expensive, especially for large number of players. In the remainder of this section, we will show that the payoff function of the rate game is unimodal. By leveraging this property, the best response can be found more efficiently by using the golden section search [16] for continuous rate space or lattice search for discrete rate space.

Unimodal payoff function

Our numerical results, an example of which is shown in Fig. 1, show that the payoff of a given user i as a function of its first packet's rate $R_{f,i}$ is unimodal in the interval $[R_{min}, R^0]$ where R_0 is the minimum rate at which $P_e(R) = 1$.

The unimodal property of the payoff function is analytically shown for the non-atomic rate game presented as follows.
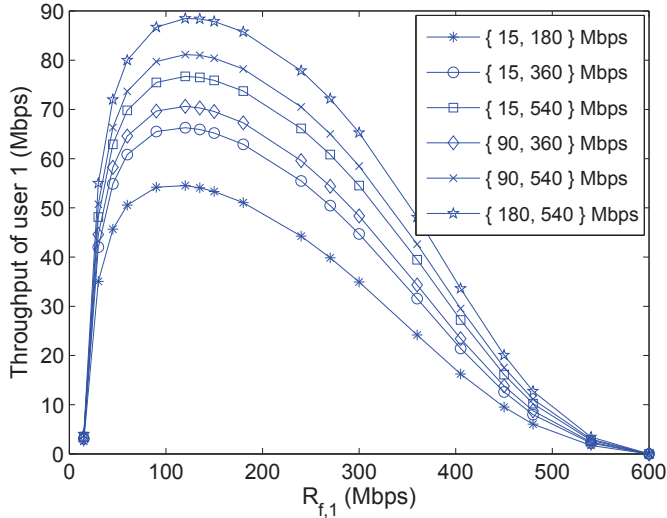
Fig. 1: Throughput of user 1 under the policy I as a function of the rate of its first packet ($R_{f,1}$), at different rates used by the remaining users ($R_{f,-1} = \{R_{f,2}, R_{f,3}\}$). ($N = 3$, $L_r = L_f = 800\,B$, A_i = set of data rates in 802.11n $\in [15, 600]\,Mbps$, $T_b = 0.5\,ms$, $P_e(R) = \min(1, A(e^{B \cdot R} - 1) + C)$ with $A = 0.1$, $B = 1.1/(A.R_{\max})$, $R_{\max} = 600\,Mbps$ and $C = 1\%$.)

1) Non-atomic rate game: Here we consider the non-atomic rate game, which is the rate game in which the number of players is large enough so that the single player has no influence on the outcome of the game but the aggregate behavior of a large set of players can change the outcome [17].

Besides, we also consider symmetric game, which is the game where players have the same action space, payoff function and packet error probability function.

Then, the non-atomic rate game G has the set of players Ψ being the unit interval $[0, 1]$ endowed with Lebesgue measure [17] and the payoff function (1) determined from the following fixed point model of failure probability and attempt probability. Moreover, for tractability, we consider the asymptotic case where retry limit m is infinite.

The failure probability (2) can be rewritten as follows.

$$F_i = 1 - G(\boldsymbol{\tau})(1 - P_e(R_{f,i})), \quad \forall i \in \Psi \qquad (10)$$

where $G(\boldsymbol{\tau})$ is a decreasing function of $\boldsymbol{\tau} = (\tau_j)_{j\in\Psi}$, which represents the probability that no other stations transmit in a given slot. Here we assume that for any set i of Lebesgue measure 0, $G(\boldsymbol{\tau}) = G(\tau_i; \tau_{-i})$ is independent of τ_i.

The packet corruption probability at bitrate R is

$$P_e(R) = \min(1, h(R)). \qquad (11)$$

Besides, the attempt probability is given by

$$\tau_i = \begin{cases} \dfrac{2}{CW}\dfrac{1 - 2F_i}{1 - F_i} & F_i < 1/2 \\ 0 & F_i \geq 1/2 \end{cases} \qquad (12)$$

which approximates (3) at $m = \infty$.

2) Payoff function in non-atomic rate game: The following theorem, proved in Appendix A, states that for the non-atomic rate game presented above, the payoff function is unimodal.

Theorem 1: Under the wireless model (6), (10), (11) with $h(R)$ being a convex function and (12), the payoff function S_i of each player in the non-atomic rate game under the policy I is unimodal.

Based on the numerical results in Figure 1 for the atomic game, and the above theorem for the non-atomic game, we conjecture that the payoff function for the atomic game is also unimodal. In light of that conjecture, the numerical results in the following section use a golden section search to find the Nash equilibrium.

V. NUMERICAL RESULTS

In this section, we will use the analytical model proposed in Section III to investigate system performance. In particular, we are interested in three performance measures: the price of anarchy, the *gain* given by the ratio of the throughput at SO to that at the myopic rate, and the bitrate of the first packet. Note that a rate vector R_f is a Nash equilibrium of $\mathcal{G}$ if for a user i

$$S_i(R_{f,i}, R_{f,-i}) \geq S_i(R'_{f,i}, R_{f,-i}), \qquad \forall R'_{f,i} \in A_i, \quad (13)$$

or a social optimum if it maximizes the system throughput:
$SO = \max_{R_f} \sum_{i=1}^{N} S_i(R_{f,i}, R_{f,-i})$.

Given a maximum bitrate $R_{\max}$, we consider the set of rates $A_i = \frac{R_{\max}}{100}\{1, 2, \dots, 100\}$ Mbps, $\forall i \in \Psi$, where $R_{\max} \in \{11, 54, 250, 600, 1000, 2500, 7000\}$ Mbps. The minimum and maximum values of $R_{\max}$ are based on the 802.11b and 802.11ad standards that support maximum bitrates of 11 Mbps and 7 Gbps, respectively [18]. We also consider cases in which A_i is the exact set of rates in the standards. We evaluate the model when the packet corruption probability at bitrate R is

$$P_e(R) = \min(1, A(e^{B \cdot R} - 1) + C) \qquad (14)$$

where $A = 0.1$, $B = \ln(1.1/A)/R_{\max}$, and $C \in \{1\%, 50\%\}$ as such error functions are frequently observed in practice [3]. Note that this error function generalizes the one used in [19].

We assume that packets can be fragmented to fit in a burst. We present results for both the U and I policies.

Adapting TXOP with the Bitrate: As the maximum bitrate supported by a system increases, users expect improvement in delay. Consequently, we decrease TXOP with the maximum bitrate $R_{\max}$ as

$$T(R_{\max}) = \frac{T_i}{(K \cdot R_{\max})^{3/4}},$$

where $T_i = 10$ ms is the TXOP used at 11 Mbps (which allows one packet transmission at the lowest bitrate of 1 Mbps) and $K = 1/(11\,\text{Mbps})$ is a scaling factor. With this adaptation, TXOP decreases at a slower rate than the increase rate in bitrate. This ensures that system efficiency continues to

improve as bitrates increase while reducing delay. Of course, we could improve the efficiency further at the cost of added delay.

A. Gain, POA, and Rate

Figure 2 shows the gain, POA, and the normalized rate (i.e. rate divided by the corresponding maximum rate) of the initial packet for $C=1\%$ at $N=2$. Observe that under the policy I, throughput gains are 11%, 17%, and 53% when the maximum bitrate is 600 Mbps, 1 Gbps and 7 Gbps, respectively. These rates correspond to the 802.11n, 802.11ac, and 802.11ad standards [18]. On more lossy links (e.g. when $C=50\%$), gains increase faster and are 43%, 57%, and 95%, respectively.

The gains improve because as we increase the bitrate, contention becomes more expensive leading to a higher penalty of losing a burst. The policy I adjusts by using a lower rate for the initial packet while continuing to use the myopic rate for the rest. With the policy U, since all packets use the same rate, users have less flexibility as lowering the rate for the first packet lowers the rate for the rest of the burst, which can degrade throughput. Consequently, users end up using a higher rates for the initial packet (and for the rest) and experience more losses. This leads to lower throughput compared to the policy I at the SO as well as at the NE. The gains are more at higher loss rates (e.g., when all rates experience at least 50% loss rate, as sometimes occurs in practice [3]) because successive packet losses lead to exponential backoffs, which increases the penalty of losing a burst further. Note that when the exact rates specified by the standards are used, we get similar results.

The POA generally remains less than 1.1 except when exact rates are used. With exact rates (as specified in the standards), the separation between rates is no longer uniform, which can either increase or decrease the POA. For instance, when $C=1\%$ and the maximum bitrate is 11 Mbps, POA under the policy I is 1.34, which is much higher than with uniform rates. However, when $C=50\%$, POA under the policy I is smaller with exact rates (see Figure 2).

Note that the same qualitative phenomena are observed in Figure 3 for higher number of users (e.g. $N=3$).

Besides, Figure 4 shows the gain and POA for $C=1\%$ and $C=50\%$ as a function of the number of stations N. As can be seen, when the number of stations increase, the gain decreases and POA increases for both policies U and I. Besides, the gain of the policy I is higher than that of the policy U while the POA of the policy I is smaller than that of the policy U for $C=1\%$. This shows the advantages of the policy I over the policy U.

B. Ordering of Rates and Throughput

From our results, we observe the following properties about the ordering of rates and throughput under different policies for uniform rate set.

We first define $R_{X,Y}$ as the rate at the operating point $X \in \{SO, NE\}$ under the policy $Y \in \{U, I\}$. Also recall that R_M is the myopic rate. Besides, let $S_{X,Y}$ be the system throughput at X under policy Y and S_M is the system throughput at the myopic rate.

The first observation is described as follows.

$$R_{SO,I} < R_{SO,U} < R_M,$$
$$S_{SO,I} > S_{SO,U} > S_M.$$

The rate ordering above is intuitive because R_M does not take into account the successful transmission probability of the subsequent packets in a burst while both $R_{SO,U}$ and $R_{SO,I}$ consider this. Therefore, R_M is the highest among three rates and S_M is the smallest. Moreover, different from the policy U, the policy I allows independence between the rate of the first packet and that of subsequent packets; therefore, $R_{SO,I}$ is smaller than $R_{SO,U}$ to obtain higher chance of a whole burst transmission while still guaranteeing $S_{SO,I} > S_{SO,U}$ due to having subsequent packets to be sent with higher rate.

Another observation is about the relation between the NE of the policy I and that of the policy U as shown below.

$$R_{NE,I} < R_{NE,U},$$
$$S_{NE,I} > S_{NE,U}.$$

In a game, rational players try to reduce the rate of the first packet to obtain higher chance to access channel; however, the rate should not be too low to affect the throughput of subsequent packets. Under the policy I, the fact that the rate of the first packet is independent of the rate of the subsequent packets allows stations to reduce the rate of the first packet to a smaller value than under the policy U. The higher chance of channel access as a result of lower rate of the first packet, together with using a higher rate for the subsequent packets, the throughput $S_{NE,I}$ under the policy I at the NE is higher.

The third observation is about the relation between the SO and the NE of the same policy, given by

$$R_{SO,Y} > R_{NE,Y},$$
$$S_{SO,Y} > S_{NE,Y}.$$

Different from the first two observations where the rate ordering is reverse of throughput ordering, the rate and throughput ordering are the same in this observation. The lower rate at NE comes from the incentive of rational players to choose a lower rate to gain higher channel access. This leads to the reduction in the remaining time for subsequent packets and longer collision duration, which causes inefficiency and hence makes throughput less.

Noticeably, we also observe that the throughput at the NE under I can be higher than at the SO under U.

We expect that the above ordering holds generally when users have symmetric channel conditions, but the formal proof remains an open problem.
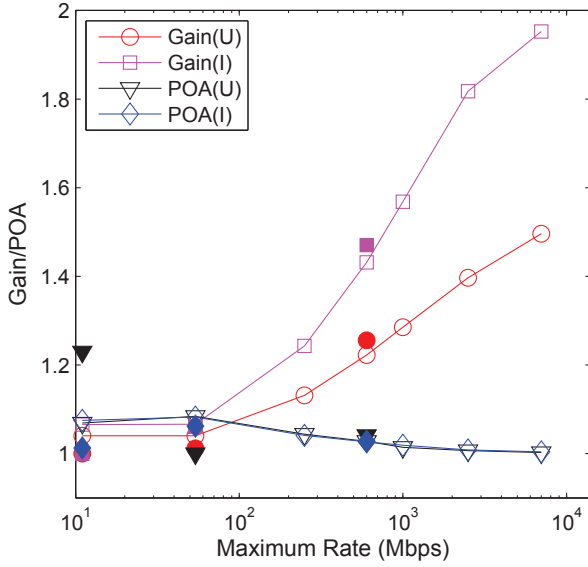
VI. CONCLUSION

In this paper, we developed a game model to study the rate at which the first packet should be sent when nodes employ burst transmissions with block acknowledgments. Our results show that stations should be allowed to send the first packet
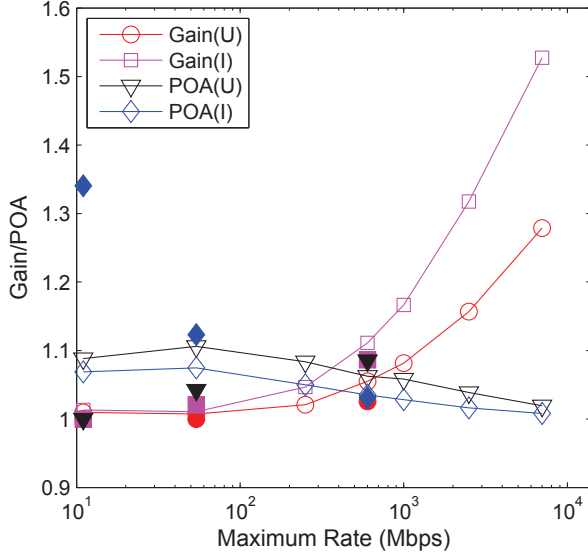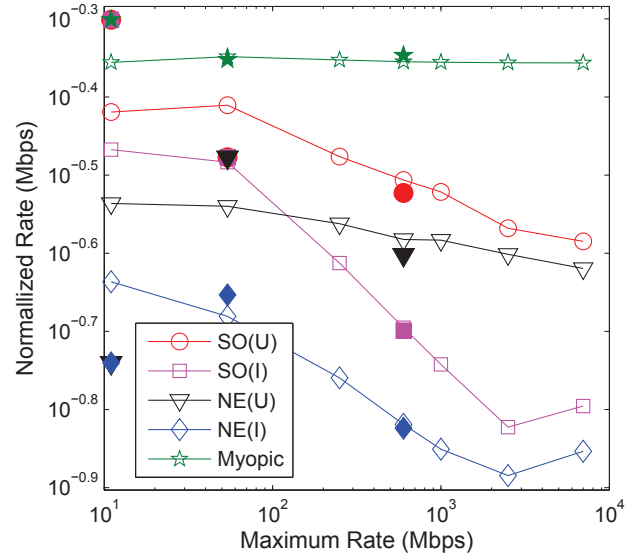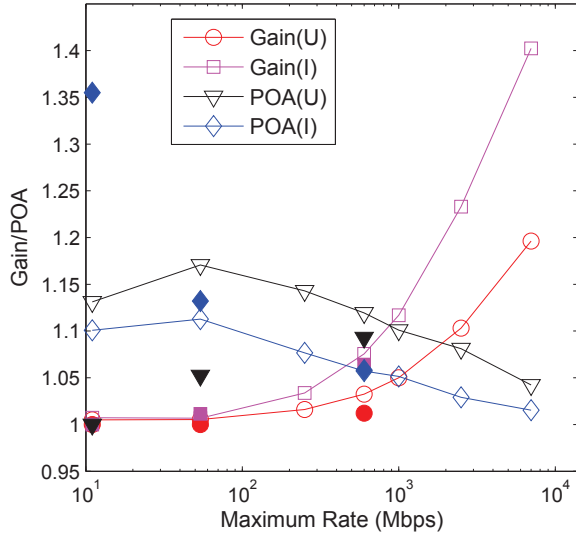
(a) C=1%
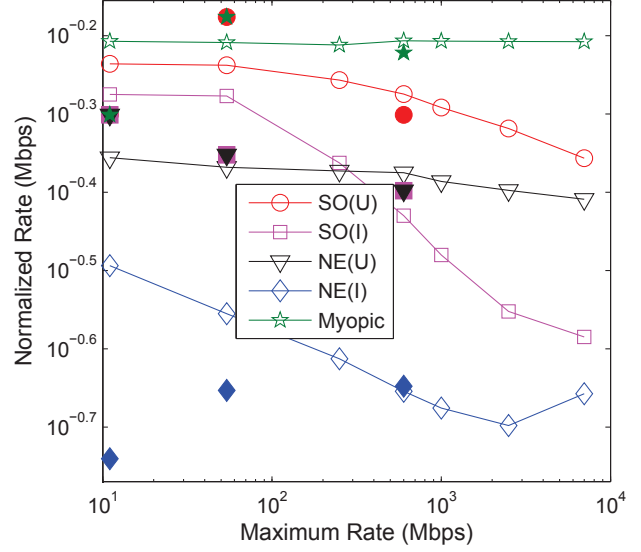
(b) C=1%

(c) C=50%

(d) C=50%

Fig. 2: Gain, POA, and the normalized rate (i.e. rate divided by the corresponding maximum rate) of the first packet under both policies U and I as a function of the maximum bitrate. The filled shapes correspond to cases when the exact set of rates specified in the 802.11b/g/n standards are used. ($N = 2$, $L_r = L_f = 1000\,B$.)

at different rate from the rest of the burst as this results in a better Nash Equilibrium than otherwise. Moreover, we show that rates that maximize the per-packet throughput can result in performance that is far from the social optimum. Since these are commonly used by rate adaptation algorithms, it would be useful to design mechanisms to achieve the social optimum. Furthermore, the analysis in this paper can be extended to study the effect of quantization error in the number of packets per burst and the change in network performance with asymmetric users.
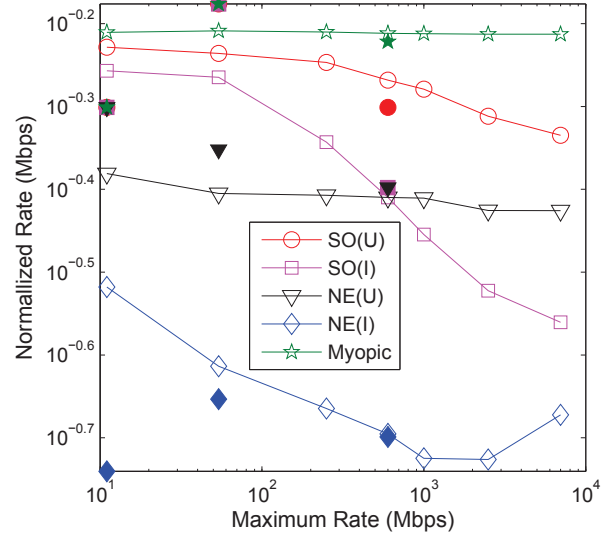
APPENDIX A
PROOF OF THEOREM 1

Proof: In the following, we will prove that S_i as a function of $R_{f,i}$ is unimodal before staying constant at 0 at high rate.
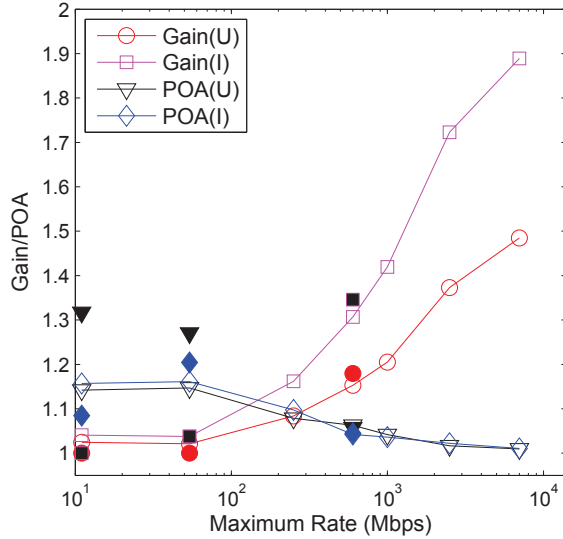
Note that a function $f(x)$ is a unimodal function on the interval $[a, b]$ if for some value m, it is monotonically increasing for $a \leq x \leq m$ and monotonically decreasing for $b \geq x \geq m$.
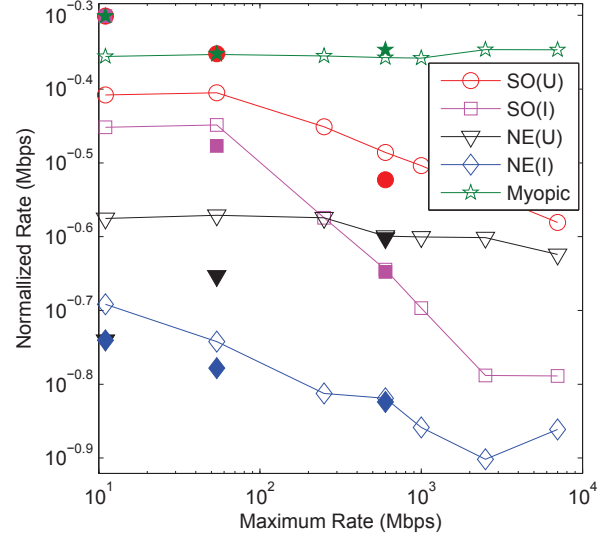
(a) C=1%

(b) C=1%

(c) C=50%

(d) C=50%

Fig. 3: Gain, POA, and the normalized rate (i.e. rate divided by the corresponding maximum rate) of the first packet under both policies U and I as a function of the maximum bitrate. The filled shapes correspond to cases when the exact set of rates specified in the 802.11b/g/n standards are used. ($N = 3$, $L_r = L_f = 1000 \, B$.)

Substituting (12) into (6) gives

$$S_i = \begin{cases} \dfrac{\tau_i(1 - F_i)\Big(1 + (H_2 - L_f/R_{f,i})H_3\Big)}{E[Y]} & F_i < 1/2 \\ 0 & F_i \geq 1/2 \end{cases} \tag{15}$$

Because F_i is a non-decreasing function of $R_{f,i}$, there exists a rate $R_{f,i}^0$ at which $F_i = 1/2$ such that $S_i = 0$ for $R_{f,i} \geq R_{f,i}^0$.

We will now show that S_i as a function of $R_{f,i}$ is unimodal on the interval $[R_{min}, R_{f,i}^0]$. This is equivalent to showing that $C_i = ln(S_i)$ is a strictly concave function for $R_{f,i} \in$ $[R_{min}, R_{f,i}^0]$. We prove this by showing that $\frac{dC_i}{dR_{f,i}}$ is a strictly decreasing function of $R_{f,i}$, which is shown as follows.

From (15), we have

$$C_i = \ln(\tau_i(1 - F_i)) + \ln(1 + (H_2 - L_f/R_{f,i})H_3) - \ln(E[Y]) \tag{16}$$

which is also given by

$$\begin{aligned} C_i = {} & \ln(\frac{2}{CW}(1 - 2F_i)) + \ln(1 + (H_2 - L_f/R_{f,i})H_3) \\ & - \ln(E[Y]) \end{aligned} \tag{17}$$

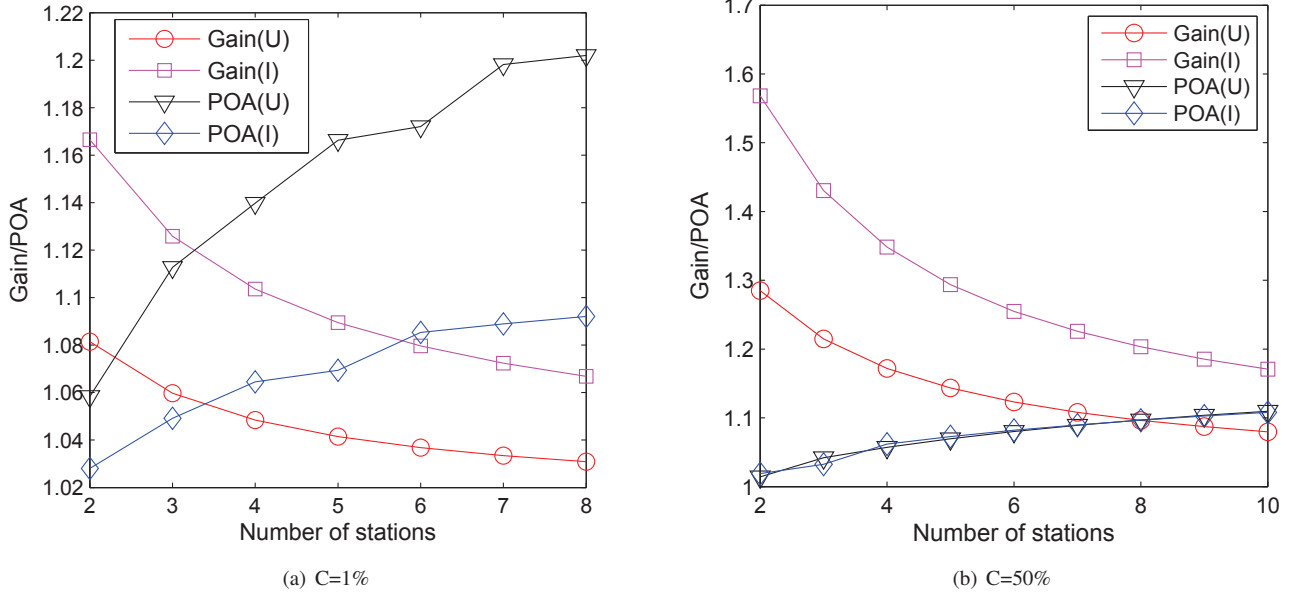due to $\tau_i(1 - F_i) = \frac{2}{CW}(1 - 2F_i)$ from (12).

(a) C=1%

(b) C=50%

Fig. 4: Gain and POA under both policies U and I as a function of the number of stations N. ($R_{\max} = 1\,Gbps$, $L_r = L_f = 1000\,B$, $T_b = 0.34\,ms$.)

Note that here we consider non-atomic games; hence, $E[Y]$ does not change when $R_{f,i}$ changes. Besides, H_2 and H_3 are also constants. Then, taking derivative of (17) gives

$$\frac{dC_i}{dR_{f,i}} = \frac{-2\frac{dF_i}{dR_{f,i}}}{1 - 2F_i} + \frac{L_f H_3 (1/R_{f,i}^2)}{1 + (H_2 - L_f/R_{f,i})H_3} \quad (18)$$

The first term of (18) is a decreasing function of $R_{f,i}$ because from (10),

- F_i is an increasing function of $R_{f,i}$ due to $P_e(R_{f,i})$ increasing with $R_{f,i}$.
- $\frac{dF_i}{dR_{f,i}}$ is also an increasing function of $R_{f,i}$. This is because $\frac{dF_i}{dR_{f,i}} = \frac{dP_e(R_{f,i})}{dR_{f,i}}$ increases with $R_{f,i}$, which comes from the hypothesis that $P_e(R_{f,i})$ is a convex function of $R_{f,i}$.

Moreover, it is clear that the second term of (18) is a strictly decreasing function of $R_{f,i}$.

Therefore, it can be concluded that for $R_{f,i} \in [R_{min}, R_{f,i}^0]$, $\frac{dC_i}{dR_{f,i}}$ is a strictly decreasing function of $R_{f,i}$, which means that C_i is a strictly concave function of $R_{f,i}$. This implies that S_i is a unimodal function. ∎

REFERENCES

[1] Godfrey Tan and John Guttag, "The 802.11 MAC protocol leads to inefficient equilibria", in *IEEE INFOCOM*, 2005.
[2] Starsky H. Y. Wong, Hao Yang, Songwu Lu, and Vaduvur Bharghavan, "Robust rate adaptation for 802.11 wireless networks", in *MobiCom'06*, 2006.
[3] John C. Bicket, "Bit-rate selection in wireless networks", Master's thesis, Massachusetts Institute of Technology, Feb 2005.
[4] A. MacKenzie and L. A. DaSilva, *Game Theory for Wireless Engineers*, Morgan & Claypool Publishers, 2006.
[5] Y. Xiao, X. Shan, and Y. Ren, "Game theory models for IEEE 802.11 DCF in wireless ad hoc networks", *IEEE Communications Magazine*, vol. 43, no. 3, pp. S22–S26, 2005.
[6] "IEEE standard for information technology - telecommunications and information exchange between systems - local and metropolitan area networks - specific requirements - part 11: Wireless LAN medium access control (MAC) and physical layer (PHY) specifications", 2012.
[7] N. Nisan, T. Roughgarden, E. Tardos, and V. V. Vazirani, *Algorithmic Game Theory*, Cambridge University Press, 2007.
[8] P. K. Dutta, *Strategies and Games: Theory and Practice*, MIT Press, 2nd edition, 1999.
[9] Z. Han, D. Niyato, W. Saad, T. Basar, and A. Hjrungnes, *Game Theory in Wireless and Communication Networks: Theory, Models, and Applications*, Cambridge University Press, 2012.
[10] Mythili Vutukuru, Hari Balakrishnan, and Kyle Jamieson, "Cross-layer wireless bit rate adaptation", in *ACM SIGCOMM 2009*.
[11] Starsky H. Y. Wong, Hao Yang, Songwu Lu, and Vaduvur Bhargha-van, "Robust rate adaptation for 802.11 wireless networks", in *ACM Mobicom 2006*.
[12] Ioannis Pefkianakis, Yun Hu, Starsky H.Y. Wong, Hao Yang, and Songwu Lu, "Mimo rate adaptation in 802.11n wireless networks", in *ACM Mobicom 2010*.
[13] G. Bianchi, "Performance analysis of the IEEE 802.11 distributed coordination function", *IEEE Journal on Selected Areas in Communications*, vol. 18, no. 3, pp. 535 –547, March 2000.
[14] Hyewon Lee, Ilenia Tinnirello, Jeonggyun Yu, and Sunghyun Choi, "A performance analysis of block ACK scheme for IEEE 802.11e networks", *Comput. Netw.*, vol. 54, pp. 2468–2481, October 2010.
[15] "IEEE Std., IEEE 802.11e, Part 11: Wireless LAN Medium Access Control (MAC) and Physical Layer (PHY) Specifications: Medium Access Control Quality of Service Enhancements", September 2005.
[16] S. Zlobec, *Stable Parametric Programming*, Kluwer Academic Publishers, 2001.
[17] D. Schmeidler, "Equilibrium points of nonatomic games", *Journal of Statistical Physics*, vol. 7, pp. 295–300, 1973.
[18] "Toward a gigabit Wi-Fi Nirvana: 802.11ac and 802.11ad", http://www.networkworld.com/news/tech/2011/021411-gigabit-wifi.html.
[19] P. Chaporkar, A. Proutiere, and B. Radunoviac, "Rate adaptation games in wireless LANs: Nash equilibrium and price of anarchy", in *IEEE INFOCOM*, 2010.