Large System Performance of Second-Order Linear Multistage CDMA Receivers

Louis G. F. Trichard, *Member, IEEE*, Jamie S. Evans, *Member, IEEE*, and Iain B. Collings, *Senior Member, IEEE*

***Abstract*—In this paper, we analyze the performance of a second-order linear multistage multiuser code-division multiple-access receiver. The receiver's filtered output is designed to converge to that of the linear minimum mean-squared error solution as the number of stages increase. Our analysis is based on a related second-order stationary iterative solution method. We derive the large system output signal to interference-plus-noise ratio for each stage. We use this result to perform a numerical optimization with respect to the two second-order parameters of our receiver. Within this iterative framework, we can achieve performance extremely close to the optimal linear multistage multiuser receiver.**

***Index Terms*—Large system analysis, linear iterative multiuser detection, linear minimum mean-squared error (MMSE) detection, random spreading.**

I. Introduction

Linear iterative solution methods provide a host of useful results for designing linear iterative multiuser receiver structures for code-division multiple-access (CDMA) systems [1]–[3]. These receiver structures are of great interest due to their ability to asymptotically attain linear minimum mean-squared error (LMMSE) performance. The popularity of the LMMSE and related receivers is due to their vastly reduced complexity compared with the optimal maximum-likelihood receiver [4], and to their ability to suppress multiple-access interference, especially compared with the conventional single-user matched filter (SUMF) receiver [5]. Unfortunately, even the direct LMMSE receiver can often be too computationally demanding, paving the way for linear multistage multiuser receivers with even less computational complexity. Our paper focuses on a second-order linear multistage multiuser (MSMU) receiver.

Linear MSMU receivers fall within the wider class of multistage interference cancellation (IC) multiuser receivers, for which there are numerous linear and nonlinear implementations [2], [3], [6]–[19]. They are of particular interest even though nonlinear MSMU receivers have been shown to perform better in certain situations. This is due to many nonlinear proposals employing the LMMSE receiver as a frontend filter (for examples, see [20]–[22]), where efficient implementations of the LMMSE receiver are desirable. As an example, the direct LMMSE receiver in a K-user-long spreading sequence CDMA system, (where adaptive receivers are not applicable) with processing gain N has a typical computational cost of $\mathcal{O}(K^3)$ or $\mathcal{O}(N^3)$ per symbol.[1] In contrast, linear multistage IC receivers will have a computational cost of $\mathcal{O}(mK^2)$ or $\mathcal{O}(mN^2)$ per symbol, where m is the number of stages. We are interested in finding linear MSMU receivers which attain near-LMMSE performance for $m \ll K$.

In this paper, we make use of the theory of linear iterative solution methods which are commonly used to numerically solve systems of linear equations [1], [2]. In particular, we consider the second-order stationary linear iterative solution method [1], [2]. This is an important extension on our previous work which considered a first-order stationary linear iterative solution method (for more details, see [3], [19], and [23]). It turns out that in the second-order case, the data estimates at the output of each stage of the receiver converge significantly faster to the LMMSE solution when compared with data estimates from the first-order receiver of [3] and [23] as the number of stages m increase (see also [24]).

In order to analyze the performance of the second-order linear MSMU receiver, we will utilize so-called large system analysis. We define a large system by taking the CDMA system parameters N and K to infinity but keeping their ratio ($\alpha = K/N$) held fixed. By doing this, a range of useful mathematical results can be applied (see [25] and [26]). In related work, large system analysis has previously been used to determine the signal-to-interference plus noise ratio (SIR) of linear CDMA receivers including the conventional SUMF, the decorrelator and the direct LMMSE receiver [27], [28]. As well, there also exist many other useful analytical techniques such as random matrix analysis, (for more details see [29] and [30]), which can be useful in the analysis of multiuser receivers [31]. This paper focuses on large system analysis.

The main contribution in this paper is the use of large system analysis to derive an expression for the large system SIR of the second-order linear MSMU receiver considered. We show that the large system SIR expression only depends on the system loading (α), the signal-to-noise ratio (SNR), the two second-order receiver parameters, and the number of stages m. Importantly, the large system SIR is independent of the realizations of

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L. G. F. Trichard is with the Kohno Laboratory, Division of Physics, Electrical, and Computer Engineering, Graduate School of Engineering, Yokohama National University, Yokohama, 240-8501, Japan (e-mail: louis@kohnolab.dnj.ynu.ac.jp).

J. S. Evans is with the ARC Special Research Centre for Ultra-Broadband Information Networks, The Department of Electrical and Electronic Engineering, University of Melbourne, Parkville, Victoria 3010, Australia (e-mail: jse@ee.mu.oz.au).

I. B. Collings is with the Telecommunications Laboratory, School of Electrical and Information Engineering, University of Sydney, Sydney NSW 2006, Australia (e-mail: iain@ee.usyd.edu.au).

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[1]If K users are detected jointly, then the per-user, per-symbol cost is a factor of K less for both the LMMSE receiver and linear MSMU receivers.

the signature sequences. In addition, we also give empirical evidence which suggests that for any particular realization the fluctuations around the large system SIR are proportional to $1/N$, (a similar result has recently been proved for the decorrelator and the LMMSE receiver [32]).

Further, we numerically maximize the large system SIR of the second-order linear MSMU receiver with respect to the two second-order receiver parameters. We will show the surprising result, namely, that the optimized second-order linear MSMU receiver can, for a practical range of stages, achieve performance with order close to the linear MSMU receiver given in [7] and [33] often called the optimal linear MSMU receiver. This optimal linear MSMU receiver is "optimal" in the sense of maximizing the total large system SIR over all users for a particular stage m [33]. In contrast to the two parameters of our second-order receiver, it requires an mth-order optimization at each stage m.

This paper is organized as follows. The CDMA signal model is described in Section II. We describe the optimal linear MSMU receiver followed by the second-order linear MSMU receiver in Sections III and IV. We derive the large system SIR for our receiver in Section V. We show that the second-order linear MSMU receiver can attain performance close to that of the optimal linear MSMU receiver in Section VI. In Section VII, we briefly discuss the effects of unequal power users. Finally, we conclude with a summary in Section VIII.

Note: As discussed above, this paper focuses on linear receiver structures. For the remainder of this paper, we omit the word "linear" when discussing receivers so as to ease readability. All receivers are linear unless otherwise stated.

II. Signal Model

In this paper, we consider a K user synchronous direct sequence code-division multiple-access (DS-CDMA) communication system with a processing gain of N. We shall consider the standard real baseband signal model (as used in [13]).

The N-dimensional chip matched filter vector for each symbol interval is given by

$$\mathbf{r} = \sum_{k=1}^{K} \sqrt{P_k} b_k \mathbf{s}_k + \mathbf{n} \tag{1}$$

where b_k is the data bit of user k, P_k is the power of user k, $\mathbf{s}_k$ is the N-dimensional spreading sequence of user k, and $\mathbf{n}$ is additive white Gaussian noise (AWGN) with zero mean and covariance $\sigma^2\mathbf{I}$. The $N \times K$ spreading sequence matrix is $\mathbf{S} = [\mathbf{s}_1 \ldots \mathbf{s}_K]$. We shall define the $N \times (K-1)$ spreading sequence matrix excluding the spreading sequence of user k as $\mathbf{S}_k = [\mathbf{s}_1, \ldots, \mathbf{s}_{k-1}, \mathbf{s}_{k+1}, \ldots, \mathbf{s}_K]$.

We assume that all users employ (baseband) binary antipodal modulation. We shall also use a random spreading model where the elements of $\mathbf{S}$ are independent and identically distributed (i.i.d.) random variables taking values of $\pm 1/\sqrt{N}$ with an equal probability. The random spreading assumption is needed for the large system analysis, which follows in Section V. We also assume that the spreading sequences are known at the receiver, the received user powers are equal with common power $P_k = P$ (for $k = 1, \ldots, K$), and we assume perfect estimation of the received user powers and the noise variance σ^2. We refer to P/σ^2 as the SNR.

At the receiver, a linear filter for the kth user is characterized by the N-dimensional filter coefficient vector $\mathbf{c}_k \in \Re^N$. The soft data estimate at the output of this filter for user k is

$$\hat{b}_k = \mathbf{c}_k^T \mathbf{r}. \tag{2}$$

The SIR at the output of the linear filter $\mathbf{c}_k$ is

$$\mathrm{SIR}_k^{(N)} = \frac{P(\mathbf{c}_k^T \mathbf{s}_k)^2}{\mathbf{c}_k^T (P\mathbf{S}_k\mathbf{S}_k^T + \sigma^2 \mathbf{I})\mathbf{c}_k}. \tag{3}$$

Note: This is a random variable which is dependent on the realizations of the spreading sequences. The superscript $^{(N)}$ indicates the dependence on the processing gain.

In this paper, we will derive an expression for the second-order MSMU receiver's large system SIR. We shall be comparing the large system SIR performance of the second-order MSMU receiver with that of the optimal MSMU receiver of (4) and [33]. In order to do this, we shall first describe the optimal MSMU receiver in the following section.

III. Optimal MSMU Receiver

The general MSMU receiver for K users and stage m can, after the mth stage, be expressed in the form [7]

$$\mathbf{C}_m = \mathbf{S} \sum_{i=0}^{m} a_i \left(\mathbf{S}^T\mathbf{S}\right)^i \tag{4}$$

where $\mathbf{C}_m = [\mathbf{c}_{1,m}, \ldots, \mathbf{c}_{K,m}]$ and a_i are the polynomial coefficients. The soft data estimate for K users at the output of stage m is

$$\hat{\mathbf{b}}_m = \mathbf{C}_m^T \mathbf{r} \tag{5}$$

where $\hat{\mathbf{b}}_m = [\hat{b}_{0,m}, \ldots, \hat{b}_{K,m}]^T$.

When the polynomial coefficients a_i are chosen so as to minimize the mean squared error $E[(\mathbf{b} - \hat{\mathbf{b}}_m)^2]$ for stage m the receiver in (4) is also known as the constrained LMMSE receiver [7]. As the number of stages increases, the output of this receiver can be shown to converge to the LMMSE solution. In a finite-sized system, only $K-1$ stages are needed to attain LMMSE performance; however, the computational load is the same as the direct LMMSE receiver. This MSMU receiver has been analyzed in [33]; however, in that case, the coefficients a_i were chosen to maximize the total large system SIR over all users for a particular stage m. It is for this receiver that we reserve the term optimal MSMU receiver.

In related work, a MSMU receiver which is based on the steepest descent method [34] also has m polynomial coefficients for stage m, which in [34] are called partial cancellation factors. The calculation of the m partial cancellation factors involves an m-dimensional matrix inversion and a sorting routine ensuring a decreasing monotonic mean squared error at the output of each stage. By appropriately optimizing this MSMU receiver, the same performance as the optimal MSMU receiver can be achieved. In fact, it can also be shown that the MSMU receiver of [34] is similar in form and performance to an MSMU receiver based on a quite general first-order nonstationary linear iterative solution method (for more details, see [1]–[3]).

Finally, we recall that these multistage receivers ([7], [33], [34], and the second-order linear MSMU receiver we are considering here) are suitable techniques which avoid the K (or N)-dimensional matrix inversion of the LMMSE receiver. For the receivers of [7], [33], and [34], the calculation of the polynomial coefficients requires an optimization over m parameters for an m stage receiver. In contrast, the second-order linear MSMU receiver is based on the second-order *stationary* linear iterative solution method where only two parameters are needed to ensure fast convergence to the LMMSE solution. We will show that the second-order linear MSMU receiver, when its two parameters are chosen to maximize the large system SIR, can achieve performance close to the optimal MSMU receiver for a practical range of stages.

IV. Second-Order MSMU Receiver Structure

The second-order MSMU receiver considered in this paper is based on the general second-order stationary linear iterative solution method of [1]. This receiver has two parameters and uses data estimates from the preceding two stages to estimate the data for the current stage. Our linear iterative receiver structure can be efficiently implemented in a recursive fashion.

We start by considering the N-dimensional direct LMMSE receiver for user k

$$\mathbf{c}_k = P^{-1/2}\left(\mathbf{S}\mathbf{S}^T + \frac{\sigma^2}{P}\,\mathbf{I}\right)^{-1}\mathbf{s}_k. \tag{6}$$

This receiver involves an $N \times N$ matrix inversion. Using the matrix-inversion lemma [13] and substituting into (6), we have the direct LMMSE data estimate

$$\hat{b}_k = \mathbf{c}_k^T\mathbf{r} = \xi_k P^{-1/2}\mathbf{s}_k^T\left(\mathbf{S}_k\mathbf{S}_k^T + \frac{\sigma^2}{P}\,\mathbf{I}\right)^{-1}\mathbf{r} \tag{7}$$

where the direct LMMSE receiver is now given by $\mathbf{c}_k = \xi_k P^{-1/2}(\mathbf{S}_k\mathbf{S}_k^T + (\sigma^2/P)\,\mathbf{I})^{-1}\mathbf{s}_k$ and $\xi_k = 1/(1+\mathbf{s}_k^T(\mathbf{S}_k\mathbf{S}_k^T + (\sigma^2/P)\,\mathbf{I})^{-1}\mathbf{s}_k)$. Now, by substituting the direct LMMSE receiver, $\mathbf{c}_k$ given in (7), into (3), the SIR for user k is

$$\mathrm{SIR}_k^{(N)} = \mathbf{s}_k^T\left(\mathbf{S}_k\mathbf{S}_k^T + \frac{\sigma^2}{P}\,\mathbf{I}\right)^{-1}\mathbf{s}_k. \tag{8}$$

In this paper, we focus our attention on MSMU receiver implementations which are computationally less intensive than the direct LMMSE receiver in (6) and (7), but which achieve near-LMMSE performance. The main computational cost in implementing the LMMSE receiver, $\mathbf{c}_k$ given in (7), comes from the inversion of the $N \times N$ matrix $(\mathbf{S}_k\mathbf{S}_k^T + (\sigma^2/P)\,\mathbf{I})$.

Observe from (7) that the LMMSE estimate is given by $\hat{b}_k = P^{-1/2}\mathbf{s}_k^T\mathbf{x}_k$, where $\mathbf{x}_k$ is the solution to the linear equation

$$\left(\mathbf{S}_k\mathbf{S}_k^T + \frac{\sigma^2}{P}\,\mathbf{I}\right)\mathbf{x}_k = \mathbf{r}. \tag{9}$$

The second-order stationary linear iterative solution method of [1] is a technique for efficiently solving (9). Successive estimates for $\mathbf{x}_k$ are obtained from the following iteration

$$\mathbf{x}_{k,m} = (\kappa\mathbf{I} - \beta\mathbf{Z}_k)\mathbf{x}_{k,m-1} + (1-\kappa)\mathbf{x}_{k,m-2} + \beta\mathbf{r} \tag{10}$$

where m is the number of iterations (or stages), $\mathbf{Z}_k = \mathbf{S}_k\mathbf{S}_k^T + (\sigma^2/P)\,\mathbf{I}$, and $(\kappa,\,\beta)$ are real valued parameters which we call the second-order parameters. The iteration is initialized with $\mathbf{x}_{k,\,0} = \beta\mathbf{r}$ and $\mathbf{x}_{k,\,-1} = \mathbf{0}$. From [1], we know that this linear iterative solution method will converge to the LMMSE receiver for user k, given in (7), as the number of stages increase provided that the pair $(\kappa,\,\beta)$ are in the range $1 < \kappa \le 2$ and $0 < \beta < 2\kappa/(\lambda_{\max} + (\sigma^2/P))$, where $\lambda_{\max}$ is the maximum eigenvalue of $\mathbf{S}_k\mathbf{S}_k^T$. Similar convergence properties exist for other types of linear and nonlinear interference cancellation receivers, some of which may be found in [2], [3], [15], and [17].

The second-order expression of (10) can be rewritten in a state space form, giving

$$\begin{bmatrix}\mathbf{x}_{k,\,m}\\ \mathbf{x}_{k,\,m-1}\end{bmatrix} = \begin{bmatrix}(\kappa\mathbf{I}-\beta\mathbf{Z}_k) & (1-\kappa)\mathbf{I}\\ \mathbf{I} & \mathbf{0}\end{bmatrix}\begin{bmatrix}\mathbf{x}_{k,\,m-1}\\ \mathbf{x}_{k,\,m-2}\end{bmatrix} + \beta\begin{bmatrix}\mathbf{r}\\ \mathbf{0}\end{bmatrix}. \tag{11}$$

The data estimate for user k and stage m at the output of the second-order MSMU receiver is

$$\hat{b}_k = \mathbf{s}^T\mathbf{x}_{k,\,m} = \mathbf{c}_{k,\,m}^T\mathbf{r} \tag{12}$$

where $\mathbf{c}_{k,\,m}$ is the linear filter coefficient vector which characterizes the second-order MSMU receiver. This linear filter coefficient vector can be written in direct form for user k and stage m as

$$\mathbf{c}_{k,\,m} = \beta\sum_{i=0}^{m}\mathbf{M}_k(i)\mathbf{s}_k \tag{13}$$

where $\mathbf{M}_k(i)$ is derived from (11) and is given by the following recursion:

$$\begin{aligned}\mathbf{M}_k(0) &= \mathbf{I}\\ \mathbf{M}_k(1) &= (\kappa\mathbf{I}-\beta\mathbf{Z}_k)\\ \mathbf{M}_k(i) &= (\kappa\mathbf{I}-\beta\mathbf{Z}_k)\mathbf{M}_k(i-1) + (1-\kappa)\mathbf{M}_k(i-2).\end{aligned} \tag{14}$$

In summary, we have now defined the N-dimensional chip rate filter coefficients of the second-order MSMU receiver for stage m and user k.

Now if there were a straightforward relationship between $(\kappa,\,\beta)$ and the polynomial coefficients $(a_0,\,\ldots,\,a_m)$ of the linear multistage receiver of (4) [7], then we could simply substitute for $(a_0,\,\ldots,\,a_m)$ into the SIR expression of [33] and [35], and we would have the SIR for our second-order MSMU receiver. However, no such closed form expression exists. In the following section, we directly derive the large system SIR of our second-order MSMU receiver.

V. Analysis of the Large System SIR

In this section, we use large system analysis to analyze the performance of the second-order MSMU receiver. In a standard approach, we define a large system by taking the CDMA system parameters N and K to infinity but keeping their ratio held fixed. By substituting (13) and (14) into (3), it can be seen that the key terms of interest in our analysis are random variables of the form $\mathbf{s}_k^T(\mathbf{S}_k\mathbf{S}_k^T)^i\mathbf{s}_k$ for $0 \le i \le 2m+1$. Consequently, we present the following lemmas.

Lemma 1: If $N \to \infty$ with $\alpha = K/N$ held fixed, the random variable $\mathbf{s}_k^T(\mathbf{S}_k\mathbf{S}_k^T)^i\mathbf{s}_k$ converges in probability to the deterministic moment $\psi_i(\alpha)$ given as follows:

$$\mathbf{s}_k^T(\mathbf{S}_k\mathbf{S}_k^T)^i\mathbf{s}_k \xrightarrow{p} \psi_i(\alpha) = \int \lambda^i dG(\lambda)$$

where $G(\lambda)$ is the limiting empirical distribution function of the eigenvalues of $\mathbf{S}_k\mathbf{S}_k^T$ (see [25], [28], and [36]). The ith moment of the limiting empirical distribution function $\psi_i(\alpha)$ can be calculated recursively as

$$\psi_i(\alpha) = \frac{1}{i+1}\left[(2i-1)(1+\alpha)\psi_{i-1}(\alpha) - (i-2)(1-\alpha)^2\psi_{i-2}(\alpha)\right] \quad (15)$$

where $\psi_0(\alpha) = 1$ and $\psi_1(\alpha) = \alpha$.

Proof: See [3] and [36].

Lemma 2: The direct form of the matrix recursion of $\mathbf{M}_k(i)$ given in (14) is

$$\mathbf{M}_k(i) = \sum_{j=0}^{i} a_{j,i}(\sigma^2, P, \kappa, \beta)(\mathbf{S}_k\mathbf{S}_k^T)^j \quad (16)$$

where $a_{j,i}(\sigma^2, P, \kappa, \beta) \equiv a_{j,i}$ can be calculated recursively as

$$\begin{aligned} a_{0,0} &= 1 \\ a_{j,i} &= \left(\kappa - \beta\frac{\sigma^2}{P}\right)a_{j,i-1} + (1-\kappa)a_{j,i-2} - \beta a_{j-1,i-1} \end{aligned} \quad (17)$$

with $a_{j,i} \triangleq 0$ if $j < 0$, $i < 0$ or $i - j < 0$.

Proof: Substitute (16) in (14), and (17) follows directly.

The following theorem presents the main theoretical result of this paper, namely, an expression for the large system SIR of our second-order MSMU receiver.

Theorem 1: Let $N, K \to \infty$, with $0 < \alpha = (K/N) < \infty$ held fixed. Then, the SIR of the mth stage of the second-order MSMU receiver converges in probability to a deterministic scalar SIR_m given by

$$\mathrm{SIR}_{k,m}^{(N)} \xrightarrow{p} \mathrm{SIR}_m = \frac{\left[\sum_{i=0}^{m} f_i\left(\alpha, \sigma^2, P, \kappa, \beta\right)\right]^2}{\sum_{i=0}^{m}\sum_{j=0}^{m} g_{i,j}\left(\alpha, \sigma^2, P, \kappa, \beta\right)} \quad (18)$$

where

$$f_i(\alpha, \sigma^2, P, \kappa, \beta) = \sum_{j=0}^{i} a_{j,i}\psi_j(\alpha) \quad (19)$$

$$g_{i,j}(\alpha, \sigma^2, P, \kappa, \beta) = \sum_{k=0}^{i}\sum_{l=0}^{j} a_{k,i}a_{l,j} \cdot \left[\psi_{k+l+1}(\alpha) + \frac{\sigma^2}{P}\psi_{k+l}(\alpha)\right] \quad (20)$$

where the moment $\psi_j(\alpha)$ is given in Lemma 1 and $a_{k,i}(\sigma^2, P, \kappa, \beta) \equiv a_{k,i}$ is given in Lemma 2.

Proof of Theorem 1: Let $\mathrm{SIR}_{k,m}^{(N)}$ be the SIR for user k at stage m of the second-order MSMU receiver. Note, that $\mathrm{SIR}_{k,m}^{(N)}$ is a random variable due to the random spreading sequence model assumed for the large system analysis. Substituting (13) in (3) gives

$$\mathrm{SIR}_{k,m}^{(N)} = \frac{\left(\mathbf{s}_k^T\sum_{i=0}^{m}\mathbf{M}_k(i)\mathbf{s}_k\right)^2}{\mathbf{s}_k^T\sum_{i=0}^{m}\sum_{j=0}^{m}\mathbf{M}_k(i)\mathbf{M}_k(j)\left(\mathbf{S}_k\mathbf{S}_k^T + \frac{\sigma^2}{P}\mathbf{I}\right)\mathbf{s}_k}$$

where $\mathbf{M}_k(i)$ is an $N \times N$ matrix given in direct form in (16) of Lemma 2.

Now, consider the numerator of $\mathrm{SIR}_{k,m}^{(N)}$ and define

$$\eta_{k,m}^{(N)} = \mathbf{s}_k^T\sum_{i=0}^{m}\mathbf{M}_k(i)\mathbf{s}_k = \sum_{i=0}^{m}\sum_{j=0}^{i} a_{j,i}\mathbf{s}_k^T(\mathbf{S}_k\mathbf{S}_k^T)^j\mathbf{s}_k$$

where $a_{j,i}(\sigma^2, P, \kappa, \beta) \equiv a_{j,i}$. The limit, as $N \to \infty$ with the ratio $\alpha = K/N$ held fixed shall be taken. From Lemma 1 it follows that:

$$\eta_{k,m}^{(N)} \xrightarrow{p} \eta_m = \sum_{i=0}^{m} f_i(\alpha, \sigma^2, P, \kappa, \beta) \quad (21)$$

for $0 \le \psi_j(\alpha) < \infty$, where $f_i(\alpha, \sigma^2, P, \kappa, \beta)$ is given in (19). We call η_m the large system limit of the sequence of random variables (r.v.s.) $\eta_{k,m}^{(N)}$.

Now, consider the denominator of $\mathrm{SIR}_{k,m}^{(N)}$ and define

$$\begin{aligned} \nu_{k,m}^{(N)} &= \mathbf{s}_k^T\sum_{i=0}^{m}\sum_{j=0}^{m}\mathbf{M}_k(i)\mathbf{M}_k(j)\left(\mathbf{S}_k\mathbf{S}_k^T + \frac{\sigma^2}{P}\mathbf{I}\right)\mathbf{s}_k \\ &= \sum_{i=0}^{m}\sum_{j=0}^{m}\sum_{k=0}^{i}\sum_{l=0}^{j} a_{k,i}a_{l,j} \\ &\quad \cdot \left[\mathbf{s}_k^T\left(\mathbf{S}_k\mathbf{S}_k^T\right)^{k+l+1}\mathbf{s}_k + \frac{\sigma^2}{P}\mathbf{s}_k^T\left(\mathbf{S}_k\mathbf{S}_k^T\right)^{k+l}\mathbf{s}_k\right]. \end{aligned}$$

Similarly, the limit as $N \to \infty$ with the ratio $\alpha = K/N$ held fixed is taken. Then

$$\nu_{k,m}^{(N)} \xrightarrow{p} \nu_m = \sum_{i=0}^{m}\sum_{j=0}^{m} g_{i,j}(\alpha, \sigma^2, P, \kappa, \beta) \quad (22)$$

for $0 \le \psi_j(\alpha) < \infty$, where $g_{i,j}(\alpha, \sigma^2, P, \kappa, \beta)$ is given in (20). We call ν_m the large system limit of the sequence of r.v.s. $\nu_{k,m}^{(N)}$.

Now, since $\eta_{k,m}^{(N)} \xrightarrow{p} \eta_m$ and $\nu_{k,m}^{(N)} \xrightarrow{p} \nu_m$, then $\mathrm{SIR}_{k,m}^{(N)} \xrightarrow{p} \mathrm{SIR}_m$ with SIR_m given in (18). □

A significant point to note is that we have given the large system SIR expression for the second-order MSMU receiver in terms of only the number of stages, the system loading, the two second-order parameters (κ, β), and the SNR.

In addition, note that it has recently been shown that as the system size increases, the bit-error rate (BER) of the direct LMMSE receiver converges almost surely to $Q(\sqrt{\mathrm{SIR}'_m})$ (constrained to antipodal signaling, equal power users in synchronous CDMA), where SIR'_m is the large system SIR of the LMMSE receiver and $Q(\cdot)$ is the Gaussian Q-function [37]. This result has also been found to be applicable for linear multistage receivers [35]. Therefore, we are able to couple these results with our large system SIR expression to calculate the BER of the second-order MSMU receiver.

VI. Numerical Results

In this section, we empirically show that the variance of the SIR of the second-order MSMU receiver decreases proportionally to $1/N$, as N increases. We then focus on two suitable second-order MSMU receiver parameters denoted $\kappa_{\rm ACF}^{\infty}$ and $\beta_{\rm ACF}^{\infty}$ which minimize the asymptotic convergence factor as $m \to \infty$ (to be discussed later on). It is found that these parameters result in good SIR performance at the output of the second-order MSMU receiver for a finite number of stages $m \ll \infty$. We then numerically optimize our receiver to maximize the large system SIR expression with respect to the second-order MSMU receiver parameters κ and β. Finally, we compare the result with the large system SIR of the optimal MSMU receiver.

Average Empirical Squared Error

We proved in Theorem 1 that $\mathrm{SIR}_{k,m}^{(N)}$ converges in probability to the large system SIR of the second-order MSMU receiver, SIR_m, as N and K increase with $\alpha = K/N$ held fixed. We shall now demonstrate this convergence in another manner with the empirical mean square error, which is the mean squared error between SIR_m and $\mathrm{SIR}_{k,m}^{(N)}$, denoted $\epsilon^{(N)} = \overline{(\mathrm{SIR}_m - \mathrm{SIR}_{k,m}^{(N)})^2}$. This gives an indication of the relationship between the variance of the SIR and N. An example of this convergence is illustrated in Fig. 1, where we have plotted $\epsilon^{(N)}$ using 1000 samples of $\mathrm{SIR}_{k,m}^{(N)}$ for each value of N in the range $8 \le N \le 368$ (incrementing in steps of four). For each sample of $\mathrm{SIR}_{k,m}^{(N)}$ we randomly generated the spreading sequences. This plot uses $m = 4$, $\alpha = 0.75$, $8 \le N \le 368$ and $\mathrm{SNR} = 12$ dB. We empirically found that $\epsilon^{(N)} \approx \hat{\epsilon}^{(N)} = 140N^{-1.1}$ showing that the empirical mean-squared error of the SIR decreases with a $1/N$ relationship. This complements the results presented in [32], where it has recently been proved that in the case of the decorrelator and direct LMMSE receiver the fluctuations around the large system SIR are proportional to $1/N$.

Optimization of the Second-Order MSMU Receiver

In this section, we numerically optimize the large system SIR of the second-order MSMU receiver SIR_m in terms of the second-order parameters (κ, β) for a finite number of stages. We compare the optimized parameters with related theoretical values $(\kappa_{\rm ACF}, \beta_{\rm ACF})$ which are known to minimize the asymptotic convergence factor [1].

Second-Order MSMU Receiver Parameters: The asymptotic convergence factor (ACF) is the spectral radius of the iteration matrix of the respective iterative solution method [1], [2]. In the case of the second-order stationary linear iterative solution method, it is minimized when

$$\kappa = \kappa_{\rm ACF} = \frac{2}{1 + \sqrt{(1-\rho_0^2)}}$$

and

$$\beta = \beta_{\rm ACF} = \frac{2\kappa_{\rm ACF}}{\mu_{\min} + \mu_{\max}} \tag{23}$$

where in our case, $\mu_{\min}/\mu_{\max}$ are the minimum/maximum eigenvalues of $(\mathbf{S}_k\mathbf{S}_k^T + (\sigma^2/P)\mathbf{I})$ and $\rho_0 = [1 - (\mu_{\min}/\mu_{\max})]/[1 + (\mu_{\min}/\mu_{\max})]$ [1]. Using this expression for $(\kappa_{\rm ACF}, \beta_{\rm ACF})$ ensures convergence to a solution as $m \to \infty$. However, it is not certain how this relates to a finite number of stages. For a discussion on the asymptotic convergence factor, see [1] and [2].

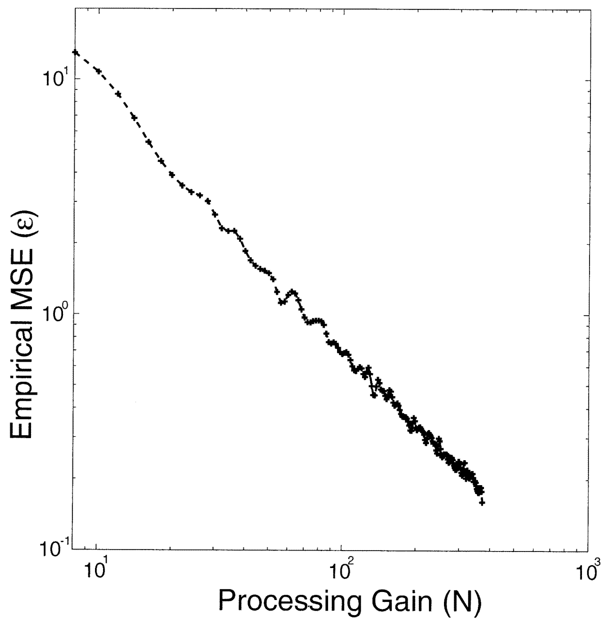


Fig. 1. Empirical mean-squared error versus N ($m = 4, \alpha = 0.75, \mathrm{SNR} =$ 12 dB).

If we consider a large system where $N \to \infty$ with $0 < \alpha = K/N \le 1$ held fixed, then the extreme eigenvalues of the matrix $(\mathbf{S}_k\mathbf{S}_k^T + (\sigma^2/P)\mathbf{I})$ converge with probability one to two deterministic scalars

$$\mu_{\min} \to \frac{\sigma^2}{P} \quad \text{and} \quad \mu_{\max} \to (\sqrt{\alpha}+1)^2 + \frac{\sigma^2}{P}. \tag{24}$$

See [2] and the references therein.

Therefore, in a large system (with random spreading), we have approximately

$$\kappa_{\rm ACF}^{\infty} = \frac{2\left(\left(\sqrt{\alpha}+1\right)^2 + \frac{2\sigma^2}{P}\right)}{\left(\sqrt{\alpha}+1\right)^2 + \frac{2\sigma^2}{P} + \sqrt{\frac{2\sigma^2}{P}\left(2\left(\sqrt{\alpha}+1\right)^2 + \frac{2\sigma^2}{P}\right)}}$$

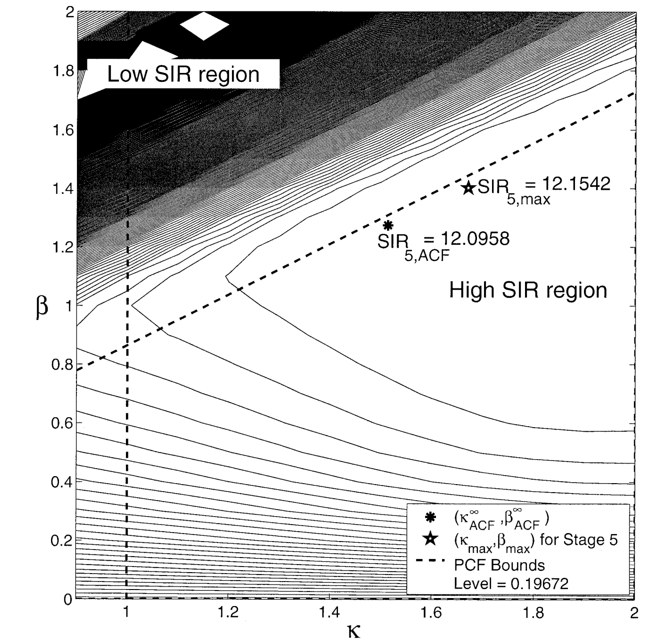
and

$$\beta_{\rm ACF}^{\infty} = \frac{2\kappa_{\rm ACF}^{\infty}}{\left(\sqrt{\alpha}+1\right)^2 + \frac{2\sigma^2}{P}} \tag{25}$$

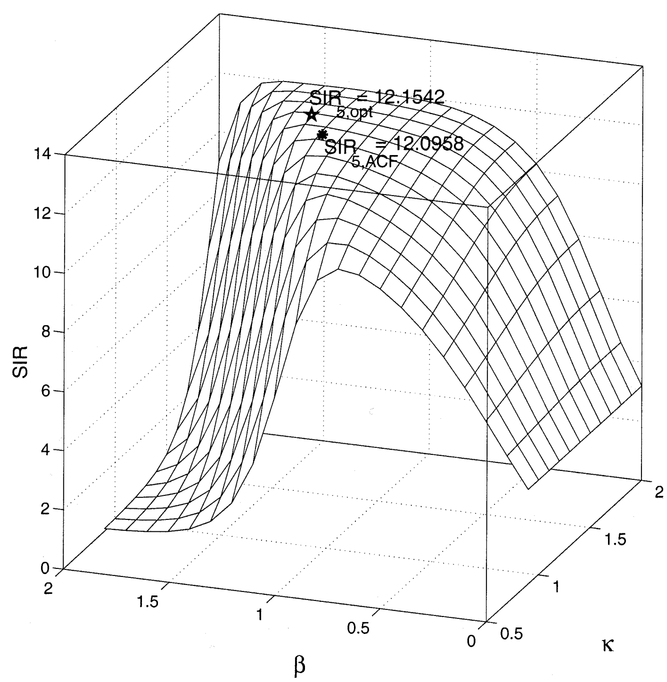
with minimal asymptotic convergence factor, where ∞ indicates the large system result.

Two-Dimensional (2-D) Optimization: We shall now numerically maximize SIR_m with respect to (κ, β) giving optimized parameters denoted $(\kappa_{\max}, \beta_{\max})$. An example is shown in Fig. 2 (*note the SIR is not in decibels*). This plot uses $\mathrm{SNR} = 12$ dB and $\alpha = 0.25$ for Stage 5. We have plotted the large system SIR for Stage 5 as κ and β varies, where the spacing between contour levels is 0.197 and the optimized large system SIR for Stage 5 is 12.1542 (not in decibels). The optimized large system SIR when $(\kappa, \beta) = (\kappa_{\max}, \beta_{\max})$ is indicated by a ★, and $*$ indicates the large system SIR when $(\kappa, \beta) = (\kappa_{\rm ACF}^{\infty}, \beta_{\rm ACF}^{\infty})$.

The bounds for (κ, β) which ensure convergence to the direct LMMSE solution as the number of stages m increases were previously stated in Section IV. In a large system, these bounds for



(a)



(b)

Fig. 2. Large system SIR as κ and β varies ($\alpha = 0.25$ and $\mathrm{SNR} = 12$ dB). (a) SIR contours of κ versus β.

our second-order MSMU receiver may be given by $1 < \kappa < 2$ and $0 < \beta < 2\kappa/((\sqrt{\alpha}+1)^2+\sigma^2/P)$. These bounds are shown by the dashed lines in Fig. 2. It can be seen that the parameters $(\kappa_{\max}, \beta_{\max})$ and $(\kappa^{\infty}_{\mathrm{ACF}}, \beta^{\infty}_{\mathrm{ACF}})$ are within these bounds. Outside the bounds, the SIR performance severely degrades, as can be seen by the closeness of the contour levels. As the number of stages increases it has been observed that the plateau of the high SIR region expands within the bounded area. Any (κ, β) within this region would eventually give large system LMMSE SIR performance; however, we are primarily interested in a finite number of stages and hence focus on the maximum point of the region.

TABLE I
NUMERICALLY OPTIMIZED VALUES OF κ, β AND SIR_m (IN dB) IN THE FORMAT $\binom{(\kappa,\beta)}{\mathrm{SIR}_m}$ FOR STAGE 5 (VARIOUS α AND SNR)

$\alpha = \frac{K}{N}$	SNR (dB) 3	6	9	12	15
0.125	(1.16, 0.84) 2.63	(1.30, 1.12) 5.55	(1.462, 1.410) 8.50	(1.65, 1.68) 11.46	(1.82, 1.72) 14.43
0.25	(1.19, 0.75) 2.26	(1.33, 0.98) 5.08	(1.50, 1.21) 7.94	(1.67, 1.40) 10.85	(1.82, 1.56) 13.77
0.375	(1.22, 0.70) 1.88	(1.37, 0.91) 4.58	(1.55, 1.11) 7.34	(1.74, 1.29) 10.14	(1.91, 1.44) 12.97
0.5	(1.25, 0.66) 1.50	(1.41, 0.85) 4.06	(1.60, 1.03) 6.67	(1.79, 1.20) 9.31	(1.99, 1.34) 11.90
0.625	(1.26, 0.62) 1.11	(1.43, 0.79) 3.53	(1.62, 0.96) 5.94	(1.82, 1.11) 8.32	(2.00, 1.24) 10.55
0.75	(1.28, 0.59) 0.73	(1.44, 0.74) 2.98	(1.63, 0.89) 5.17	(1.81, 1.02) 7.24	(1.97, 1.12) 9.04
0.875	(1.29, 0.56) 0.36	(1.45, 0.70) 2.44	(1.62, 0.83) 4.39	(1.78, 0.93) 6.14	(1.91, 1.01) 7.54
1	(1.29, 0.53) -0.01	(1.45, 0.66) 1.90	(1.60, 0.77) 3.63	(1.74, 0.85) 5.07	(1.84, 0.91) 6.15

The numerically optimized values of κ, β and, therefore, SIR_5 (in decibels) for Stage 5 are shown in Table I for a range of SNRs and system loadings, $\alpha = K/N$. We have found that for the large system second-order parameters $(\kappa^{\infty}_{\mathrm{ACF}}, \beta^{\infty}_{\mathrm{ACF}})$ the second-order MSMU receiver gives near-LMMSE performance between Stages 5 and 8 for most system loadings. As well, the optimized values of (κ, β) for Stage 5 may be used for $m \leq 8$ with negligible degradation.

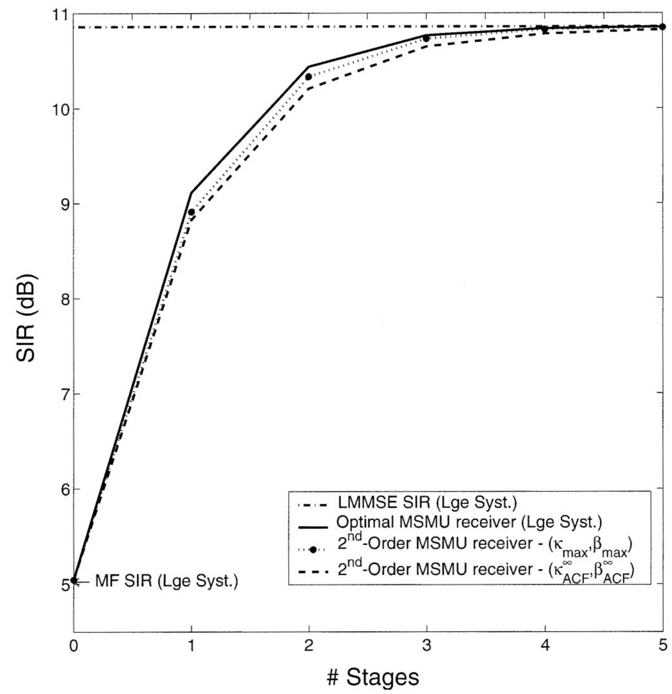
Performance Comparison

In this section, we compare the performance of our numerically optimized over (κ, β) second-order MSMU receiver with that of the mth-order optimal MSMU receiver, of [33] discussed previously. The results are shown in Fig. 3(a) and (b) for up to Stage 5, where $\alpha = 0.25$ and 0.75, respectively, and $\mathrm{SNR} = 12$ dB. The second-order MSMU receiver has been numerically optimized specifically for Stage 5 and the optimal MSMU receiver is separately optimized for each stage (each plotted point) up to Stage 5. We indicate the large system LMMSE SIR from [28] by the dashed-dot line. The dashed line indicates the large system SIR of our receiver when $(\kappa, \beta) = (\kappa^{\infty}_{\mathrm{ACF}}, \beta^{\infty}_{\mathrm{ACF}})$. The large system SIR of our numerically optimized receiver is indicated by the dotted line (with $*$ marks) when $(\kappa, \beta) = (\kappa_{\max}, \beta_{\max})$. The optimal MSMU receiver's large system SIR is the solid line.

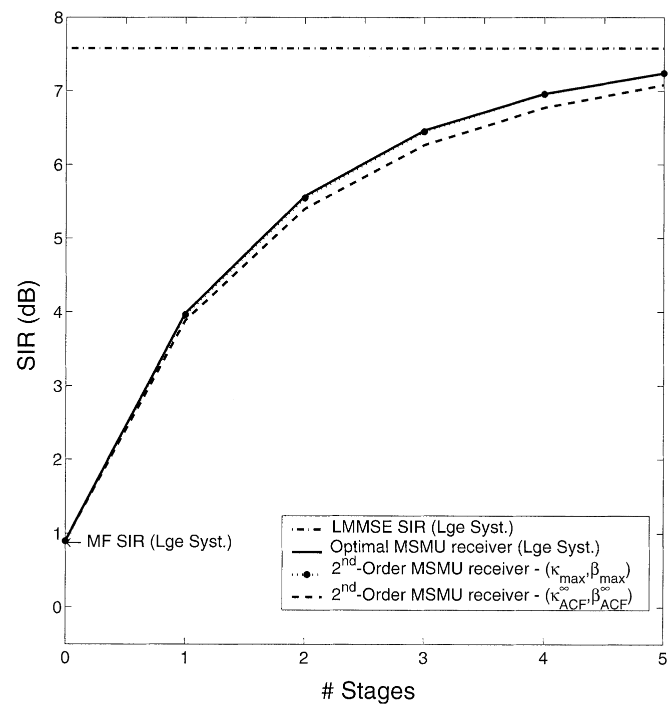
The surprising result is that the optimized second-order MSMU receiver gives very similar performance to the optimal MSMU receiver, which is considerably more complex to optimize. Also, in the initial stages the SIR performance for $(\kappa^{\infty}_{\mathrm{ACF}}, \beta^{\infty}_{\mathrm{ACF}})$ is only slightly degraded than for the numerically optimized parameters (as seen in Fig. 3). However, as the system loading increases the SIR degradation for $(\kappa^{\infty}_{\mathrm{ACF}}, \beta^{\infty}_{\mathrm{ACF}})$ slightly increases compared with that of the optimal MSMU receiver.

Optimization Guidelines

In this section, we decrease the computational load of the 2-D optimization problem by expressing one of the second-order MSMU receiver parameters (κ, β) as a simple function of the other, which leads to a simpler one-dimensional (1-D) optimization problem.

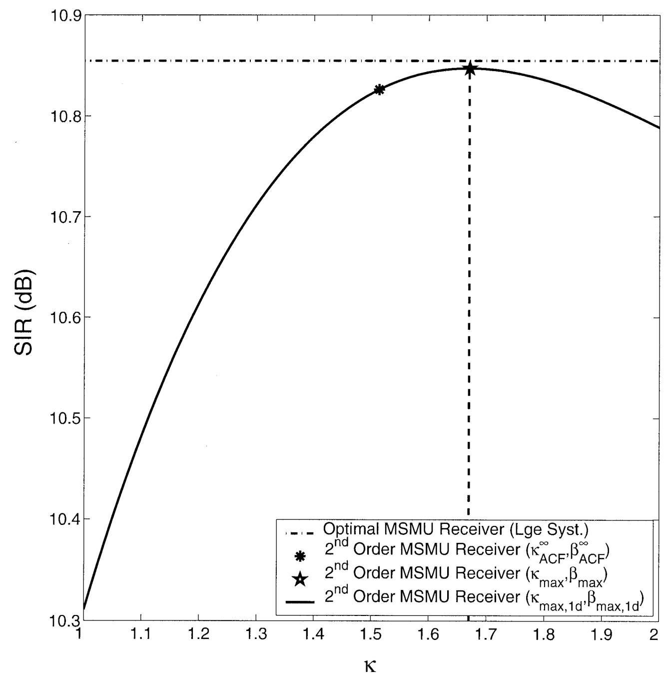


(a)

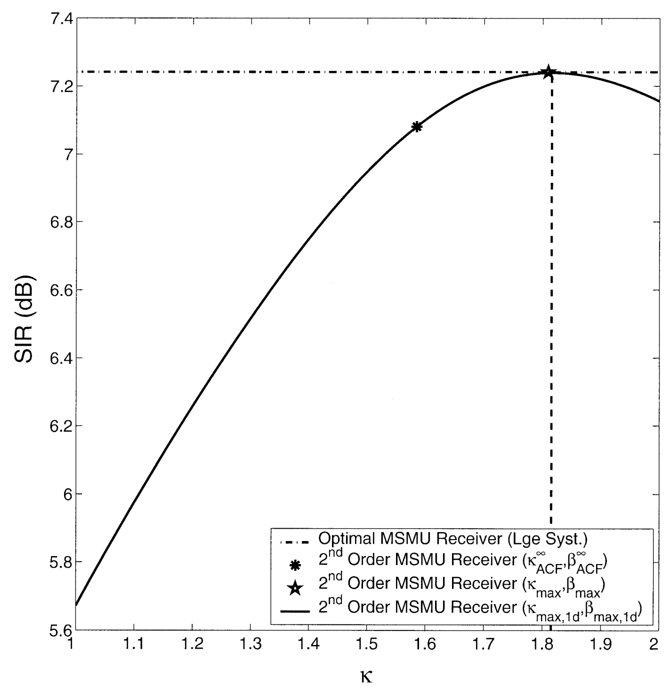


(b)

Fig. 3. Large system SIR versus Stages ($\mathrm{SNR} = 12$ dB, $m = 5$). (a) $\alpha = 0.25$. (b) $\alpha = 0.75$.



(a)



(b)

Fig. 4. Large system SIR versus κ ($\mathrm{SNR} = 12$ dB and $m = 5$). (a) $\alpha = 0.25$. (b) $\alpha = 0.75$.

One-Dimensional Optimization: We have found by taking the line with the slope of the boundary line $\beta = 2\kappa/((\sqrt{\alpha}+1)^2 + \sigma^2/P)$, which passes through the point $(\kappa_{\mathrm{ACF}}^{\infty}, \beta_{\mathrm{ACF}}^{\infty})$, that a 1-D optimization is possible.

This line is given by

$$\beta = \frac{2}{(\sqrt{\alpha}+1)^2 + \frac{\sigma^2}{P}}\kappa + \left(\beta_{\mathrm{ACF}}^{\infty} - \frac{2\kappa_{\mathrm{ACF}}^{\infty}}{(\sqrt{\alpha}+1)^2 + \frac{\sigma^2}{P}}\right). \tag{26}$$

Substituting this expression for β into the large system SIR, we performed a simple 1-D numerical search to optimize the large system SIR with respect to κ giving $(\kappa_{\mathrm{max},\,1d}, \beta_{\mathrm{max},\,1d})$, where substituting $\kappa = \kappa_{\mathrm{max},\,1d}$ into (26) gives $\beta_{\mathrm{max},\,1d}$.

We show the value for $\kappa_{\mathrm{max},\,1d}$ which results in the maximum large system SIR along the line given by (26) in Fig. 4(a) and (b). The vertical dashed line indicates $\kappa_{\mathrm{max},\,1d}$. These plots use $\alpha = 0.25$ and 0.75, $\mathrm{SNR} = 12$ dB, and $m = 5$. We observed from an extensive investigation that the degradation

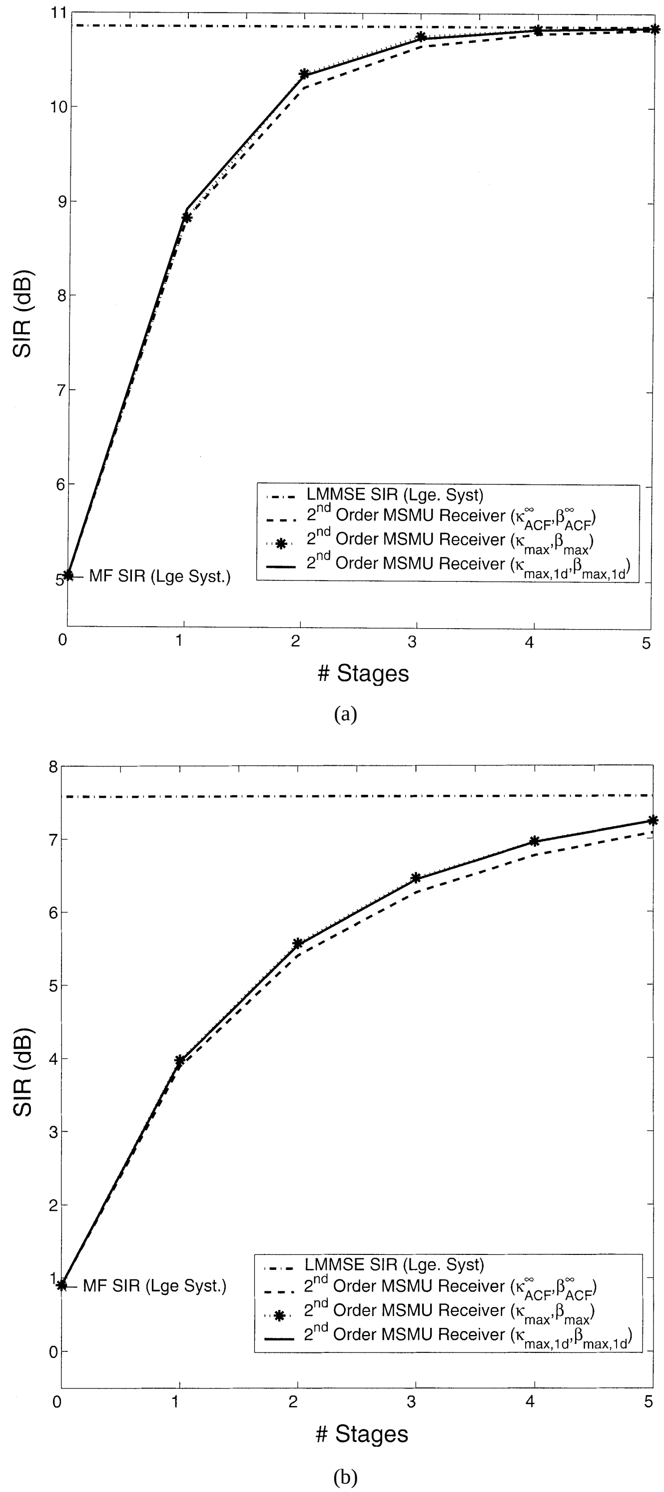


Fig. 5. Comparison of Large system SIR versus Stages, for 1-D and 2-D numerical optimization ($\mathrm{SNR} = 12$ dB and $m = 5$). (a) $\alpha = 0.25$. (b) $\alpha = 0.75$.

using this approximation is negligible over most system parameters of interest.

Finally, we show the large system SIR performance using the 1-D numerical optimization compared with that using the previous 2-D numerical optimization as the number of stages increase in Fig. 5(a) and (b). These plots use $\alpha = 0.25$ and 0.75, respectively, $\mathrm{SNR} = 12$ dB, and $m = 5$. As can be seen, using the 1-D approximated values $(\kappa_{\max,\,1d},\,\beta_{\max,\,1d})$ results in negligible performance degradation as the stages increase compared with that of the 2-D optimized values $(\kappa_{\max},\,\beta_{\max})$.

VII. Impact of Unequal Power Users

So far, our discussion has focused on a model where the users have equal received powers. It is quite straightforward to apply a similar large system SIR analysis to that of Section V for the N-dimensional receiver for unequal received powers. The main difference in the large system analysis will be the key terms of interest which are now random variables of the form $\mathbf{s}_k^T(\mathbf{S}_k\mathbf{P}_k\mathbf{S}_k^T)^i\mathbf{s}_k$. Here, $\mathbf{P}_k = \mathrm{diag}[P_1,\,\ldots,\,P_{k-1},\,P_{k+1},\,\ldots,\,P_K]$ is a $K-1 \times K-1$ dimensional matrix of received user powers excluding the received power of user k. Now, taking the limit as $N \to \infty$ with $\alpha = K/N$ held fixed, if the empirical distribution of the eigenvalues of $\mathbf{P}_k$ converges in probability to a limiting empirical distribution function denoted by $F(P)$, then $\mathbf{s}_k^T(\mathbf{S}_k\mathbf{P}_k\mathbf{S}_k)^i\mathbf{s}_k$ converges in probability to a deterministic scalar $\psi_i(\alpha,\,F(P))$. No simple closed-form expressions are available for $\psi_i(\alpha,\,F(P))$; however, methods for calculating these eigenvalue moments can be found in [36] and [38]. Following on with the analysis will give a large system SIR expression for unequal power users, similar to (18), in terms of $\psi_i(\alpha,\,F(P))$. A recent and related analysis [39] of linear multiuser receivers incorporates $\sqrt{P_k}$ into the spreading sequence $\mathbf{s}_k$ giving the unnormalized spreading sequence $\sqrt{P_k}\mathbf{s}_k$ for user k. This results in an alternative derivation of the limiting deterministic moments and will similarly result in an equivalent large system SIR expression.

While the analysis of the standard receiver as discussed above is fairly straightforward, the actual performance that results can be quite poor in situations where the powers of the users are dramatically different. The key iteration matrix becomes $(\mathbf{S}_k\mathbf{P}_k\mathbf{S}_k^T + \sigma^2\mathbf{I})$ and when there are big differences in the entries of the diagonal power matrix $\mathbf{P}_k$ this matrix is poorly conditioned. Practically, this would mean slow convergence, as the number of stages increase to the LMMSE receiver.

In order to explore this issue further and to hint at a solution to the problem, it is convenient to work with an alternative form of the LMMSE receiver and corresponding second-order iterative implementation. For the received signal model of (1) the linear MMSE receiver produces the estimate

$$\hat{\mathbf{b}} = \mathbf{P}^{-1/2}(\mathbf{S}^T\mathbf{S} + \sigma^2\mathbf{P}^{-1})^{-1}\mathbf{S}^T\mathbf{r} \tag{27}$$

where $\hat{\mathbf{b}} = [\hat{b}_1,\,\ldots,\,\hat{b}_K]^T$ is the $K \times 1$ vector of LMMSE receiver estimates and $\mathbf{P} = \mathrm{diag}[P_1,\,\ldots,\,P_K]$ is a $K \times K$ diagonal matrix of received user powers, which are assumed unequal. The (K-dimensional) second-order MSMU Receiver structure that aims to iteratively implement the above receiver is

$$\hat{\mathbf{b}}_m = (\kappa\mathbf{I} - (\mathbf{S}^T\mathbf{S} + \sigma^2\mathbf{P}^{-1}))\hat{\mathbf{b}}_{m-1} + (1-\kappa)\hat{\mathbf{b}}_{m-2} + \beta\mathbf{y} \tag{28}$$

where $\mathbf{y} = \mathbf{S}^T\mathbf{r}$ is the $K \times 1$ vector of the received matched filtered signal. When κ and β are appropriately chosen, then this receiver will converge to the LMMSE receiver estimate given above (scaled by $\mathbf{P}^{1/2}$).

Denote $\mu_{\min}^{UEP}/\mu_{\max}^{UEP}$ as the minimum/maximum eigenvalues of the iteration matrix $(\mathbf{S}^T\mathbf{S}+\sigma^2\mathbf{P}^{-1})$. It can be inferred from [1, Th. C.1] that

$$\mu_{\min}^{UEP} \le 1+\frac{\sigma^2}{P_{j,\max}} \quad \text{and} \quad \mu_{\max}^{UEP} \ge 1+\frac{\sigma^2}{P_{j,\min}}$$

where $P_{j,\min}$ and $P_{j,\max}$ are the minimum and maximum received powers. If the powers vary significantly, as they would in Rayleigh fading for example, then the condition number (the ratio of the maximum and minimum eigenvalues) has the potential to be very large leading to instability or to slow convergence of the iterative algorithm.

A solution to this problem is to use the more general iterative receiver

$$\hat{\mathbf{b}}_m = (\kappa\mathbf{I}-\beta\mathbf{U}\,(\mathbf{S}^T\mathbf{S}+\sigma^2\mathbf{P}^{-1}))\hat{\mathbf{b}}_{m-1}+(1-\kappa)\hat{\mathbf{b}}_{m-2}+\beta\mathbf{U}\mathbf{y} \tag{29}$$

where the receiver of (28) has been modified by the nonsingular $K\times K$ preconditioning matrix $\mathbf{U}$ [1]. Our numerical investigations indicate [40], [41] that a good choice for $\mathbf{U}$ is the diagonal matrix $[\mathbf{I}+\sigma^2\mathbf{P}^{-1}]^{-1}$, which minimizes the eigenvalue spread. The large system analysis of this modified receiver is the subject of current research.

VIII. Conclusion

In this paper, we derived an expression for the large system SIR of the second-order MSMU receiver. We have shown that the large system SIR only depends on the number of stages, the system loading, two second-order MSMU receiver parameters, and the SNR. We have shown experimentally that as N, K increase with their ratio held constant, the average empirical mean squared error between the large system SIR and the SIR of our receiver decreases with a $1/N$ relationship. Further, using simple numerical methods the large system SIR can be optimized in terms of both the second-order parameters. We have shown that it is possible to attain the performance of the optimal MSMU receiver with only two parameters to select compared with selecting the m coefficients of the optimal MSMU receiver. Finally, we have found that the line with slope $2/((\sqrt{\alpha}+1)^2+\sigma^2/P)$ (parallel to boundary line), which passes through the point $(\kappa_{\mathrm{ACF}}^{\infty}, \beta_{\mathrm{ACF}}^{\infty})$ reduces the computational cost of the numerical optimization with negligible overall performance degradation.

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Louis G. F. Trichard (S'98–M'03) was born in Cape Town, South Africa, in 1973. He received the B.S. degree in computer science and mathematics and the B.E. degree in electrical engineering (information systems engineering) from the University of Sydney, Sydney, Australia, in 1996 and 1998, respectively. He was awarded the Telstra Australia Prize in electrical engineering upon graduation. He continued his studies in the Telecommunications Laboratory in the School of Electrical and Information Engineering, University of Sydney, where he submitted his Ph.D. thesis in 2001 and received the Ph.D. degree in 2002 in electrical engineering.

In December 2001, he moved to Tokyo, Japan, where he is currently a JSPS Postdoctoral Research Fellow at the Kohno Laboratory, Division of Physics, Electrical and Computer Engineering, Graduate School of Engineering, Yokohama National University. His current research interests include multiuser detection, wireless communications, and ultrawideband communications.

Jamie S. Evans (S'93–M'98) was born in Newcastle, Australia, in 1970. He received the B.S. degree in physics and the B.E. degree in computer engineering from the University of Newcastle, Callaghan, Australia, in 1992 and 1993, respectively, and received the University Medal upon graduation. He received the M.S. and the Ph.D. degrees from the University of Melbourne, Parkville, Victoria, Australia, in 1996 and 1998, respectively, both in electrical engineering, and was awarded the Chancellor's Prize for Excellence for his Ph.D. thesis.

From March 1998 to June 1999, he was a Visiting Researcher in the Department of Electrical Engineering and Computer Science, University of California, Berkeley. He returned to Australia to take up a position as Lecturer at the University of Sydney, Sydney, Australia, where he stayed until July 2001. Since that time, he has been a Senior Lecturer in the Department of Electrical and Electronic Engineering, University of Melbourne. His research interests are in communications theory, information theory, and statistical signal processing with current focus on wireless communications networks.

Dr. Evans currently serves on the Editorial Board of the IEEE TRANSACTIONS ON WIRELESS COMMUNICATIONS.

Iain B. Collings (S'92–M'95–SM'02) was born in Melbourne, Australia, in 1970. He received the B.E. degree in electrical and electronic engineering from the University of Melbourne, Parkville, Victoria, Australia, and the Ph.D. degree in systems engineering from the Australian National University (ANU), Canberra, in 1992 and 1995, respectively.

In 1995, he was a Research Fellow in the Australian Cooperative Research Center for Sensor Signal and Information Processing, ANU, where he worked in the area of radar signal processing. From 1996 to 1999, he was a Lecturer at the University of Melbourne, and since 1999, he has been a Senior Lecturer in the School of Electrical and Information Engineering, University of Sydney, Sydney, Australia. His current research interests include synchronization, channel estimation, equalization, and multicarrier modulation for time-varying and frequency-selective channels.

Dr. Collings currently serves as an Editor for the IEEE TRANSACTIONS ON WIRELESS COMMUNICATIONS.