Angle-of-Arrival-Dependent Interference Modeling in Rician Massive MIMO

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***Abstract*—In this paper, we study the uplink in a single-cell massive multiple-input–multiple-output system. The base station (BS) is equipped with three antenna arrays, each covering one third of the cell area. Each antenna array comprises a large yet finite number of antennas. The single-antenna users are randomly and uniformly distributed in the cell, transmitting to the BS utilizing full channel inversion power control. All users experience Rician fading. Receiver maximum-ratio-combining is performed at the BS. Under such a setting, we focus on one cell sector and analyze the intrasector interference in a realistic situation where the number of BS antennas is not extremely large compared with the user number. In particular, we show that, due to the line-of-sight (LoS) component of the channel, the interference is partially determined by the angles of arrival of the signals. We approximate the LoS component interference by a Beta mixture. The interference in Rician fading is then modeled as a noncentral chi-square distribution with a random noncentrality parameter, corresponding to the LoS component. The approximate interference distribution can be used to compute signal-to-interference-ratio-dependent metrics such as outage probability and average throughput.**

***Index Terms*—Multiuser detection, phased arrays, Rician channels.**

I. Introduction

A. Motivation and Scope

MASSIVE multiple-input-multiple-output (MIMO), an enabler of future broadband networks, has attracted a lot of research interest. In particular, massive MIMO uses a number of base station (BS) service-antennas to communicate with multiple active user terminals [1]. The technology relies on signal processing techniques such as beamforming (BF) at the BS antennas. Maximum-ratio-combining (MRC) is a preferred BF scheme due to its simplicity [2].

In massive MIMO systems, determining the required number of BS antennas is an important problem, the answer depending upon all of the system parameters and performance requirements [3]. One approach to this problem is to investigate how many antennas are needed to achieve the satisfactory performance given a certain number of users in the system [4]. Admittedly, a large number of BS antennas asymptotically eliminate the cochannel user interference [3], [5]. However, in reality, the feasible BS antenna number is limited, and mutually orthogonal channel conditions are generally not fully satisfied. Hence, the interuser interference cannot be completely eliminated, and there is room for interference studies. Moreover, massive MIMO does not rely on asymptotic results, hence, it is necessary to analyze a system with large but finite number of antennas and users [1]. Characterizing the interference is essential for network performance analysis, and is extremely useful for the investigation of how many users the system can support while offering satisfactory quality of service and maximizing spectral efficiency.

Most of the analyses on massive MIMO assume Rayleigh fading channels [3], [4], [6]–[10]. Despite its practical significance, interference in Rician massive MIMO is rarely investigated. The majority of the analyses on Rician (massive) MIMO system performance [11]–[18] are performed in terms of the signal-to-noise ratio (SNR), where the Wishart distribution is a commonly used tool [19], [20].

Therefore, in this paper, we model the interference in a Rician MIMO system with *large yet finite* numbers of antennas at the BS. To be specific, we study the interference in the uplink of a single cell, where the single-antenna users transmit to a multiantenna BS. All users experience Rician fading. MRC is performed at the BS. We assume the BS has perfect channel estimation of all users and the effect of pilot contamination [3] is not considered. We will see that, even with these simplifying assumptions, the inclusion of angle-of-arrival (AoA) in the modeling leads to an interesting and challenging interference analysis.

B. State of the Art and the Challenge

One common approach to approximate a Rician MIMO channel is to model the channel entries as independently and identically distributed (i.i.d.) complex random variables with Nakagami-m distributed amplitudes and uniformly distributed phases in $[0, 2\pi)$ [21]–[24]. However, this model is invalid for colocated linear antenna arrays in Rician channels, since

1) the Nakagami-m distribution approximates the *amplitude* of a Rician channel [25, p. 88] and does not contain the phase information;

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2) the phase and amplitude distributions in a Rician channel are not separable [25, p. 83], hence, a Nakagami-m distributed amplitude with a uniformly distributed phase does not represent a Rician channel;
3) under BF schemes, due to the LoS component, the channel phases at a linear antenna array partially determine the signal strength [26], hence, the phases at each antenna element cannot be assumed i.i.d..

In this paper, we model the Rician channel as an LoS component with a Rayleigh fading scattering component, as in [27] and [28].

In Rician channels, due to the LoS component, the phase of the received signal at the antenna depends on the location of the transmitter, which determines the AoA of the signal. As a result, in Rician massive MIMO systems, the interference power depends on the locations (especially the directions) of both the target user and the interferer. The authors of [29] point out that with a finite BS antenna number, when two users are located very close together, their LoS channels will not be mutually orthogonal. Hence, the interference induced by the LoS channel component is nonnegligible and constitutes an ultimate challenge to our work, as well as to the works listed later.

In [30], the authors derive the SNR distribution in an MIMO system deploying zero forcing. The transmitters, target receiver, and nontarget receivers are all equipped with multiple antennas. The scenarios of Rician–Rayleigh fading (where the target receiver experiences Rician fading and nontarget receivers experience Rayleigh fading), Rayleigh–Rician fading, and Rician–Rician fading are all considered. In the Rician-Rician scenario, due to the LoS channel component, the complicated noncentral Wishart distribution is involved. The authors have shown that when a channel-mean correlation condition is satisfied, the SNR follows a central Wishart distribution. The SNR distribution is, hence, obtained based on this condition. However, when the channel-mean correlation condition is not satisfied, the SNR distribution is not addressed due to the complexity of the noncentral Wishart distribution. More recently, in [31], the authors further study a point-to-point zero-forcing Rician MIMO system. The transmitter and the receiver are both equipped with multiple antennas, and the channel is considered to be rank-1 LoS Rician fading. Each transmitter antenna transmits a different data stream, and the receiver antenna array performs zero-forcing beamforming to separate the streams. The SNR of one data stream is studied. The authors derive the exact distributions of the SNR and its moment generating function, and propose a novel algebraic method to compute it. However, the rank-1 channel setting implies that all the single-antenna interferers and the target transmitter are collocated. The scenario of randomly located interferers is not addressed.

In [27], the authors propose an LoS-based BF scheme in a single cell Rician massive MIMO system, where a multiantenna BS serves multiantenna users. BF is performed by both the BS and the users. Only the LoS component is considered as the useful signal and the fading component is treated as noise. The authors study both the uplink and downlink. Asymptotic signal-to-interference-and-noise ratio and rate expressions are derived when the BS or user antenna number goes to infinity. The authors point out that if the BS antenna number is finite, even though each user has a very large antenna array, the interference cannot be eliminated due to the LoS component. However, the interference analysis has not been addressed in this paper.

The authors of [28] study artificial noise-aided jamming design in massive MIMO Rician channels. The authors consider a transmitter equipped with a large antenna array, with the single-antenna receiver and eavesdroppers experiencing Rician channels, and study the secrecy outage probability. The secrecy outage occurs when the secrecy capacity with respect to *at least one* eavesdropper is below the threshold. In particular, it is pointed out that the *locations* of the eavesdropper and the receiver are crucial in the secrecy capacity analysis, due to the LoS channel component. The authors provide the exact expression for the cumulative distribution function (cdf) of the signal leakage to one eavesdropper [28, eq. (7)], based on its *location* distribution. The authors then study the secrecy outage caused by an eavesdropper from different directions, and compare the performances of uniform jamming and directional jamming. The signal leakage can be equivalently viewed as the single-user interference contributed by the LoS component. The cdf provided accurately describes the single-user interference distribution. However, it involves numerically solving the inverse of a sinc-like function at each single value. As a result, it requires large computation power and makes multiuser interference analysis difficult.

In our work, we endeavour to provide a simple yet reasonable approximation for the Rician interference incorporating the LoS component. This approximate interference distribution enables us to analyze multiuser interference in an efficient manner.

C. Our Approach and Contribution

In this paper, we consider a single cell uplink system, where the single-antenna users transmit to a multiantenna BS. Full channel inversion uplink power control is employed to overcome path loss. The BS deploys sector antenna arrays and performs receiver MRC. We focus on one cell sector, in which the users are randomly distributed. All users experience Rician fading. By randomly choosing a target user, we investigate the interference within the sector.

Modeling the interference induced by the LoS component is essential to understanding the Rician interference, and constitutes an ultimate challenge. Hence, we first study the interference under a pure LoS channel model, and then, extend it to the Rician case, *without performing repeated analysis*. Finally, we investigate some system design problems using the Rician interference distribution.

In the LoS massive MIMO channel, since we are interested in large yet finite BS antenna numbers, we simplify the interference expression asymptotically. With the simplification, we present an approach to resolve the dependence on locations, which leads to reasonable approximations for the interference distribution in an LoS massive MIMO system. In Rician massive MIMO systems, we first derive the interference distribution conditioned on a known LoS component (the channel mean), and then, uncondition the LoS interference.

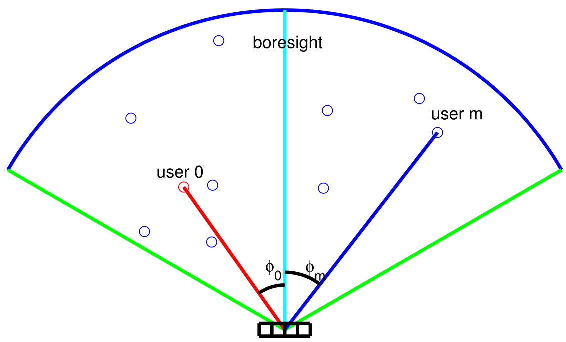


Fig. 1. Network model illustration: black-white grids denoting BS antenna array; red circle denoting random target user; blue circles being interferers; green lines draw the boundary of the sector of interest.

Finally, we make the following contributions:

1) the single-user effective LoS interference is modeled by a Beta distribution, whose parameters do *not* depend on the BS antenna number [26];
2) the total interference induced by the LoS component is modeled by a Beta mixture;
3) the Rician interference is modeled by a noncentral Chi-square distribution, whose noncentrality parameter is the LoS interference.

As will be shown, although the interference model is derived based on the asymptotic results, it is indeed a good approximation even for moderate antenna numbers. The approximate interference distribution is then applied to study SIR-related performance metrics such as coverage probability and average throughput.

D. Paper Organization

Section II describes our system model, defines performance metrics, and introduces the LoS and Rician channel models. Section III analyzes the interference in pure LoS massive MIMO systems. Section IV-C extends the analysis to the interference in Rician massive MIMO systems. Section V applies the Rician interference distribution to outage and average throughput computation. Finally, Section VI concludes this paper.

II. System Model

We study the uplink in a single cell. The cell is modeled as a disk. The BS is equipped with three directional linear antenna arrays. One array serves one-third of the cell, covering $\frac{2\pi}{3}$ radians in the angular domain. We focus on one BS antenna array and the corresponding cell sector, as illustrated in Fig. 1.

The BS antenna array consists of N omnidirectional antenna elements. There are $M+1$ single-antenna users randomly located within the sector, transmitting to the BS simultaneously on the same channel. Assuming that the BS can perfectly estimate the channels of all users, receiver MRC is performed at the BS. Without loss of generality, considering user 0 as the target user, the received signal at the BS after BF is

$$y_0 = \mathbf{w}_0^H s_0 \sqrt{P_0 r_0^{-\alpha}}\mathbf{h}_0 + \mathbf{w}_0^H \sum_{m=1}^{M} s_m \sqrt{P_m r_m^{-\alpha}}\mathbf{h}_m + n_0$$

where $(\cdot)^H$ denotes Hermitian transpose, s_m and P_m are the data symbol and the transmit power of user m, r_m is the distance from user m to the BS, α is the path-loss factor, and n_0 is the thermal noise. In addition, $\mathbf{h}_m$ denotes the channel vector from user m to the BS and $\mathbf{w}_m$ denotes the BF weight used to detect the signal of user m.

Assuming MRC, we have $\mathbf{w}_m = \dfrac{\mathbf{h}_m}{||\mathbf{h}_m||}$, where $||\cdot||$ is the Euclidean norm. We consider an interference-limited system, where the thermal noise is negligible compared to the received interference, and hence, the performance metric is the received signal-to-interference ratio (SIR) at the BS. We further assume full channel inversion uplink power control to compensate the path loss, i.e., $P_m = \frac{1}{N} r_m^{\alpha}$ for all m. The received (RX) signal power at the BS is denoted as

$$S = \frac{1}{N}|\mathbf{w}_0^H \mathbf{h}_0|^2$$

and the interference power in the system is denoted as

$$\mathcal{I} = \sum_{m=1}^{M} I_m \tag{1}$$

where I_m is the interference exerted by interferer m, defined as

$$I_m \triangleq \frac{1}{N}|\mathbf{w}_0^H \mathbf{h}_m|^2. \tag{2}$$

Then, the SIR, given N BS antennas and M interferers, is simply written as $\mathrm{SIR} = \dfrac{S}{\mathcal{I}}$. The analysis of the interference in (1) is the main challenge of this paper. In the remainder of this section, we describe the LoS and Rician channel models, respectively.

A. Pure LoS Channel Model

In this paper, we denote the direct LoS channel vector from user m to the BS antenna array as $\overline{\mathbf{h}}_m$, and the corresponding BF weight as $\overline{\mathbf{w}}_m$. Assuming the size of the antenna array is negligible compared to the distance between the BS and the user, $\overline{\mathbf{h}}_m$ is given by [32]

$$\overline{\mathbf{h}}_m = \left[\overline{h}_{m1}, \ldots, \overline{h}_{mn}, \ldots, \overline{h}_{mN}\right]^T \tag{3}$$

where

$$\overline{h}_{mn} = \exp\left(j\Psi_{m,n}\right)$$
$$\Psi_{m,n} = \underbrace{-2\pi\frac{r_m}{\lambda}}_{\text{distance phase shift}} \underbrace{+2\pi\frac{d}{\lambda}(n-1)\sin(\phi_m)}_{\text{antenna array phase shift}}$$

$m = 1, \ldots, M$ and $n = 1, \ldots, N$. The phase shift $\Psi_{m,n}$ has two contributing factors: The propagation distance between the user and the antenna array, and the spacing between antenna elements. In the aforementioned definition, λ is the carrier wavelength, d is the antenna spacing, and ϕ_m is the AoA from user m to the BS, measured from the antenna array broadside direction, as illustrated in Fig. 1. Since the users are randomly placed in the sector, ϕ_m is independently and identically distributed (i.i.d.) according to a uniform distribution over $\left[-\frac{\pi}{3}, \frac{\pi}{3}\right]$.

B. Rician Channel Model

The Rician channel is comprised of an LoS component and a Rayleigh fading component. We assume all the users have the same LoS amplitude A and fading variance σ^2, and the Rician channel from user m to the BS can be expressed as

$$\mathbf{h}_m = [h_{m,1}, \ldots, h_{m,n}, \ldots h_{m,N}]^T$$
$$h_{m,n} \sim \mathcal{CN}\left(\sqrt{\frac{K}{K+1}}\overline{h}_{m,n}, \frac{1}{K+1}\right) \tag{4}$$

in which $\mathcal{CN}(\cdot)$ denotes a complex Gaussian random variable, $\overline{h}_{m,n}$ follows the definition in (3), and $K = \frac{A^2}{\sigma^2}$ is the Rician K factor. When $K \to \infty$, or $K = 0$, the Rician channel degrades to a pure LoS or Rayleigh channel, respectively. For the remainder of this paper, $\mathbf{h}_m$ and $h_{m,n}$ denote the Rician channel vector and the Rician channel coefficients, unless specified otherwise.

In the next two sections, we analyze the interference under LoS and Rician channel models.

III. Pure LoS Interference

In pure LoS channels, the received signal strength is a constant under MRC. The only uncertainty is the interference power.

To differentiate from the Rician case, the total interference power in the LoS scenario is denoted by $\mathcal{I}^L$. Following (1), given N BS antennas, the interference exerted by the M interferers in the system is written as

$$\mathcal{I}^L = \sum_{m=1}^{M} I_m^L \tag{5}$$

where I_m^L is the LoS single-user interference, given by [32]

$$I_m^L = \frac{1}{N}|\overline{\mathbf{w}}_0^H \overline{\mathbf{h}}_m|^2 = \begin{cases} \frac{1}{N^2}\frac{1-\cos(N2\pi\Delta\theta_{m,0})}{1-\cos(2\pi\Delta\theta_{m,0})}, & \theta_{m,0} \neq 0 \\ 1, & \theta_{m,0} = 0 \end{cases} \tag{6}$$

in which $\theta_{m,0}$ is the *angular separation* between the AoAs of interferer m and the target user 0, defined as

$$\theta_{m,0} \triangleq \sin(\phi_m) - \sin(\phi_0)$$

and Δ is the *normalized antenna spacing*, defined as

$$\Delta \triangleq d/\lambda.$$

In this section, we show that $\mathcal{I}^L$ can be well approximated by a Beta-mixture distribution by taking the following three steps.

1) The single-user interference I_m^L is composed of a mainlobe surrounded by many small sidelobes and the shape is determined by the antenna number N (see an example in Fig. 2). We approximate the effective interference by its mainlobe only.
2) For moderate and large BS antenna numbers, the effective single-user interference can be modeled by one Beta distribution, whose parameters do *not depend on the actual BS antenna numbers*. The overall single-user interference distribution is approximated by a Dirac Delta function plus the Beta-distribution.

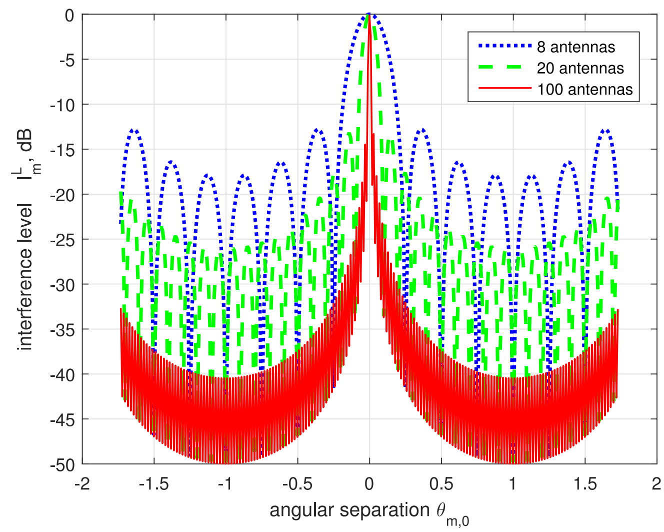


Fig. 2. LoS single-user interference I_m^L versus $\theta_{m,0}$, $\Delta = 1/2$.

3) The total interference is then approximated by a Beta-mixture distribution.

The details are addressed in the sequel.

A. Mainlobe-only Approximation

According to (6), we plot I_m^L as a function of $\theta_{m,0}$ in Fig. 2, and observe that:

1) I_m^L is nonmonotonic and nonperiodic with respect to $\theta_{m,0}$, which makes it intractable to analyze;
2) major interference appears in the mainlobe (defined by the first nulls); the relative sidelobe interference level is significantly lower. For large arrays, the power ratio between the mainlobe peak and the first sidelobe is 13.26 dB [33, p. 11].

From (6), the first nulls of I_m^L appear at $\theta_{m,0} = \pm\frac{1}{N\Delta}$, thus the mainlobe width (MLW) is $\frac{2}{N\Delta}$. Hence, we make our first approximation by ignoring the sidelobes, and have the following expression.

Approximation 1: (Effective Interference Approximation): The LoS single-user interference can be simplified as

$$I_m^L \approx J_m^{(N)} \cdot \mathbb{1}\{|\Theta_{m,0}| \leqslant 1\} \tag{7}$$

where

$$\Theta_{m,0} \triangleq N\Delta\theta_{m,0} = N\Delta\left[\sin(\phi_m) - \sin(\phi_0)\right] \tag{8}$$

is the *normalized angular separation* over a half MLW, such that when $\Theta_{m,0} \in [-1, 1]$, interferer m falls within the mainlobe. In addition, $\mathbb{1}\{\cdot\}$ is the indicator function and

$$J_m^{(N)} \triangleq \begin{cases} \frac{1}{N^2}\frac{1-\cos(2\pi\Theta_{m,0})}{1-\cos\left(\frac{2\pi}{N}\Theta_{m,0}\right)}, & 0 < |\Theta_{m,0}| \leq 1 \\ 1, & \Theta_{m,0} = 0 \end{cases} \tag{9}$$

is the *effective interference* from interferer m given N BS antennas.

∎

B. LoS Single-User Interference Distribution

To obtain the single-user interference, we need to find out:

1) the probability that an arbitrary interferer is an effective interferer;
2) the distribution of effective interference.

Since our regime of interest is a large yet finite BS antenna array, we focus on the behavior of the two aforementioned quantities when the antenna number N is large, and have the following two lemmas.

Lemma 1: (Effective Interferer Probability): As N goes large, the probability that user m is an effective interferer is

$$p = \frac{9}{\pi^2}\text{arctanh}\left(\frac{\sqrt{3}}{2}\right)\frac{1}{N\Delta} + O\left(\frac{1}{N^2}\right)$$
$$\approx \frac{9}{\pi^2}\text{arctanh}\left(\frac{\sqrt{3}}{2}\right)\frac{1}{N\Delta} \tag{10}$$

■

Proof: See Appendix A.

Lemma 2: (Asymptotic Effective Interference Expression): As N goes large, given an effective interferer, its interference converges to

$$J_m^* \triangleq \lim_{N\to\infty} J_m^{(N)} \xrightarrow{d} \text{sinc}^2(\pi\Theta),\ \ \Theta \sim U[-1,1] \tag{11}$$

where $U[a,b]$ denotes uniform distribution on $[a,b]$, $\xrightarrow{d}$ denotes convergence in distribution, and Θ is a dummy variable.

■

Proof: See Appendix B.

We note that J_m^* does not depend on N or Δ. However, since the sinc function is not invertible, it is intractable to find the distribution of J_m^* analytically. Hence, we propose a *parameter estimation* method (see also the Gamma approximations in [34]) to approximate the effective interference distribution. Hereafter, we will show that for moderate and large BS antenna numbers, given an effective interferer, its interference can be modeled by the same Beta-distribution *regardless of* N and Δ.

Approximation 2: (The Beta Approximation [26]) The asymptotic effective interference is well approximated by the Beta distribution

$$J_m^* \sim \text{Beta}(0.43, 0.51). \tag{12}$$

■

Justification. Based on the aforementioned simulations, we propose that the normalized histogram of J_m^* can be well described by a Beta distribution. The Beta distribution is a family of curves defined on $[0,1]$, following the probability density function (pdf)

$$f(x;A,B) = \frac{x^{A-1}(1-x)^{B-1}}{\int_0^1 u^{A-1}(1-u)^{B-1}du} \tag{13}$$

with $0 \le x \le 1$ and two shape parameters $A, B > 0$. Since

1) the single-user interference is strictly confined within $[0,1]$, coinciding with the domain of the Beta distribution;

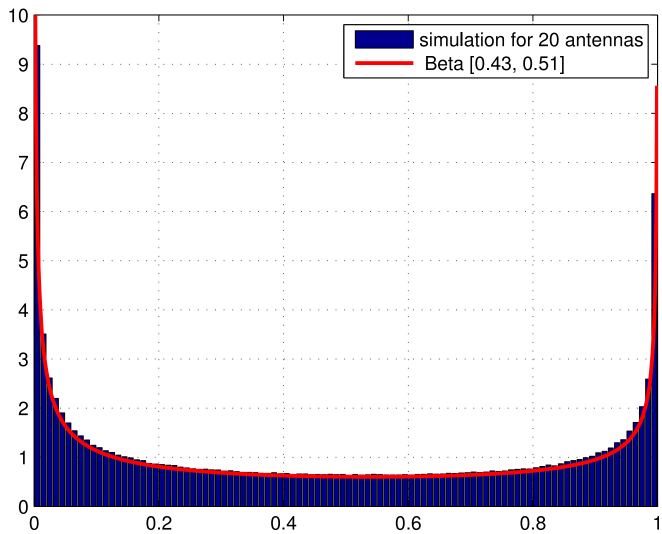


Fig. 3. LoS single-user effective interference pdf, $N = 20$, $\Delta = 1/2$

2) the two shape parameters of a Beta distribution allow fine tuning the shape of the curve;

the Beta appears to be a good candidate distribution for the approximation. We simulate 100 000 000 samples of the mainlobe interference according to (11) for ten times. Based on those data points, we conduct a maximum likelihood estimation (MLE). The estimated shape parameters $\hat{A}$ and $\hat{B}$ are given by [35, p. 231]

$$\psi(\hat{A}) - \psi(\hat{A}+\hat{B}) = q^{-1}\sum_{i=1}^{q}\ln y_i$$
$$\psi(\hat{B}) - \psi(\hat{A}+\hat{B}) = q^{-1}\sum_{i=1}^{q}\ln(1-y_i) \tag{14}$$

where $\psi(x) = \Gamma'(x)/\Gamma(x)$, with $\Gamma(x)$ and $\Gamma'(x)$ being the Gamma function and the derivative of Gamma function, y_i are the data points, and q is the total number of simulations. We resort to the Newton method to solve the system of equations in (14), yielding (12).

■

Remark 1: It is important to note that this interference model is derived based on the asymptotic result, hence, its parameters do not depend on the spacing and number of BS antennas. Given the antenna array is large, this approximation matches well to the simulations and so we do not need to perform any additional fits.

In Fig. 3 , we simulate the single-user effective interference according to (9) when the BS antenna number is moderate ($N = 20$), and compare it with the Beta approximation in (12). A Kolmogorov–Smirnov (KS) goodness-of-fit test is also conducted. Drawing 1000 samples of the mainlobe interference when $N = 20$, there is no evidence to reject the Beta distribution at 5% significance level. As demonstrated, although our derivation assumes infinite N, the Beta-distribution is a reasonable approximation even when N is moderate. Therefore, for all BS antenna numbers (many tens to a few hundreds), the mainlobe interference can be modeled by the same Beta distribution.

Combining (7), (10), and (12) yields the following proposition.

Proposition 1: (LoS Single-User Interference Distribution) The LoS single-user interference distribution can be described by

$$f_{I_m^L}(x) = (1-p)\,\delta_0(x) + pf(x;\alpha,\beta) \tag{15}$$

where $\delta_0(x)$ is the Dirac Delta function representing null interference, $f(x;\alpha,\beta)$ is the pdf of a Beta-distribution following (13), with $\alpha = 0.43$, $\beta = 0.51$. ■

To this end, we have another remark.

Remark 2: Admittedly, as $N \to \infty$, the effective interferer probability converges to 0. With a large BS antenna array, a single-user's interference is possibly negligible. However, when there are tens of users in the system, even though each user has a very small probability (inversely proportional to the antenna number) to contribute effective interference, the total interference can be significant. Our result in (15) provides an insight to the single-user interference distribution, when the antenna number is finite. Based on (15), we study the total interference in the sequel.

C. LoS Total Interference Distribution

Based on the previous discussion, the total LoS interference can be rewritten as

$$\mathcal{I}^L = \sum_{m=1}^{M} I_m^L, \quad I_m^L = J_m^* \cdot \mathbb{1}\{|\Theta_{m,0}| \leqslant 1\}.$$

Assuming that for all interferers, their normalized angular separations from the target user are independent, we analyze the total interference distribution by answering the following two questions.

1) If there are multiple effective interferers, what is their resultant effective interference?
2) Given M potential interferers, what is the distribution of the number of effective interferers, and the total interference?

Proposition 2: (Multiple Effective Interferers Interference): Given k independent effective interferers ($k = 1, 2, \ldots$), each incurring i.i.d. Beta distributed interference $J_m^* \sim \text{Beta}(\alpha, \beta)$, their resultant interference $\mathcal{J}_k = \sum_{m=1}^{k} J_m^*$ can then be *approximated* by a scaled Beta distribution [35, p. 209], with pdf

$$f_{\mathcal{J};k}(x) = \frac{1}{k} f\left(\frac{x}{k}; \alpha_k, \beta_k\right) \tag{16}$$

where

$$\alpha_k = \frac{\alpha\,[k(\alpha+\beta+1)-1]}{\alpha+\beta}, \quad \beta_k = \frac{\beta\,[k(\alpha+\beta+1)-1]}{\alpha+\beta}.$$

■

Proposition 3: (The Beta Mixture Model): Given M interferers, the pdf of the total LoS interference is given by

$$f_{\mathcal{I}^L}(x) = P(0)\delta_0(x) + \sum_{k=1}^{M} P(k) f_{\mathcal{J};k}(x) \tag{17}$$

where $f_{\mathcal{J};k}(x)$ follows (16), and

$$P(k) = \binom{M}{k} p^k\,(1-p)^{M-k} \tag{18}$$

is the probability of having k effective interferers out of the M potential interferers, in which the effective user probability p follows (10). ■

To simplify the calculation, we further investigate how many terms in (17) are significant. Based on the Chernoff bound [36, p. 53] of the Binomial distribution, given a small percentage t, we have

$$P\left(k > \mu + \sqrt{-3\mu \ln t}\right) \le t$$

where μ is the expectation of the Binomial distribution, given by

$$\mu = Mp = \frac{M}{N}\frac{1}{\Delta}\frac{9}{\pi^2}\text{arctanh}\left(\frac{\sqrt{3}}{2}\right).$$

We define the *cutoff number*

$$C \triangleq \left\lceil \mu + \sqrt{-3\mu \ln t} \right\rceil$$

where $\lceil x \rceil$ denotes the ceiling of x. The probability of having more than C effective interferer is not more than t. We choose a small t and ignore the probability of having interference contributed by $C+1$ or more effective interferers. For example, under massive MIMO settings, given $\Delta = 1/2$ and $t = 5\%$, C is usually not more than 5. This simplification significantly reduces the computation time. The pdf of the total interference can be simplified as

$$\begin{aligned} f_{\mathcal{I}^L}(x) &= P(0)\delta_0(x) + \sum_{m=1}^{C} P(m)\frac{1}{m} f\left(\frac{x}{m}; \alpha_m, \beta_m\right) \\ &\quad + \left(1 - \sum_{m=1}^{C} P(m)\right)\frac{1}{C+1} f\left(\frac{x}{C+1}; \alpha_{C+1}, \beta_{C+1}\right) \end{aligned} \tag{19}$$

and the cdf of the total interference follows

$$\begin{aligned} F_{\mathcal{I}^L}(t) &= P(0) + \sum_{m=1}^{C} P(m) F\left(\frac{t}{m}; \alpha_m, \beta_m\right) \\ &\quad + \left(1 - \sum_{m=1}^{C} P(m)\right) F\left(\frac{t}{C+1}; \alpha_{C+1}, \beta_{C+1}\right) \end{aligned} \tag{20}$$

where $F(x; A, B)$ is the cdf of the Beta distribution with parameters A and B. Fig. 4 compares the simulated interference pdf and cdf with the theoretical results in (19) and (20).

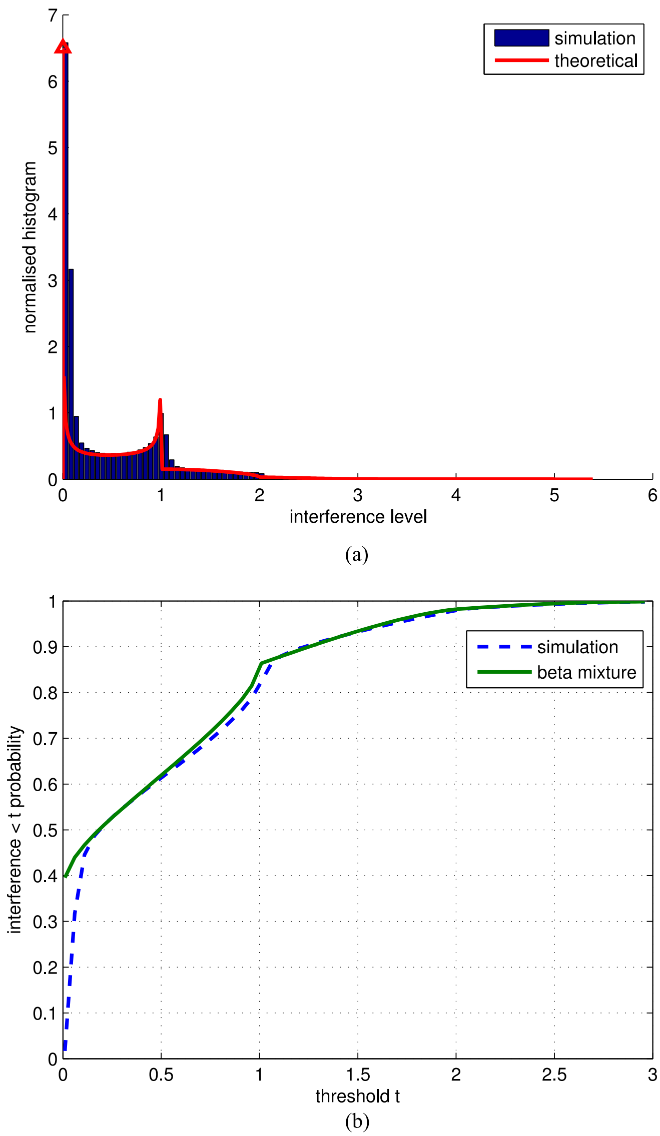


Fig. 4. LoS total interference distribution comparison, $N = 60$, $M = 25$, $\Delta = 1/2$.

In Fig. 4(a), the leftmost impulse represents the null interference. The distribution within $[0, 1]$ resembles the shape of a Beta distribution as in Fig. 3, and is mainly contributed by the most significant interferer. The interference level is significant up to not more than 3, indicating our definition of the cutoff number is sensible. The simulation slightly mismatches the theoretical results at some points, which is contributed by the sidelobe interference. This figure also indicates that the pure LoS interference needs to be described by a very special distribution, such as our Beta mixture.

The gap in the leftmost of Fig. 4(b) is incurred by the approximation that we ignore all the sidelobes. Sidelobe interference dominates the very low interference region. When all the interferers are in the sidelobes, we consider that there are no effective interferers and the interference is zero. While in reality, a small amount of interference still exists. Therefore, we overestimate the cdf in the low-interference region. We conduct a KS goodness-of-fit test on the region $t \in [0.1, \infty)$. Drawing 500 samples of the total interference with $N = 60$, $M = 25$, there is no evidence to reject the Beta-mixture distribution at 5% significance level. Nevertheless, it is still a reasonable approximation overall.

IV. Rician Interference

The LoS interference analysis in Section III enables us to study the Rician interference. In this section, we will show that under massive MIMO assumptions, the Rician interference distribution can be well approximated by a noncentral Chi-square distribution, with a noncentrality parameter following the Beta-mixture model of the LoS interference distribution.

The single-user interference and total interference in Rician channels are denoted by I_m and $\mathcal{I}$, respectively, following the same definitions as in (2) and (1). In our analysis, we assume that all the users in the system have the same Rician K factor. In Section IV-A, we first study the behavior of the BF weight $\mathbf{w}_m$ when the BS antenna number is large. Next, in Section IV-B, we will show that when the LoS channel component is given, the Rician interference distribution can be approximated by a noncentral Chi-square distribution with a fixed noncentrality parameter. Finally, we uncondition the LoS component using the result obtained in Section III and present the model for total interference distribution in Section IV-C. Although in the derivation, we use some asymptotic results assuming $N \to \infty$, the final result is a good approximation even when the antenna number is moderate.

A. Rician Beamforming Weight in Massive MIMO

Recall, that for any m, the beamforming weight is

$$\mathbf{w}_m = \frac{\mathbf{h}_m}{||\mathbf{h}_m||} = \left[\frac{h_{m,0}}{||\mathbf{h}_m||}, \ldots, \frac{h_{m,n}}{||\mathbf{h}_m||}, \ldots, \frac{h_{m,N-1}}{||\mathbf{h}_m||}\right]$$

where

$$h_{m,n} \sim \mathcal{CN}\left(\sqrt{\frac{K}{K+1}} \exp\left(j\Psi_{m,n}\right), \frac{1}{K+1}\right)$$
$$m = 1, \ldots, M, \ n = 1, \ldots, N.$$

Due to the LoS component (nonzero mean) of $h_{m,n}$, the numerator and denominator of $\mathbf{w}_m$ are correlated, and the entries of $\mathbf{w}_m$ are also correlated, hence, it is intractable to accurately describe $\mathbf{w}_m$ statistically. However, since we consider massive MIMO, we can exploit the advantage of having large N, and approximate $\mathbf{w}_m$ using its asymptotic behavior.

Approximation 3: (Rician Beamforming Weight): When N is large, the beamforming weight can be approximated as

$$\mathbf{w}_m \approx \frac{\mathbf{h}_m}{\sqrt{N}}. \tag{21}$$

■

Justification. See Appendix C.

■

B. Rician Interference Given Fixed LoS Component—The Noncentral Chi-square Model

In this subsection, we present the Rician interference distribution conditioned on the LoS component is given.

Combining (21) and (2) yields the approximate single-user Rician interference power

$$I_m \approx \left| \sum_{n=0}^{N-1} \frac{1}{N} h_{0,n}^* h_{m,n} \right|^2.$$

According to the definition of $h_{m,n}$, given the LoS component is known, all the $h_{0,n}^*$ and $h_{m,n}$ are mutually independent. Hence, their product $h_{0,n}^* h_{m,n}$ forms a *double complex Gaussian* [37]. Based on this result, we have the following total interference distribution.

Approximation 4: (Total Rician Interference Distribution Conditioned on Fixed LoS Component): When N is large and the LoS channel components are known, the total interference power $\mathcal{I}$ from M interferers is modeled as

$$\frac{2N(K+1)^2}{2K+1}\mathcal{I} \sim \chi_V^2(\delta); \quad V = 2M, \quad \delta = \frac{2NK^2}{2K+1}\mathcal{I}^L \quad (22)$$

where $\chi_V^2(\delta)$ denotes the noncentral Chi-square distribution with degrees of freedom V and noncentrality parameter δ, and $\mathcal{I}^L$ is the LoS total interference, as studied in Section III-C. ■

Justification. See Appendix D. ■

C. Rician Interference Distribution—The Noncentral Chi-square Mixed With Beta Mixture

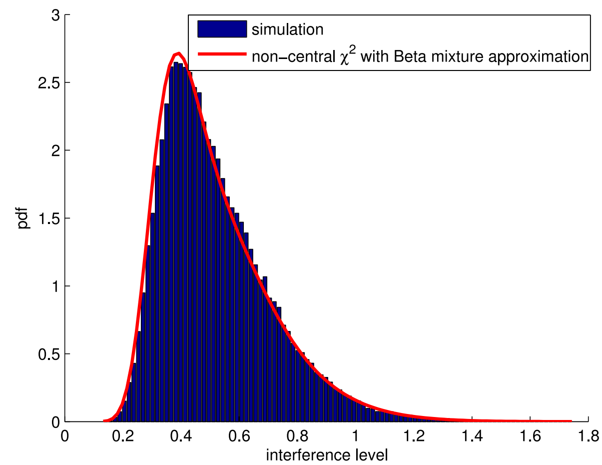
Unconditioning the LoS total interference in (22) yields the following proposition.

Proposition 4: (Total Rician Interference Distribution): The pdf of the total Rician interference can be easily obtained as

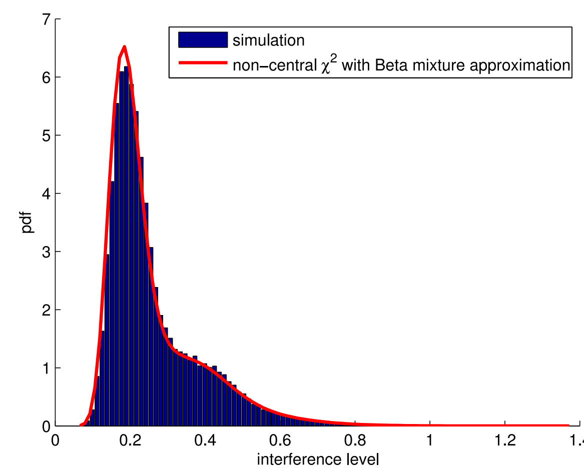
$$f_{\mathcal{I}}(t) = \int_0^M \rho_{(N,K)} f_{\chi^2}\left(\rho_{(N,K)}t; 2M, \varrho_{(N,K)}x\right) f_{\mathcal{I}^L}(x)dx$$
$$\rho_{(N,K)} = \frac{2N(K+1)^2}{2K+1}, \; \varrho_{(N,K)} = \frac{2NK^2}{2K+1} \quad (23)$$

where $f_{\chi^2}(t; V, \delta)$ is the pdf of noncentral Chi-square distribution with degrees of freedom V and noncentrality parameter δ, and $f_{\mathcal{I}^L}(x)$ follows (19). ■

Fig. 5 compares the simulated Rician interference pdf with the computation in (23). Fig. 5 plots the total interference from 20 interferers with 40 BS antennas and Fig. 5(b) plots 20 interferers with 80 antennas. The heavy tail in the figure corresponds to the effective LoS interference profile. With increasing antenna numbers, a user is less likely to exert effective LoS interference, hence, the pdf is more concentrated in the low interference region. As demonstrated, the interference distribution is a good approximation even when the antenna number is moderate.



(a)



(b)

Fig. 5. Rician total interference pdf, $M = 20$, $K = 1$, $\Delta = 1/2$.

V. Application to System Design and Discussions

With the derived interference distribution, we can analyze some system performance metrics dependent on the SIR, such as determining the user number based on outage and throughput requirements. In Rician channels, due to the fading component, the received power is also a random variable. Hence, the SIR analysis depends on both the received signal power and the interference. In this section, we first present the pdf of the received signal strength. Next, we study the SIR-based outage and throughput, and provide two system design examples based on the quality of service (QoS) requirement. In particular, we comment on the regions of outage and throughput where the proposed interference approximation performs well and where it slightly loses accuracy.

A. Rician Received Signal

Recall that the received signal power is

$$S = \frac{1}{N}|\mathbf{w}_0^H \mathbf{h}_0|^2 = \sum_{n=0}^{N-1} \left| \frac{1}{\sqrt{N}} h_{0,n} \right|^2.$$

Following the Rician channel definition in (4), separating the real and imaginary parts of $\frac{1}{\sqrt{N}}h_{0,n}$ yields

$$2N(K+1)\sum_{n=0}^{N-1}\left|\frac{1}{\sqrt{N}}h_{0,n}\right|^2 \sim \chi^2_{2N}(2NK).$$

The noncentral χ^2 distribution can further be closely approximated by a scaled central χ^2 distribution [38, Sec. III], and we have

$$\rho_{(N,K)}S \sim \chi^2_{\rho_{(N,K)}} \tag{24}$$

where χ^2_v denotes a central χ^2 distribution with v degrees of freedom.

B. Coverage/Outage Probability and QoS

The interference $\mathcal{I}$ depends on the location differences between the target user and the interferers, while the signal strength S does not. Hence, S and $\mathcal{I}$ are independent, and we have the following proposition.

Proposition 5: (Rician Coverage Probability): Under Rician channel conditions, the coverage probability P_c is given by

$$\begin{aligned} P_c &\triangleq P(\text{SIR} > T) \\ &= \int_{x=0}^{M} F_{\mathcal{F}}\left(\frac{\rho_{(N,K)}}{2M}\frac{1}{T}; 2M, \rho_{(N,K)}, \varrho_{(N,K)}x\right) f_{\mathcal{I}^L}(x)dx \end{aligned} \tag{25}$$

in which $F_{\mathcal{F}}(x; V_1, V_2, \delta)$ is the cdf of a noncentral F-distribution with degrees of freedom V_1, V_2 and noncentrality parameter δ, and $f_{\mathcal{I}^L}(x)$ follows (19).

Proof: See Appendix E. ■

Although we did not obtain a closed-form expression, the numerical calculation can be easily performed. The outage probability is given by the complement of the coverage probability, i.e., $P_{\text{out}} = 1 - P_c$.

Fig. 6 compares the computed outage probability with the simulation. The number of interferers is fixed as 20, and the outage is plotted for various antenna numbers. Fig. 6(a) plots with $K = 1$ (0 dB), and Fig. 6(b) plots with $K = 5$ (7 dB), as in the WINNER II indoor office and urban/rural macrocell scenarios [39, p. 47]. Despite all the approximations made, our result matches well to the simulation.

Comparing the two plots, when K factor is larger, a gap between the computed result and the simulation appears at the high outage (high SIR) region. The gap is mainly induced by the approximation that we ignore all the side-lobe interference for the LoS component, as explained for Fig. 4. For the same reason, if we calculate the outage for pure LoS case, our result will diverge from the simulation at very high SIR threshold; see [26]. While in Rician case, the fading component mitigates the difference between our approximation and the simulation. This is because, when all the LoS component interferers are in the sidelobe, although we consider no effective LoS interference, we still take into account the interference incurred by the fading component. Hence, with a larger K factor, we expect the gap to be wider. On the other hand, with a smaller K factor, the

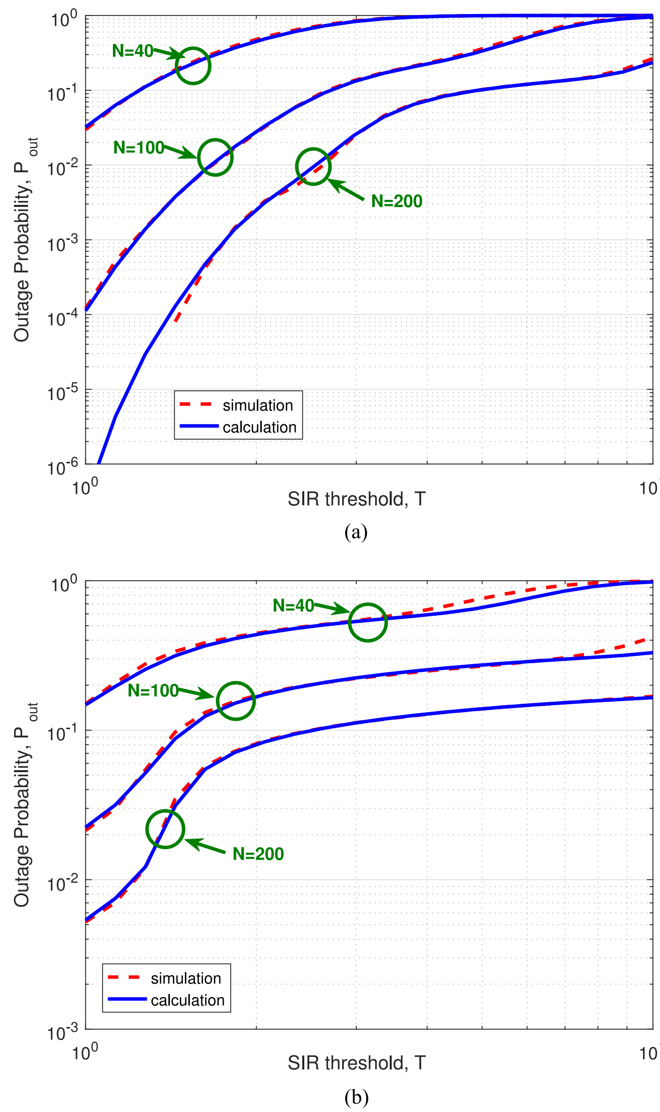


Fig. 6. Rician outage probability, $M = 20$, $\Delta = 1/2$.

channel condition is closer to Rayleigh fading, the theoretical result will match better to the simulation. However, in terms of outage probability, the region of interest is usually between 0.1% and 20%. Our approximation matches very well to the simulation in the region of interest, even when K is large.

The aforementioned result can be used to determine the number of users that the same time–frequency resource can accommodate given the BS antenna number and required SIR threshold. In Fig. 7, we present the outage probability versus the interferer number, with solid lines representing our analytical results and dash lines the simulation results. The cyan lines represent $1\%, 2\%, 5\%$, and 10% outage level.

The figure provides an intuitive interpretation of how many *extra* users the same resource block can support, given the required QoS. The key findings are summarized in Table I. If the QoS requires 1% outage at $T = 10$ dB, even 200 BS antennas can only support two users on the same frequency.

To this end, we have observed that when the SIR threshold decreases, the increment in the number of users that the system can accommodate is larger when the K factor is smaller. As

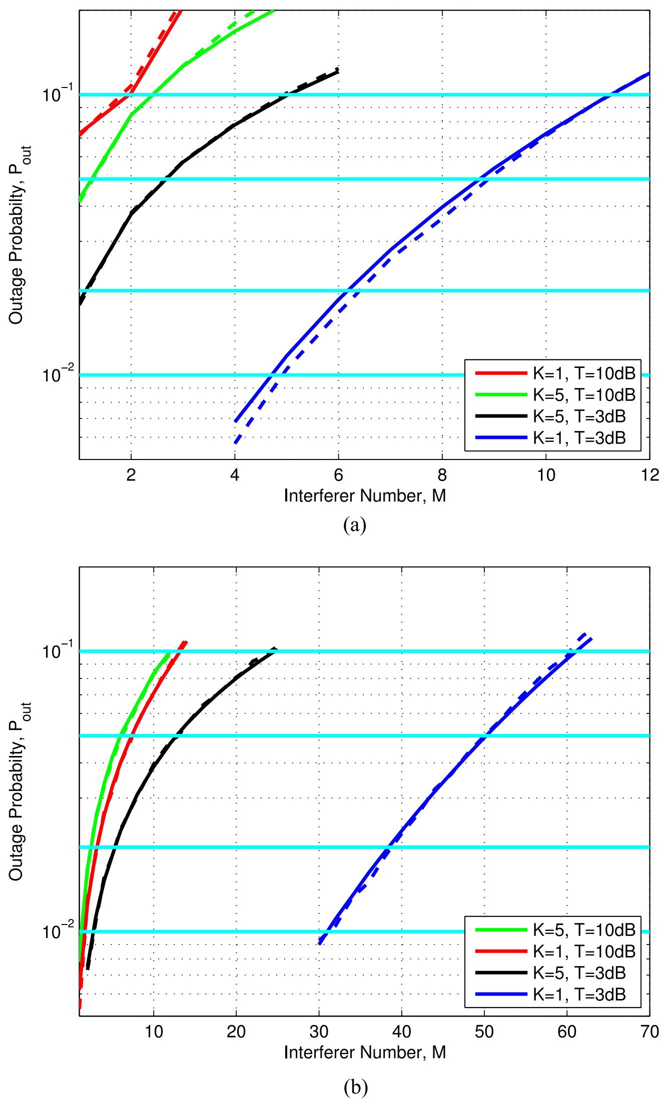


Fig. 7. Rician QoS, max number of interferer.

TABLE I
MAX INTERFERER NUMBER

$T = 3$ dB	$N = 40$				$N = 100$				$N = 200$			
max M	$K = 1$		$K = 5$		$K = 1$		$K = 5$		$K = 1$		$K = 5$	
P_{out}	simu	calc	simu	calc	simu	calc	simu	calc	simu	calc	simu	calc
10%	11	11	5	5	29	29	12	12	60	60	24	24
5%	8	8	2	2	24	24	6	6	50	50	13	13
2%	6	6	1	1	18	18	2	2	38	38	5	5
1%	5	4	NA	NA	15	14	1	1	31	31	3	3
$T = 10$ dB	$N = 40$				$N = 100$				$N = 200$			
max M	$K = 1$		$K = 5$		$K = 1$		$K = 5$		$K = 1$		$K = 5$	
P_{out}	simu	calc	simu	calc	simu	calc	simu	calc	simu	calc	simu	calc
10%	1	1	2	2	6	6	6	6	12	13	12	12
5%	NA	NA	1	1	3	3	3	3	7	7	6	6
2%	NA	NA	NA	NA	1	1	1	1	3	3	2	2
1%	NA	NA	NA	NA	NA	NA	NA	NA	1	1	1	1

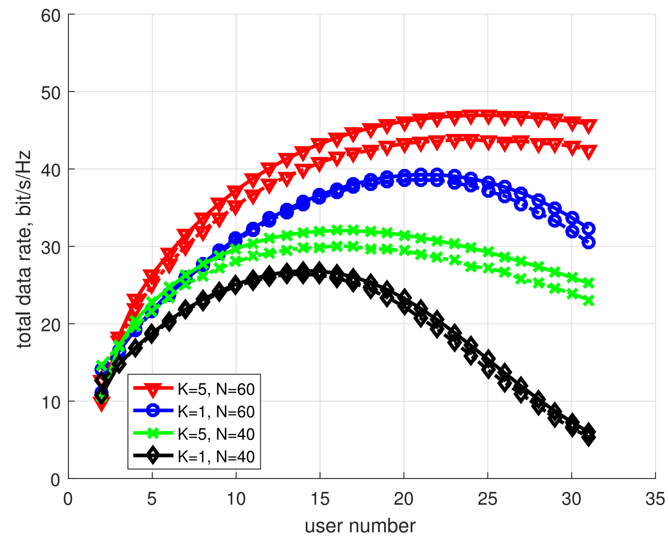


Fig. 8. Rician average throughput, SIR$= 3$ dB, $\Delta = 1/2$, varying K and N

shown in Fig. 7, as T deceases from 10 to 3 dB, comparing the cases where $K = 1$ and $K = 5$, we observe that large K allows fewer additional users. Equivalent phenomenon can be observed in Fig. 6: With the same interferer number, at lower SIR threshold, large K gives higher outage probability. This is because that the channel with a larger K factor has a smaller variance, hence it is less likely to yield extremely small or large interference. As a result, compared to the channel with a smaller K factor, larger K yields worse performance at low SIR threshold while better at high SIR threshold.

C. Maximum Average Throughput

To guarantee the user experience, let us consider a user transmits successfully if its received SIR is above the threshold T; otherwise it transmits no effective data. Under this requirement, we define the *average throughput* of the system as

$$\begin{aligned} R &\triangleq (M+1)E\left[\log_2(1+\text{SIR})\cdot \mathbf{1}(\text{SIR} > T)\right] \\ &= (M+1)\int_T^{+\infty} \log_2(1+t) f_{\text{SIR}}(t)dt \end{aligned}$$

where f_{SIR} is the pdf of the SIR and is a function of the user number in the system. The f_{SIR} can be easily obtained as

$$f_{\text{SIR}}(t) = \int_{x=0}^{M} \frac{\rho_{(N,K)}}{2Mt^2} f_{\mathcal{F}}\left(\frac{\rho_{(N,K)}}{2M}\frac{1}{t}; 2M, \rho_{(N,K)}, \varrho_{(N,K)}x\right) f_{\mathcal{I}^L}(x)dx$$

where $f_{\mathcal{F}}(x; V_1, V_2, \delta)$ denotes the pdf of a noncentral F-distribution.

With this result, we can numerically find out the optimal number of users in the system in terms of maximizing the average throughput. Fig. 8 illustrates the optimal user number in a cell sector given moderate antenna numbers (60 and 40), required SIR threshold $T = 3$ dB and when $K = 1$ and 5. Solid lines represents the theoretical calculation, dash lines the simulation.

It can be observed that larger K factor increases the system's capacity of accommodating more users. On the other hand, when K factor is smaller, the whole system's throughput decays faster when the user number exceeds the optimal number.

However, when K factor is large ($K = 5$), the theoretical throughput becomes slightly inaccurate. This is because the average throughput involves the high SIR region (which is very likely to result in high outage), where our proposed result slightly loses accuracy, as discussed for Fig. 6. This is caused by the approximation that we ignore all the LoS sidelobe interference. Since our approximation tends to underestimate the interference, it overestimates the system's capability of accommodating users.

Nonetheless, our result on throughput still provides reasonable system design insights such as estimating the optimal user number in terms of maximizing the throughput. Especially, the accuracy of the proposed model does not rely on extremely large antenna numbers.

VI. Conclusion

In this paper, we study the interference in an uplink single-cell massive MIMO system with large yet finite BS antenna numbers, for LoS and Rician channels, respectively. Both the target user and the interferers experience channel conditions with an LoS component. The LoS component induces interference dependence on the AoAs of the users. We present a novel method to resolve this dependence. We first provide a good approximation for the interference distribution in the pure LoS case, and then, exploit this approximation to derive the Rician interference distribution. With the interference distribution, we further analyze the outage probability and throughput for the Rician case. Our result is very useful in dimensioning how many users a system can support, given the BS antenna number and the required QoS. We also point out that in which cases the proposed interference model performs well, and where it loses accuracy.

Future work includes the investigations on the interference in a multicell system, the performance gain by using other receiver beamforming techniques such as zero forcing, and the impact of users experiencing various K factors.

Appendix A
Derivation of Effective Interferer Probability

The probability that user m is an effective interferer is

$$p \triangleq P\left(-1 \le \Theta_{m,0} \le 1\right). \tag{26}$$

Recall that all the users are randomly placed in the sector, their AoAs ϕ_m and ϕ_0 are, hence, independently and uniformly distributed over $\left[-\frac{\pi}{3}, \frac{\pi}{3}\right]$. From (8), the probability density function (pdf) of $\Theta_{m,0}$ when having N BS antennas can be obtained as

$$f^{(N)}_{\Theta_{m,0}}(z) = \int_{y_L}^{y_U} \frac{1}{N\Delta}\left(\frac{3}{2\pi}\right)^2 \frac{1}{\sqrt{1-(\frac{z}{N\Delta}+y)^2}} \frac{1}{\sqrt{1-y^2}} dy$$
$$|z| \leqslant \sqrt{3}N\Delta \tag{27}$$

where the integral limits are

$$y_L = \max\left\{-\frac{\sqrt{3}}{2}, -\frac{\sqrt{3}}{2} - \frac{z}{N\Delta}\right\},$$
$$y_U = \min\left\{\frac{\sqrt{3}}{2}, \frac{\sqrt{3}}{2} - \frac{z}{N\Delta}\right\}.$$

Following (26), for any user

$$p = \int_{-1}^{1} f^{(N)}_{\Theta_{m,0}}(z)dz.$$

Applying the Taylor expansion of $f^{(N)}_{\Theta_{m,0}}(z)$ yields (10), completing the derivation.

∎

Appendix B
Derivation of Asymptotic Effective Interference Expression

We first recall the asymptotic effective interference expression in (9), and apply the Taylor expansion around 0 to the denominator $\cos\left(\frac{2\pi}{N}\Theta_{m,0}\right)$, yielding

$$\lim_{N\to\infty} J^{(N)}_m = \mathrm{sinc}^2(\pi\Theta_{m,0}),\ -1 \le \Theta_{m,0} \le 1 \tag{28}$$

where $\mathrm{sinc}(x) = \frac{\sin(x)}{x}$. This implies when the BS antenna number is large, the effective interference converges point wise to the square of a sinc function of $\Theta_{m,0}$ within $[-1, 1]$.

Next, we study the conditional distribution of the normalized angular separation $\Theta_{m,0}$ given it is within $[-1, 1]$. Applying Bayes' rule, the *conditional* pdf of $\Theta_{m,0}$ given $\Theta_{m,0} \in [-1, 1]$ is

$$f^{(N)}_{\Theta_{m,0}\big| |\Theta_{m,0}|\le 1}(t) = \frac{f^{(N)}_{\Theta_{m,0}}(t)}{\int_{-1}^{1} f^{(N)}_{\Theta_{m,0}}(z)dz},\ |t| \le 1. \tag{29}$$

Combining (27) and (30) yields

$$\lim_{N\to\infty} f^{(N)}_{\Theta_{m,0}\big| |\Theta_{m,0}|\le 1}(t) = 1/2. \tag{30}$$

This implies that the *conditional* pdf of $\Theta_{m,0}$ in (30) converges to a uniform distribution over $[-1, 1]$. Then, by combining (31) and (29), we have (11), completing the derivation.

∎

Appendix C
Derivation of Rician Beamforming Weight

Recall that the beamforming weight is given by

$$\mathbf{w}_m = \frac{\mathbf{h}_m}{||\mathbf{h}_m||}$$

where $||\mathbf{h}_m|| = \sqrt{\sum_{n=0}^{N-1} |h_{m,n}|^2}$. Each $h_{m,n}$ is complex Gaussian distributed with various means, while their amplitudes $|h_{m,n}|$ are i.i.d. Rician distributed, given the K factors are the same. As the antenna number N goes large, according to the weak law of large numbers [40, p. 185]

$$\frac{1}{N}\sum_{n=0}^{N-1}|h_{m,n}|^2 \xrightarrow{p} E\left[|h_{m,n}|^2\right] = 1$$

where $\xrightarrow{p}$ denotes convergence in probability, and $E\left[|h_{m,n}|^2\right]$ is obtained as the second moment of the Rician distribution. According to the continuity theorem [40, p. 155]

$$\frac{||\mathbf{h}_m||}{\sqrt{N}} = \sqrt{\frac{1}{N}\sum_{n=0}^{N-1}|h_{m,n}|^2} \xrightarrow{p} 1$$

yielding (21). ∎

Appendix D
Derivation of Rician Interference Given Fixed LoS Component

We denote the single-user *interference signal* as

$$z \triangleq \sum_{n=0}^{N-1}\frac{1}{N}h_{0,n}^* h_{m,n}$$

and its mean and variance can be obtained, respectively, as [37]

$$E[z] = \begin{cases} \dfrac{K}{K+1}\dfrac{1}{N}e^{j\frac{2\pi}{\lambda}(R_0-R_m)}\dfrac{1-\exp\left(jN2\pi\Delta\theta_{m,0}\right)}{1-\exp\left(j2\pi\Delta\theta_{m,0}\right)}, & \theta_{m,0}\neq 0\\ \dfrac{K}{K+1}, & \theta_{m,0}=0 \end{cases}$$

$$\text{Var}\left[z\right] = \frac{2K+1}{N(K+1)^2}.$$

The single-user *interference power* can be written as

$$I_m = \Re\{z\}^2 + \Im\{z\}^2.$$

According to the analysis in [37], the real and imaginary parts of z are correlated, and the correlation is induced by the nonzero mean. However, to make the following analysis tractable, we make the approximation that for large N, the real and imaginary parts of z are independently Gaussian distributed with the same variance, i.e.

$$\Re\{z\} \sim \mathcal{N}\left(\Re\{E[z]\}, \sigma_R^2\right), \quad \Im\{z\} \sim \mathcal{N}\left(\Im\{E[z]\}, \sigma_R^2\right)$$

where

$$\sigma_R^2 = \frac{\text{Var}[z]}{2} = \frac{2K+1}{2N(K+1)^2}.$$

Based on this assumption, I_m is the sum of the square of two independent Gaussian random variables. Hence, I_m can be modeled by the following noncentral Chi-square distribution

$$\frac{I_m}{\sigma_R^2} \sim \chi^2_{V_1}(\delta_1); \quad V_1 = 2, \quad \delta_1 = \frac{2NK^2}{2K+1}I_m^L$$

where $\chi^2_V(\delta)$ denotes the noncentral Chi-square distribution, with degrees of freedom V and noncentrality parameter δ, and I_m^L is the LoS single-user interference in (6).

Following the same method, the results in (22) can be obtained. ∎

Appendix E
Derivation of Rician SIR Coverage Probability

The coverage probability is defined as $P_c(T) = \mathbb{P}(\text{SIR} > T)$, where T is the threshold. The received signal strength S and interference $\mathcal{I}$ are independent. From (22)–(24), we have

$$\rho_{(N,K)}\mathcal{I} \sim \chi^2_{2M}\left(\varrho_{(N,K)}\mathcal{I}^L\right), \ \rho_{(N,K)}S \sim \chi^2_{\rho_{(N,K)}}.$$

Therefore, the ratio between the interference and the received signal power follows a scaled noncentral F distribution [38]

$$\frac{\mathcal{I}}{S}\frac{\rho_{(N,K)}}{2M} \sim \mathcal{F}(2M, \rho_{(N,K)}, \varrho_{(N,K)}\mathcal{I}^L) \tag{31}$$

where $\mathcal{F}(v_1, v_2, \delta)$ denotes a noncentral F distribution with v_1 and v_2 degrees of freedom and noncentrality parameter δ and (25) can consequently be computed as

$$\begin{aligned} P_c(T) &= \mathbb{P}\left(\frac{\mathcal{I}}{S} < \frac{1}{T}\right)\\ &= \mathbb{E}_{\mathcal{I}^L}\left[\mathbb{P}\left(\frac{\mathcal{I}}{S} < \frac{1}{T}|\mathcal{I}^L\right)\right]\\ &= \int_{x=0}^{M} F_{\mathcal{F}}\left(\frac{\rho_{(N,K)}}{2M}\frac{1}{T}; 2M, \rho_{(N,K)}, \varrho_{(N,K)}x\right) f_{\mathcal{I}^L}(x)dx \end{aligned}$$

in which $F_{\mathcal{F}}(x; v_1, v_2, \delta)$ is the cdf of the noncentral F distribution, and

$$\rho_{(N,K)} = \frac{2N(K+1)^2}{2K+1}, \ \varrho_{(N,K)} = \frac{2NK^2}{2K+1}.$$

As a remark, the scenario that the target user experiences Rician fading and interferers experience Rayleigh fading is a special case of our result, i.e., the signal strength S follows a noncentral Chi-square distribution, the interference $\mathcal{I}$ follows Chi-square distribution, and thus, the SIR can also be modeled as a noncentral F distribution. ∎

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