V. Conclusion

In this paper, the PRADA framework for solving the RAN overload problem in 3GPP LTE-A networks due to MTC has been analyzed. We have mathematically derived and compared the access success probabilities of systems applying EAB and the PRADA framework. The theoretical analyses have been shown to match the simulation results very well.

Appendix

The problem of deriving the expected number of successful RA requests per PRACH slot can be formulated as a bins-and-balls problem. Let Y be the number of preambles that are chosen exactly by one UE and X_i, $i = 1, \ldots, M$, be indicators that if the ith preamble is chosen exactly once, $X_i = 1$; otherwise, $X_i = 0$. Then, $Y = X_1 + X_2 + \cdots + X_M$. By taking expectation on both sides, $\mathbb{E}[Y] = \mathbb{E}[X_1 + X_2 + \cdots + X_M] \stackrel{(a)}{=} \mathbb{E}[X_1] + \mathbb{E}[X_2] + \cdots + \mathbb{E}[X_M] \stackrel{(b)}{=} M\mathbb{E}[X_1]$, where (a) follows from the linearity property, and (b) is due to $\mathbb{E}[X_1] = \mathbb{E}[X_2] = \cdots = \mathbb{E}[X_M]$. For each UE, preamble 1 has probability $1/M$ of being chosen, and only one out of N UEs chooses preamble 1. Therefore, $\mathbb{E}[X_1] = (N/M)(1 - 1/M)^{N-1}$. Finally, $\mathbb{E}[Y] = N(1 - 1/M)^{N-1}$.

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On the Bit Error Probability of Optimal Multiuser Detectors in Cooperative Cellular Networks

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***Abstract*—In this paper, we present new theoretical analysis on the uncoded bit error probability (BEP) of optimal multiuser detectors in cooperative cellular networks. We consider the uplink of a cooperative network where an arbitrary number of receivers jointly detect the signals transmitted from multiple transmitters. For such a network, we derive accurate upper and lower bounds on the BEP with independent Rayleigh fading and arbitrary path loss. We observe that our lower bound accurately approximates the BEP at low signal-to-noise ratios (SNRs), whereas the upper bound is accurate at high SNRs. We further evaluate our bounds asymptotically to explicitly characterize the cooperative diversity order and array gains in the high-SNR regime.**

***Index Terms*—Base station cooperation, bit error probability (BEP), multiuser detection.**

I. Introduction

A major barrier in realizing theoretical minimum error rates for uplink transmission in cellular networks is intercell interference [1]. The conventional approach to manage intercell interference is to adopt a sparse frequency reuse pattern such that users in neighboring cells communicate over orthogonal frequency bands. This is problematic in dense network areas (e.g., commercial districts and city centers) where the capacity requirement is very high. Base station cooperation is a promising approach to exploit interference by treating antennas of multiple cells as a virtual multiple antenna array [2], [3]. In so doing, base station cooperation effectively treats intercell interference as useful information and significantly improves the network reliability and spectral efficiency.

The concept of a joint processing global receiver that has access to all the received signals was initially proposed in [4] and [5]. The Wyner model in [4] was further extended to incorporate more realistic characteristics in cellular networks such as fading [6] and finite-capacity backhaul links [7]. Considering the bit error probability (BEP), a belief propagation algorithm for base station cooperation was proposed in [8], where local message passing was shown to provide a globally near-optimum error rate. Recently, an upper bound on the symbol error probability (SEP) of cooperative cellular networks has been considered in [9], where a union bound on the SEP is derived by adding the pairwise error probabilities over all possible transmit–receive symbols. The results were valid for distinct path-loss values between the transmitters and the receivers.

In this paper, we present new theoretical analysis on the BEP of cooperative base stations for both binary phase-shift keying (BPSK) and quadrature phase-shift keying (QPSK). We consider the uplink of a cooperative cellular network with an arbitrary number of transmitters

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and receivers for which we study the joint maximum *a posteriori* probability (MAP) detection. Our contributions are given here.

1) We derive accurate upper and lower bounds on the BEP of any given transmitter in cooperative multicell networks with independent Rayleigh fading and arbitrary path loss. In contrast to [9], our general expressions include the important case of base stations with colocated receivers.
2) We apply the principles of multiuser detection in [10] to a general cooperative scenario. As such, we take into account similarities in the probabilities, which significantly reduce the number of summations in the final BEP expressions compared with the approach in [9].
3) We derive closed-form expressions for the BEP at high signal-to-noise ratios (SNRs). Based on our asymptotic bounds, we explicitly characterize the diversity order and array gains of the network. Specifically, we show that the diversity order is equal to the total number of cooperating receivers.

Through numerical examples, we observe that the lower bound accurately approximates the exact BEP at low SNRs, whereas the upper bound is accurate in the medium- to high-SNR regime. We highlight that the error performance improves with increasing number of cooperating receivers. Interestingly, we find that the upper bound approaches the lower bound when the number of cooperating receivers is large.

II. System Model

We consider the uplink of a cooperative multicell network with N transmitters and M cooperating receivers. The receivers are connected via high-capacity delay-free links to a central processor at which all transmitted bits are jointly detected. As such, the signal received at receiver m is given by

$$y_m = \sum_{n=1}^{N} h_{mn} g_{mn} b_n + w_m \tag{1}$$

where $h_{mn} \sim \mathcal{CN}(0,1)$ is the Rayleigh-distributed complex channel gain between transmitter n and receiver m, g_{mn} is the fixed path gain between transmitter n and receiver m, b_n is the BPSK modulated bit[1] from transmitter n, and $w_m \sim \mathcal{CN}(0,\sigma^2)$ is the complex Gaussian noise at receiver m. This is a general model that applies to a wide range of cellular scenarios where the signals from multiple users are detected by multiple cooperating base stations equipped with colocated receivers.[2]

For such a network, we consider the optimal MAP detector at the central processor to jointly detect the transmitted bits in such a way that the *a posteriori* probability $P[b_1, b_2, \ldots, b_N | y_1, y_2, \ldots, y_M]$ is maximized. Perfect channel state information is assumed at all the receivers. Given equiprobable transmitted bits, we find that the *a posteriori* probability is maximized by the set of bits that minimizes

$$\Delta(b_1, b_2, \ldots, b_N) = \sum_{m=1}^{M} \left| y_m - \sum_{n=1}^{N} h_{mn} g_{mn} b_n \right|^2 . \tag{2}$$

It is important to note that the minimum BEP is different for each transmitter due to the fact that we consider nonidentical path gains between the transmitters and the receivers. In the following, we will analyze the BEP for an arbitrary transmitter i. As such, we first define the set of transmitted bits from the N transmitters as $B = \{b_1, b_2, \ldots, b_N\}$ and the set of detected bits as $\hat{B} = \{\hat{b}_1, \hat{b}_2, \ldots, \hat{b}_N\}$. We then define $\mathcal{Z}_n^i$ as the general set of all possible transmitted–detected bits $(B, \hat{B})$, where the bit from transmitter i, $\hat{b}_i$, and any other $n-1$ bits are detected in error. As such, $\mathcal{Z}_n^i$ contains a total of n erroneous bits including $\hat{b}_i$. Note that the sets $\mathcal{Z}_1^i$ to $\mathcal{Z}_N^i$ together represent all the possible ways that the detector can make an error in bit $\hat{b}_i$. Thus, our BEP bounds for a given transmitter i is derived based on the probability of all sets $\mathcal{Z}_n^i$, where $n \in \{1, 2, \ldots, N\}$.

[1]We consider BPSK modulation in the following derivations before extending our results to QPSK modulation in Section V.

[2]Multiple receivers can be located at the same base station with independent channel gains h_{mn} and identical path gains g_{mn}.

III. Bit Error Probability Bounds Conditioned on Fading

Here, we present new analytical bounds on the BEP of a given transmitter i conditioned on fading. We express the conditional probability that the detector makes an error in the ith bit as

$$P_{b|h}^i = \sum_{n=1}^{N} P\left[\mathcal{Z}_n^i | h\right] \tag{3}$$

where $P[\mathcal{Z}_n^i | h]$ is the probability conditioned on fading that n bits including bit b_i are detected in error. To find the exact value of $P[\mathcal{Z}_n^i | h]$, a multidimensional Gaussian distribution should be integrated over a polytope, which cannot be written either in closed form or in terms of the Q-function [10]. As the exact BEP is intractable, we proceed to derive accurate upper and lower bounds for $P[\mathcal{Z}_n^i | h]$.

A. Upper Bound Conditioned on Fading

Let us first consider the upper bound for $P[\mathcal{Z}_n^i | h]$. Recall that the MAP detector will always detect the set of bits that minimizes the Δ term in (2). Thus, detecting n bits including bit b_i in error occurs when the observations are closer to $\hat{B}$ than to B for any $(B, \hat{B}) \in \mathcal{Z}_n^i$. As such, the detected bit sequence $\hat{B}$ substituted into (2) must be less than the corresponding transmitted bit sequence B substituted into (2). Accordingly, an upper bound on the BEP is given by

$$P\left[\mathcal{Z}_n^i | h\right] \leq \sum_{(B,\hat{B}) \in \mathcal{Z}_n^i} P\left[\Delta(B) > \Delta(\hat{B}) | B \text{ transmitted}\right] P[B] \tag{4}$$

where $P[B]$ is the probability that bit sequence B is transmitted. The inequality in (4) arises because the observations may be closer to $\hat{B}$ than to B, but $\hat{B}$ might not be closest to the observation. Expressing $\Delta(B)$ and $\Delta(\hat{B})$ according to (2) and y_m according to (1) results in

$$\begin{aligned} P\left[\mathcal{Z}_n^i | h\right] &\leq \frac{1}{2^N} \sum_{(B,\hat{B}) \in \mathcal{Z}_n^i} P\left[\sum_{m=1}^{M} \left| w_m \right|^2 > \sum_{m=1}^{M} \left| h_{mi} g_{mi} b_i \right.\right. \\ &\qquad + \sum_{x=1}^{n-1} h_{mj_x} g_{mj_x} b_{j_x} \\ &\qquad + w_m - h_{mi} g_{mi} \hat{b}_i \\ &\qquad \left.\left. - \sum_{x=1}^{n-1} h_{mj_x} g_{mj_x} \hat{b}_{j_x} \right|^2 \right] \end{aligned} \tag{5}$$

where $P[B] = 1/2^N$ given equiprobable transmitted bits. We observe that all the terms except for the n erroneous terms are canceled out since $\hat{b}_i \neq b_i$, $\hat{b}_{j_x} \neq b_{j_x}$, $\forall x \in \{1, 2, \ldots, n-1\}$, and $\hat{b}_l = b_l$, $\forall i \neq j_x \neq l$. This observation significantly reduces the number of summations in our final results compared with the approach in [9].

To evaluate the probability in (5) over the whole set of $\mathcal{Z}_n^i$, we first fix the values of i and j_x and note that the rest of the bits can take 2^{N-n} combinations of values. This leads to (6), shown at the bottom of the page, where C_n^i is the set of all sets of n transmitters in error including transmitter i, $C_n^i(\rho) = \{\rho^{th} \text{ set of } C_n^i\}$, $(j_1, j_2, \ldots, j_{n-1}) \in C_n^i(\rho)$, and

$$\Psi_m\left(b_i, b_{j_1}, b_{j_2}, \ldots, b_{j_{n-1}}\right) = h_{mi}g_{mi}b_i + \sum_{x=1}^{n-1} h_{mj_x}g_{mj_x}b_{j_x}. \quad (7)$$

We note that (6) contains a summation of 2^n probability terms representing all combinations of n erroneous bits. Given that w_m follows a Gaussian distribution, we reexpress (6) in terms of the Q-function as

$$P\left[\mathcal{Z}_n^i|h\right] \leq \frac{1}{2^{n-1}} \times \sum_{C_n^i(\rho)\in C_n^i}\left[\mathcal{Q}\left(\frac{\sqrt{2\sum_{m=1}^M |\Psi_m(+1,+1,\ldots,+1)|^2}}{\sigma}\right) + \ldots + \mathcal{Q}\left(\frac{\sqrt{2\sum_{m=1}^M |\Psi_m(+1,-1,\ldots,-1)|^2}}{\sigma}\right)\right]. \quad (8)$$

The factor of $1/2^{n-1}$ in (8) arises by observing that the number of probability terms in (6) can be halved by considering channel symmetry.[3] Substituting (8) into (3), our upper bound on the BEP of transmitter i conditioned on fading is derived as

$$P_{b|h}^i \leq \sum_{n=1}^N \frac{1}{2^{n-1}} \times \sum_{C_n^i(\rho)\in C_n^i}\left[\mathcal{Q}\left(\frac{\sqrt{2\sum_{m=1}^M |\Psi_m(+1,+1,\ldots,+1)|^2}}{\sigma}\right) + \ldots + \mathcal{Q}\left(\frac{\sqrt{2\sum_{m=1}^M |\Psi_m(+1,-1,\ldots,-1)|^2}}{\sigma}\right)\right]. \quad (9)$$

B. Lower Bound Conditioned on Fading

Our lower bound assumption is based on the scenario where the receiver has knowledge of all the interference. As such, this known interference can be subtracted from the received signal, which leads to a system similar to a single-transmitter scenario with no interference, i.e., $N = 1$, where an error occurs only when $b_i \neq \hat{b}_i$. Thus, $\Delta(b_1, b_2, \ldots, b_N)$ in (2) reduces to $\Delta(b_i) = \sum_{m=1}^M |y_m - h_{mi}g_{mi}b_i|^2$. After some mathematical manipulations, a lower bound on the BEP conditioned on fading is

$$P_{b|h}^i \geq \mathcal{Q}\left(\frac{\sqrt{2\sum_{m=1}^M |h_{mi}g_{mi}|^2}}{\sigma}\right). \quad (10)$$

IV. Bit Error Probability Bounds Averaged Over Fading

Here, we present new analytical bounds on the BEP of a given transmitter i averaged over independent Rayleigh fading channels between the transmitters and the receivers. Our BEP bounds are derived by integrating the upper and lower bounds in (9) and (10), respectively, over their corresponding fading distributions.

A. Upper Bound Averaged Over Fading

Let us first consider the upper bound of $P[\mathcal{Z}_n^i|h]$ in (9) averaged over fading. We note that each Q-function in (9) contains an addition of M terms with independent and nonidentical Rayleigh distributions. As such, integrating (9) over the fading distribution results in

$$P\left[\mathcal{Z}_n^i\right] \leq \frac{1}{2^{n-1}} \times \sum_{C_n^i(\rho)\in C_n^i}\left[\underbrace{\int_0^\infty \cdots \int_0^\infty}_{\text{M-fold}} \mathcal{Q}\left(\frac{\sqrt{2\sum_{m=1}^M \beta_m^2}}{\sigma}\right) \times f_{\beta_1}\ldots f_{\beta_M}\, d\beta_1 \ldots d\beta_M + \cdots + \underbrace{\int_0^\infty \cdots \int_0^\infty}_{\text{M-fold}} \mathcal{Q}\left(\frac{\sqrt{2\sum_{m=1}^M \gamma_m^2}}{\sigma}\right) \times f_{\gamma_1}\ldots f_{\gamma_M}\, d\gamma_1 \ldots d\gamma_M\right] \quad (11)$$

where $P[\mathcal{Z}_n^i]$ represents $P[\mathcal{Z}_n^i|h]$ averaged over fading, f_{β_m} is the probability density function (pdf) of $\beta_m = |\Psi_m(+1,+1,\ldots,+1)|$, and f_{γ_m} is the pdf of $\gamma_m = |\Psi_m(+1,-1,\ldots,-1)|$. We note that the expression in (11) contains a summation of 2^{n-1} M-fold integrals.

Observing the first M-fold integral in (11), we have expressed the joint pdf of $f_{\beta_1,\ldots,\beta_M}$ as a product of individual pdf's f_{β_1} to f_{β_M} due to the independence between the channels. Based on (7), the pdf of β_m can be written as

$$f_{\beta_m} = \frac{2\beta_m}{\Omega_{mn}} \exp\left(\frac{-\beta_m^2}{\Omega_{mn}}\right) \quad (12)$$

[3]Due to the symmetry in the channel, the probability of transmitting B and detecting $\hat{B}$ is the same as the probability of transmitting $\hat{B}$ and detecting B.

$$P\left[\mathcal{Z}_n^i|h\right] \leq \frac{1}{2^N} \sum_{C_n^i(\rho)\in C_n^i}\left[2^{N-n} P\left[\sum_{m=1}^M |w_m|^2 > \sum_{m=1}^M |w_m - 2\Psi_m(+1,+1,\ldots,+1)|^2\right] + \ldots + 2^{N-n} P\left[\sum_{m=1}^M |w_m|^2 > \sum_{m=1}^M |w_m - 2\Psi_m(-1,-1,\ldots,-1)|^2\right]\right] \quad (6)$$

where $\Omega_{mn} = g_{mi}^2 + \sum_{x=1}^{n-1} g_{mj_x}^2$, and β_m follows a Rayleigh distribution having real and imaginary parts that are Gaussian distributed with zero mean and variance $\Omega_{mn}/2$. Similarly, the pdf of γ_m and all other terms in (11) follow the same distribution as f_{β_m}. Substituting f_{β_m} into the pdf's in (11), and reexpressing the Q-function as $Q(x) = (1/\pi)\int_0^{\pi/2} \exp(-x^2/2\sin^2\theta)d\theta$ [10, eq. (3.50)] allows us to separate the M-fold integrals in (11) into a multiplication of M separate integrals. We then proceed to solve the integration term by term using the integral identity in [11, eq. (3.326/2)], which results in a concise expression given by

$$P\left[\mathcal{Z}_n^i\right] \leq \sum_{C_n^i(\rho)\in C_n^i} \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \prod_{m=1}^{M} \left(\frac{\sigma^2\sin^2\theta}{\sigma^2\sin^2\theta + \Omega_{mn}}\right) d\theta. \tag{13}$$

By taking the summation of (13) over all n values, our new upper bound on the minimum BEP of transmitter i averaged over fading is given by

$$P_b^i \leq \sum_{n=1}^{N} \sum_{C_n^i(\rho)\in C_n^i} \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \prod_{m=1}^{M} \left(\frac{\sigma^2\sin^2\theta}{\sigma^2\sin^2\theta + \Omega_{mn}}\right) d\theta. \tag{14}$$

We note that the product term in (14) can be further simplified using partial fraction expansion. To do so, we define Υ as the set of $L \leq M$ distinct values of Ω_{mn} terms in the product. As such, the product term in (14) can be reexpressed as

$$\prod_{m=1}^{M} \left(\frac{\sigma^2\sin^2\theta}{\sigma^2\sin^2\theta + \Omega_{mn}}\right) = \prod_{l=1}^{L} \left(\frac{\chi}{\chi + \Omega_l}\right)^{k_l} \tag{15}$$

where $\chi = \sigma^2\sin^2\theta$, k_l denotes the number of Ω_{mn} terms in the product that are equal to $\Omega_l \in \Upsilon$, and $\sum_{l=1}^{L} k_l = M$. By employing partial fraction expansion, (15) can be reexpressed as

$$\prod_{l=1}^{L} \left(\frac{\chi}{\chi + \Omega_l}\right)^{k_l} = \left[1 + \sum_{l=1}^{L}\sum_{k=1}^{k_l} \frac{A_{lk}}{(\chi + \Omega_l)^k}\right] \tag{16}$$

where $A_{lk} = (1/(k_l - k)!)(d^{k_l-k}/d\chi^{k_l-k})(\chi^M/\prod_{p=1,p\neq l}^{L}(\chi + \Omega_p)^{k_p})|_{\chi=-\Omega_l}$. Substituting (16) into (14), we solve the resulting integral by applying the identity in [11, eq. (3.682)], which results in

$$P_b^i \leq \sum_{n=1}^{N} \sum_{C_n^i(\rho)\in C_n^i} \left[\frac{1}{2} + \sum_{l=1}^{L}\sum_{k=1}^{k_l} \frac{A_{lk}}{2}\Omega_l^{-k}\, {}_2F_1\left[\frac{1}{2}, k; 1; \frac{-\sigma^2}{\Omega_l}\right]\right] \tag{17}$$

where ${}_2F_1(a, b; c; z)$ denotes the Gauss hypergeometric function [11, eq. (9.1)]. We will show that our upper bound provides an accurate approximation of the exact BEP at medium to high SNRs.

B. Lower Bound Averaged Over Fading

A lower bound of P_b^i is obtained by integrating (10) over the corresponding fading distribution. This results in

$$P_b^i \geq \underbrace{\int_0^{\infty} \cdots \int_0^{\infty}}_{\text{M-fold}} \mathcal{Q}\left(\frac{\sqrt{2\sum_{m=1}^{M}\alpha_m^2}}{\sigma}\right) f_{\alpha_1}\ldots f_{\alpha_M}\, d\alpha_1 \ldots d\alpha_M \tag{18}$$

where f_{α_m} is the pdf of $\alpha_m = |h_{mi}g_{mi}|$, which follows a Rayleigh distribution having real and imaginary parts that are Gaussian distributed with zero mean and variance $g_{mi}^2/2$. Following the same steps as the upper bound, we solve the integrals in (18) term by term to obtain a lower bound on the BEP of transmitter i averaged over fading given by

$$P_b^i \geq \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \prod_{m=1}^{M} \left(\frac{\sigma^2\sin^2\theta}{\sigma^2\sin^2\theta + g_{mi}^2}\right) d\theta. \tag{19}$$

As the integrand in the lower bound in (19) takes the same form as the integrand in the upper bound in (14), we employ partial fraction to simplify the lower bound. We define Φ as the set of $R \leq M$ distinct values of g_{mi} in the product term in (19). The resultant simplified expression of the BEP lower bound for cooperation is given by

$$P_b^i \geq \frac{1}{2}\left[1 + \sum_{r=1}^{R}\sum_{q=1}^{q_r} A'_{rq} g_r^{-2q}\, {}_2F_1\left[\frac{1}{2}, r; 1; -\frac{\sigma^2}{g_r^2}\right]\right] \tag{20}$$

where $A'_{rq} = (1/(q_r - q)!)(d^{q_r-q}/d\chi^{q_r-q})(\chi^M/\prod_{p=1,p\neq r}^{R}(\chi + g_p^2)^{q_p})|_{\chi=-g_r^2}$, q_r denotes the number of g_{mi} terms in the product that are equal to $g_r \in \Phi$ and $\sum_{r=1}^{R} q_r = M$. We find that this lower bound provides an accurate approximation of the exact BEP at low SNRs.

V. Main Results

Here, we highlight important observations based on our analytical BEP bounds of optimal multiuser detectors in cooperative cellular networks. To obtain further insights, we evaluate our BEP bounds asymptotically to characterize the cooperative diversity order and array gain at high SNRs. Finally, we extend our analysis to consider the BEP with QPSK modulation.

A. Upper and Lower BEP Bounds of BPSK

Based on (17) and (20), our theoretical bounds on the BEP of transmitter i is given by

$$\begin{aligned}
&\frac{1}{2}\left[1 + \sum_{r=1}^{R}\sum_{q=1}^{q_r} A'_{rq} g_r^{-2q}\, {}_2F_1\left[\frac{1}{2}, r; 1; -\frac{\sigma^2}{g_r^2}\right]\right] \\
&\leq P_b^i \\
&\leq \sum_{n=1}^{N} \sum_{C_n^i(\rho)\in C_n^i} \left[\frac{1}{2} + \sum_{l=1}^{L}\sum_{k=1}^{k_l} \frac{A_{lk}}{2}\Omega_l^{-k} \right.\\
&\qquad\qquad \left. \times\, {}_2F_1\left[\frac{1}{2}, k; 1; \frac{-\sigma^2}{\Omega_l}\right]\right].
\end{aligned} \tag{21}$$

We note that (21) consists of a fixed number of finite sums, providing a computationally efficient alternative to Monte Carlo simulation. Based on (21), we present the following BEP bounds for the scenario of distinct path-loss values, which result in a simple closed-form expression.[4]

Cooperation With Distinct Path Loss: When the path loss from a given transmitter to each receiver is distinct, we can set $L = R = M$

[4]Note that the BEP bounds for the scenario of identical path-loss values can be also obtained from (21) but have not been included due to page limitations.

and $k_l = q_r = 1$ in (21), which reduces to

$$\frac{1}{2}\left[1-\sum_{m=1}^{M}\frac{(g_{mi})^{2M-1}}{\sqrt{\sigma^2+g_{mi}^2}\prod\limits_{z=1,z\neq m}^{M}(g_{mi}^2-g_{zi}^2)}\right]$$
$$\leq P_b^i \leq \sum_{n=1}^{N}\sum_{C_n^i(\rho)\in C_n^i}\frac{1}{2}$$
$$\times\left[1-\sum_{m=1}^{M}\frac{(\Omega_{mn})^{\frac{2M-1}{2}}}{\sqrt{\sigma^2+\Omega_{mn}}\prod\limits_{z=1,z\neq m}^{M}(\Omega_{mn}-\Omega_{zn})}\right] \quad (22)$$

where $g_{mi}^2 \neq g_{zi}^2$, $\Omega_{mn} \neq \Omega_{zn}$, $\forall(m,z) \in \{1,2,\ldots,M \;\&\; m\neq z\}$, and $\Omega_{zn} = g_{zi}^2 + \sum_{x=1}^{n-1} g_{zj_x}^2$. Note that our upper bound in (22) yields the same result as [9, eq. (20)] for BPSK. However, (22) is a more computationally efficient expression due to our multiuser detection approach, which takes into account similarities in probabilities and channel symmetry. In fact, the upper bound in [9, eq. (20)] involves a summation of $2^N(2^N - 2^{N-1})$ terms compared with only 2^{N-1} terms in our result in (22).

B. Asymptotic Results

We find that our BEP bounds can be evaluated asymptotically, leading to further design insights. We consider the high-SNR behavior of the BEP by evaluating the Taylor series expansion of the bounds as $\sigma^2 \to 0^+$. Let us first consider the asymptotic lower bound of the BEP. Preserving the dominant term in the Taylor series expansion of the integrand in (19) as $\sigma^2 \to 0^+$ results in

$$\prod_{m=1}^{M}\left(\frac{\sigma^2\sin^2\theta}{\sigma^2\sin^2\theta+g_{mi}^2}\right)=\frac{\sin^{2M}\theta}{\prod_{m=1}^{M}g_{mi}^2}(\sigma^2)^M+o\left((\sigma^2)^M\right) \quad (23)$$

where M is the first nonzero order term, and $o(\cdot)$ represents the higher order terms (i.e., we write $f(x) = o[g(x)]$ as $x \to x_0$ if $\lim_{x\to x_0}(f(x)/g(x)) = 0$). Integrating (23) with respect to θ results in the asymptotic lower bound of the BEP given by

$$P_b^i \geq \frac{1}{\pi}\int_0^{\frac{\pi}{2}}\frac{\sin^{2M}\theta}{\prod_{m=1}^{M}g_{mi}^2}(\sigma^2)^M d\theta$$
$$\approx \frac{(2M)!}{M!M!2^{2M+1}\prod_{m=1}^{M}g_{mi}^2}(\sigma^2)^M. \quad (24)$$

The same approach is applied for the asymptotic upper bound of the BEP in (14), which results in

$$P_b^i \leq \frac{1}{\pi}\sum_{n=1}^{N}\sum_{C_n^i(\rho)\in C_n^i}\int_0^{\frac{\pi}{2}}\frac{\sin^{2M}\theta}{\prod_{m=1}^{M}\Omega_{mn}}(\sigma^2)^M$$
$$\approx \sum_{n=1}^{N}\sum_{C_n^i(\rho)\in C_n^i}\frac{(2M)!}{M!M!2^{2M+1}\prod_{m=1}^{M}\Omega_{mn}}(\sigma^2)^M. \quad (25)$$

By reexpressing (24) and (25) as $P_b^i \approx (G_a\sigma^2)^{G_d}$ [12], where G_d is the diversity order and G_a includes the array gain, our asymptotic expressions reveal that a diversity order of M is achieved. Furthermore, we observe that the gap between the asymptotic upper and lower BEP bounds can be characterized as a ratio of their respective array gains.

C. Upper and Lower BEP Bounds of QPSK

We now present the upper and lower BEP bounds of optimal multiuser detection with QPSK modulation when Gray code bit mapping is employed [13]. We first note that the exact BEP of QPSK is obtained by averaging the BEP of the in-phase and quadrature bits of the QPSK symbol [13, eq. (16)]. We also note that the path-loss values for the in-phase and quadrature bits are the same. This means that the BEP of the in-phase and quadrature bits averaged over fading are the same. As such, the BEP of transmitter i with QPSK modulation can be calculated based on the BEP of the in-phase bit or the quadrature bit. Following the same approach outlined in Sections III and IV, the BEP of a given transmitter i with QPSK modulation can be expressed as

$$\frac{1}{2}\left[1+\sum_{r=1}^{R}\sum_{q=1}^{q_r}A'_{rq}g_r^{-2q}{}_2F_1\left[\frac{1}{2},r;1;-\frac{\sigma^2}{g_r^2}\right]\right]$$
$$\leq P_b^i$$
$$\leq \sum_{n=1}^{2N}\sum_{\varphi_n^i(\rho)\in\varphi_n^i}\left[\frac{1}{2}+\sum_{l=1}^{L}\sum_{k=1}^{k_l}\frac{A_{lk}}{2}\Omega_l^{-k}{}_2F_1\left[\frac{1}{2},k;1;\frac{-\sigma^2}{\Omega_l}\right]\right] \quad (26)$$

where φ_n^i is the set of all sets of transmitter indices with n bits in error including the in-phase bit (or the quadrature bit) of transmitter i, and $\varphi_n^i(\rho) = \{\rho^{th}$ set of $\varphi_n^i\}$. Based on (26), the asymptotic BEP of QPSK is given by

$$\frac{(2M)!}{M!M!2^{2M+1}\prod_{m=1}^{M}g_{mi}^2}(\sigma^2)^M$$
$$\leq P_b^i$$
$$\leq \sum_{n=1}^{2N}\sum_{\varphi_n^i(\rho)\in\varphi_n^i}\frac{(2M)!}{M!M!2^{2M+1}\prod_{m=1}^{M}\Omega_{mn}}(\sigma^2)^M. \quad (27)$$

VI. Numerical Examples

Here, we present numerical examples to validate the accuracy of our upper and lower BEP bounds and to highlight the BEP advantage of optimal multiuser detection in cooperative cellular networks. We first consider a two-cell network, as shown in Fig. 1, with four randomly placed transmitters (i.e., Tx_1, Tx_2, Tx_3, and Tx_4) and two base stations with two receivers in the cell center (i.e., Rx_1, Rx_2, Rx_3, and Rx_4). For each simulation trial, fading and noise components are drawn from an independent complex Gaussian distribution. The channel fading is complex Gaussian with zero mean and unit variance, and the additive noise is complex Gaussian with zero mean and variance σ^2. In the simulations, we consider a path-loss exponent of 4 with a relative path loss of 100 dB at 100 m.

In Figs. 2 and 3, we compare the BEP of noncooperation where a single base station (i.e., Rx_1 and Rx_2) is involved in the detection,

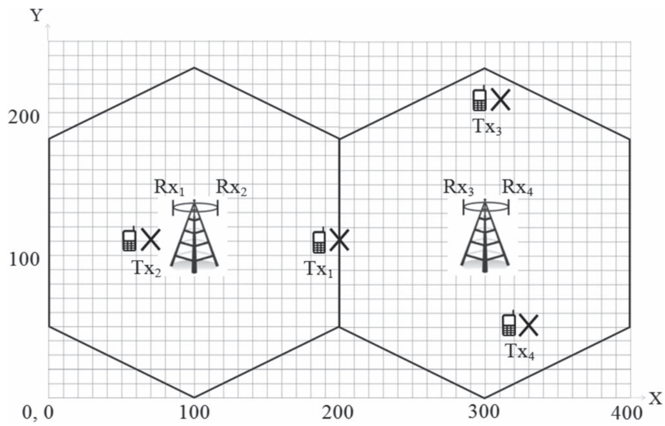

Fig. 1. Network model with four transmitters (Tx_1-Tx_4) and four receivers (Rx_1-Rx_4).

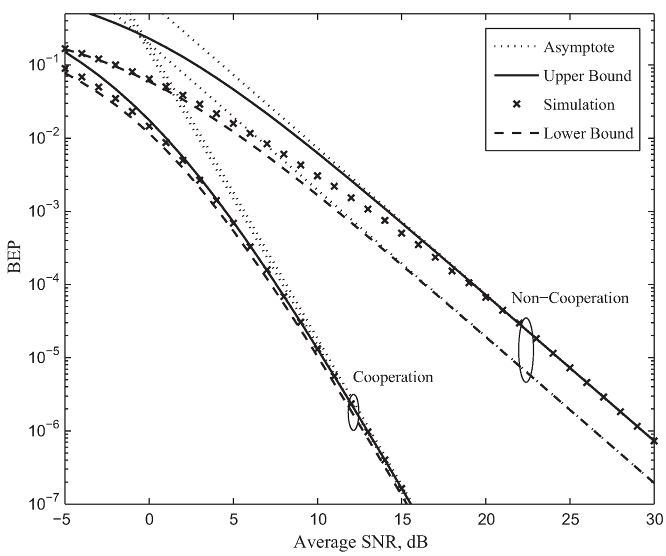

Fig. 2. BEP of Tx_1 versus average received SNR of Tx_1 for $N = 4$ transmitters.

with the BEP of cooperation where both base stations (i.e., Rx_1, Rx_2, Rx_3, and Rx_4) are involved in the detection. The upper and lower BEP bounds are generated using (21). The asymptotic upper and lower BEP bounds are generated using (24) and (25), respectively. The exact simulation points are generated using Monte Carlo simulations.

Fig. 2 plots the BEP of Tx_1 located at the cell edge versus the average received SNR of Tx_1. At low SNRs, we observe that the BEP simulation is close to the lower bound. This is because, at low SNRs, the noise power dominates, making the interference negligible. Hence, the BEP simulation approaches the lower bound corresponding to the single-transmitter limit. At high SNRs, the interference dominates, and we observe that the BEP simulation approaches the upper bound. We also highlight that a significant BEP improvement is gained through cooperation. For example, at $\text{BEP} = 10^{-5}$, cooperation provides an SNR gain of approximately 14 dB over noncooperation.

Fig. 3 plots the BEP of Tx_2 located close to the cell center versus the average received SNR of Tx_2. Comparing Figs. 2 and 3, we clearly see that the gap between cooperation and noncooperation for Tx_2 is much smaller than the gap between cooperation and noncooperation for Tx_1. For example, at $\text{BEP} = 10^{-5}$ in Fig. 3, we observe that cooperation only provides an SNR gain of approximately 3 dB over noncooperation. This highlights that cooperation provides more benefit to cell-edge users compared with users located closer to the cell center.

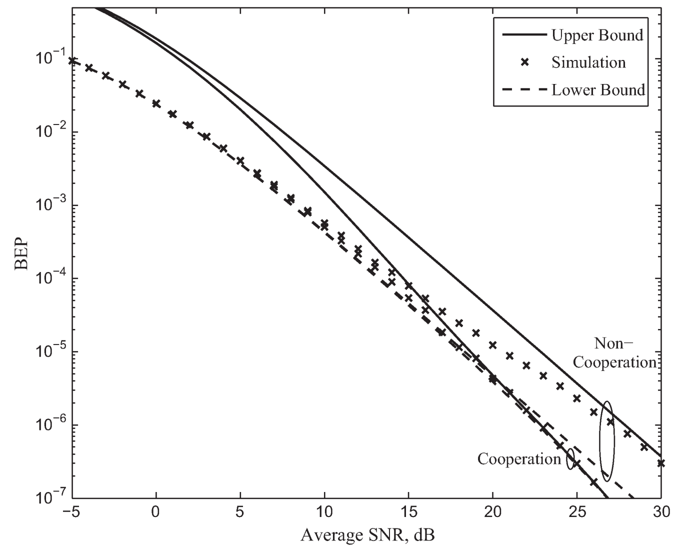

Fig. 3. BEP of Tx_2 versus average received SNR of Tx_2 for $N = 4$ transmitters.

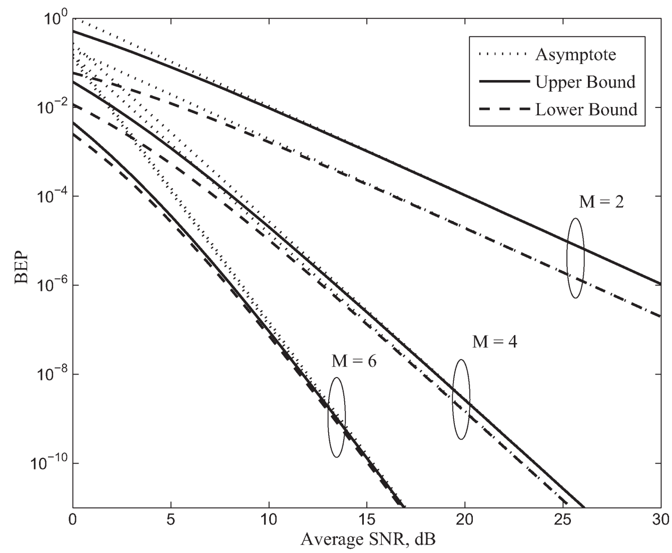

Fig. 4. BEP of Tx_1 versus average received SNR with QPSK modulation as the number of cooperating receivers increases.

Fig. 4 plots the BEP of Tx_1 as the number of receivers at each base station increases from 1 to 3. The analytical curves in Fig. 4 are obtained for QPSK modulation based on our result in (26) and (27). We observe that the BEP decreases with the increasing number of cooperating receivers. For example, at $\text{BEP} = 10^{-5}$, a total of six cooperating receivers provide an SNR gain of approximately 17 dB over two cooperating receivers. Interestingly, we find that the gap between the asymptotic upper and lower bounds diminishes to zero when the number of cooperating receivers is large. This means that the BEP approaches the single-transmitter lower bound, highlighting that optimal multiuser detection in cooperative cellular networks properly exploits interference.

VII. Conclusion

We have presented a new theoretical analysis for upper and lower bounds on the BEP of optimal multiuser detectors in cooperative cellular networks. Results were obtained for a general network model with an arbitrary number of transmitters and receivers, subjected to

Rayleigh fading and path loss. We demonstrated that the lower bound provides an accurate approximation of the exact BEP at low SNRs, whereas the upper bound is accurate at medium to high SNRs. Our analytical expressions are further evaluated asymptotically to characterize the cooperative diversity order in the high-SNR regime.

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On Optimality of Myopic Policy for Opportunistic Access With Nonidentical Channels and Imperfect Sensing

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***Abstract*—We consider the access problem in a multichannel opportunistic communication system with imperfect sensing, where the state of each channel evolves as a nonidentical and independently distributed Markov process. This problem can be cast into a restless multiarmed bandit (RMAB) problem, which is intractable for its exponential computation complexity. A promising approach that has attracted much research attention is the consideration of an easily myopic policy that maximizes the immediate reward by ignoring the impact of the current policy on future reward. Specially, we formalize a family of generic functions, which is referred to as *g*-regular functions, characterized by three axioms, and then establish a set of closed-form conditions for the optimality of the myopic policy and illustrate the engineering implications behind the obtained results.**

***Index Terms*—Imperfect detection, myopic policy, opportunistic spectrum access, restless multiarmed bandit (RMAB).**

I. INTRODUCTION

We consider an opportunistic multichannel communication system with nonidentical but independent channels in which a user is limited to sense and transmit only on a subset of them each time. Given that the detection of a channel state is not perfect, the fundamental optimization problem we address is how the user can exploit past imperfect detection information and the stochastic properties of channels to maximize its utility (e.g., expected throughput) by switching among channels opportunistically.

A. General Context and Related Work

The decision problem can be cast into a restless multiarmed bandit (RMAB) problem, which is proved to be pSPACE-hard [1], and very few results are reported on the structure of the optimal policy due to its high complexity. Thus, the myopic strategy with a simple and tractable structure has recently attracted extensive research attention, which consists of sensing the channels to maximize the expected immediate reward, also called the greedy policy, while ignoring the impact of the current decision on future reward.

Following the research line on the myopic policy, for the case of perfect sensing, Zhao *et al.* [2] established the structure of the myopic policy, analyzed the performance, and partly obtained the optimality for the case of independent and identically distributed (i.i.d.) channels. Ahmad *et al.* [3] derived the optimality of the myopic sensing policy for the positively correlated i.i.d. channels when the user is limited

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