Optimal Energy Harvesting Protocols for Wireless Relay Networks

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***Abstract*—In this paper, we consider a relay network over a flat-fading channel, where the relay has no fixed power supply and thus needs to replenish energy via wireless energy harvesting (EH) from the signals transmitted by the source. We propose a novel hybrid protocol, which is a combination of existing EH protocols, such as power splitting (PS) and time switching (TS). We formulate the optimization problems and derive some explicit results. In particular, we derive the optimal PS and TS ratios at the relay for all three EH protocols to achieve the maximum throughput for information transfer from the source to the destination for both decode-and-forward and amplify-and-forward relaying schemes. We show that the proposed hybrid protocol outperforms both PS and TS protocols.**

***Index Terms*—Energy harvesting, hybrid, power splitting, relay networks, time switching.**

I. Introduction

Wireless communications has advanced tremendously in the past decades to become an essential and inseparable part of our daily life. Given the growth trends of smart wireless devices that can support advanced data intensive applications, we can anticipate major challenges to arise in wireless communications due to high energy consumption and environmental impact. Wireless sensor networks (WSNs) are essential when wireless communication techniques are applied smartly for future networks in military, health, automobiles, agriculture and mining. Thus, a focus towards green communications has been increased on WSNs operated in remote areas. One of the key challenges with future communication devices in such networks is how to supply sufficient energy to remote wireless networks. Powering sensor nodes by magnetic induction, wind or solar leads to many challenges due to constant unavailability, limited space, implementation overhead, or requirement of large scale infrastructure. Motivated by this, an important focus is given on wireless energy harvesting (EH) techniques. For example, in sensor networks, it is costly to replace sensor batteries and EH relying on natural energy sources is also inconvenient due to their intermittent nature. Thus, widespread low-power devices can be charged wirelessly [1]–[3].

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In meeting the ever increasing demand for covering every corner of the globe, cooperative communications, in particularly relay networks, have recently been promoted as a viable solution [4]. The deployment of relays has proven especially helpful in eliminating coverage holes in wireless networks. Due to the random positions and high mobility of nodes, relays need to be opportunistically deployed where most needed. When access to a main power supply is not possible, it is more convenient for relays to use self-sustainable radio signal energy. However, relay operation in energy harvesting may be considerably different from the traditional self-powered relay [5] where the relay simply forwards its received signal to the destination with hard decoding such as decode-and-forward (DF) relaying or without hard decoding such as amplify-and-forward (AF) relaying. While the majority of the research has considered EH in point-to-point communication systems [6]–[10], some of the research has considered EH in wireless relaying networks which has also gained much more interest recently [5], [11]–[20]. Energy harvesting in relay networks is the focus of this paper.

A. Related Work

In multihop relay-assisted EH networks, the operation of intermediate relays may help in two ways: i) Only energy transfer – the energy from the source node is transferred to the destination via intermediate relays. Thus, only the source is the non-EH node. This topic has received less attention as no information transfer takes place in the relay network which may limit the possible applications. However, the destination may use the harvested energy for its feedback information or for a separate network which is independent from source and relay nodes [21]; and ii) Both energy and information transfer – intermediate relays assist with both energy and information transfer. Relays may transfer both simultaneously, or relays may first harvest energy from the source signal, and then use that harvested energy to forward the source information to the destination. Thus, the source and destination are non-EH nodes [12]. Among these two ways, the later has garnered significant interest in the literature because it allows information exchange between far apart source-destination pairs. However, designing EH protocols at the relaying stage is challenging when combined with the information processing such as DF or AF.

At the relay, the same antenna may be shared for both information decoding and EH. Mainly, two EH protocols have been introduced in the literature: i) the time-switching (TS) protocol and ii) the power-splitting (PS) protocol [22]. The TS protocol is implemented with a simpler switch which helps independent EH followed by information transfer, and the PS protocol is implemented with a power splitter which splits the received signal into two signals for simultaneous EH and information transfer. For example, TS and PS protocols have been considered for point-to-point networks in [8], and for relay networks, the TS protocol is considered in [11], [12], and [23] and the PS protocol is considered in [11]–[13]. Further, these EH protocols are studied for DF relaying in [5] and [14]–[17] and AF relaying in [12] and [18]–[20]. Especially in relay networks, it is important to note that the EH time of the TS protocol or the power-splitting ratio of the PS protocol is important because it directly affects network performance, and thus designing optimal EH protocols is an interesting and timely research problem.

In [12] and [24], PS and TS protocols with AF and DF relaying are studied, and the outage probability is derived with Rayleigh fading for high signal-to-noise ratio (SNR). Based on this, optimal TS and PS ratios are numerically calculated because of the analytical complexity of throughput expressions. Reference [25] shows that there is a trade-off between EH time and data transmission time, and this trade-off is discussed for AF relaying with half-duplex and full-duplex. Further, the optimal TS ratio is derived by using approximations. In [15], the relay's strategies to distribute the harvested energy among the multiple users are investigated for DF relaying, and the PS ratio is chosen based on the targeted data rate at each link. Similar criteria for PS and TS protocols are also considered for multihop AF and DF relay networks (the number of relays may be more than one) in [26]. However, this criterion does not maximize the overall network throughput. In [23] and [27], optimal TS and PS ratios are analytically derived in order to maximize the throughput when source to relay and relay to destination channels are Rayleigh and AWGN, respectively. References [28] and [29] propose noncoherent EH protocols based on the AF and DF relaying (PS and TS noncoherent AF and DF) which do not require any instantaneous channel state information. This paper highlights with some numerical examples that unique optimal values of the PS or TS ratio minimizing the error rate exist. Reference [30] proposes a TS EH protocol with continuous and discrete time modes, in which the TS ratio is chosen to enable relay transmission with preset fixed transmission power. In [13], antenna selection and the PS ratio are jointly optimized to maximize the achievable rate for AF relay networks, and a two-stage procedure (algorithm) is proposed to determine the optimal values of a the non-convex problem. In [31], an interference aided EH scheme is proposed for DF relay networks with both PS and TS protocols, where the relay harvests energy from the received information signal and co-channel interference signals. Since [31] focuses on deriving analytical expressions for the ergodic capacity and the outage capacity, a rigorous analytical framework is not provided for optimal EH protocols.

B. Motivation for a New EH Protocol

It is clear that the majority of work uses PS and/or TS protocols. Recent research gives similar attention on to both as they are equally important. Further, there is no clear-cut answer as to which one is better. For example, the TS protocol outperforms the PS protocol at low SNR, and the PS protocol outperforms the TS protocol at high SNR [31]. In contrast to traditional PS and TS protocols, alternative PS and TS protocols which may have less implementation complexity, require less channel knowledge, improve performance, etc. are also available in the literature [28]–[30]. But such protocols do not show significant superiority over the traditional PS or TS protocol for all network conditions.

However, recently, a time power switching based relaying (TPSR) protocol is proposed in [32] for AF relay networks with simultaneous energy harvesting and information processing. The communication block is divided into two slots for source to relay and relay to destination transmissions, which are not necessarily of equal lengths. Further, the first time slot performs EH with both PS and TS modes and also information decoding. Thus, this setup has some drawbacks which may not help in real implementation: i) the source and relay may need different transmission rates as time-slot lengths are not necessarily equal for the source to relay and relay to destination transmissions, thus this may not help in AF relaying (although the paper [32] uses AF relaying with no comments on it); ii) PS mode always depends on TS mode, and this protocol cannot be reduced to the PS mode, however it can be reduced to TS mode. Therefore, this does not provide a general setup. Further, this paper considers only AF relaying but not DF relaying, and there is no analytical framework to optimize PS and TS ratios jointly which may be essential for such a combined protocol.

C. Contribution

This paper proposes a novel *hybrid protocol* for energy harvesting which is a combination of PS and TS protocols. This is applied to relay networks, and it outperforms both PS and TS protocols. Further, the proposed hybrid protocol has some unique characteristics, and helps to develop a general analytical framework:

- It is implemented with independent PS and TS modes with the PS mode followed by the TS mode. Thus, the hybrid protocol can operate as all three protocols, i.e., PS, TS or hybrid.
- This protocol allocates the same time-slot lengths for the source to relay and relay to destination transmissions. Therefore the source and relay can use the same transmission rate, and it can easily be applied for both AF and DF relaying leading to less implementation complexity. However, this protocol can be extended for networks with variable transmission rates at the source and relay stages.

Since this is the first work investigating such hybrid protocols, first we provide unified system and analytical models which help to discuss PS, TS or hybrid protocol for DF or AF relaying. Second, this paper focuses on deriving the optimal hybrid protocol in which optimal EH time and power-spitting ratio are derived by maximizing the throughput for both DF and AF relaying. Since the hybrid protocol may reduce to the PS protocol or may closely approach the TS protocol,

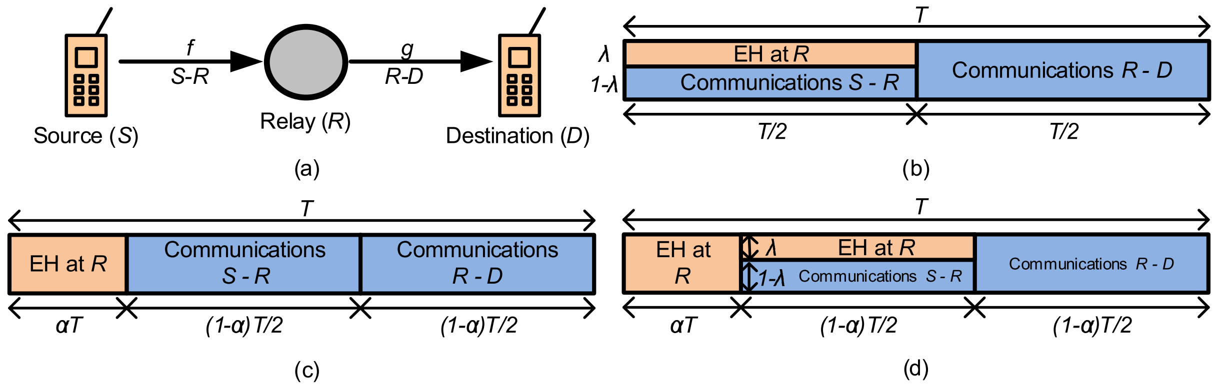


Fig. 1. Traditional single-relay network with source (S), relay (R) and destination (D), and three different EH protocols.

we derive optimal PS and TS protocols separately for each relaying schemes as well.

The rest of this paper is organized as follows. Section II discusses the system model. Section III and Section IV analyze optimal EH protocols for DF relaying and AF relaying, respectively. Section V presents numerical and simulation results, followed by concluding remarks in Section VI. Related proofs are provided in the Appendix.

II. System Model

Consider a wireless relay network with a source (S) sending information to its destination (D) via a relay (R), as shown in Fig. 1(a).

Irrespective of relay and destination locations, we assume that the processing power required for the relaying circuitry is negligible compared to the power used for signal transmission from the relay to the destination. Each node has a single antenna. The power budget is P for the source and Q for the relay. The fading coefficients from the source to the relay channel ($S-R$) and from the relay to the destination channel ($R-D$) are denoted by f and g, respectively. All channels are assumed to be independent. However, this assumption does not affect the analysis in Section III and Section IV because our analysis is based on instantaneous channel conditions. The path losses of $S-R$ and $R-D$ are denoted Ω_f and Ω_g, respectively, which include effects of the carrier frequency, antenna heights, and distances between nodes. For numerical examples in Section V, we use the Lee's area-to-area model which is used to predict a path loss over flat terrain [33]. There is no direct link between the source and the destination. All channels are assumed to be independent. However, this assumption does not effect the analysis in Sections III and IV because our analysis is based on instantaneous channel conditions. Communications take place in half-duplex mode. The transmission block period is T. In contrast to the two-step communications protocol of the traditional relay network, different protocols for EH relay networks, namely i) power-splitting (PS), ii) time-switching (TS), and iii) hybrid, are shown in Fig. 1. Their operations are explained below.

PS protocol follows two steps (which are also shown in Fig. 1(b)):

- *Step 1:* the source communicates with the relay over $T/2$ time duration. A fraction ($\lambda \in [0,\ 1]$) of the received signal is for EH at the relay, and the remaining fraction $(1-\lambda)$ of the received signal is for $S-R$ communications.
- *Step 2:* the relay communicates with the destination over $T/2$ time duration.

TS protocol follows three steps (which are also shown in Fig. 1(c)):

- *Step 1:* the relay harvests energy from the source's RF signals for αT time duration, where $0 \leq \alpha \leq 1$;
- *Step 2:* the source communicates with the relay over $(1-\alpha)T/2$ time duration;
- *Step 3:* the relay communicates with the destination over $(1-\alpha)T/2$ time duration.

Hybrid protocol which is proposed in this paper follows three steps (which are also shown in Fig. 1(d)):

- *Step 1:* the relay harvests energy from the source's RF signals for αT time duration, where $0 \leq \alpha \leq 1$;
- *Step 2:* the source communicates with the relay over $(1-\alpha)T/2$ time duration. A fraction λ, where $0 \leq \lambda \leq 1$, of the received signal is also for EH at the relay, and the remaining fraction $(1-\lambda)$ of the received signal is for $S-R$ communications.
- *Step 3:* the relay communicates with the destination over $(1-\alpha)T/2$ time duration.

When $\alpha = 0$ or $\lambda = 0$, the hybrid protocol is equivalent to the PS or TS protocol, respectively. Thus, the following analytical model is developed based on the hybrid protocol.

A. Analytical Model

Denote the information symbol of the source as s, which has unit average energy. The received signal at the relay can be written as $y_r = \sqrt{\frac{P}{\Omega_f}} fs + n_{r,a} + n_{r,c}$, where $n_{r,a}$ and $n_{r,c}$ are additive noises at the relay antenna and at the down-converter, respectively, which are assumed to be independent and identically distributed (i.i.d.) complex Gaussian random variables with zero-mean, and $N_{r,a}$ and $N_{r,c}$ variances, i.e., $n_{r,a} \sim \mathcal{CN}(0, N_{r,a})$ and $n_{r,c} \sim \mathcal{CN}(0, N_{r,c})$.

For EH, we neglect energy associated with received signal noise. There are two EH models: i) a linear model which applies when the received powers at the energy harvester is constant [3]; and ii) a non-linear model which applies when the received power at the energy harvester is dynamic [34]. We assume that the total harvested energy at the relay is linearly and directly proportional to the received RF power. Thus, the energy conversion efficiency η is independent of the input power level at the relay, and we can use the conventional linear EH model. The relay harvests energy i) for αT time duration using the RF signal y_r, and ii) for $(1-\alpha)T/2$ time duration using the fraction λ of the RF signal power, i.e., $\sqrt{\lambda} y_r$, with the rectification efficiency η, $0 < \eta \leq 1$. Thus, the harvested energy at the relay is $E = \eta P \alpha T \frac{|f|^2}{\Omega_f} + \eta \lambda P \frac{|f|^2}{\Omega_f} \frac{(1-\alpha)T}{2} = \eta P \frac{|f|^2}{\Omega_f} \frac{[2\alpha+\lambda(1-\alpha)]T}{2}$. We assume that perfect channel state information (CSI) is available only at the relay which decides λ and/or α values. Estimation of CSI, calculation of optimal α and/or λ and feedback processes consume some power at the relay which may be a constant power level irrespective of other network parameters. However, it is reasonable to assume that the processing power required for the relaying circuitry is negligible compared to the power used for signal transmission from the relay to the destination.

By assuming that the relay utilizes all harvested energy E during the third step, the average transmit power of the relay can be given as $Q = \frac{E}{(1-\alpha)T/2}$, which can be written as[1]

$$Q = \eta P \frac{|f|^2}{\Omega_f} \left(\frac{2\alpha}{1-\alpha} + \lambda \right). \tag{1}$$

For data transmission, the received signal at the relay can be written as

$$\hat{y}_r = \sqrt{1-\lambda} \left(\sqrt{\frac{P}{\Omega_f}} f s + n_{r,a} \right) + n_{r,c}, \tag{2}$$

which has $(1-\lambda) P \frac{|f|^2}{\Omega_f}$ amount of signal power and $(1-\lambda) N_{r,a} + N_{r,c}$ amount of noise power. Thus, the receive signal-to-noise ratio (SNR) at the relay can be given as

$$\gamma_1(\lambda) = \frac{P}{N_{r,a} + N_{r,c}} \frac{|f|^2}{\Omega_f} \frac{\left(1 + \frac{N_{r,c}}{N_{r,a}}\right)(1-\lambda)}{\left(1 + \frac{N_{r,c}}{N_{r,a}}\right) - \lambda}. \tag{3}$$

This is valid for $\lambda \in [0, 1]$, and we have $\gamma_1(0) = \frac{P}{N_{r,a}+N_{r,c}} \frac{|f|^2}{\Omega_f}$ and $\gamma_1(1) = 0$.

B. DF Relaying

We assume that the relay decodes the source information correctly. The received signal at the destination is given as $y_d = \sqrt{\frac{Q}{\Omega_g}} g \hat{s} + n_{d,a} + n_{d,c}$ where $\hat{s}$ which has unit average energy is the re-encoded signal of s, and $n_{d,a}$ and $n_{d,c}$ are the additive noise at the destination antenna and at the down-converter, respectively, with $n_{d,a} \sim \mathcal{CN}(0, N_{d,a})$ and $n_{d,c} \sim \mathcal{CN}(0, N_{d,c})$. This has $\eta P \frac{|f|^2}{\Omega_f} \frac{|g|^2}{\Omega_g} \left(\frac{2\alpha}{1-\alpha} + \lambda \right)$ signal power and $N_{d,a} + N_{d,c}$ noise power. Thus, the receive SNR at the destination can be given as

$$\gamma_2(\lambda, \alpha) = \eta \frac{P}{N_{d,a} + N_{d,c}} \frac{|f|^2}{\Omega_f} \frac{|g|^2}{\Omega_g} \left(\frac{2\alpha}{1-\alpha} + \lambda \right). \tag{4}$$

This is valid for $\lambda \in [0, 1]$. However, $\lim_{\alpha \to 1} \gamma_2(\lambda, \alpha) \to \infty$ which is not realistic because there is no time for information transfer via $R - D$ when $\alpha \to 1$.

C. AF Relaying

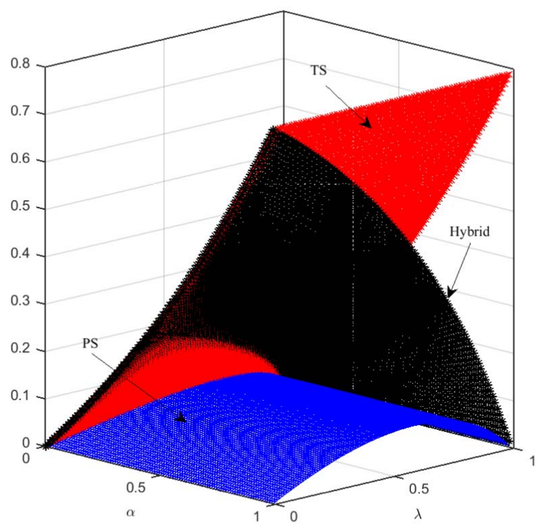
By assuming that the relay has knowledge of instantaneous CSI f, the coherent power coefficient of the AF relay is $W = \frac{1}{N_{r,c} + (1-\lambda)(N_{r,a} + P\frac{|f|^2}{\Omega_f})}$ [35]. Thus, the received signal at the destination is $y_d = \sqrt{\frac{WQ}{\Omega_g}} \hat{y}_r g + n_{d,a} + n_{d,c}$ where $\hat{y}_r$ is given in (2). This has signal power $WQ(1-\lambda) P \frac{|f|^2}{\Omega_f} \frac{|g|^2}{\Omega_g}$ and noise power $WQ \frac{|g|^2}{\Omega_g} [(1-\lambda) N_{r,a} + N_{r,c}] + (N_{d,a} + N_{d,c})$. Thus, the receive SNR at the destination can be simplified as

$$\gamma_d(\lambda, \alpha) = \frac{\gamma_1(\lambda) \gamma_2(\lambda, \alpha)}{\gamma_1(\lambda) + \gamma_2(\lambda, \alpha) + 1}. \tag{5}$$

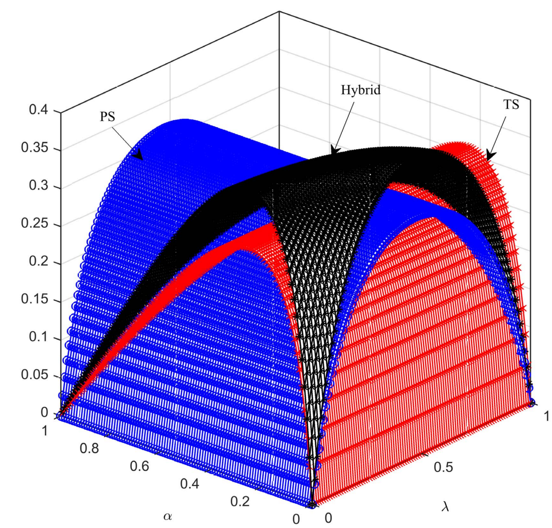
This is valid for $\lambda \in [0, 1]$ and $\alpha \in [0, 1]$. At extreme values of λ and α, we have: i) For PS protocol ($\alpha = 0$), $\lim_{\lambda \to 0} \gamma_d(\lambda, 0) = \lim_{\lambda \to 1} \gamma_d(\lambda, 0) = 0$; ii) For TS protocol ($\lambda = 0$), $\lim_{\alpha \to 0} \gamma_d(0, \alpha) = 0$ and $\lim_{\alpha \to 1} \gamma_d(0, \alpha) = \frac{P}{N_{r,a}+N_{r,c}} \frac{|f|^2}{d_f^m}$; and iii) For hybrid protocol, $\lim_{(\lambda,\alpha)\to(0,0)} \gamma_d(\lambda, \alpha) = \lim_{(\lambda,\alpha)\to(1,1)} \gamma_d(\lambda, \alpha) = 0$.

For both relaying schemes, performance metrics such as SNR, SNR outage (an outage occurs if the received SNR drops below a predetermined SNR threshold) or throughput depends on λ and α. For example, we consider behaviors of SNR, $\gamma_d(\lambda, \alpha)$, and throughput, $\tau = (1-\alpha) \log[1 + \gamma_d(\lambda, \alpha)]/2$, of AF relaying, and illustrate these performance metrics in Fig. 2 when $\frac{P}{N_{r,a}+N_{r,c}} = 10\,\mathrm{dB}$, $\frac{|f|^2}{\Omega_f} = \frac{|g|^2}{\Omega_g} = 0.5$, $\frac{N_{r,c}}{N_{r,a}} = 1$ and $\eta = 0.9$. Note that the purpose of Fig. 2 is only to show the monotonic behaviors of SNR and throughput expressions, but not to show the performance limit. Therefore, we remove values in the vertical axis. As the example in Fig. 2: i) For PS protocol, when λ varies from 0 to 1, SNR or throughput has a maximum in $\lambda \in [0, 1]$; ii) For TS protocol, when α varies from 0 to 1, SNR monotonically increases from 0 to its maximum, but throughput has a maximum in $\alpha \in [0, 1]$; and iii) For hybrid protocol, when $\alpha \to 1$ and $\lambda \to 0$, while SNR approaches its maximum, throughput approaches to 0, but throughput has a maximum in $\lambda \in (0, 1)$ and $\alpha \in (0, 1)$. Therefore, maximizing SNR does not maximize the throughput. Since we focus on information transfer, it is reasonable to consider the throughput rather than the SNR. In general, maximizing SNR is not a useful way to select λ or α. Each protocol has a maximum throughput for a particular $\lambda = \lambda^*$, $\alpha = \alpha^*$ or $(\alpha, \lambda) = (\alpha^*, \lambda^*)$ pair for PS, TS or hybrid protocol, respectively. We define the *optimal EH protocol* for each protocol when any particular protocol maximizes the throughput. For example, the TS protocol can operate at any α. If we can design α in order to achieve maximum throughput at $\alpha = \alpha^*$, we call that

[1]If the the power efficiency of the power amplifier is ϑ, $0 < \vartheta \leq 1$, the average transmit power of the relay may be given as $Q = \vartheta \eta P \frac{|f|^2}{\Omega_f} \left(\frac{2\alpha}{1-\alpha} + \lambda \right)$. Since ϑ is another proportional term, one can write $\xi = \vartheta \eta$, $0 < \xi \leq 1$. Thus, we can re-write the average transmit power of the relay as $Q = \xi P \frac{|f|^2}{\Omega_f} \left(\frac{2\alpha}{1-\alpha} + \lambda \right)$ which is similar to (1).



(a)



(b)

Fig. 2. For all three EH protocols PS, TS and hybrid of AF relaying: (a) variation of SNR versus TS ratio α and PS ratio λ; and (b) variation of throughput versus TS ratio α and PS ratio λ.

particular TS protocol with that specific α^* as the optimal TS EH protocol. Similar definitions are applied for PS and hybrid protocols. Section III and Section IV discuss how optimal EH protocols can be implemented for DF relaying and AF relaying, respectively.

D. Notations

For the presentation simplicity with few number of parameters, we use convenient definitions as described follows. As our main focus is on deriving optimal λ and α values, we mainly focus on these two parameters. Since we do not consider noise estimation errors or noise uncertainty, it is reasonable to assume that total effective noise powers at relay and destination are the same [36], i.e., $N_{r,c} + N_{r,a} = N_{d,c} + N_{d,a}$. However, it is not necessary to have the same variance for each noise term, i.e., $N_{r,c}$, $N_{r,a}$, $N_{d,c}$ or $N_{d,a}$. Thus, we define the ratio between down-converter and antenna noise variances as $\sigma \triangleq \frac{N_{r,c}}{N_{r,a}}$ where $\sigma > 0$. Since P is the average transmit power, we name $\rho = \frac{P}{N_{r,c}+N_{r,a}} = \frac{P}{N_{d,c}+N_{d,a}}$ as the *average SNR* with respect to transmit power and effective noise power at the receiver. Further, $S-R$ and $R-D$ channel effects including path loss are denoted as $\gamma_f = \frac{|f|^2}{\Omega_f}$ and $\gamma_g = \frac{|g|^2}{\Omega_g}$, respectively. We use two definitions for parameters a and b such as $a \triangleq \rho\gamma_f$ and $b \triangleq \eta\gamma_g$ where $a, b > 0$. Now we can write (3)-(5) in compact forms as follows:

$$\gamma_1(\lambda) = (1+\sigma)a\left(\frac{1-\lambda}{1+\sigma-\lambda}\right), \tag{6}$$

$$\gamma_2(\lambda,\alpha) = ab\left(\frac{2\alpha}{1-\alpha}+\lambda\right), \tag{7}$$

$$\gamma_d(\lambda,\alpha) = \frac{(1+\sigma)a^2b\left(\frac{2\alpha}{1-\alpha}+\lambda\right)\left(\frac{1-\lambda}{1+\sigma-\lambda}\right)}{ab\left(\frac{2\alpha}{1-\alpha}+\lambda\right)+(1+\sigma)a\left(\frac{1-\lambda}{1+\sigma-\lambda}\right)+1}. \tag{8}$$

III. Optimal EH Protocols for DF Relaying

In this section, we provide an analytical framework to find the optimal PS, TS and hybrid EH protocols by evaluating α, λ and (α, λ) pair in order to maximize the throughput.

With substitutions $x = 1+\sigma-\lambda$ and $y = 1-\alpha$, we can write SNRs in (6) and (7) as

$$\gamma_1(x) = (1+\sigma)a - \frac{(1+\sigma)\sigma a}{x}$$

and

$$\gamma_2(x,y) = \frac{2ab}{y} - abx - ab(1-\sigma), \tag{9}$$

respectively, where $x \in [\sigma, 1+\sigma]$ and $y \in [0,1]$. Using these receive SNRs, throughput of $S-R$ and $R-D$ links can be given as

$$\tau_1(x,y) = \frac{y}{2}\log\left[1+\gamma_1(x)\right]$$

and

$$\tau_2(x,y) = \frac{y}{2}\log\left[1+\gamma_2(x,y)\right], \tag{10}$$

respectively. For DF relaying, the effective throughput $\tau_{df}(x,y)$ is the minimum of $\tau_1(x,y)$ and $\tau_2(x,y)$, which can be given as

$$\tau_{df}(x,y) = \min\{\tau_1(x,y), \tau_2(x,y)\}. \tag{11}$$

Note that $\min\{i,j\}$ or $\max\{i,j\}$ gives the minimum or the maximum value of i and j, respectively. In the following, we analyze three EH protocols separately.

A. PS Protocol in DF Relaying

We have PS protocol when $\alpha = 0$, i.e., $y = 1$. In this case, we find optimal PS ratio, λ_{df}^{ps}, by using the optimal x which can be given as

$$x_{df}^{ps} = \arg\max_{x\in[\sigma,1+\sigma]} \min\{\tau_1(x,1), \tau_2(x,1)\} \tag{12}$$

where $\tau_1(x,1) = \frac{1}{2}\log\left[1+(1+\sigma)a - \frac{(1+\sigma)a}{x}\right]$ and $\tau_2(x,1) = \frac{1}{2}\log\left[1+(1+\sigma)ab - abx\right]$. When x varies from σ to $1+\sigma$, $\tau_1(x,1)$ monotonically increases from 0 to $\frac{1}{2}\log(1+a)$, and $\tau_2(x,1)$ monotonically decreases from $\frac{1}{2}\log(1+ab)$ to 0. Thus, optimal value can be calculated when $\tau_1(x,1) = \tau_2(x,1)$, which can be given as

$$x_{df}^{ps} = \frac{(b-1)(1+\sigma) + \sqrt{1+\sigma}\sqrt{(b+1)^2\sigma + (b-1)^2}}{2b}, \tag{13}$$

then $\lambda_{df}^{ps} = 1+\sigma - x_{df}^{ps}$. To calculate λ_{df}^{ps}, the relay needs only the $R-D$ channel knowledge, i.e., $|g|$ and Ω_g. It is important to note that λ_{df}^{ps} is designed in order to achieve the same throughput capability in both $S-R$ and $R-D$ links, i.e., $\tau_1(x_{df}^{ps},1) = \tau_2(x_{df}^{ps},1)$. Thus, the corresponding optimal throughput can be given as

$$\tau_{df}^{ps} = \frac{1}{2}\log\Bigg[1+\frac{1}{2}a[(b+1)(n+1) - \sqrt{n+1}\sqrt{(b+1)^2 n + (b-1)^2}]\Bigg]. \tag{14}$$

B. TS Protocol in DF Relaying

We have TS protocol when $\lambda = 0$, i.e., $x = 1+\sigma$. In this case, we find optimal TS ratio, α_{df}^{ts}, by using optimal y which can be given as

$$y_{df}^{ts} = \arg\max_{y\in[0,1]} \min\{\tau_1(1+\sigma,y), \tau_2(1+\sigma,y)\} \tag{15}$$

where $\tau_1(1+\sigma,y) = \frac{y}{2}\log[1+a]$ and $\tau_2(1+\sigma,y) = \frac{y}{2}\log\left[1-2ab+\frac{2ab}{y}\right]$. We can completely solve this problem for y_{df}^{ts} as follows. The detailed analytical discussion and proof are given in Appendix A. Then, the optimal value can be calculated as

$$y_{df}^{ts} = \begin{cases} y_1 \triangleq \dfrac{2b}{2b+1} : \{\mathcal{C}_1 < 0 \text{ and } \mathcal{C}_2 > 0\} \text{ or} \\ \quad \{\mathcal{C}_1 > 0 \text{ and } \mathcal{C}_2 < 0\} \text{ or } \{\mathcal{C}_1 = 0 \text{ and } \mathcal{C}_3 \geq 0\} \\ y_2 \triangleq \dfrac{1}{e} : \ \{\mathcal{C}_1 = 0 \text{ and } \mathcal{C}_3 < 0\} \\ y_3 \triangleq \dfrac{2ab\mathcal{W}\left(\frac{2ab-1}{e}\right)}{(2ab-1)\left(1+\mathcal{W}\left(\frac{2ab-1}{e}\right)\right)} : \text{otherwise.} \end{cases} \tag{16}$$

Further, in this case, we need to consider three terms $\mathcal{C}_1 - \mathcal{C}_3$ which are defined as:

$$\mathcal{C}_1 \triangleq 2ab-1; \quad \mathcal{C}_2 \triangleq \mathcal{W}\left(\frac{2ab-1}{e}\right) - \frac{2ab-1}{a+1}$$

and

$$\mathcal{C}_3 \triangleq b - \frac{1}{2(e-1)} \tag{17}$$

where $\mathcal{W}(\cdot)$ is the LambertW function [37]. Then, $\alpha_{df}^{ts} = 1 - y_{df}^{ts}$. To calculate α_{df}^{ts}, the relay needs knowledge of both $S-R$ and $R-D$ channels such as $|f|$, Ω_f, $|g|$ and Ω_g.

Thus, the corresponding optimal throughput can be given as

$$\tau_{df}^{ts} = \begin{cases} \tau(y_1) = \dfrac{b\log(1+a)}{2b+1} : \{\mathcal{C}_1 < 0 \text{ and } \mathcal{C}_2 > 0\} \text{ or} \\ \quad \{\mathcal{C}_1 > 0 \text{ and } \mathcal{C}_2 < 0\} \text{ or } \{\mathcal{C}_1 = 0 \text{ and } \mathcal{C}_3 \geq 0\} \\ \tau(y_2) = \dfrac{\log(1+2(e-1)ab)}{2e} : \{\mathcal{C}_1 = 0 \text{ and } \mathcal{C}_3 < 0\} \\ \tau(y_3) = \dfrac{abW\left(\frac{2ab-1}{e}\right)}{2ab-1} : \text{otherwise.} \end{cases} \tag{18}$$

It is important to note that i) when $y_{df}^{ts} = y_1$, we have $\tau_{df}^{ts} = \tau_1(1+\sigma, y_{df}^{ts}) = \tau_2(1+\sigma, y_{df}^{ts})$; and ii) otherwise (i,e,. $y_{df}^{ts} = y_2$ or $y_{df}^{ts} = y_3$), we have $\tau_1(1+\sigma, y_1) > \tau_2(1+\sigma, y_1)$, and thus $\tau_{df}^{ts} = \tau_2(1+\sigma, y_{df}^{ts})$. Thus, we may not have the same throughput in both $S-R$ and $R-D$.

C. Hybrid Protocol in DF Relaying

An optimal hybrid protocol can be achieved by maximizing the network throughput in (11), denoted as the optimal throughput τ^*, which can be given as

$$\tau^* = \max_{x\in[\sigma,1+\sigma], y\in[0,1]} \min\{\tau_1(x,y), \tau_2(x,y)\}.$$

Thus, optimal values of x and y are

$$(x_{df}^*, y_{df}^*) = \arg\max_{\substack{x\in[\sigma,1+\sigma] \\ y\in[0,1]}} \min\{\tau_1(x,y), \tau_2(x,y)\}. \tag{19}$$

This is a more complicated problem than previous two cases as we have two parameters, x and y, to be optimized. We can solve this problem for x_{df}^* and y_{df}^* semi-analytically as follows. The detailed analytical discussion and proof are given in Appendix B. In this case, the relay needs knowledge of $|f|$, Ω_f, $|g|$ and Ω_g. Then, we can develop the following terms $\mathcal{C}_4$, $\mathcal{X}(\mathcal{Y})$, $G(\mathcal{Y})$ and $\mathcal{J}(\mathcal{Y})$:

$$\begin{aligned} \mathcal{C}_4 &\triangleq \log\left[\frac{1}{2}(2+a(1+\sigma+b(1+\sigma)-\kappa))\right] \\ &\quad + \frac{2ab(1+\sigma-b(1+\sigma)+\kappa)}{\kappa(a(\kappa-(1+\sigma)(1+b))-2)} \\ \mathcal{X}(\mathcal{Y}) &\triangleq \frac{b\left[\frac{2}{\mathcal{Y}}-(1-\sigma)\right]-(\sigma+1)+G(\mathcal{Y})}{2b}, \\ G(\mathcal{Y}) &\triangleq \sqrt{\left[b\left(\frac{2}{\mathcal{Y}}+(\sigma-1)\right)+(\sigma+1)\right]^2+4b\sigma(\sigma+1)}, \\ \mathcal{J}(\mathcal{Y}) &\triangleq \log\left[1+\frac{a}{2}[1+\sigma-b(1-\sigma)+\frac{2b}{\mathcal{Y}}-G(\mathcal{Y})]\right] \\ &\quad - \frac{2ab(\mathcal{Y}G(\mathcal{Y})+\mathcal{Y}(1+\sigma+b(1-\sigma))-2b)}{\mathcal{Y}G(\mathcal{Y})(2ab+(2+a(1+\sigma+b(1-\sigma)))\mathcal{Y}-a\mathcal{Y}G(\mathcal{Y}))}, \end{aligned} \tag{20}$$

where $\kappa = \sqrt{(n+1)\left[(b+1)^2\sigma+(b-1)^2\right]}$. Further, $\mathcal{Y}^*$ is the solution of $\mathcal{J}(\mathcal{Y}) = 0$ for $\mathcal{Y} > 0$, and $\mathcal{X}^* = \mathcal{X}(\mathcal{Y}^*)$.

Then, we have optimal values for x and y as

$$(x_{df}^*, y_{df}^*) = \begin{cases} (x_{ps}^*, 1) : \mathcal{C}_4 \geq 0 \\ (\mathcal{X}^*, \mathcal{Y}^*) : \mathcal{C}_4 < 0. \end{cases} \tag{21}$$

This is a semi-analytical solution because $\mathcal{Y}^*$ which is the solution of $\mathcal{J}(\mathcal{Y}) = 0$ may be difficult to solve in a closed form, but can be easily calculated numerically. Further, the corresponding optimal throughput can be given as

$$\tau^* = \begin{cases} \tau_{ps}^* : \mathcal{C}_4 \geq 0 \\ \min\left[\tau_1(\mathcal{X}^*, \mathcal{Y}^*); \tau_2(\mathcal{X}^*, \mathcal{Y}^*)\right] : \mathcal{C}_4 < 0. \end{cases} \tag{22}$$

It is important to note that the hybrid protocol reduces to PS protocol when $\mathcal{C}_4 \geq 0$. However it is not necessary to reduce to TS protocol because the hybrid protocol may adjust x and y in order to get $\tau_1(x, y) = \tau_2(x, y)$ for any a and b. This phenomena will be further discussed in Section V with numerical examples.

IV. Optimal EH Protocols for AF Relaying

In this section, we provide analysis to obtain optimal EH protocols under AF relaying. Similarly to Section III, with substitutions $x = 1 + \sigma - \lambda$ and $y = 1 - \alpha$, we can re-write the end SNR in (8) as

$$\gamma_d(x, y) = \frac{(1+\sigma)a^2 b(\frac{2}{y} - x - 1 + \sigma)(1 - \frac{\sigma}{x})}{ab(\frac{2}{y} - x - 1 + \sigma) + (1+\sigma)a\left(1 - \frac{\sigma}{x}\right) + 1} \tag{23}$$

where $x \in [\sigma, 1+\sigma]$ and $y \in [0, 1]$. Then, for given x and y, the effective throughput for AF relaying can be given as

$$\tau_{af}(x, y) = \frac{y}{2}\log\left[1 + \gamma_d(x, y)\right]. \tag{24}$$

A. PS Protocol in AF Relaying

When $y = 1$, we can find the optimal value for x as

$$x_{af}^{ps} = \arg\max_{x\in[\sigma, 1+\sigma]} \gamma_d(x, 1)$$

where $\gamma_d(x, 1) = \frac{(1+\sigma)a^2 b(1+\sigma-x))(1-\frac{\sigma}{x})}{ab(1+\sigma-x)+(1+\sigma)a(1-\frac{\sigma}{x})+1}$. Since we have $\gamma_d(\sigma, 1) = \gamma_d(1+\sigma, 1) = 0$ and $\gamma_d(x, 1)$ is positive for $x \in (\sigma, 1+\sigma)$, there may be local maximum(s) in the interval $x \in (\sigma, 1+\sigma)$ as there is no discontinuity in $x \in (\sigma, 1+\sigma)$. Such maximum(s) may be evaluated by $\frac{\partial \gamma_d(x,1)}{\partial x} = 0$. Then, we have

$$[\sigma ab - (1+\sigma)a - 1]x^2 + 2\sigma(1+\sigma)a(1-b)x + \sigma(1+\sigma)[(1+\sigma)ab - \sigma a + 1] = 0.$$

Since this is a quadratic equation, there cannot be more than two real roots for x. This concludes that $\gamma_d(x, 1)$ has only one local maximum in the interval $x \in [\sigma, 1+\sigma]$,[2] which can be given as

$$x_{af}^{ps} = \begin{cases} x_1 = \sqrt{\frac{(a+1)\sigma(\sigma+1)(ab+1)}{(a(-b)\sigma+a\sigma+a+1)^2}} - \frac{a(b-1)\sigma(\sigma+1)}{a(1-(b-1)\sigma)+1} : \\ \qquad \{0 < b \leq 1 \text{ and } 0 < \sigma \text{ and } a > 0\} \text{ or} \\ \qquad \{b > 1 \text{ and } 0 < \sigma \leq \frac{1}{b-1} \text{ and } a > 0\} \text{ or} \\ \qquad \{b > 1 \text{ and } \sigma > \frac{1}{b-1} \text{ and } 0 < a < \frac{1}{(b-1)\sigma-1}\} \\ x_2 = -\frac{a(b-1)\sigma(\sigma+1)}{a(1-(b-1)\sigma)+1} - \sqrt{\frac{(a+1)\sigma(\sigma+1)(ab+1)}{(a(-b)\sigma+a\sigma+a+1)^2}} : \\ \qquad \{b > 1 \text{ and } \sigma > \frac{1}{b-1} \text{ and } a > \frac{1}{(b-1)\sigma-1}\} \\ x_3 = \frac{ab+1}{2a(b-1)} + \frac{\sigma}{2} : \\ \qquad \{b > 1 \text{ and } \sigma > \frac{1}{b-1} \text{ and } a = \frac{1}{(b-1)\sigma-1}\}. \end{cases} \tag{25}$$

Thus, optimal power splitting ratio for AF relaying can be calculated as $\lambda_{af}^{ps} = 1 + \sigma - x_{af}^{ps}$. Unlike the PS protocol under DF relaying, in this case, we need both channel knowledge at the relay because it has the coherent power coefficient. Then, we can calculate the corresponding optimal throughput by using (24) as $\tau_{af}^{ps} = \tau_{af}(x_i, 1)$ given in (25), where $i = 1, 2$ or 3.

B. TS Protocol in AF Relaying

When $x = 1 + \sigma$, we can find the optimal value for y as $y_{af}^{ts} = \arg\max_{y\in[0,1]} \tau_{af}(1+\sigma, y)$ where $\tau_{af}(1+\sigma, y) = \frac{y}{2}\log\left[1 + \frac{2a^2 b\left(\frac{1}{y}-1\right)}{2ab\left(\frac{1}{y}-1\right)+a+1}\right]$. For $a, b > 0$, we have $\tau_{af}(1+\sigma, y \to 0) \to 0, \tau'_{af}(1+\sigma, 0) > 0$, $\tau''_{af}(1+\sigma, 0) < 0$ and $\tau_{af}(1+\sigma, 1) = 0$, $\tau'_{af}(1+\sigma, 1) < 0$, $\tau''_{af}(1+\sigma, 1) < 0$. Further, $y \in (0, 1)$, we have $\tau''_{af}(1+\sigma, y)$ which satisfies

$$\frac{2a^3 b^2(a(4b(y-1) - y) - 2y)}{(y - 2ab(y-1))^2(a(y - 2b(y-1)) + y)^2} < 0 \tag{26}$$

as $a(4b(y-1) - y) - 2y < 0$. Thus $\tau_{af}(1+\sigma, y)$ has a maximum in $y \in [0, 1]$. This problem can easily be solved with standard optimization tools. In order to provide an analytical solution, we may consider $\tau'_{af}(1+\sigma, y) = 0$, which satisfies

$$\log\left[1 + \frac{2a^2 b(y-1)}{(a(2b(y-1) - y) - y)}\right] = \frac{2a^2 by}{(a(2b(y-1) - y) - y)(2ab(y-1) - y)}. \tag{27}$$

Due to the complexity of the involved expression, deriving a closed-form (or a general) solution for the optimization problem may be difficult. But, the optimal values can be calculated numerically.

In order to simplify the optimization problem, and be able to provide an analytical solutions for optimal values, we consider a tight upper bound for $\tau_{af}(1+\sigma, y)$ using the fact that $\frac{wz}{w+z+1} \leq \min(w, z)$; $\forall w, z \geq 0$. This approximation has

[2] In this case, it may be difficult to show that $\frac{\partial^2 \gamma_d(x,1)}{\partial x^2} < 0$ for $x \in [\sigma, 1+\sigma]$.

been widely used in performance analysis in relay networks. In this case, we have an upper bound: $\tau_{af}(1+\sigma, y) \leq \frac{y}{2}\log\left[1+a\min\left(1, b\left(\frac{2}{y}-2\right)\right)\right]$.[3] An approximation for y_{af}^{ts} can be found by maximizing this upper bound, i.e.,

$$y_{af}^{ts} \approx \arg\max_{y\in[0,1]} \frac{y}{2}\log\left[1+a\min\left(1, b\left(\frac{2}{y}-2\right)\right)\right].$$

1) If $1 \leq b\left(\frac{2}{y}-2\right)$: This means that $y \leq \frac{2b}{1+2b}$, and we find an approximation as

$$y_{af}^{ts} \approx \max_{y\in[0,2b/(1+2b)]} y\log[1+a] \Rightarrow y_{af}^{ts} \approx \frac{2b}{1+2b}.$$

2) If $1 \geq b\left(\frac{2}{y}-2\right)$: This means that $y \leq \frac{2b}{1+2b}$, and we find an approximation as

$$y_{af}^{ts} \approx \max_{y\in[2b/(1+2b),1]} y\log\left[1+ab\left(\frac{2}{y}-2\right)\right].$$

Using a similar argument used at the beginning of this section, we can easily show that there is a maximum for the function $y\log\left[1+ab\left(\frac{2}{y}-2\right)\right]$ in $y \in [0,1]$, which can be calculated by solving $\log\left[\frac{y+2b(1-y)}{y}\right] = \frac{2b}{y+2b(1-y)}$. After some manipulations, we can write this as

$$\begin{aligned}\frac{2b-1}{e} &= \frac{(2b-1)y}{y+2b(1-y)}e^{\frac{(2b-1)y}{y+2b(1-y)}}\\ y &= \frac{2b\mathcal{W}\left(\frac{2b-1}{e}\right)}{(2b-1)\left(1+\mathcal{W}\left(\frac{2b-1}{e}\right)\right)}.\end{aligned} \tag{28}$$

Combing these two possibilities, we can conclude that

$$y_{af}^{ts} \approx \begin{cases}\dfrac{2b}{1+2b} : 1 \geq 2b-2\mathcal{W}\left(\dfrac{2b-1}{e}\right)\\ \dfrac{2b\mathcal{W}\left(\frac{2b-1}{e}\right)}{(2b-1)\left(1+\mathcal{W}\left(\frac{2b-1}{e}\right)\right)} : \text{otherwise}\end{cases} \tag{29}$$

Accuracy of this approximation is shown with numerical examples in Section V.

C. Hybrid Protocol in AF Relaying

An optimal hybrid protocol for an AF relay can be found by maximizing the throughput in (24). Thus optimal values of x and y can be calculated as

$$(x_{af}^*, y_{af}^*) = \arg\max_{x\in[\sigma,1+\sigma];\ y\in[0,1]} \frac{y}{2}\log\left[1+\gamma_d(x,y)\right]. \tag{30}$$

Due to the complexity of the involved expression, deriving a closed-form (or a general) solution of the optimization problem may be difficult. But, the optimal values can be calculated numerically as we discussed in Section IV-B. Thus, we omit the detailed discussion.

In order to simplify the optimization problem and be able to provide analytical solutions for optimal values, we can consider the same upper bound used in Section IV-B, i.e., From (5) or (8), we have $\frac{\gamma_1(x)\gamma_2(x,y)}{\gamma_1(x)+\gamma_2(x,y)+1} \leq \min(\gamma_1(x), \gamma_2(x,y))$

[3]The similar technique is also used in [25]. Unlike this paper, [25] assumes that the relay node has no energy supply and harvests energy from the surrounding (solar, vibration etc).

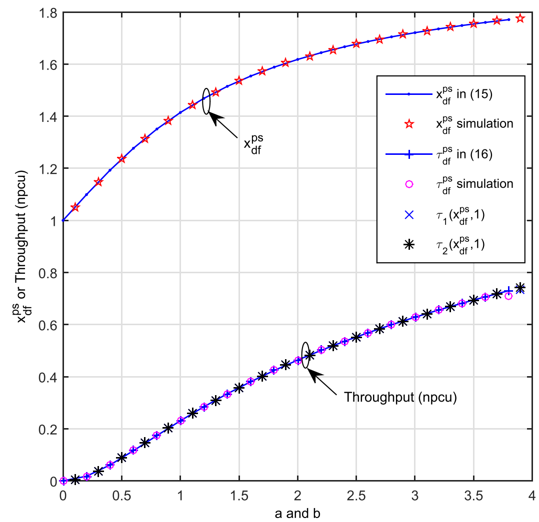


Fig. 3. Variation of throughput and PS ratio versus a and b for the PS Protocol in DF Relaying (Section III-A).

where $\gamma_1(x)$ and $\gamma_2(x,y)$ are given in (9). Approximations for optimal values of x and y can be calculated as

$$(x_{af}^*, y_{af}^*) \approx \arg\max_{\substack{x\in[\sigma,1+\sigma]\\ y\in[0,1]}} y\log\left[1+\min(\gamma_1(x), \gamma_2(x,y))\right]. \tag{31}$$

1) When $\gamma_2(x,y) \leq \gamma_1(x)$: In this case, we can find an approximation by solving

$$(x_{af}^*, y_{af}^*) \approx \arg\max_{\substack{x\in[\sigma\,1+\sigma]\\ y\in[0,1]}} y\log\left[1+\frac{2ab}{y}-abx-ab(1-\sigma)\right].$$

This is exactly same to the case we considered in Section III-C, and details are also given in Appendix B1.

2) When $\gamma_1(x) \leq \gamma_2(x,y)$: In this case, we can find an approximation by solving

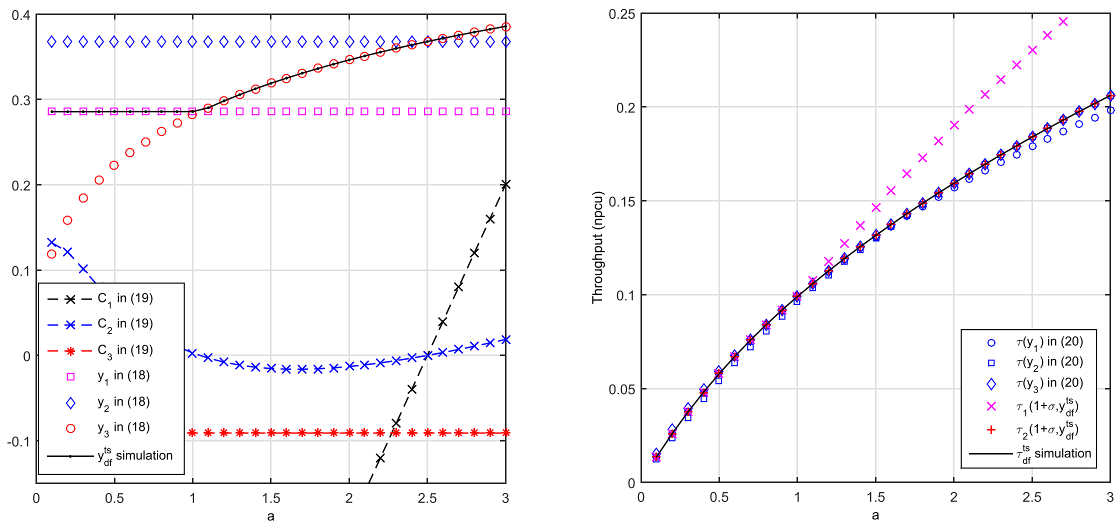
$$(x_{af}^*, y_{af}^*) \approx \arg\max_{\substack{x\in[\sigma\,1+\sigma]\\ y\in[0,1]}} y\log\left[1+(1+\sigma)a-\frac{(1+\sigma)\sigma a}{x}\right].$$

This is exactly same to the case we considered in Section III-C, and details are also given in Appendix B2.

It is important to note that optimal values of x and y in the hybrid protocol with DF relaying are approximated optimal values of x and y in hybrid protocol with AF relaying. However, the accuracy of these approximations depends on $S-R$ and $R-D$ channel conditions, which will be discussed with numerical examples in Section V.

V. Numerical and Simulation Results

This section provides numerical results based on the analysis in Section III and Section IV, and simulation results based on the system model in Section II. We derive the optimal EH protocols in terms of $a(=\frac{P}{N_{r,a}+N_{r,c}}\frac{|f|^2}{\Omega_f}$ or $\frac{P}{N_{d,a}+N_{d,c}}\frac{|f|^2}{\Omega_f})$, $b(=\eta\frac{|g|^2}{\Omega_g})$ and σ (the ratio between down-converter noise variance and antenna noise variance) which include effects of transmit power, noise



(a) (b)

Fig. 4. For the TS Protocol in DF Relaying (Section III-B), (a) variation of three terms $\mathcal{C}_1 - \mathcal{C}_3$ and TS ratio versus a when $b = 0.2$; and (b) variation of throughput versus a when $b = 0.2$.

variance, path loss, multi-path fading and rectification efficiency. Further, a, b and σ are unitless positive parameters. Numerical examples verify the analysis in Section V-A, and discuss the system performance in Section V-B. We calculate the throughput in the *nat per channel use (npcu)* as we use natural log in the analysis.

A. Validation of the Analysis

In this section, we verify analytical framework in Section III and Section IV by using Figs. 3-7. It is important to note that we may not use practical values in Figs. 3-7 because we show the variations with a and/or b when $\sigma = 1$. If we use practical values, we may not be able to discuss and verify concisely all possible conditions derived in Section III and Section IV.

1) Optimal EH Protocols With DF Relaying: In Section III-A, we analytically derived the optimal PS ratio for DF relaying ($x_{df}^{ps} = 1 + \sigma - \lambda_{df}^{ps}$). Fig. 3 shows variations of numerically calculated x_{df}^{ps} in (13) and τ_{df}^{ps} in (14); and simulated x_{df}^{ps}, τ_{df}^{ps} and individual link throughput of $S - R$ and $R - D$ links, i.e., $\tau_1(x_{df}^{ps}, 1)$ and $\tau_2(x_{df}^{ps}, 1)$ when a and b vary from 0 to 4. All analytical results exactly match with simulation results which confirm the validity of our analysis. Further, optimal PS protocol with DF relaying is achieved when throughput values of $S - R$ and $R - D$ links are equal which is also validated from Fig. 3, i.e., $\tau_{df}^{ps} = \tau_1(x_{df}^{ps}, 1) = \tau_2(x_{df}^{ps}, 1)$.

In Section III-B, we analytically derive the optimal TS ratio for DF relaying ($y_{af}^{ts} = 1 - \alpha_{af}^{ts}$), which is confirmed with a numerical example in Fig. 4. Depending on the behaviors of three terms, $\mathcal{C}_1$, $\mathcal{C}_2$ and $\mathcal{C}_3$ in (17), we may have three different y_{af}^{ts} values denoted as y_1, y_2 and y_3 in (16). Fig. 4a shows variations of numerically calculated $\mathcal{C}_1$, $\mathcal{C}_2$, $\mathcal{C}_3$ and y_1, y_2, y_3 when a varies from 0 to 3 at $b = 0.2$. Further, Fig. 4a also shows the optimal y_{af}^{ts} obtained from simulation. When $a \leq 1$, we have $\mathcal{C}_1 < 0$ and $\mathcal{C}_2 > 0$, and thus y_{af}^{ts} follows y_1. When $1 < a < 2.5$ or $2.5 < a$, we have $\mathcal{C}_1 < 0$ and $\mathcal{C}_2 < 0$ or $\mathcal{C}_1 > 0$ and $\mathcal{C}_1 > 0$, and thus y_{af}^{ts} follows y_3. When $a = 2.5$, we have $2ab = 1$ and $y_{af}^{ts} = y_2$.

For the same parameters used in Fig. 4a, the optimal throughput of the TS protocol for DF relaying (τ_{df}^{ts}) is plotted in Fig. 4b by using simulation, and the analytical result in (18). Further, individual link throughput values at optimal y_{af}^{ts} [$\tau_1(1 + \sigma, y_{af}^{ts})$ and $\tau_2(1 + \sigma, y_{af}^{ts})$] For the TS protocol, there are three possibilities for τ_{af}^{ts} as in (18), which are calculated numerically and also plotted in Fig. 4b. We can notice that i) when $a \leq 1$: $\tau_{af}^{ts} = \tau_1(1+\sigma, y_{af}^{ts}) = \tau_2(1+\sigma, y_{af}^{ts}) = \tau(y_1)$ where both links have the same throughput as the PS protocol; ii) when $a \geq 1$ and $a \neq 2.5$: $\tau_{af}^{ts} = \tau_2(1 + \sigma, y_{af}^{ts}) = \tau(y_3)$ where overall throughput depends on $R - D$ link; and iii) when $a = 2.5$: $\tau_{af}^{ts} = \tau_2(1 + \sigma, y_{af}^{ts}) = \tau(y_2)$ where overall throughput also depends on the $R - D$ link. Moreover, simulation results exactly matches with analytical results for all cases considered in Fig. 4, which confirm our analysis in Section III-B.

In Section III-C, we derive an optimal hybrid protocol for DF relaying with optimal TS and PS ratios (x_{df}^*, y_{df}^*) semi-analytically. Depending on the behavior of term $\mathcal{C}_4$ in (20), we may have two different sets of (x_{df}^*, y_{df}^*) as given in (21). Fig. 5 shows variations of numerically calculated $\mathcal{C}_4$, $\mathcal{X}^*$ and $\mathcal{Y}^*$ when a and b varies from 1 to 3. Recalling that $\mathcal{Y}^*$ is the solution for $\mathcal{J}(\mathcal{Y}) = 0$ and $\mathcal{X}^* = \mathcal{X}(\mathcal{Y}^*)$ which are given in (20). Further, Fig. 5 shows the optimal (x_{df}^*, y_{df}^*) obtained from simulations. We can notice that i) when $a, b \leq 1.8$: we have $\mathcal{C}_4 < 0$, and (x_{df}^*, y_{df}^*) follow ($\mathcal{X}^*, \mathcal{Y}^*$); and ii) when $a, b > 1.8$, we have $\mathcal{C}_4 \geq 0$, then the hybrid protocol acts as the PS protocol, and (x_{df}^*, y_{df}^*) follow ($x_{df}^{ps}, 1$). This is exactly what we discuss in Section III-C, and thus it confirms our analytical results. Unlike in the TS protocol, the hybrid protocol designs x_{df}^* and y_{df}^*) in order to achieve the same link throughput values over the simulated region, i.e.,

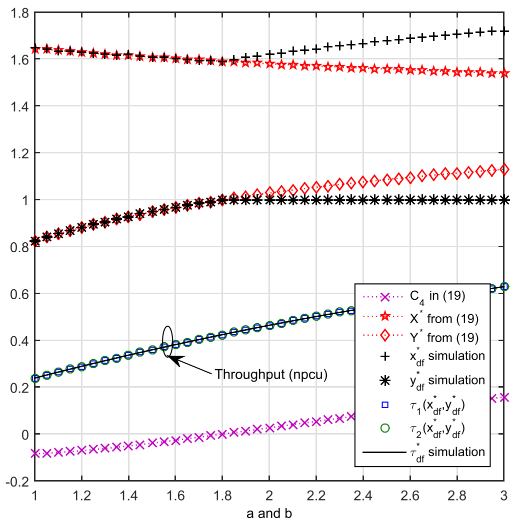


Fig. 5. Variation of term $\mathcal{C}_4$, PS ratio, TS ratio and throughput versus a and b for the hybrid Protocol in DF Relaying (Section III-C).

$\tau_{df}^* = \tau_1(x_{df}^*, y_{df}^*) = \tau_2(x_{df}^*, y_{df}^*)$ where $\tau_1(x_{df}^*, y_{df}^*)$ and $\tau_2(x_{df}^*, y_{df}^*)$ are $S-R$ and $R-D$ throughput values at optimal x_{df}^* or y_{df}^*, respectively.

2)Optimal EH Protocols With AF relaying: For AF relaying, while we provide exact and closed-form solutions for optimal values of the PS protocol in Section IV-A, we provide semi-analytical solutions and approximations for optimal values of TS and hybrid protocols in Sections IV-B and IV-C, respectively.

In Section IV-A, we analytically derive the optimal PS ratio for AF relaying, which is confirmed with a numerical example in Fig. 6. Numerically calculated x_1, x_2, x_3 in (25) and simulated optimal x_{af}^{ps} are shown in Fig. 6a. This shows that x_{af}^{ps} follows x_1 when $0 < b < 3.43$; x_2 when $b = 3.43$; and x_3 when $3.43 < b$. As $a = 0.7$, we can notice that all conditions in (25) are satisfied for these different b regions. Variations of numerically calculated $\tau_{af}(x_i, 1)$ in (24) where x_i in (25) and simulated optimal throughput are shown in Figs. 6b. Similar variations noticed in Fig. 6a can also be seen from corresponding throughput values. This confirms our analysis in Section IV-A.

Fig. 7 compares optimal throughput (simulation) and its approximation (analytical) for both TS and hybrid protocols when a and b vary from 0. For the TS protocol, analytical results perfectly match the simulation results for a low a and b. For the TS protocol for a moderate a and b or the hybrid protocol, analytical results match closely with the simulations, e.g., the difference between optimal throughput and the throughput at approximated ratios at $a = b = 2$ is 0.0193 npcu or 0.0301 npcu for TS or hybrid protocols, respectively. This confirms the accuracy of our approximations in Section IV-C and they are also good lower bounds.

B. Discussion

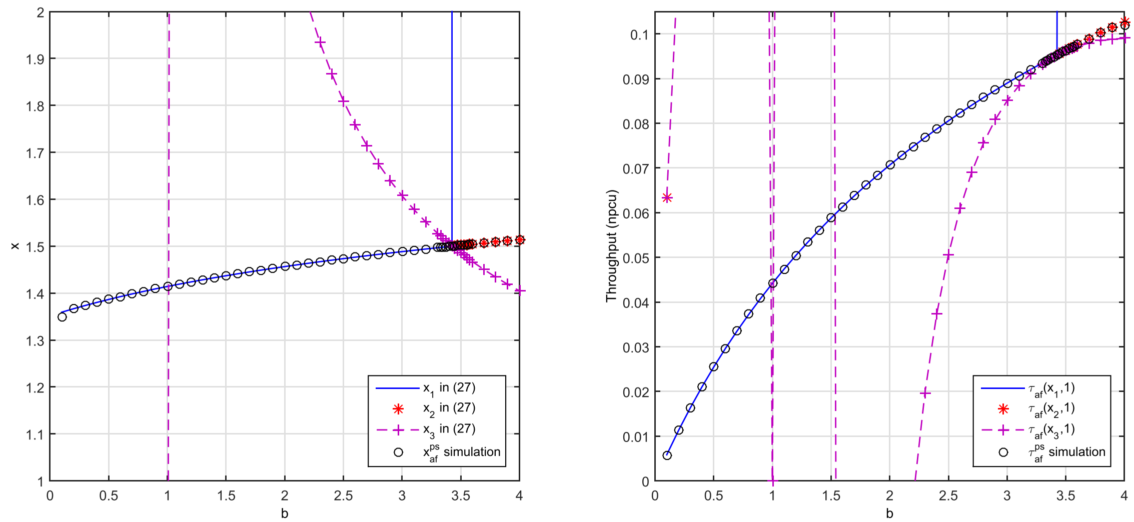
We have already confirmed our analytical results with two main parameters a and b in Figs. 3-7. In this section, we discuss how network parameters affect throughput of different EH protocols under non-fading and fading environments. In Figs. 8-9, we use practical values available in the literature, especially for multi-path fading and path loss. Several highly useful empirical path loss models for macro-cellular systems have been obtained by curve fitting to experimental data. Two of the more useful models for cellular systems are Hata's model and Lee's model. In Figs. 8-9, we use Lee's area-to-area model which calculates path loss over flat terrain [33, eq. (2.333)].[4] For numerical examples, we use some practical values given in [33, Sec. 2.7.3.2 and Table 2.2] such as source or relay antenna height of 30.48 m, destination antenna height equal to 3 m, carrier frequency of 900 MHz, empirical measurements obtained from an open area, e.g., path loss exponent of 4.35. Further, we use source transmit power equal to 10 W, whenever necessary. Since we do not consider noise effects (e.g., estimation errors or noise uncertainty), we assume σ equal to 1.

1) Non-Fading Environment: In this case, we assume that the channel is fixed for the whole communication period. Thus, this is equivalent to the instantaneous perform even in the fading scenario. Therefore, we performance simulations with one channel realization with $|f|^2 = |g|^2 = 0.7$ and $\eta = 0.9$. Path loss is calculated according to Lee's area-to-area model described above with $d_f = d_g = 0.6$ km. Fig. 8a shows the throughput versus SNR (ρ) for all three EH protocols PS, TS and hybrid of DF relaying and AF relaying. We have three main observations:

- For the simulation region, DF relaying outperforms AF relaying because we assume that DF relay decodes the received signal correctly (however, DF relay performance may vary according to the decoding error rate at the relay);
- For each relay scheme, the hybrid protocol outperforms both PS and TS protocols. While the TS protocol approaches the hybrid protocol in low SNR, the PS protocol approaches the hybrid protocol in high SNR. The low-SNR variation is also shown in the magnified portion. For DF relaying, the throughput differences between the TS and hybrid protocols and PS and hybrid protocols are 0.00056 npcu and 0.00807 npcu at -2 dB, respectively, and -0.16867 npcu and 0[5] at 13 dB, respectively. It is important to note that the PS protocol closely follows the performance of the hybrid protocol from moderate SNR to high SNR region.;
- As the TS protocol outperforms the PS protocol at low SNR and vice verse at high SNR, there is a crossover point which is clearly shown in the magnified portion. When $\rho \geq 3.5$ dB with DF relaying and $\rho \geq 9.3$ dB with AF relaying, the PS protocol outperforms the TS protocol.

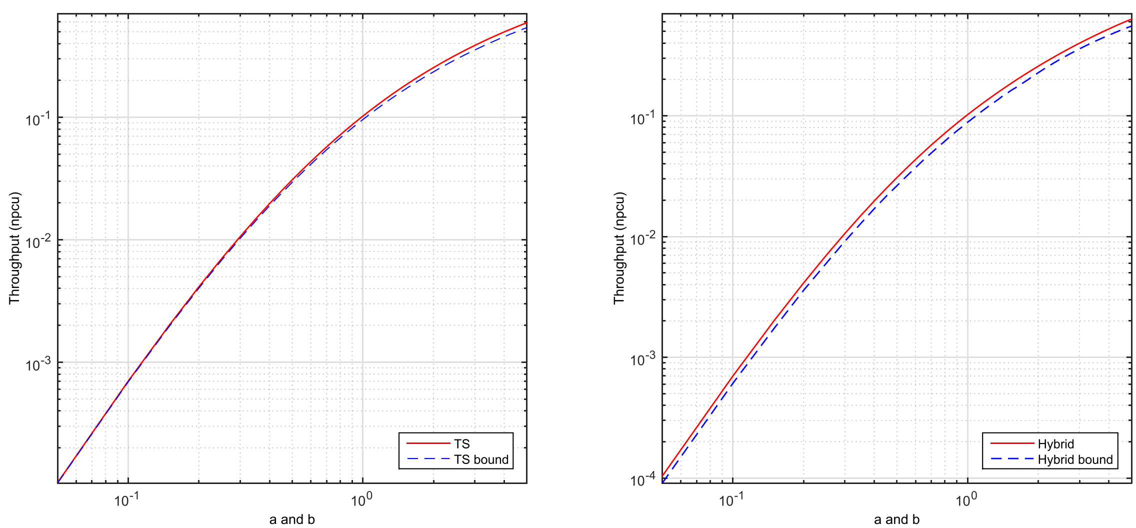
[4]The received signal power at distance d can be expressed as $\mu_{\Omega_p}(d) = \mu_{\Omega_p}(d_0)\left(\frac{d}{d_0}\right)^{-\beta}\left(\frac{f}{f_0}\right)^{-n}\alpha_0$, where $\mu_{\Omega_p}(d_0)$ is the power at reference distance d_0 and reference frequency f_0, β is the path loss exponent, α_0 is a correction factor used to account for different antenna heights, transmit power and antenna gains (see [33, eq. (2.335)]), and n depends upon the carrier frequency and the geographic area.

[5]In DF relaying, the hybrid protocol may reduce to the PS protocol as discussed in Section III-A.



(a) (b)

Fig. 6. For the PS Protocol in AF Relaying (Section IV-A), (a) variation of PS ratio versus b when $a = 0.7$; and (b) variation of throughput versus b when $a = 0.7$.

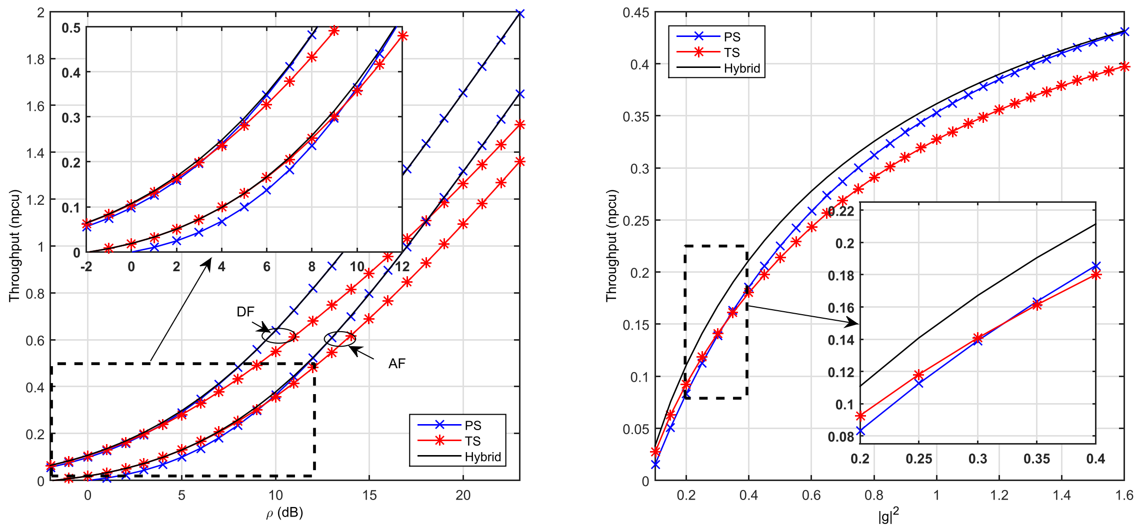


(a) (b)

Fig. 7. Variation of throughput versus a and b in AF relaying: (a) for the TS Protocol (Section IV-B); and (b) for the hybrid Protocol (Section IV-C).

As another example, we consider DF relaying in Fig. 8b which shows the throughput vs. $|g|^2$ for all three EH protocols PS, TS and hybrid. We set $\rho = 5\,\text{dB}$, $|f|^2 = 0.7$, and $\eta = 0.9$. When $|g|^2 \geq 0.3$ the PS protocol outperforms the TS protocol. This point may be important when we consider the implementation complexity of the hybrid protocol. If our hardware is only capable of supporting PS and TS protocols, we may use the crossover point as a switching point, i.e., we have the TS protocol for $|g|^2 < 0.3$ and the PS protocol for $0.3 \leq |g|^2$. However, we may lose 0.028 npcu throughput which is approximately 15.76% loss over the hybrid protocol. We can further improve the performance by switching between three protocols for a given throughput limit. For example, we require to guarantee the throughput $\tau \geq \tau^* - \epsilon$ where τ^* is throughput of the optimal hybrid protocol. If $\epsilon = 0.025$ npcu, we may switch EH protocols: i) TS when $|g|^2 < 0.30$; ii) hybrid when $0.30 \leq |g|^2 \leq 0.45$ iii) PS when $0.45 < |g|^2$. However, in this paper, we do not provide any criterion to select switching points when we are given network parameters. The analysis, however, is more mathematically complex and will be a part of our future work.

2) Fading Environment: So far we consider non-fading scenarios, and variations of throughput also correspond to instantaneous performance. Now we consider the fading effect assuming complex Gaussian (Rayleigh fading) channels. In this paper, we assume that we have all CSI requirements, and we decide optimal λ and α based on these instantaneous values as discussed in Section III and Section IV. Thus, when we have multi-path fading, we do not need the average throughput expression over fading channels in order to decide optimal λ and α because they are decided based on instantaneous values in each time period T with instantaneous



(a) (b)

Fig. 8. For all three EH protocols PS, TS and hybrid: (a) variation of throughput versus SNR ρ for DF relaying and AF relaying when $|f|^2 = |g|^2 = 0.7$; and (b) variation of throughput versus $|g|^2$ for DF relaying when $\rho = 5\,\mathrm{dB}$, $|f|^2 = 0.7$, and $\eta = 0.9$.

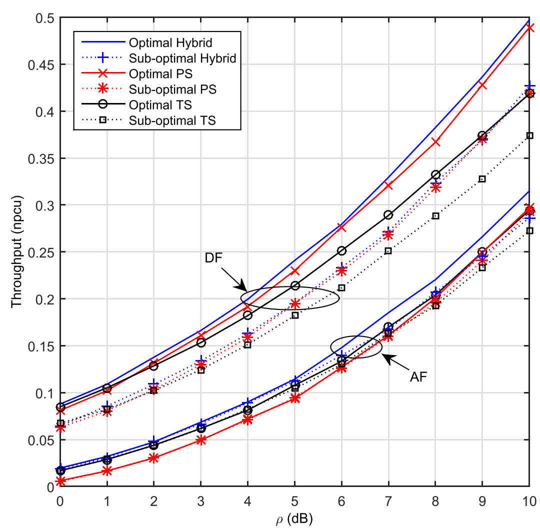


Fig. 9. Variation of average throughput over Rayleigh fading channels versus ρ for all three EH protocols PS, TS and hybrid of DF relaying and AF relaying when average channel gains of $S-R$ and $R-D$ channels are unity and $\eta = 0.95$.

CSI knowledge. However, if we do not have instantaneous channel knowledge, but we have average channel knowledge such as mean and variance, then we may decide λ and/or α values based on the average throughput expression which may give sub-optimal solutions. In Fig. 9, we considers both cases:

- Case 1: We decide optimal λ and α based on instantaneous throughput values where channel coefficients f and g are complex Gaussian random variables following $f,\ g \sim \mathcal{CN}(0,1)$. For a given time interval T, f and g are constant, but they change in every time interval T. In this case, we calculate optimal λ and α values for each set of complex Gaussian channel realizations based on Section III and Section IV, and we calculate the corresponding instantaneous throughput values for each T. Then, we calculate the average throughput over 10^5 channel realizations. This is the average throughput over Rayleigh fading for the proposed optimization problems in Section III and Section IV. This is possible when we have instantaneous CSI requirements, which is one of assumptions in this paper.
- Case 2: We plot the average throughput over Rayleigh fading when PS and TS ratios are selected based on the average throughput expression, e.g., the average throughput of the hybrid protocol under AF relaying is

$$\hat{\tau}_{af}(x,y) = \int\int \tau_{af}(x,y) f_{\gamma_f}(\gamma_f) f_{\gamma_g}(\gamma_g) d\gamma_f d\gamma_g \quad (32)$$

where $f_h(h)$ is the probability density function of random variable h. In this paper, we do not provide analytical results under such fading because it needs a different analytical frame work which may be a future extension. Thus, we calculate average throughput by using numerical techniques. Then, we can calculate PS and TS ratios which maximize the average throughput, e.g., we may select PS and TS ratios of the hybrid protocol under AF relaying based on the average throughput as

$$(\hat{x}_{af}, \hat{y}_{af}) = \arg \max_{x\in[1,2];\ y\in[0,1]} \hat{\tau}_{af}(x,y). \quad (33)$$

Since $\hat{\tau}_{af}(x,y)$ depends only on average channel parameters, we may calculate $(\hat{x}_{af}, \hat{y}_{af})$ numerically without instantaneous channel knowledge. Then, we use the same set of $(\hat{x}_{af}, \hat{y}_{af})$ for every set of channel realizations to find corresponding instantaneous throughput values, and we calculate the average throughput over 10^5 channel realizations. This is the average throughput over Rayleigh fading when PS and TS ratios are selected based on the average throughput expression.

Our proposed optimal EH protocols are coherent type as they need instantaneous channel knowledge. While such optimal protocols have superior performance, their implementation complexity may be high as they need to estimate the channel at every instant. Sub-optimal protocols which are based on average throughput have less implementation complexity. However, we may not be able to achieve the same performance obtained from optimal protocols. These two cases are compared in Fig. 9.

Fig. 9 shows the average throughput of three protocols verses the average SNR over Rayleigh fading channels for DF and AF relaying. We consider both optimal (i.e., the optimal PS and TS ratios are selected according to Sections III and IV) and sub-optimal (i.e., sub-optimal PS and TS ratios are selected by maximizing the average throughput, e.g., (33)) protocols. For both relaying, optimal EH protocols outperform sub-optimal EH protocols for all three EH protocols. For example, with DF relaying, throughput differences between optimal and sub-optimal protocols at 9 dB are 0.058, 0.052 and 0.037 npcu for hybrid, PS and TS, respectively, which are around 17%, 16% and 13% throughput loss, respectively. With AF relaying, throughput differences are 0.015, 0.002 and 0.005 npcu which are around 8%, 1% and 3% throughput loss for hybrid, PS and TS, respectively. Thus, we notice that we have a significant performance loss with sub-optimal solutions, specially with DF relaying. Thus, as an extension, a novel sub-optimal hybrid protocol can be proposed in order to reduce the implementation complexity while achieving close to optimal performance, which may be another interesting research direction.

VI. Conclusion

In this paper, we have proposed a novel EH protocol–the *hybrid protocol* – which can be applied for relay networks. The hybrid protocol is a more general setup because it can switch to either existing PS or TS protocols. Thus, for each DF or AF relaying, we provide unified system and analytical models which are valid for PS, TS or hybrid protocols. Further, we derive the optimal PS, TS and hybrid protocols in which optimal EH time and power-spitting ratio are derived by maximizing the throughput. Our analytical results are rigorous and require less computational complexity. These optimal protocols are coherent type because optimal EH time and power-spitting ratio depend on the knowledge of channels. We show that the hybrid protocol may reduce to the PS protocol or may closely approach the TS protocol. Many extensions are also possible for the work presented in this paper: i) Performance can be analyzed for different fading scenarios; ii) Non-coherent type optimal/sub-optimal EH protocols can be proposed; iii) Alternative hybrid protocols can be introduced, e.g. the relay transmission rate can be adaptively varied according to the network performance, and thus the source and relay may have different transmission rates; and iv) Criteria for hybrid protocol can be designed to switch between TS to hybrid or hybrid to PS (or vice versa) in order to reduce the computational complexity while maintaining the required quality of service (e.g., achievable throughput).

Appendix

A. Proof of (16)

Based on (15), we consider two cases: i) $\tau_1(1+\sigma, y) \leq \tau_2(1+\sigma, y)$; and ii) $\tau_1(1+\sigma, y) \geq \tau_2(1+\sigma, y)$.

1) When $\tau_1(1+\sigma, y) \leq \tau_2(1+\sigma, y)$: This occurs when $a \leq -2ab + \frac{2ab}{y}$ which gives $y \leq \frac{2b}{1+2b}$ for $a > 0$. In this case, we denote the solution for (15) as y_1 which can be given as

$$y_1 = \arg \max_{y \in [0, \frac{2b}{1+2b}]} \tau_1(1+\sigma, y) = \frac{2b}{1+2b}.$$

2) When $\tau_2(1+\sigma, y) \leq \tau_1(1+\sigma, y)$: This occurs when $y \geq \frac{2b}{1+2b}$ for $a > 0$. In this case, the solution for (15) can be calculated as $y = \arg\max_{y \in [\frac{2b}{1+2b}, 1]} \tau_2(1+\sigma, y)$. Although this case is valid for $y \in [\frac{2b}{1+2b}, 1]$, to solve this problem, we first consider the solution for y which maximizes the function $\tau_2(1+\sigma, y)$ when $y \in [0, 1]$. For $a, b > 0$, we have

$$\begin{aligned}
&\frac{\partial \tau_2(1+\sigma, y)}{\partial y}\Big|_{y=0} > 0, \quad \frac{\partial^2 \tau_2(1+\sigma, y)}{\partial y^2}\Big|_{y=0} < 0;\\
&\frac{\partial \tau_2(1+\sigma, y)}{\partial y}\Big|_{y=1} < 0, \quad \frac{\partial^2 \tau_2(1+\sigma, y)}{\partial y^2}\Big|_{y=1} < 0;\\
&\tau_2(1+\sigma, y \to 0) \to 0, \quad \tau_2(1+\sigma, y \to 1) \to 0.
\end{aligned}$$

For $y \in [0, 1]$, we have $\frac{\partial^2 \tau_2(1+\sigma, y)}{\partial y^2} = \frac{-4b^4}{y[y-2b^2(y-1)]^2} < 0$. Thus, $\tau_2(1+\sigma, y)$ has a maximum for $y \in [0, 1]$. The corresponding y value can be found by solving $\frac{\partial \tau_2(1+\sigma, y)}{\partial y} = 0$, which satisfies

$$\log\left[\frac{2ab-(2ab-1)y}{y}\right] = \frac{2ab}{2ab-(2ab-1)y}. \tag{34}$$

- For $2ab = 1$, (34) can be written as $\log\left[\frac{1}{y}\right] = 1$. We denote the solution for $\log\left[\frac{1}{y}\right] = 1$ as y_2 which can be given as $y_2 = \frac{1}{e}$. However, $\tau_2(1+\sigma, y) \leq \tau_1(1+\sigma, y)$ is valid $y \geq \frac{2b}{1+2b}$, and thus this solution is possible when $b - \frac{1}{2(e-1)}$.
- For $2ab \neq 1$, after some manipulations, we can write (34) as
$$\begin{aligned}
\frac{2ab-1}{e} &= \frac{(2ab-1)y}{2ab-(2ab-1)y} e^{\frac{(2ab-1)y}{2ab-(2ab-1)y}}\\
\mathcal{W}\left(\frac{2ab-1}{e}\right) &= \frac{(2ab-1)y}{2ab-(2ab-1)y}
\end{aligned} \tag{35}$$
where the second equality comes from properties of the LambertW function, $\mathcal{W}(\cdot)$, i.e., $w(z) = \mathcal{W}(p)$ is the solution of the equation $w(z)e^{w(z)} = p$. We denote the solution for (35) as y_3 which can be given as $y_3 = \frac{2ab\mathcal{W}\left(\frac{2ab-1}{e}\right)}{(2ab-1)\left(1+\mathcal{W}\left(\frac{2ab-1}{e}\right)\right)}$. Note that, for $a, b > 0$, we have $\left(1+\mathcal{W}\left(\frac{2ab-1}{e}\right)\right) \neq 0$. Further, $\tau_2(1+\sigma, y) \leq \tau_1(1+\sigma, y)$ is valid $y \geq \frac{2b}{1+2b}$, and thus this solution is possible when i) $2ab > 1$ and $\mathcal{W}\left(\frac{2ab-1}{e}\right) < \frac{2ab-1}{a+1}$; or ii) $2ab < 1$ and $\mathcal{W}\left(\frac{2ab-1}{e}\right) > \frac{2ab-1}{a+1}$.

By combining all possible cases and conditions, the optimal y, y_{df}^{ts}, can be given as in (16).

B. Proof of (21)

Based on (19), we consider two cases: i) $\tau_1(x, y) \leq \tau_2(x, y)$; and ii) $\tau_1(x, y) \geq \tau_2(x, y)$, where $\tau_1(x, y)$ and $\tau_2(x, y)$ are given in (10).

1) When $\tau_1(x, y) \geq \tau_2(x, y)$: This satisfies when $\frac{2ab}{y} - abx - ab(1-\sigma) \leq (1+\sigma)a - \frac{(1+\sigma)\sigma a}{x}$. For $a > 0$, we have

$$F(x, y) = bx^2 + [1 + \sigma + b(1-\sigma) - \frac{2b}{y}]x - \sigma^2 - \sigma \geq 0. \tag{36}$$

This inequality satisfies for two regions when $x \geq x_1(y)$ or $x \leq x_2(y)$ where $x_1(y)$ and $x_2(y)$ are solutions of the quadratic equation $F(x, y) = 0$ and $x_2(y) \leq x_1(y)$. However, it can be shown that $x_1(y) \geq 0$ and $x_2(y) \leq 0$ for $b > 0$. Since $x \in [\sigma, 1+\sigma]$, we can drop $x_2(y)$ and we are only interest in $x_1(y)$ which can be given as

$$x_1(y) = \frac{b\left[\frac{2}{y} - (1-\sigma)\right] - (\sigma+1) + G(y)}{2b} \tag{37}$$

where $G(y) = \sqrt{\left[b\left(\frac{2}{y} + \sigma - 1\right) + (\sigma+1)\right]^2 + 4b\sigma(\sigma+1)}$. For the brevity, we use x_1 instead $x_1(y)$. Now different regions for x_1 are considered.

Case I: if $0 < x_1 < \sigma \overset{(37)}{\Longrightarrow} y < 0$ or $y > 2$. Since $x \in [\sigma, 1+\sigma]$ and $y \in [0, 1]$, this case does not exist.

Case II: if $\sigma \leq x_1 \leq 1+\sigma \overset{(37)}{\Longrightarrow} \frac{2b}{2b+1} \leq y \leq 2$, and we may have solution which can be evaluated as

$$\begin{aligned}
&\max_{x,y} \min\left[\tau_1(x, y), \tau_2(x, y)\right] \\
&= \max_{x,y} \ \tau_2(x, y) = \max_{y} \ \tau_2(x, y|x = x_1(y)) \\
&\overset{(10)}{=} \max_{y} \ y \log\left[1 - abx_1(y) + \frac{2ab}{y} - ab(1-\sigma)\right] \\
&\overset{(37)}{=} \max_{y} \ \underbrace{y \log\left[1 + \frac{a}{2}[\sigma + 1 - b(1-\sigma)] + \frac{ab}{y} - \frac{aG(y)}{2}\right]}_{\mathrm{T}_2(y)}.
\end{aligned} \tag{38}$$

For $y \geq 0$, it can be shown that $\frac{\partial^2 \mathrm{T}_2(y)}{\partial y^2} < 0$. Thus, there is a maximum value at $y = y_2^*$ which can be calculated by solving the equation $\frac{\partial \mathrm{T}_2(y)}{\partial y} = \mathrm{T}_2'(y) = 0$, where

$$\begin{aligned}
\mathrm{T}_2'(y) &= \log\left[1 + \frac{a}{2}[1 + \sigma - b(1-\sigma)] + \frac{ab}{y} - \frac{aG(y)}{2}\right] \\
&- \frac{2ab(yG(y) + y(1 + \sigma + b(1-\sigma)) - 2b)}{yG(y)(2ab + (2 + a(1 + \sigma + b(1-\sigma)))y - ayG(y))}
\end{aligned} \tag{39}$$

This equation has complicated form which does not help to derive a closed-form solution for y_2^*. Thus, numerical calculation may be needed. However, in this case, we interest $y \in [\frac{2b}{2b+1}, 1]$. Noting that $\frac{\partial^2 \mathrm{T}_2(y)}{\partial y^2} < 0$ for $y \geq 0$, we can come up with following cases for optimal y, y^*, considering the behavior of $\mathrm{T}_2'(y)$ at boundaries when $a, b > 0$, i.e., $\mathrm{T}_2'\left(\frac{2b}{2b+1}\right)$ and $\mathrm{T}_2'(1)$. Since $\mathrm{T}_2'\left(\frac{2b}{2b+1}\right) > 0$, we have to consider possibilities for $\mathrm{T}_2'(1)$ where

$$\begin{aligned}
\mathrm{T}_2'(1) &= \log\left[\frac{1}{2}(2 + a(1 + \sigma + b(1+\sigma) - \kappa))\right] \\
&+ \frac{2ab(1 + \sigma - b(1+\sigma) + \kappa)}{\kappa(a(\kappa - (1+\sigma)(1+b)) - 2)}
\end{aligned} \tag{40}$$

and $\kappa = \sqrt{(n+1)\left[(b+1)^2\sigma + (b-1)^2\right]}$. Then, we can get optimal value for y as

- $\mathrm{T}_2'(1) \geq 0 \Rightarrow y^* = 1$ because this happens when $y_2^* > 1$. When $y^* = 1$, the hybrid protocol is equivalent to the PS protocol, and the solution for this case is already discussed under the PS protocol in Section III-A where we can find x_{df}^*.
- $\mathrm{T}_2'(1) < 0 \Rightarrow y^* = y_2^*$ where y_2^* can be calculated numerically by using $\mathrm{T}_2'(y) = 0$.

Then, $x^* = x_1(y^*)$ where $x_1(y)$ is in (37), and $\tau^* = \tau_2(x^*, y^*)$.

2) When $\tau_1(x, y) \leq \tau_2(x, y)$: This case satisfies when $F(x, y) \leq 0$ where $F(x, y)$ is in (36). Then, we have $0 \leq x \leq x_1(y)$ where $x_1(y)$ is in (37). Similar to Appendix B1, different regions for x_1 are considered. For $\tau_1(x, y) \leq \tau_2(x, y)$, $0 < x_1 < \sigma$ does not exist as discussed under **Case I** in Appendix B1.

Case III: When $\sigma \leq x_1 \leq 1+\sigma \Rightarrow \frac{2b}{2b+1} \leq y \leq 2$, and thus we can find solution as

$$\begin{aligned}
&\max_{x,y} \min\left[\tau_1(x, y), \tau_2(x, y)\right] \\
&\quad = \max_{x,y} \ \tau_1(x, y) = \max_{y} \ \tau_1(x, y|x = x_1(y)) \\
&\quad \overset{(10)}{=} \max_{y} \ y \log\left[1 + (1+\sigma)a - \frac{(1+\sigma)\sigma a}{x_1(y)}\right] \\
&\quad = \max_{y} \ T_2(y)
\end{aligned} \tag{41}$$

which can be solved as discussed under **Case II** in Appendix B1.

3) When $x_1(y) \geq 1 + \sigma$: Under both scenarios, i.e., $\tau_1(x, y) \leq \tau_2(x, y)$ or $\tau_2(x, y) \leq \tau_1(x, y)$, we may have $x_1 > 1 + \sigma$ which corresponds $0 < y < \frac{2b}{2b+1}$. Since $x \in [\sigma, 1+\sigma]$, we can select $x^* = 1 + \sigma$, which is equivalent to the TS protocol. The optimal y, y^* can be analyzed as discussed in Section III-B. But in the TS protocol, we notice that we always have $y_{af}^{ts} \geq \frac{2b}{2b+1}$. Thus, the hybrid protocol does not exactly equivalent to the TS protocol.

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