Decentralized Relay Selection in Multi-User Multihop Decode-and-Forward Relay Networks

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Abstract—This paper analyzes the performance of a multi-user multihop relay network using a low complexity decentralized relay selection (DRS) scheme for decode-and-forward cooperative networks. We carry out a rigorous diversity order analysis, with Nakagami-m fading and pathloss and show that the DRS scheme achieves full diversity while maintaining a complexity that is quadratic in the number of users, quadratic in the number of relays and independent of the number of hops. For a special case of two-user networks we derive exact closed-form expressions for the outage probability by considering the order statistics. Furthermore, we extend our analysis to consider interfering relay networks and derive an accurate lower bound on the outage of an arbitrary network user. Based on the lower bound we also show how the outage probability saturates in the high signal-to-interference-plus-noise ratio regime. Extensive numerical examples are used to illustrate the accuracy of the analysis and to highlight the use of the DRS scheme in multi-user multihop relay networks.

Index Terms—Decode-and-forward, multihop networks, outage probability, relay selection.

I. INTRODUCTION

COOPERATIVE communication networks - comprised of multiple nodes distributed across the network forming virtual multiple antenna arrays - have recently emerged as a promising technique to broaden network coverage, enhance network reliability, and mitigate the effect of fading in wireless communication networks [1]–[7]. Cooperative communication networks with multiple relay nodes, known as cooperative relay networks, typically arise in sensor mesh and ad hoc communications. For such networks, the careful selection of relays is extremely important in order to achieve full spatial diversity with low complexity and overhead. As a result, relay selection (RS) in cooperative relay networks has gained considerable research attention during the past few years.

There exists a rich body of literature on RS for two-hop networks where each source experiences a link from the source to relay and relay to destination [8]–[17]. There are various protocols proposed to choose the serving relay from among a collection of available relays, such as *best relay selection* where the relay whose path has the maximum signal-to-noise ratio (SNR) is selected [8], [16], *nearest neighbor selection* where the relay that is nearest to the base station is selected [10], *best harmonic mean selection* where the relay with the largest harmonic mean is selected [9]. In [17], authors consider a two-path relay channel where two relays take turns to decode-and-forward (DF) the transmissions from source to destination. All these works consider networks with single source-destination pairs. In [18], the authors extended RS to multi-user networks where, due to the user competition for relays, the RS problem becomes more challenging when compared to the single source-destination case. An optimal RS (ORS) algorithm which maximizes the minimum end-to-end received SNR of all users is investigated. The complexity of ORS is quadratic in both the number of relays and the number of users. The authors also propose a suboptimal RS (SRS) scheme with complexity linear in the number of relays and quadratic in the number of users. Based on amplify-and-forward (AF) relaying, the authors derive the diversity order of both the ORS and SRS schemes and compare it against the single source-destination case.

Recently, there has been increasing interest in multihop cooperative relay networks where the signal from source to the destination traverses through multiple clusters of relay nodes [19]–[25]. It is a promising technique to increase the transmission range in wireless networks. In [19], a novel optimization metric is derived to select the best relay on a per-hop basis. A suboptimal hop-by-hop RS is proposed that utilizes the mean channel state information (CSI) of the current hop. In [20], the authors present three RS schemes, *optimal*, *adhoc* and *N-hop*, to achieve the full diversity gain provided by the cooperation between relays in each hop. Similar to the dual-hop network, in ORS, serving relays were selected to maximise the received SNR at the destination while in adhoc and N-hop schemes, RS is conducted independently in each hop and set of N hops, respectively. Based on DF relaying, the outage analyses of the proposed three RS schemes were conducted for Rayleigh fading channels. In [21], an approximate lower bound on the average bit error rate (BER) for ORS was derived based on the probability density function (PDF) of a max-min exponential random variable. More recently in [22], the exact BER of ORS in Rayleigh fading was analyzed based

on a convergent power series of gamma functions and a tight approximation of the Gaussian Q-function. It is again noted that the ORS is a centralized algorithm that selects the best relay path with a full diversity performance at the cost of high complexity and high signaling overheads to obtain CSI of all links. In [23]–[25], authors consider the interesting application of multihop relaying in cognitive radio networks and analyse the end-to-end outage probability. However, all aforementioned works over multihop relaying consider only a single source-destination pair.

When we have multiple source-destination pairs the amount of research work available in the open literature is limited. For such networks, spectral-efficient half-duplex transmission schemes with interference alignment were proposed in [26] to minimize the impact of the interference between the relays. However, the authors did not provide a RS scheme to select the multihop relay paths for each user pair. In [27], a low-complexity decentralized RS scheme is proposed for a two-user multihop relay network. Based on DF relaying, the authors derive exact closed-form expressions for the outage probability of each user and show that the proposed decentralized RS scheme can achieve full diversity. However, the results and the analysis is limited to the case with only two source-destination pairs. Thus, there remains a critical need to design new multihop RS algorithms for a more general multiuser network to select the best relays in each hop and assign these relays to each user pair. Finding such optimal paths is very challenging and our motivation is to design a multihop multiple RS strategy that can support multiple source-destination users with low overheads while still achieving a full diversity order.

In this paper, we consider a multi-user multihop relay network where each hop is equipped with multiple relays that assist users to communicate with their designated destinations. We assume that at each hop, each user can only be helped by a single relay and one relay can help at most one user. The main contributions of this paper are listed as follows.

- We propose a decentralized RS (DRS) algorithm for multi-user multihop relay networks, that maximizes the minimum received SNR or signal-to-interference-plus-noise ratio (SINR) for all users. In the ORS algorithm where the path through the relay network for each user is found by an exhaustive search, the end-to-end network performance is maximized. However, it requires global CSI and a joint optimization of all possible paths through the network, which adds significant processing complexity. Our proposed DRS is a more practical hop-by-hop routing scheme where RS is performed independently in each hop and the complexity is quadratic with the number of relays but independent of the number of hops.
- The diversity order of the proposed DRS algorithm is analyzed theoretically based on the DF relaying protocol. Considering a general Nakagami-m fading model, it is shown that the DRS algorithm can achieve full spatial diversity, i.e., for a network with N_s source nodes and N relays, all source nodes have diversity order mN, when receiving nodes have complete interference cancellation capability. Thus, with the DRS algorithm user
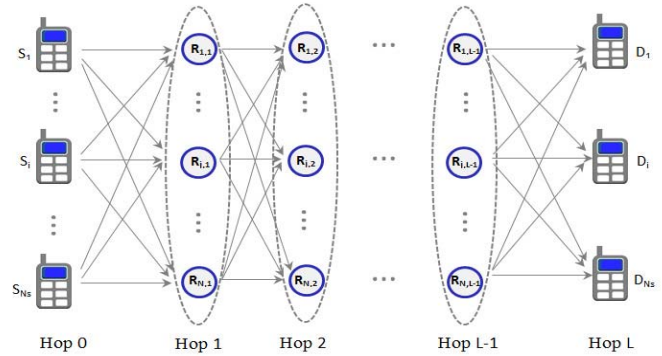

Fig. 1.   A multi-user multihop relay network.

competition for relays does not affect diversity order. Furthermore, we derive a closed-form expression for the array gain of the system. For the special case of two-user networks, an exact closed-form expression for the outage probability is derived using order statistics.
- We extend our analysis of the DRS algorithm to interfering multi-user multihop relay networks where the signals transmitted by active nodes in each hop interfere with each other. The introduction of interference causes the SINRs of users to be correlated with each other resulting in a more challenging mathematical problem. Based on the DRS algorithm we derive a tight lower bound on the outage probability of interfering users. We clearly illustrate how interference degrades the outage probability performance and using the lower bound we show that the outage probability enters a saturation regime at high SINRs.
- Extensive numerical examples are used to simulate the outage probability performance, illustrating the accuracy of our analytical derivations. Furthermore, the performance of our proposed DRS algorithm is compared against three performance benchmarks that were identified based on well-known RS algorithms, namely, optimal, naive and random RS [18]. Interestingly, we reveal that the performance gap between DRS and ORS is small for a low number of relays and hops.

The rest of the paper is organized as follows. The system model is presented in Section II. The RS schemes, including the ORS, naive RS and the proposed sub-optimal DRS, are introduced and detailed in Section III. In Section IV the analysis for DRS without user interference is provided along with the special case of two-user relay networks. The case with interfering relay networks is investigated in Section V. Finally, the numerical examples are provided in Section VI followed by the concluding remarks in Section VII.

II. SYSTEM MODEL

We consider a multihop wireless relay network, shown in Fig. 1, where N_s source nodes $(S_1, S_2, \ldots S_{N_s})$ send information to N_s corresponding destination nodes $(D_1, D_2, \ldots D_{N_s})$ via a general L-hop network with N DF relays in each hop. We have N_s user pairs, denoted as user u, $u \in \{1, 2, \ldots N_s\}$, where user u corresponds to the $S_u - D_u$ pair. There are $(L + 1)$ node clusters: a source

cluster, $(L-1)$ relay clusters and a destination cluster. We assume that each user pair is assisted by only one relay in each cluster (and each relay assists at most one user) to minimize the synchronization requirement and avoid creating too much processing complexity in any single relay. Thus, we need $N \geq N_s$ relays in each cluster and the communication takes place between nodes in two neighboring clusters at a time. Each node is equipped with a single antenna and operates in half-duplex mode with a power budget of P for each transmission. We denote the ith node of the lth hop as $R_{i,l}$ where $i = 1, 2, \ldots N_s$ for $l = 0$ and L, and $i = 1, \cdots, N$ for $l = 1, 2, \cdots, L - 1$. Here, the 0th hop and Lth hop correspond to the source and destination clusters, respectively. In the first part of the paper, we consider an interference free network. As such, we assume either the active nodes in each hop are assigned orthogonal channels or receiving nodes have complete interference cancellation capabilities. In Section V, this assumption is removed allowing the users to interfere.

We consider DF relaying at the relays, and each relay has a sufficient SNR level for error-free decoding. Let $s_{i,l}$ denote the information symbol for transmission at $R_{i,l}$ which has unit average energy. The transmit signal undergoes both multipath fading and distance-dependent path loss.[1] The channel coefficient and distance between node $R_{i,l-1}$ and node $R_{j,l}$ are $h_{ij,l}$ and $d_{ij,l}$, respectively. The received signal at node $R_{j,l}$ from node $R_{i,l-1}$ can be written as

$$y_{j,l} = \sqrt{\frac{P}{d_{ij,l}^\beta}} \, h_{ij,l} s_{i,(l-1)} + n_{j,l} \qquad (1)$$

where β is the path loss exponent, and $n_{j,l}$ is the additive noise at $R_{j,l}$, which is assumed to be independent and identically distributed (i.i.d.) complex Gaussian with zero mean and σ^2 variance. We assume that the distance between hops is much larger than the distance between the relay nodes in each hop. Therefore, the channel gains between transmit nodes in the $(l-1)$th hop and the receiving nodes in the lth hop are i.i.d. However, the channel gains between different consecutive hops are independently but non-identically distributed., i.e., the channel gains between transmit nodes in the $(l-1)$th hop and the receiving nodes in the lth hop and the channel gains between transmit nodes in the $(k-1)$th hop and the receiving nodes in the kth hop, where $l \neq k$, are non-identically distributed.

The corresponding received SNR is given as

$$\tilde{\gamma}_{ij,l} = \frac{P g_{ij,l}}{d_{ij,l}^\beta \sigma^2} \qquad (2)$$

where $g_{ij,l} = |h_{ij,l}|^2$. This SNR is valid for any hop $l = 1, \cdots, L$, and the SNR matrix of the each hop which includes all possible connections can be given as

$$\tilde{\Gamma}_l = \begin{cases} (\tilde{\gamma}_{ij,l}) \in \mathbb{R}^{N_s \times N}; & l = 1 \\ (\tilde{\gamma}_{ij,l}) \in \mathbb{R}^{N \times N}; & l = 2, \cdots, L-1 \\ (\tilde{\gamma}_{ij,l}) \in \mathbb{R}^{N \times N_s}; & l = L. \end{cases} \qquad (3)$$

[1] We model large scale fading as a pure distance based path loss model. The absence of shadow fading is motivated by the fact that in a multihop relay network, relay nodes can be placed in such a way that shadowing is avoided.

We assume that the amplitude of the channel gain, $|h_{ij,l}|$, follows a Nakagami-m distribution and all wireless channels are i.i.d. Then, $g_{ij,l}$ follows a gamma distribution, and the corresponding cumulative distribution function (CDF) of $\tilde{\gamma}_{ij,l}$ can be given as

$$F_{\tilde{\gamma}_{ij,l}}(x) = \frac{\gamma\left(m, \frac{mx}{\tilde{\Omega}_l}\right)}{\Gamma(m)} \qquad (4)$$

where m is a parameter related to the Nakagami fading channel, $\tilde{\Omega}_l = P/(d_{ij,l}^\beta \sigma^2)$, and $\Gamma(.)$ and $\gamma(.,.)$ are the gamma and incomplete gamma functions, respectively [28].

III. Multi-Hop Relay Selection Schemes

The main challenge in a multi-user multihop relay network is to select the best path for each source to traverse through the network to the corresponding destination. In this section, we discuss possible RS schemes for a multi-user multihop network with multiple DF relays by focusing on the outage probability performance. It is important to note that for DF relaying the outage probability of a given path is limited by the minimum SNR of L hops [20, eq. (3)]. Therefore, to minimize the outage probability of each user, the RS scheme should select the paths by maximising the minimum SNR of each user. Also, the RS scheme used for selecting relay nodes in each hop should be designed in such a way that user fairness is maintained while keeping the network complexity low. Considering the energy constraints at the relay node we make the practical assumption that relay nodes cannot be shared between users, i.e., a given relay node cannot serve more than one source node. In the following, we define RS schemes that take into account the individual performance as well as user fairness.

A. Optimal RS Scheme and Challenges

The ORS involves an exhaustive search through all possible paths from source nodes to destination nodes. If we consider user 1 who needs to pass through L hops to reach its destination, we can find N^{L-1} possible paths to traverse. Thus, for N_s users we have $N_s N^{L-1}$ possible paths from source nodes to destination nodes. Since a relay cannot be shared between both users, for a given path assigned to one user, there are $(N-1)^{L-1}$ possible paths for the other user. Thus, we have $\left[\prod_{i=0}^{N_s-1}(N-i)\right]^{L-1}$ paths for N_s users, and we need to find the N_s paths that maximize the minimum end-to-end SNR of all users. Although this exhaustive search approach leads to an ORS scheme, it has an exponential complexity with the number of hops. For example, a small network with $N_s = 2$, $N = 3$ and $L = 4$, results in 216 possible path pairs. Furthermore, the ORS scheme should be implemented by a centralized system with a master node (which may be an external node) that has access to global CSI of all the links in the network. This increases the complexity and overhead for channel training/estimation making the implementation of the ORS scheme virtually impossible.

B. Naive RS Scheme

In the naive RS, user 1 first selects the best path from among all possible paths. Then, user 2 selects its best path that does not conflict with user 1 and so on. In each hop, user i selects one relay from among $(N - i)$ relays that are not used by the first $(i-1)$ users, i.e., user 1 to user $(i-1)$. As such, the user fairness is compromised, and in a system with Nakagami-m fading channels user i can only achieve a maximum diversity order of $(N-i) \times m$. However, the complexity of the selection algorithm is lower than the ORS scheme as we only have N^{L-1} paths for user 1 and $(N - 1)^{L-1}$ paths for user 2 and so on.

C. Proposed Decentralized RS (DRS) Scheme

In this paper, we consider a DRS scheme which performs RS independently at each hop. As opposed to the optimal and naive RS schemes, this scheme achieves user fairness while maintaining lower network complexity.

Note that, the $(l - 1)$th hop has a set of N relays which we denote as $\mathcal{R}_{l-1} = \{R_{1,l-1}, \cdots, R_{N,l-1}\}$. After the RS at the $(l - 1)$th hop, a set of N_s relays is selected from $\mathcal{R}_{l-1}$ and we denote this as $\mathcal{R}_{l-1}^* = \{R_{1,l-1}^{(1)}, \cdots, R_{u,l-1}^{(u)}, \cdots, R_{N_s,l-1}^{(N_s)}\}$ where $R_{u,l-1}^{(u)}$ is the selected relay for the user u at the $(l-1)$th hop. Then, the SNR matrix from $\mathcal{R}_{l-1}^*$ to $\mathcal{R}_l$ can be formed by first extracting the corresponding rows from $\tilde{\mathbf{\Gamma}}_l$ in eq. (3), and then arranging them according to the user index u. Thus, the RS matrix at the lth hop may be given as

$$\mathbf{\Gamma}_l = (\gamma_{uj,l}) \in \mathbb{R}^{N_s \times N}; \quad l = 1, \cdots, L. \tag{5}$$

It is important to note that $\mathbf{\Gamma}_1 = \tilde{\mathbf{\Gamma}}_1$; for $l = 2, \cdots, L - 1$, the entry $\gamma_{uj,l}$ represents the SNR at $R_{j,l}$ given that $R_{u,l-1}^{(u)}$ is selected for the user u at the $(l - 1)$th hop which can be extracted from entries in $\tilde{\mathbf{\Gamma}}_l$; and $\mathbf{\Gamma}_L = \tilde{\mathbf{\Gamma}}_L^T$ which is the transpose of $\tilde{\mathbf{\Gamma}}_L$. Based on $\mathbf{\Gamma}_l$, we perform the following steps.

- *RS at the lth hop ($l = 1, \cdots, L - 2$)*: We need to select N_s relays at the lth hop that will maximize the minimum SNR of N_s users at the lth hop. To do so, we apply the optimal single-hop RS algorithm proposed in [15] and extended in [18], in which the worst-case complexity is $\mathcal{O}(N_s^2 N^2)$. While this RS algorithm is detailed for a dual-hop ($L = 2$) relay network in [15], for the sake of completeness, we include the main steps applicable for a general multi-hop ($L \geq 2$) relay network, as given in Algorithm 1.
  We apply this selection criterion for each SNR matrix sequentially starting from $\mathbf{\Gamma}_1$ to $\mathbf{\Gamma}_{L-2}$. As such, the complexity of each selection is quadratic in the number of relays but independent of the number of hops.
- *RS at the $(L-1)$th hop*: In the final two clusters of relays (i.e., the $(L - 1)$th and Lth hop), we perform a joint RS by using $\mathbf{\Gamma}_{L-1}$ in (5) and $\tilde{\mathbf{\Gamma}}_L$ in (3) and construct $\mathbf{\Gamma}_{L-1,L}$ as

$$\mathbf{\Gamma}_{L-1,L} = \min\left(\gamma_{uj,L-1}, \tilde{\gamma}_{ju,L}\right) \in \mathbb{R}^{N_s \times N}.$$

  We repeat the algorithm for the $(L - 1)$th hop by using $\mathbf{\Gamma}_{L-1,L}$. The motivation for the joint RS is because if we apply the RS algorithm directly to $\mathbf{\Gamma}_{L-1}$, we will select

N_s relays in the $(L-1)$th hop, and the selected relays will directly communicate with the corresponding destination nodes in the Lth hop. As such, $\tilde{\mathbf{\Gamma}}_L$ is not involved in the selection which results in a degradation in the diversity order [20].

Algorithm 1 Steps of the DRS Scheme for $l = 1, \cdots, L - 2$

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1. Generate $\mathbf{\Gamma}_l$ using (5).
2. Sort all $\gamma_{uj,l}$'s in descending order as $\gamma_{(1),l} \geq \gamma_{(2),l} \geq \cdots \geq \gamma_{(N_s N),l}$, where $\gamma_{(k),l}$ denotes the kth largest element among $N_s N$ independent entries in $\mathbf{\Gamma}_l$.
3. Generate a temporary matrix $\hat{\mathbf{\Gamma}}_l$ with N_s rows and N columns. Fill $\hat{\mathbf{\Gamma}}_l$ starting from $\gamma_{(1),l}$ until each row and at least N_s columns have entries. Having an entry in each row make sure that there is a selection available for each user. Having entries in at least N_s columns make sure that the relays are not shared between users. This filling process results in maximizing the minimum SNR assigned to each user.
4. Assign the last entry for the corresponding user and remove the row and the column.
5. Repeat step 4 until all users have a relay assigned. If an assignment cannot be made go back to step 3 and add the next entry to $\hat{\mathbf{\Gamma}}_l$.
6. Repeat steps 4 and 5 until all users have assigned relays.

Our proposed DRS scheme can be implemented in a decentralized manner where a master node is assigned in each hop (which may be one of the relays) that has access to local CSI of the links in the given hop. Thus, the DRS scheme will lead to a significant decrease in the complexity and overhead compared with the ORS scheme based on exhaustive search.

D. Examples

We consider a network with $N_s = 2$, $N = 2$ and $L = 3$. Each relay cluster has two relays $N = 2$. For one channel realization, we assume the SNR matrices for three hops $\tilde{\mathbf{\Gamma}}_1$, $\tilde{\mathbf{\Gamma}}_2$ and $\tilde{\mathbf{\Gamma}}_3$, respectively, are given by

$$\begin{bmatrix} 0.52 & 0.69 \\ 0.40 & 0.78 \end{bmatrix}; \begin{bmatrix} 0.58 & 0.97 \\ 0.07 & 0.45 \end{bmatrix}; \text{ and } \begin{bmatrix} 0.03 & 0.62 \\ 0.10 & 0.44 \end{bmatrix}.$$

- *Optimal RS*: Applying an exhaustive search, we have selections $S_1 - R_{2,1} - R_{2,2} - D_1$ and $S_2 - R_{1,1} - R_{1,2} - D_2$. Then, the effective end-to-end SNRs are 0.10 for user 1 and 0.40 for user 2, where the minimum SNR of the network is 0.10.
- *Naive RS*: Applying the naive RS scheme, we have selections $S_1 - R_{1,1} - R_{2,2} - D_1$ and $S_2 - R_{2,1} - R_{1,2} - D_2$. Then, the effective end-to-end SNRs are 0.10 for user 1 and 0.07 for user 2, where the minimum SNR of the network is 0.07.
- *Proposed DRS*: To maximize the minimum SNR of $\mathbf{\Gamma}_1$, we select $R_{1,1}$ for S_1 and $R_{2,1}$ for S2. For the last two hops, we maximize the minimum SNR by using the joint selection based on $\mathbf{\Gamma}_2$ and $\tilde{\mathbf{\Gamma}}_3$. Then, we have selections $S_1 - R_{1,1} - R_{2,2} - D_1$ and $S_2 - R_{2,1} - R_{1,2} - D_2$.

The effective end-to-end SNRs are 0.10 for user 1 and 0.07 for user 2, where the minimum SNR is 0.07.

- *Proposed DRS without joint selection*: To maximize the minimum SNR of Γ_1 and Γ_2, we select $R_{1,1}$ for S_1 and $R_{2,1}$ for S_2; and $R_{1,2}$ for $R_{1,1}$ and $R_{2,2}$ for $R_{2,1}$ which means, so far, we have selections $S_1 - R_{1,1} - R_{1,2}$ and $S_2 - R_{2,1} - R_{2,2}$. Since there are no remaining options in the last hop, $R_{1,2}$ transmits to D_1 and $R_{2,2}$ transmits to D_2. As such, the effective end-to-end SNRs are 0.03 for user 1 and 0.44 for user 2, where the minimum SNR is 0.03. This highlights the performance advantage of joint selection in the last two hops.

Interestingly, if we consider the channel realizations of the SNR matrices $\tilde{\Gamma}_1$, $\tilde{\Gamma}_2$ and $\tilde{\Gamma}_3$, respectively, are given by

$$\begin{bmatrix} 0.60 & 0.89 \\ 0.82 & 0.23 \end{bmatrix}; \quad \begin{bmatrix} 0.16 & 0.52 \\ 0.69 & 0.49 \end{bmatrix}; \quad \text{and} \quad \begin{bmatrix} 0.86 & 0.85 \\ 0.11 & 0.79 \end{bmatrix},$$

a minimum SNR of the network of 0.52 is achieved for all three RS schemes. This illustrates that for some example scenarios both naive RS and proposed DRS give optimal results. But for many other example scenarios the minimum SNR of the network with naive RS or proposed DRS is smaller that that of the ORS. Hence, naive RS and proposed DRS are sup-optimal schemes.

IV. PERFORMANCE ANALYSIS FOR DRS WITHOUT USER-INTERFERENCE

In this section, we first derive the outage probability, diversity order and array gain of a general multihop network based on the DRS scheme discussed in Section III. Then, we consider the performance of a two-user network as a special case.

A. Outage Probability

For notational simplicity, let us define the effective SNR between nodes $R_{i,l-1}$ and $R_{j,l}$ as $\acute{\gamma}_{ij,l}$. Note that, for $l = 1, \cdots, L-2$, we can write that $\acute{\gamma}_{ij,l} = \tilde{\gamma}_{ij,l}$ where $\tilde{\gamma}_{ij,l}$ is in (2). In the last two hops, i.e., hop $L-1$ and L, we apply the joint selection such that the SNR of the channel between nodes $R_{i,L-2}$ and $R_{j,L}$ via $R_{k,L-1}$ can be given by $\min(\tilde{\gamma}_{ik,L-1}, \tilde{\gamma}_{kj,L})$. Recall that $j \in \{1, 2, \ldots, N_s\}$ is for the N_s destination nodes. Then, we denote the effective SNR of the $(L-1)$th hop as $\acute{\gamma}_{ij,L-1} = \min(\tilde{\gamma}_{ik,L-1}, \tilde{\gamma}_{kj,L})$. Based on (4), we can derive the corresponding CDF of $\acute{\gamma}_{ij,l}$ as

$$F_{\acute{\gamma}_{ij,l}}(x)$$
$$= \begin{cases} F_{\tilde{\gamma}_{ij,l}}(x); & l = 1, \cdots, L-2 \\ 1 - \left(1 - F_{\tilde{\gamma}_{ij,L-1}}(x)\right)\left(1 - F_{\tilde{\gamma}_{ij,L}}(x)\right); & l = L-1. \end{cases}$$
$$(6)$$

Here the expression for $l = L-1$ results due to the joint processing in the last two hops. For presentation simplicity, we assume that the $(L-1)$ relay clusters are placed between the source and destination pairs with equal distances, i.e., $d_{ij,l} = d/L; \ \forall i, j, l$ where d is the distance between the source and destination pairs. Since all channels are identical, we set $\tilde{\Omega}_l = \Omega = P/((d/L)^\beta \sigma^2)$ for $l = 1, \cdots, L$. However,

it is important to note that the following derivation applies to the general case where the $(L-1)$ relay clusters are placed randomly between the source and destination pairs such that the distance between clusters in not identical.

Since we maximize the minimum SNR for each hop in the channel matrix we need to apply the order statistics of $\acute{\gamma}_{ij,l}$. The CDF of the kth largest element among $N_s N$ independent entries in Γ_l, denoted as $\gamma_{(k),l}$, can be written as [18, eq. (8)]

$$F_{\gamma_{(k),l}}(x) = \sum_{i=0}^{k-1} \frac{(N_s N)! \binom{k-1}{i}(-1)^i F_{\acute{\gamma}_{ij,l}}(x)^{N_s N-k+i+1}}{(N_s N-k+i+1)(N_s N-k)!(k-1)!} \quad (7)$$

where $F_{\acute{\gamma}_{ij,l}}(x)$ is chosen from (6) depending on the hop. For the DF multihop relay network the end-to-end SNR of user u, $\gamma^{(u)}$, can be written as

$$\gamma^{(u)} = \min\{\gamma_{u,1}, \cdots, \gamma_{u,l}, \cdots, \gamma_{u,L-1}\}, \quad (8)$$

where $\gamma_{u,l}$ is the selected SNR for user u at the lth hop, i.e, the SNR selected from among N_s SNRs in the uth row of Γ_l. With the DRS scheme, we can write an expression for the outage probability of user u such that the end-to-end SNR falls below a certain predetermined threshold SNR γ_{th} as

$$\begin{aligned} P_o^{(u)} &= \mathbb{P}[\gamma^{(u)} \leq \gamma_{th}] \\ &= \mathbb{P}[\min\{\gamma_{u,1}, \cdots, \gamma_{u,l}, \cdots, \gamma_{u,L-1}\} \leq \gamma_{th}] \\ &= 1 - \prod_{l=1}^{L-1} \left(1 - \mathbb{P}[\gamma_{u,l} \leq \gamma_{th}]\right). \end{aligned} \quad (9)$$

Based on our DRS scheme which maximizes the minimum SNR for each Γ_l, we note that $\gamma_{u,l}$ can take $\gamma_{(1),l}, \gamma_{(2),l}, \cdots$, or $\gamma_{(N_s N-N+1),l}$. Thus, we can re-write (9) as

$$P_o^{(u)}$$
$$= 1 - \prod_{l=1}^{L-1} \left(1 - \sum_{k=1}^{N_s N-N+1} \mathbb{P}[\gamma_{u,l} = \gamma_{(k),l}]\mathbb{P}[\gamma_{(k),l} \leq \gamma_{th}]\right)$$
$$= 1 - \prod_{l=1}^{L-1} \left(1 - \sum_{k=1}^{N_s N-N+1} \mathbb{P}[\gamma_{u,l} = \gamma_{(k),l}]F_{\gamma_{(k),l}}(\gamma_{th})\right), \quad (10)$$

where $F_{\gamma_{(k),l}}(\gamma_{th})$ is defined in (7) and $\mathbb{P}[\gamma_{u,l} = \gamma_{(k),l}]$ denotes the probability of each event $\gamma_{u,l} = \gamma_{(k),l}$ where $k = 1, \cdots, N_s N - N + 1$. For the general case with N_s users, deriving analytical expressions for the probability of each event is challenging, however, these probabilities can be calculated numerically, via Monte Carlo simulations. We provide analytical expressions of $\mathbb{P}[\gamma_{u,l} = \gamma_{(k),l}]$s only for a two-user network in Section IV-C.

B. Diversity Order and Array Gain

Diversity order determines how fast the outage probability decreases in the high transmit power regime. It is defined as $\delta \triangleq \lim_{P \to \infty} \frac{\log P_o^{(u)}}{\log P}$ [29]. Then the array gain can be defined as $\epsilon = \lim_{P \to \infty} \left(P^\delta P_o^{(u)}\right)^{-1}$. In this section, we analyze the achievable diversity order and the array gain of the DRS scheme.

Theorem 1: In a multi-user multihop relay network with the DRS scheme, the achievable diversity order δ and array-gain ϵ of each user, respectively, are given by

$$\delta = mN$$

$$\epsilon = \begin{cases} \dfrac{(L-2+2^N)(N_sN)!m^{mN}\prod_{i=1}^{N-1}\frac{N-i}{N_sN-i}}{N_sN!(N_sN-N)!(m!)^N\left(\gamma_{th}\left(\frac{d}{L}\right)^\beta\sigma^2\right)^{-mN}}; & N_s < N \\[4mm] \dfrac{2(L-2+2^N)(N_sN)!m^{mN}\prod_{i=1}^{N-1}\frac{N-i}{N_sN-i}}{N_sN!(N_sN-N)!(m!)^N\left(\gamma_{th}\left(\frac{d}{L}\right)^\beta\sigma^2\right)^{-mN}}; & N_s = N, \end{cases}$$

where m is the Nakagami shape parameter and N is the number of relays in each hop.

A detailed proof of Theorem 1 is given in Appendix A. From Theorem 1 we understand that our DRS scheme achieves full diversity order.

If we do not perform joint processing in the last two hops, i.e., if the signal in the last hop is transmitted directly to the corresponding destination node without any node selection, then $\acute{\gamma}_{ij,l} = \tilde{\gamma}_{ij,l}$ for all the hops and we get $F_{\acute{\gamma}_{ij,l}}(x) = F_{\tilde{\gamma}_{ij,l}}(x), \forall l \in \{1,2,\ldots,L\}$. Thus, the outage probability expression in (10) reduces to

$$P_o^{(u)} = 1 - \left(1 - \gamma\left(m, \frac{m\gamma_{th}}{\tilde{\Omega}_l}\right)\right)$$
$$\times \prod_{l=1}^{L-1}\left(1 - \sum_{k=1}^{N_sN-N+1}\mathbb{P}[\gamma_{u,l} = \gamma_{(k),l}]F_{\gamma_{(k),l}}(\gamma_{th})\right). \tag{11}$$

Following the same steps as given in Appendix A, we can separate the higher order terms in (11) as

$$P_o^{(u)} = \frac{\left(m\gamma_{th}\left(\frac{d}{L}\right)^\beta\sigma^2\right)^m}{m!}P^{-m} + \mathcal{O}\left(\frac{1}{P^{m+1}}\right), \tag{12}$$

and observe that without joint processing the diversity order reduces to m.

C. Special Case: Two-User Networks

In this section, we consider a special case with only two users and $N \geq 2$ relays. Then, the RS matrix of the network can be written as

$$\mathbf{\Gamma}_l = \begin{bmatrix} \gamma_{11,l} & \cdots & \gamma_{1n,l} & \cdots & \gamma_{1N,l} \\ \gamma_{21,l} & \cdots & \gamma_{2n,l} & \cdots & \gamma_{2N,l} \end{bmatrix}_{2\times N}. \tag{13}$$

When $N_s = 2$, the outage probability can be written by using (10) as

$$P_o^{(u)} = 1 - \prod_{l=1}^{L-1}\left(1 - \sum_{k=1}^{N+1}\mathbb{P}[\gamma_{u,l} = \gamma_{(k),l}]F_{\gamma_{(k),l}}(\gamma_{th})\right). \tag{14}$$

Now we consider the analytical expressions for $\mathbb{P}[\gamma_{u,l} = \gamma_{(k),l}]$. Although they are presented in [27], for the sake of completeness, we briefly summarize the results as follows.

It is important to note that each event occurs with equal probability for each user pair u. Hence, we use a factor $1/2$ to calculate the outage of a particular user pair u. Using γ_{max} and γ_{min} to denote the maximum and minimum of the selected SNRs, we consider the probability of the following cases:

- $\gamma_{u,l} = \gamma_{(1),l}$ happens only if $\gamma_{max} = \gamma_{(1),l}$, and thus γ_{min} can take any value of $\gamma_{(2),l}, \gamma_{(3),l}, \ldots,$ or $\gamma_{(N+1),l}$. Subsequently, we consider the following three scenarios: i) $\gamma_{max} = \gamma_{(1),l}$ and $\gamma_{min} = \gamma_{(2),l}$ happens when $\gamma_{(1),l}$ and $\gamma_{(2),l}$ are in two distinct rows and two distinct columns of Γ. This event has probability $\frac{1}{2}\frac{N-1}{2N-1}$; ii) $\gamma_{max} = \gamma_{(1),l}$ and $\gamma_{min} = \gamma_{(3),l}$ happens when $\gamma_{(1),l}$ and $\gamma_{(2),l}$ are in the same column, or $\gamma_{(1),l}$ and $\gamma_{(2),l}$ are in the same row and $\gamma_{(3),l}$ in the other row and a column different to $\gamma_{(1),l}$. This event has probability $\frac{1}{2}\frac{N}{2(2N-1)}$; and iii) For $N > 2$, $\gamma_{max} = \gamma_{(1),l}$ and $\gamma_{min} = \gamma_{(k),l}$ $(k = 4,5,\ldots,N+1)$ happens when all $\gamma_{(2),l}, \gamma_{(3),l}, \ldots, \gamma_{(k-1),l}$ are in the same row and $\gamma_{(k),l}$ in the other row and a column different to $\gamma_{(1),l}$. This event has probability $\frac{1}{2}\frac{N-1}{2N-(k-1)}\prod_{i=1}^{k-2}\frac{N-i}{2N-i}$.
- $\gamma_{u,l} = \gamma_{(2),l}$ happens when $\gamma_{max} = \gamma_{(2),l}$ or $\gamma_{min} = \gamma_{(2),l}$. Since we have only two source nodes, γ_{max} can either be $\gamma_{max} = \gamma_{(1),l}$ or $\gamma_{max} = \gamma_{(2),l}$. Thus, $\mathbb{P}[\gamma_{max} = \gamma_{(2),l}] = \frac{1}{2} - \mathbb{P}[\gamma_{u,l} = \gamma_{(1),l}]$ which can be calculated with the aid of the previous case. Further, $\gamma_{min} = \gamma_{(2),l}$ happens only when $\gamma_{(1),l}$ and $\gamma_{(2),l}$ are in two distinct rows and two distinct columns of $\mathbf{\Gamma}_l$. This event has probability $\frac{1}{2}\frac{N-1}{2N-1}$.
- $\gamma_{u,l} = \gamma_{(3),l}$ is equivalent to $\gamma_{min} = \gamma_{(3),l}$ which happens when $\gamma_{(1),l}$ and $\gamma_{(2),l}$ are in the same column, or $\gamma_{(1),l}$ and $\gamma_{(2),l}$ are in the same row and $\gamma_{(3),l}$ is in a different row. This event has probability $\frac{1}{2}\frac{N+2}{2(2N-1)}$.
- For $N > 2$, $\gamma_{u,l} = \gamma_{(k),l}$ for $k = 4, \cdots, N+1$ is equivalent to $\gamma_{min} = \gamma_{(k),l}$ for $k = 4, \cdots, N+1$ which happens when all $\gamma_{(1),l}, \gamma_{(2),l}, \cdots, \gamma_{(k-1),l}$ are in the same row and $\gamma_{(k),l}$ is in a different row. This event has probability $\frac{1}{2}\frac{2N\binom{N}{k-1}}{(2N-(k-1))\binom{2N}{k-1}}$.

$$P_o^{(u)} = \begin{cases} 1 - \prod_{l=1}^{L-1}\left(1 - \frac{1}{3}\sum_{k=1}^{3}F_{\gamma_{(k),l}}(\gamma_{th})\right) & ;N = 2 \\[4mm] 1 - \prod_{l=1}^{L-1}\left[1 - \left(\frac{3N-2}{8N-4} + \frac{1}{2}\sum_{k=4}^{N+1}\frac{(N-1)\prod_{i=1}^{k-2}\frac{N-i}{2N-i}}{2N-(k-1)}\right)F_{\gamma_{(1),l}}(\gamma_{th}) - \frac{(N+2)F_{\gamma_{(3),l}}(\gamma_{th})}{4(2N-1)} \right. \\[4mm] \left. - \left(\frac{3N-2}{8N-4} - \frac{1}{2}\sum_{k=4}^{N+1}\frac{(N-1)\prod_{i=1}^{k-2}\frac{N-i}{2N-i}}{2N-(k-1)}\right)F_{\gamma_{(2),l}}(\gamma_{th}) - \sum_{k=4}^{N+1}\frac{2N\binom{N}{k-1}F_{\gamma_{(k),l}}(\gamma_{th})}{2(2N-(k-1))\binom{2N}{k-1}}\right] & ;N > 2. \end{cases} \tag{15}$$

By considering all possible cases and substituting the corresponding event probabilities in (14), the exact outage probability of user u, $P_o^{(u)}$, can be derived in closed-form as (15), given at bottom of the previous page. Moreover, following the similar steps as in Section IV-B, we can derive the diversity order and array-gain as

$$\delta = mN$$

$$\epsilon = \begin{cases} \dfrac{(L-2+2^N)(2N)!m^{mN}\prod_{i=1}^{N-1}\frac{N-i}{2N-i}}{(N!)^2(m!)^N\left(\gamma_{th}\left(\frac{d}{L}\right)^\beta\sigma^2\right)^{-mN}}; & N=2 \\[20pt] \dfrac{(L-2+2^N)(2N)!m^{mN}\prod_{i=1}^{N-1}\frac{N-i}{2N-i}}{2(N!)^2(m!)^N\left(\gamma_{th}\left(\frac{d}{L}\right)^\beta\sigma^2\right)^{-mN}}; & N>2, \end{cases}$$

This results is also consistent with Theorem 1 when $N_s = 2$.

V. PERFORMANCE ANALYSIS FOR DRS WITH USER-INTERFERENCE

In this section, we consider a network with interfering users. Note that so far in this paper, we have assumed that the active nodes in each hop are assigned orthogonal channels or the receiving nodes have complete interference cancellation capability. As a result, the signal received by each user at each relay node was interference free. However, in a multiple-access system or a frequency-reused cell, interference from other transmitting sources may cause performance degradation and, therefore, cannot be ignored. When the users are interfering with each other the corresponding received signal at node $R_{j,l}$ can be written as

$$y_{j,l} = \sum_{u=1}^{N_s} \sqrt{\frac{P}{(d_{uj,l}^{(u)})^\beta}}\, h_{uj,l}^{(u)} s_{u,(l-1)}^{(u)} + n_{j,l}, \qquad (16)$$

where $d_{uj,l}^{(u)}$ and $h_{uj,l}^{(u)}$ denotes the channel coefficients between node $R_{j,l}$ and $R_{u,l-1}^{(u)}$ and $s_{u,l-1}^{(u)}$ is the information symbol at $R_{u,l-1}^{(u)}$. The corresponding received SINR of user u at node j in hop l is given as

$$\gamma_{uj,l} = \frac{\omega_{uj,l}}{\sum_{q=1,q\neq u}^{N_s}\omega_{qj,l}+\sigma^2}, \qquad (17)$$

where $\omega_{qj,l} = Pg_{qj,l}^{(u)}/(d_{qj,l}^{(u)})^\beta$ with $g_{qj,l}^{(u)}=|h_{qj,l}^{(u)}|^2$. From (17), we learn that the received SINRs of different users at a given node, i.e., the SINRs in each column of (5), are now correlated due to the effect of interfering channels.

Based on the proposed DRS algorithm, the outage probability of user u can be found from (10), even in the interfering case. However, the analytical expression for $F_{\gamma_{(k),l}}(\gamma_{th})$ in (17) is no longer valid as $F_{\gamma_{(k),l}}(\gamma_{th})$ in the interfering case corresponds to the CDF of the kth largest SINR from among N_sN correlated SINRs. As discussed below, deriving an expression for $F_{\gamma_{(k),l}}(\gamma_{th})$ is mathematically challenging due to the complexity of resulting order statistics. Based on [30, eq. (5.3.1)], the CDF of the kth largest element among N_sN correlated SINRs, denoted as $\gamma_{(k),l}$, can be written as

$$F_{\gamma_{(k),l}}(x) = \sum_{j=N_sN-k+1}^{N_sN}(-1)^{j-N_sN+k-1}\binom{j-1}{N_sN-k}H_{j:j}(x),$$
$$\qquad (18)$$

where

$$H_{j:j}(x) = \sum_{1\leq i_{j+1}<\ldots<i_{N_sN}\leq N_sN} F_{\gamma_{(1),l}}^{i_{j+1},\ldots,i_{N_sN}}(x), \quad (19)$$

with $F_{\gamma_{(1),l}}^{i_{j+1},\ldots,i_{N_sN}}(x)$ denoting the CDF of the largest of j correlated random variables picked from the sample of N_sN SINRs in the lth hop. Let us denote this set of j correlated random variables picked from the sample of N_sN SINRs by $\gamma_{i_1,l},\ldots,\gamma_{i_j,l}$. The superscript notation in (19) indicates that $\gamma_{i_{j+1},l},\ldots,\gamma_{i_{N_sN},l}$ have been dropped from the sample of N_sN SINRs. By noting that $F_{\gamma_{(1),l}}^{i_{j+1},\ldots,i_{N_sN}}(x) = F_{\gamma_{i_1,l},\ldots,\gamma_{i_j,l}}(x,\ldots,x)$, where $F_{\gamma_{i_1,l},\ldots,\gamma_{i_j,l}}(x,\ldots,x)$ denotes the joint CDF of the selected j SINRs, we can reexpress (18) as

$$F_{\gamma_{(k),l}}(x) = \sum_{j=N_sN-k+1}^{N_sN}\left[(-1)^{j-N_sN+k-1}\binom{j-1}{N_sN-k}\right.$$
$$\left.\times \sum_{1\leq i_1<\ldots<i_j\leq N_sN}F_{\gamma_{i_1,l},\ldots,\gamma_{i_j,l}}(x,\ldots,x)\right]. \quad (20)$$

As such, we need to know the joint CDF of all possible subsets of SINRs in $\boldsymbol{\Gamma}_l$ in order to compute the CDFs of ordered SINRs. This is explained in detail in Appendix B using a simple example of $N_s = 2$ and $N = 2$. As a result deriving a general expression for $F_{\gamma_{(k),l}}(x)$, and therefore for outage probability, is mathematically challenging in the interfering case. As the exact results are intractable, in the following, we derive an accurate lower bound on the outage probability of an arbitrary user in interfering relay networks.

Remark: In the interfering case, all hops are processed individually, including the last two hops that were jointly analysed in the interference free case. This is because in the interfering case the SINRs in the Lth hop depends on the relay nodes that were selected in the $(L-1)$th hop. As such, the master node has to consider all possible combinations of selected relay nodes in the $(L-1)$th hop and compare the SINRs. This significantly increases the processing complexity. Furthermore, the effect of interference causes the performance to saturate in the high SINR regime. Therefore the loss of diversity gain that motivated the joint processing in the interference free case does not apply in the interfering case.

A. Lower Bound for Outage Probability

From (17), we learn that each term in a given column in $\boldsymbol{\Gamma}_l$ is correlated and the elements in a given row are independent. Let $\gamma_{(N),l}^{(u)}$ denote the best SINR from among all the SINRs in the uth row of $\boldsymbol{\Gamma}_l$, i.e., $\gamma_{(N),l}^{(u)} = \max(\gamma_{u1,l},\gamma_{u2,l},\ldots,\gamma_{uN,l})$. If $\gamma_{(N),l}^{(u)}$ is assigned for user u, $\forall u \in \{1,2,\ldots,N_s\}$ then all the users experience the best SINR at hop l. However, this happens only if $\gamma_{(N),l}^{(1)},\gamma_{(N),l}^{(2)},\ldots,\gamma_{(N),l}^{(N_s)}$ are in different columns in $\boldsymbol{\Gamma}_l$, because according to the proposed DRS scheme relays cannot be shared among users. Observing the structure of the SINR expression in (17), we argue that the probability of two SINRs of $\gamma_{(N),l}^{(1)},\gamma_{(N),l}^{(2)},\ldots,\gamma_{(N),l}^{(N_s)}$ being located in the same column

is very small in the interfering case, especially when $N \gg N_s$. This is because the received signal power that acts as the desired signal for one user becomes the interfering signal for other users. For example, for a two-user case the received SINR of user 1 at relay node j in the lth hop is given by,

$$\gamma_{2j,l} = \frac{\omega_{1j,l}}{\omega_{2j,l} + \sigma^2}, \qquad (21)$$

and the received SINR of user 2 at relay node j in the same hop is given by

$$\gamma_{2j,l} = \frac{\omega_{2j,l}}{\omega_{1j,l} + \sigma^2}. \qquad (22)$$

Note how the signal power that appears in the numerator in (21) changes to the interfering power that appears in the denominator in (22) and vice versa. This structure makes the probability of $\gamma_{1j,l}$ and $\gamma_{2j,l}$ being the two best SINRs in rows one and two, respectively, very small. As such, we can write a lower bound on the outage probability of user u based on (9) as

$$P_o^{(u)} \geq 1 - (1 - \mathbb{P}[\gamma_{u,L} \leq \gamma_{th}]) \prod_{l=1}^{L-1} \left(1 - \mathbb{P}[\gamma_{(N),l}^{(u)} \leq \gamma_{th}]\right)$$
$$\overset{(a)}{=} 1 - (1 - \mathbb{P}[\gamma_{u,L} \leq \gamma_{th}]) \prod_{l=1}^{L-1} \left(1 - (\mathbb{P}[\gamma_{uj,l} \leq \gamma_{th}])^N\right),$$
$$(23)$$

where $\gamma_{u,L}$ denotes the SINR of user u at the last hop and $j \in \{1, 2, \ldots, N\}$. The result in (a) follows from the fact that $\gamma_{(N),l}^{(u)}$ is the maximum of N i.i.d. SINRs. Based on the proof of $\mathbb{P}[\gamma_{uj,l} \leq \gamma_{th}]$ given in Appendix C, the outage probability of user u can be lower bounded as (24), given at bottom of this page, where $\Phi_l = P/(d_{uj,l}^{(u)})^\beta$ and $\Theta_l = \frac{\gamma_{th}^q \left(\frac{m}{\Phi_l}\right)^{(N_s-1)m}}{\Gamma(q+1)\Gamma((N_s-1)m+q)}$. In section VI, we illustrate that the expression in (24) provides an accurate lower bound for the outage probability of user u when the number of relays is large compared to the number of users.

B. Outage Probability Saturation

In the high SINR regime this outage probability lower bound saturates due to the effect of interference. This can be illustrated as follows. Let us reexpress the exponential function in (24) in a series expansion as

$$P_o^{(u)}$$
$$\geq 1 - \left(\sum_{q=0}^{m-1} \sum_{k=0}^{q} \sum_{r=0}^{\infty} \Upsilon \left(\frac{(d_{uj,L}^{(u)})^\beta}{P} \right)^{r+q-k} \right)$$
$$\times \prod_{l=1}^{L-1} \left(1 - \left(1 - \sum_{q=0}^{m-1} \sum_{k=0}^{q} \sum_{r=0}^{\infty} \Upsilon \left(\frac{(d_{uj,l}^{(u)})^\beta}{P} \right)^{r+q-k} \right)^N \right),$$
$$(25)$$

where $\Upsilon = \binom{q}{k} \frac{(-1)^r m^{r+q-k} \gamma_{th}^{r+q} \sigma^{2(r+q-k)} \Gamma((N_s-1)m+k)}{r!(\gamma_{th}+1)^{(N_s-1)m+k}\Gamma(q+1)\Gamma((N_s-1)m)}$ contains all the terms that are independent from P. Then we consider the lowest exponent for $1/P$ which gives $r = 0$ and $k = q$ as

$$P_o^{(u)} \geq 1 - \left(\sum_{q=0}^{m-1} \Upsilon_0 + \mathcal{O}(P^{-1}) \right)$$
$$\times \left(1 - \left(1 - \sum_{q=0}^{m-1} \Upsilon_0 + \mathcal{O}(P^{-1}) \right)^N \right)^{L-1}, \quad (26)$$

where $\Upsilon_0 = \sum_{q=0}^{m-1} \frac{\gamma_{th}^q \Gamma((N_s-1)m+q)}{(\gamma_{th}+1)^{(N_s-1)m+q}\Gamma(q+1)\Gamma((N_s-1)m)}$. Then, applying the binomial expansion to expand the powers followed by some mathematical manipulations the smallest order term of P in (26) can be separated as

$$P_o^{(u)} \geq 1 - \Upsilon_0 \left(1 - (1 - \Upsilon_0)^N \right)^{L-1} + \mathcal{O}(P^{-1}). \quad (27)$$

As can be seen by (27) the smallest order of P is zero and the outage probability lower bound has an error floor in the high SINR regime.

VI. NUMERICAL AND SIMULATION RESULTS

In this section, we present simulation results to validate our analysis and evaluate the performance of the sub-optimal DRS scheme. All nodes have the same power P which varies between 15 dBm–30 dBm ($\approx$ 30 mW–10 W), and the noise variance is set to one. We consider equal distances between the L clusters, i.e., $d_{ij,l} = d/L$, where d is the the distance between the source and the destination which is set to 2 km. The value of the path loss is 140 dB for the first kilometer with path-loss exponent $\beta = 3.8$ which corresponds to built-up areas. The SNR threshold γ_{th} is set to 5 dB unless otherwise stated.

$$P_o^{(u)} \geq 1 - \left(\exp\left(-\frac{m\gamma_{th}}{\Phi_L}\right) \sum_{q=0}^{m-1} \sum_{k=0}^{q} \binom{q}{k} \frac{\Theta_L \sigma^{2(q-k)} \Gamma((N_s-1)m+k)}{\left(\frac{m\gamma_{th}}{\Phi_L} + \frac{m}{\Phi_L}\right)^{(N_s-1)m+k}} \right)$$
$$\times \prod_{l=1}^{L-1} \left(1 - \left(1 - \exp\left(-\frac{m\gamma_{th}\sigma^2}{\Phi_l}\right) \sum_{q=0}^{m-1} \sum_{k=0}^{q} \binom{q}{k} \frac{\Theta_l \sigma^{2(q-k)} \Gamma((N_s-1)m+k)}{\left(\frac{m\gamma_{th}}{\Phi_l} + \frac{m}{\Phi_l}\right)^{(N_s-1)m+k}} \right)^N \right), \quad (24)$$
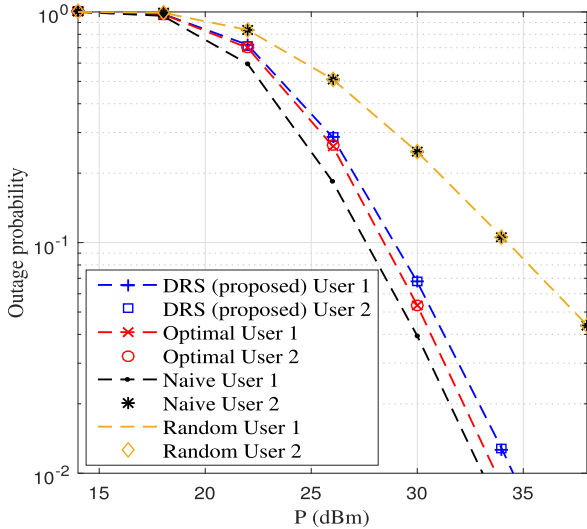
Fig. 2. User outage probability for two-user networks for ORS, DRS, naive RS, and random RS schemes when $N_s = 2$, $N = 2$, and $L = 3$.

Fig. 2 plots the simulated outage probability with transmit power for the DRS scheme with those of the optimal (exhaustive search) RS , naive RS and random RS schemes of a two-user network ($N_s = 2$) with $N = 2$ and $L = 3$ over Rayleigh fading ($m = 1$) channels. In the random RS, each user randomly chooses a relay at each hop without conflict. For the DRS, ORS, or random RS schemes, both users have the same outage probability. For the naive RS scheme, user 1 first selects the best path in each hop (similar to the DRS). Then, user 2 selects its best path that does not conflict with user 1. As such, the outage probabilities of user 1 and user 2 are different. user 1 actually achieves the performance of the single-user DRS since it has all N relays to choose from. As such, in this example the performance of user 1 with naive RS is even larger than the ORS scheme. User 2 has a worse performance since it has only $(N - 1)$ relays to choose from. This demonstrates the unfairness of the naive RS scheme. The DRS, ORS and naive RS (user 1) schemes achieve the same diversity order of two. The naive scheme (user 2) and the random RS provide the same performance and achieve a diversity order of only one because when there are only two relays, user 2 has no option for the RS as one relay has already been selected by user 1. At $P = 30$ dBm, while the ORS (user 1 or user 2) and naive RS (user 1) outperform the DRS (user 1 or user 2) by approximately 1 dB and 2.5 dB, respectively, the DRS (user 1 or user 2) outperforms naive RS (user 2) and random RS (user 1 or user 2) by approximately 8 dB. Since both users have the same outage probability, the DRS scheme provides fairness among users.

Fig. 3 plots the outage probability with transmit power for each user of a two-user network with $N = 2$, 3 or 4, and $L = 4$ over Nakagami-m fading channels when $m = 2$. The simulation curves are shown as discrete marks and the exact analytical outage probability curves generated from (11) are shown as continuous lines. The asymptotic analytical outage probability lines generated from (32) are shown as dashed lines. For the entire simulated power range, we can see that our
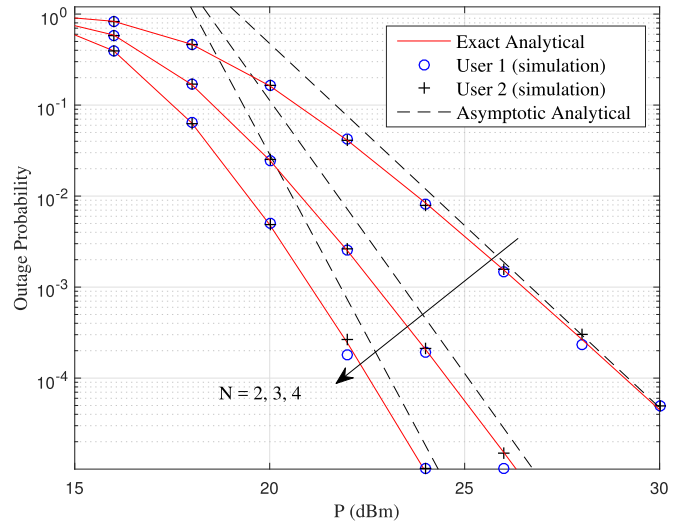

Fig. 3. User outage probability for two-user networks with $N = 2, 3, 4$, $m = 2$ and $L = 4$ for the proposed DRS scheme.
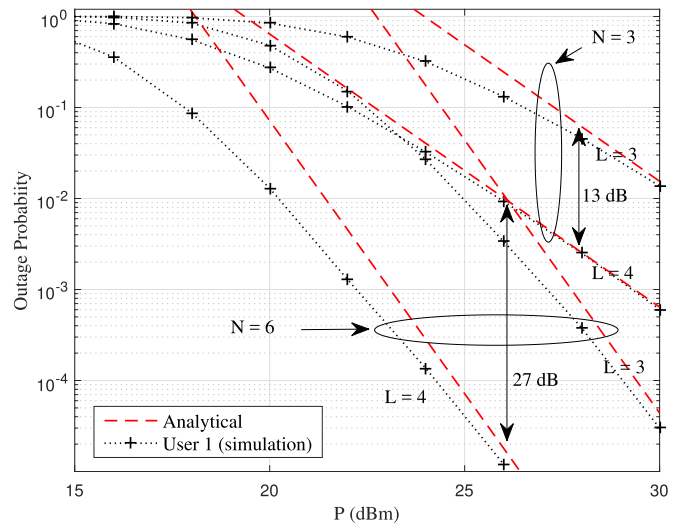

Fig. 4. User outage probability for multi-user networks with $N_s = 3$, $N = 3, 6$, and $L = 3, 4$ for the proposed DRS scheme.

exact analytical results match the simulation for all network settings and the system archives a diversity order of 2, 3 and 4 as we change $N = 2$, 3 and 4. This confirms the accuracy of our analysis for two-user multihop relay networks. This figure also shows the fairness of the DRS scheme as both users achieve similar outage probability performances.

Fig. 4 plots the outage probability with the transmit power for user 1 of a three-user network ($N_s = 3$) with $N = 3$ or 6, and $L = 3$ or 4 over Rayleigh fading ($m = 1$) channels. Only the simulated outage probability of user 1 is shown in the figure as all users have the same outage probability. The outage probability approximation in (32) is accurate for large P which confirms the validity of our analysis of the diversity order and array gain. We also observe that the DRS scheme has diversity order 3 or 6 for $N = 3$ or 6, respectively, which is the full diversity order. We further investigate the effect of the number of hops on array gain. For $N = 3$ at $P = 28$ dBm, the $L = 3$ has 13 dB loss in array gain compared with $L = 4$. Similarly, for $N = 6$ at $P = 26$ dBm, the $L = 3$ has 27 dB loss in array gain compared with $L = 4$,
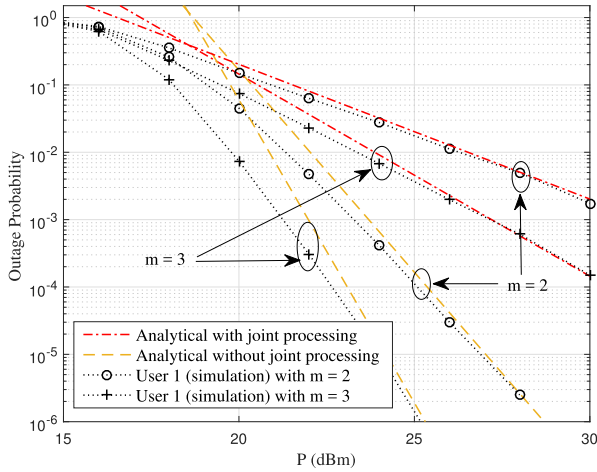
Fig. 5. User outage probability for multi-user networks with $N_s = 3$, $N = 3$, $m = 2, 3$ and $L = 4$ for the proposed DRS scheme.
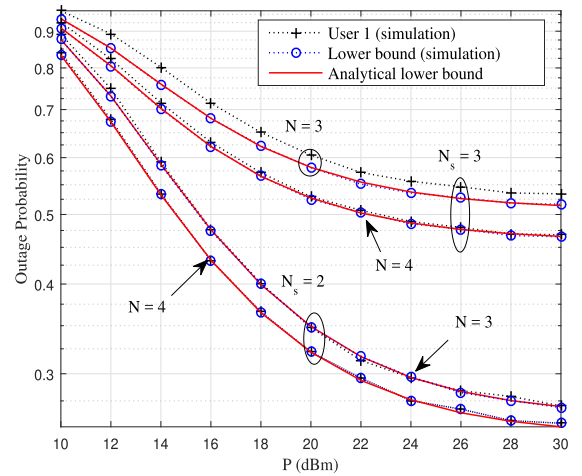

Fig. 6. User outage probability lower bound for interfering multi-user networks with $N_s = 2, 3$, $N = 3, 4$, $m = 1$ and $L = 4$ for the proposed DRS scheme.

which is a significant performance loss. For this example, when the number of relays is doubled ($N = 3$ to $N = 6$), the array gain is also approximately doubled. Furthermore, the distance dependent path-loss has a major impact on the user outage as well.

Fig. 5 plots the outage probability with the transmit power for user 1 of a three-user network ($N_s = 3$) for $N = 3$ and $L = 4$ over Nakagami-m fading channels when $m = 2$ or 3. Based on the RS at last two hops, we consider two scenarios: i) with joint processing, and ii) without joint processing. With joint processing, the DRS has diversity order 6 and 9 when $m = 2$ and 3, respectively, which is the full diversity order Nm, confirming the accuracy of (32). Without joint processing, the DRS has diversity order 2 and 3 when $m = 2$ and 3, respectively, which is a reduced diversity order m which is independent of number of relays N. This also confirms the accuracy of (12), and clearly illustrates the importance of joint processing in DRS scheme. For example, when we apply joint processing at $m = 2$ we can achieve $P_o^{(u)} = 2 \times 10^{-3}$, with approximately 7 dBm less power when compared to without joint processing. Similarly, when we apply joint processing at $m = 3$ we can achieve $P_o^{(u)} = 2 \times 10^{-3}$, with approximately 5 dBm less power when compared to without joint processing.

Fig. 6 plots the outage probability lower bound for a multi-user multihop relay network with interfering users, for $N_s = 2$ and 3, $N = 3$ and 4 and $L = 4$. Assuming a Rayleigh faded environment the Nakagami shape parameter m is set to one. The SINR threshold γ_{th} is set to -5 dB.[2] The simulation curves for the exact outage probability and the lower bound are shown as dotted lines with discrete marks and the analytical outage probability lower bound curves generated from (24) are shown as continuous lines. For the entire simulated power range, we can see that our analytical lower bound exactly matches the simulation lower bound for all network settings. This confirms the accuracy of our analysis. For $N_s = 2$, we observe that the lower bound is almost on top of the

exact outage probability curve when $N = 3$ and 4. For $N_s = 3$, we observe that the lower bound is lose when $N = 3$ but it is almost on top of the exact outage probability as soon N is as large as 4. We also observe that due to the effect of interference the outage probability performance is significantly degraded. In the high SINR regime the outage enters a saturation highlighting the importance in interference cancellation in multi-user relay networks.

VII. CONCLUSION

The relay selection problem in a network with multiple source-destination pairs with multiple hops is analysed in this paper. A sub-optimal DRS scheme is proposed and analysed under general channel conditions with Nakagami-m fading and path loss. As opposed to ORS whose complexity increases exponentially with the number of hops, the complexity of the proposed decentralized RS scheme is quadratic in the number of users, linear in the number of relays and independent of the number of hops. Based on a rigorous theoretical analysis the proposed scheme was shown to achieve full spatial diversity. For a two-user network the exact closed-form expression of the outage probability was derived and tested against simulations. Extending the analysis to interfering relay networks, an accurate lower bound on the outage probability was derived in closed-form. Due to the effect of interference the outage probability was shown to enter a saturation in the high SINR regime.

An interesting extension to this work is to apply this system model to a cognitive radio network and study routing protocols for path selection. While in [23]–[25] the authors consider multihop relaying in cognitive radio networks only a single secondary source and destination is considered. However, extending to multiple secondary sources and secondary destinations would formulate interesting research problems. The analysis in this paper was based on DF relaying. An extension to AF relaying would formulate another challenging analytical problem. While error accumulation in AF relaying limits its extension to multihop relaying, a com-

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[2]Note that the outage probability is significantly degraded due to the effect of interference. As such, we selected a lower SINR threshold to clearly illustrate the effect of interference on outage probability.

parison would lead to interesting insights. It is also desirable to extend the analysis to full-duplex relays to gain more interesting insights.

APPENDIX A
PROOF OF THEOREM 1

In this section we provide a detailed proof of Theorem 1. Note that the SNR elements of $\mathbf{\Gamma}_l$ are i.i.d.. Thus there are in total $(N_s N)!$ possible orderings of the SNR elements, each with probability $1/(N_s N)!$. For any given k, we can count the number of orderings that result in $\gamma_{u,l} = \gamma_{(k),l}$ and denote it as ε_k. Thus $\mathbb{P}[\gamma_{u,l} = \gamma_{(k),l}] = \varepsilon_k/(N_s N)!$. For the DRS scheme, only the relay ordering matters i.e., ε_k depends only on N_s, N (the dimensions of $\mathbf{\Gamma}_l$) and k, and is independent of $F_{\hat{\gamma}_{ij,l}}(x)$ which is the CDF of each element of $\mathbf{\Gamma}_l$. Therefore, $\mathbb{P}[\gamma_{u,l} = \gamma_{(k),l}]$ is independent of $F_{\hat{\gamma}_{ij,l}}(x)$, and does not include P.

Based on the previous outage probability analysis in (10), we can write

$$P_o^{(u)} = 1 - \prod_{l=1}^{L-1} \left(1 - \sum_{k=1}^{N_s N - N + 1} \sum_{i=0}^{k-1} \Psi_{lki} \right.$$
$$\left. \times \left(F_{\hat{\gamma}_{ij,l}}(\gamma_{th})\right)^{N_S N - k + i + 1}\right), \quad (28)$$

where (28) follows by substituting $F_{\gamma_{(k),l}}(\gamma_{th})$ in (7) into (10) and $\Psi_{lki} = \frac{\mathbb{P}[\gamma_{u,l} = \gamma_{(k),l}](N_s N)!\binom{k-1}{i}}{(-1)^i (N_s N - k + i + 1)(k-1)!(N_s N - k)!}$. From (28) we learn that only the term $F_{\hat{\gamma}_{ij,l}}(\gamma_{th})$ depends on P. Therefore, we proceed to reexpress $F_{\hat{\gamma}_{ij,l}}(\gamma_{th})$ based on (6) and (4) to write (29) given at bottom of this page, where we have separated the product terms in (28) based on the outage probability of the first $L-2$ hops and the last two hops. Next, we use the infinite series expansion of the incomplete gamma function, which is given by

$$\gamma\left(m, \frac{mx}{\tilde{\Omega}_l}\right) = \left(\frac{mx}{\tilde{\Omega}_l}\right)^m \exp\left(\frac{-mx}{\tilde{\Omega}_l}\right) \sum_{q=0}^{\infty} \frac{(mx/\tilde{\Omega}_l)^q}{m(m+1)...(m+q)},$$ and express the exponential function therein using [28, eq. (1.211)] to write (30) given at bottom of this page. Based on (30), we identify that the effective terms at high power region, i.e., the terms with the lowest exponent for $1/P$ as $P \to \infty$, are obtained when $t = q = i = 0$ and $k = N_s N - N + 1$. Using this, and the fact that $\tilde{\Omega}_l = \Omega = P/((d/L)^\beta \sigma^2)$ for $l = 1, \cdots, (L)$, we next apply the binomial expansion and write the product of $L-2$ terms as a summation given by (31) at bottom of this page. Note that in (31) we have only expressed the dominant terms in the high SNR regime. As $\mathbb{P}[\gamma_{u,l} = \gamma_{(N_s N - N + 1),l}] = \mathbb{P}[\gamma_{u,z} = \gamma_{(N_s N - N + 1),z}]$ for any $l \neq z$, $l \in 1, 2, \ldots, L-1$ we can further

$$P_o^{(u)} = 1 - \left[\prod_{l=1}^{L-2}\left(1 - \sum_{k=1}^{N_s N - N + 1}\sum_{i=0}^{k-1}\Psi_{lki}\left(\frac{\gamma\left(m, \frac{m\gamma_{th}}{\tilde{\Omega}_l}\right)}{\Gamma(m)}\right)^{N_S N - k + i + 1}\right)\right.$$
$$\left. \times \left(1 - \sum_{k=1}^{N_s N - N + 1}\sum_{i=0}^{k-1}\Psi_{(L-1)ki}\left(1 - \left(1 - \frac{\gamma\left(m, \frac{m\gamma_{th}}{\tilde{\Omega}_{L-1}}\right)}{\Gamma(m)}\right)\left(1 - \frac{\gamma\left(m, \frac{m\gamma_{th}}{\tilde{\Omega}_L}\right)}{\Gamma(m)}\right)\right)^{N_S N - k + i + 1}\right)\right], \quad (29)$$

$$P_o^{(u)} = 1 - \left[\prod_{l=1}^{L-2}\left(1 - \sum_{k=1}^{N_s N - N + 1}\sum_{i=0}^{k-1}\Psi_{lki}\left(\frac{\left(\frac{m\gamma_{th}}{\tilde{\Omega}_l}\right)^m \sum_{t=0}^{\infty}\sum_{q=0}^{\infty}\frac{(-1)^t\left(\frac{m\gamma_{th}}{\tilde{\Omega}_l}\right)^{t+q}}{t!m(m+1)...(m+q)}}{\Gamma(m)}\right)^{N_S N - k + i + 1}\right)\right.$$
$$\times \left(1 - \sum_{k=1}^{N_s N - N + 1}\sum_{i=0}^{k-1}\Psi_{(L-1)ki}\left(1 - \left(1 - \frac{\left(\frac{m\gamma_{th}}{\tilde{\Omega}_{L-1}}\right)^m \sum_{t=0}^{\infty}\sum_{q=0}^{\infty}\frac{(-1)^t\left(\frac{m\gamma_{th}}{\tilde{\Omega}_{L-1}}\right)^{t+q}}{t!m(m+1)...(m+q)}}{\Gamma(m)}\right)\right.\right.$$
$$\left.\left.\left. \times \left(1 - \frac{\left(\frac{m\gamma_{th}}{\tilde{\Omega}_L}\right)^m \sum_{t=0}^{\infty}\sum_{q=0}^{\infty}\frac{(-1)^t\left(\frac{m\gamma_{th}}{\tilde{\Omega}_L}\right)^{t+q}}{t!m(m+1)...(m+q)}}{\Gamma(m)}\right)\right)^{N_S N - k + i + 1}\right)\right].$$
$$(30)$$

$$P_o^{(u)} \approx \sum_{l=1}^{L-2}\frac{\mathbb{P}[\gamma_{u,l} = \gamma_{(N_s N - N + 1),l}](N_s N)! m^{mN}}{N!(N_s N - N)!(m!)^N}\left(\frac{\gamma_{th}\left(\frac{d}{L}\right)^\beta \sigma^2}{P}\right)^{mN}$$

$$+ \frac{2^N \mathbb{P}[\gamma_{u,L-1} = \gamma_{(N_s N - N + 1),L-1}](N_s N)! m^{mN}}{N!(N_s N - N)!(m!)^N}\left(\frac{\gamma_{th}\left(\frac{d}{L}\right)^\beta \sigma^2}{P}\right)^{mN} + \mathcal{O}\left(\frac{1}{P^{mN+1}}\right). \quad (31)$$

simplify (31) as

$$
P_o^{(u)}
$$

$$
\approx \underbrace{\frac{(L-2+2^N)\mathbb{P}[\gamma_{u,l} = \gamma_{(N_sN-N+1),l}](N_sN)!m^{mN}}{N!(N_sN-N)!(m!)^N \left(\gamma_{th}\left(\frac{d}{L}\right)^\beta \sigma^2\right)^{-mN}}}_{\epsilon} P^{-mN}
$$

$$
+ \mathcal{O}\left(\frac{1}{P^{mN+1}}\right), \tag{32}
$$

where

$$
\mathbb{P}[\gamma_{u,l} = \gamma_{(N_sN-N+1),l}] =
\begin{cases}
\frac{1}{N_s} \prod_{i=1}^{N-1} \frac{N-i}{N_sN-i}; & N_s < N \\
\frac{2}{N_s} \prod_{i=1}^{N-1} \frac{N-i}{N_sN-i}; & N_s = N,
\end{cases} \tag{33}
$$

for all $l \in 1, 2, \dots, L-1$ and the array gain of user u denoted as ϵ.

APPENDIX B
EXAMPLE: DERIVATION OF $F_{\gamma_{(k)},l}(x)$ IN INTERFERING RELAY NETWORKS

In this section, we provide a detailed derivation of $F_{\gamma_{(k)},l}(x)$ for a simple example. Let us consider the lth hop of a relay network with two users and two relays in each hop. Let the channel be subjected to Rayleigh fading. Assuming that the users interfere with each other we can write the received SINR matrix as

$$
\mathbf{\Gamma}_l = \begin{bmatrix} \gamma_{11,l} & \gamma_{12,l} \\ \gamma_{21,l} & \gamma_{22,l} \end{bmatrix}, \tag{34}
$$

where $\gamma_{uj,l} = \frac{\omega_{uj,l}}{\omega_{vj,l}+\sigma^2}$, $v \in \{1,2\}, v \neq u$. Based on (20) the CDF of the largest, second largest and the third largest SINRs from among the four SINRs in (34) can be written as follows. For the ease of notation let $\mathcal{S} = \{\gamma_{11,l}, \gamma_{21,l}, \gamma_{12,l}, \gamma_{22,l}\}$.

$$
F_{\gamma_{(1)},l}(x) = \sum_{\gamma_p,\gamma_q,\gamma_r,\gamma_s \in \mathcal{S}} F_{\gamma_p,\gamma_q,\gamma_r,\gamma_s}(x,x,x,x), \tag{35}
$$

$$
F_{\gamma_{(2)},l}(x) = \sum_{\gamma_p,\gamma_q,\gamma_r \in \mathcal{S}} F_{\gamma_p,\gamma_q,\gamma_r}(x,x,x)
$$

$$
- 3 \sum_{\gamma_p,\gamma_q,\gamma_r,\gamma_s \in \mathcal{S}} F_{\gamma_p,\gamma_q,\gamma_r,\gamma_s}(x,x,x,x), \tag{36}
$$

$$
F_{\gamma_{(3)},l}(x) = \sum_{\gamma_p,\gamma_q \in \mathcal{S}} F_{\gamma_p,\gamma_q}(x,x)
$$

$$
- 2 \sum_{\gamma_p,\gamma_q,\gamma_r \in \mathcal{S}} F_{\gamma_p,\gamma_q,\gamma_r}(x,x,x)
$$

$$
+ 3 \sum_{\gamma_p,\gamma_q,\gamma_r,\gamma_s \in \mathcal{S}} F_{\gamma_p,\gamma_q,\gamma_r,\gamma_s}(x,x,x,x). \tag{37}
$$

As such, we need the joint CDF of all possible combinations of two, three and four SINRs in (34) in order to find the above order statistics. The joint CDF of two independent SINRs in $\mathcal{S}$, which we denote by $F_{(\text{ind})}(x,x)$, can be evaluated following the steps in Appendix VII which results in

$$
F_{(\text{ind})}(x,x) = \left(1 - \frac{\exp(-x\sigma^2)}{1+x}\right)^2. \tag{38}
$$

The joint CDF of two correlated SINRs in $\mathcal{S}$, which we denote by $F_{(\text{corr})}(x,x)$, can also be found based on some mathematical manipulations as (39), as given in bottom of this page. Then the expressions for $F_{\gamma_{(1)},l}(x)$, $F_{\gamma_{(2)},l}(x)$ and $F_{\gamma_{(3)},l}(x)$ can be expressed in closed form in-terms of $F_{(\text{ind})}(x,x)$ and $F_{(\text{corr})}(x,x)$ as

$$
F_{\gamma_{(1)},l}(x) = \left(F_{(\text{corr})}(x,x)\right)^2 \tag{40}
$$

$$
F_{\gamma_{(2)},l}(x) = 4\left(F_{(\text{ind})}(x,x)\right)\left(F_{(\text{corr})}(x,x)\right)^2
$$

$$
- 3\left(F_{(\text{corr})}(x,x)\right)^2, \tag{41}
$$

$$
F_{\gamma_{(3)},l}(x) = 4\left(F_{(\text{ind})}(x,x)\right) + 2\left(F_{(\text{corr})}(x,x)\right)
$$

$$
- 2\left(F_{(\text{ind})}(x,x)\right)\left(F_{(\text{corr})}(x,x)\right)^2
$$

$$
+ 3\left(F_{(\text{corr})}(x,x)\right)^2. \tag{42}
$$

Note that based on (20), (38) and (39) the ordered SINRs $F_{\gamma_{(1)},l}(x), F_{\gamma_{(2)},l}(x) \dots F_{\gamma_{(N+1)},l}(x)$ required for a two-user network with N relays can be found. However, when the number of users is large deriving general expressions for order statistics is mathematically challenging as we have to consider a large number of subsets of joint CDFs.

APPENDIX C
PROOF OF $\mathbb{P}[\gamma_{uj,1} \leq \gamma_{th}]$

In this section we derive an expression for $\mathbb{P}[\gamma_{uj,1} \leq \gamma_{th}]$. First, we reexpress $\gamma_{uj,1}$ based on (17) and rewrite $\mathbb{P}[\gamma_{uj,1} \leq \gamma_{th}]$ as

$$
\mathbb{P}\left[\gamma_{uj,1} \leq \gamma_{th}\right]
$$

$$
= \mathbb{P}\left[\omega_{uj,l} \leq \gamma_{th}\left(\sum_{k=1,k\neq u}^{N_s} \omega_{kj,l} + \sigma^2\right)\right]
$$

$$
= E_\Delta\left[1-\exp\left(-\frac{m\gamma_{th}(\Delta+\sigma^2)}{\Phi_l}\right)\sum_{q=0}^{m-1} \frac{\left(\frac{m\gamma_{th}(\Delta+\sigma^2)}{\Phi_l}\right)^q}{\Gamma(q+1)}\right], \tag{43}
$$

where $E_\Delta[.]$ indicates the expectation taken over the distribution of $\Delta = \sum_{k=1,k\neq u}^{N_s} \omega_{kj,l}$. Note that $\omega_{kj,l}$ has a gamma distribution with shape parameter m and rate parameter (m/Φ_l).

$$
F_{(\text{corr})}(x,x) =
\begin{cases}
\frac{\exp(-x(3+x))}{1+x}\left((1+x)\exp\left(x^2+3x\right) + \exp\left(x + \frac{x^2(1+x)}{x-1}\right)\right. \\
\qquad\qquad \left. -2\exp\left(x^2+2x\right) - x\exp\left(x^2+3x+\frac{2x}{1-x}\right)\right); & x < 1 \\
1 - \frac{2\exp(-x)}{1+x}; & x \geq 1.
\end{cases} \tag{39}
$$

The sum of $(N_s - 1)$ random variables with such independent and identical gamma distributions again results in a new gamma distribution with shape parameter $(N_s - 1)m$ and rate parameter (m/Φ_l). As such, the expectation in (43) can be reexpressed as

$$
\mathbb{P}\left[\gamma_{uj,1} \leq \gamma_{th}\right] = 1 - \exp\left(-\frac{m\gamma_{th}\sigma^2}{\Phi_l}\right)
$$
$$
\times \sum_{q=0}^{m-1} \left[\Theta_l \times \int_0^\infty (\Delta + \sigma^2)^q \Delta^{(N_s-1)m-1} \right.
$$
$$
\left. \times \exp\left(-\frac{\Delta m}{\Phi_l}\left(1 + \gamma_{th}\right)\right) d\Delta\right]. \tag{44}
$$

Expanding $(\Delta + \sigma^2)^q$ using binomial expansion and solving the resultant integral based on [28, eq. (3.351) and (3)] results in the closed-form expression fo $\mathbb{P}\left[\gamma_{uj,1} \leq \gamma_{th}\right]$ given by

$$
\mathbb{P}\left[\gamma_{uj,1} \leq \gamma_{th}\right]
$$
$$
= 1 - \left[\exp\left(-\frac{m\gamma_{th}\sigma^2}{\Phi_l}\right)\right.
$$
$$
\left. \times \sum_{q=0}^{m-1}\sum_{k=0}^{q} \binom{q}{k} \frac{\Theta_l \sigma^{2(q-k)}\Gamma((N_s-1)m+k)}{\left(\frac{m\gamma_{th}}{\Phi_l} + \frac{m}{\Phi_l}\right)^{(N_s-1)m+k}}\right]. \tag{45}
$$

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