Exact Outage Analysis of Multiple-User Downlink With MIMO Matched-Filter Precoding

Saman Atapattu, *Member, IEEE*, Prathapasinghe Dharmawansa, *Member, IEEE*, Chintha Tellambura, *Fellow, IEEE*, and Jamie Evans, *Member, IEEE*

***Abstract*— Here, we address the problem of outage characterization of a single-cell multiple-user multiple-input-multiple-output (MU-MIMO) network with matched-filter precoding at the base-station (BS). In particular, we derive an exact expression for the outage probability of any user. This expression is valid for an arbitrary number of BS antennas and mobile users. Since the expression contains an infinite sum, a tight truncation error bound has been derived to facilitate precise numerical evaluations. Furthermore, asymptotic expressions are provided for high BS transmit power and massive MIMO scenarios.**

***Index Terms*— Massive MIMO, MF precoding, MIMO, multiple-user network, outage probability.**

I. Introduction

Multiple-input multiple-output (MIMO) systems exploit spatial diversity not only to increase throughput but also to enhance the reliability of the wireless channel [1]. As MIMO evolves from 3GPP (High-Speed Packet Access plus and Long Term Evolution) to fifth generation (5G) wireless, the number of antennas at a BS is expected to increase dramatically from a few (conventional MIMO) to tens and even hundreds (massive MIMO) of times the number of active users [2]. Due to the multiple-user downlink, inter-user interference critically affects both the sum-rate and outage of multiple-user MIMO (MU-MIMO) systems. To alleviate this, the BS employs matched-filter (MF) precoding [3]. In MF precoding, the BS pre-multiplies the downlink symbols of each user by the Hermitian of the channel vector between the user and BS. MF precoding offers low computational complexity, robustness, and high asymptotic performance for massive MIMO systems [4].

Performance analysis of MU-MIMO systems with linear precoding is extensive Since exact performance characterization is often intractable with a few exceptions, limiting results and approximations are common. It turns out that such approximations are often valid for certain parameter values (e,g., number of antennas and users) only [5]–[10]. Also, for small-scale MU-MIMO systems, performance has been analyzed in the high-SNR regime, because of the analytical complexity of the signal-to-interference plus noise ratio (SINR) [5], [6]. The block-error probability is investigated for linear receivers in [5]. The SINR distribution, outage and diversity gain are approximately analyzed for different linear precoders in [6] and [7]. In contrast, for large-scale or massive-MIMO systems, an asymptotic deterministic equivalence model may be applied by considering a large number of transmit antennas. By applying the law of large numbers, the overall effect is modeled in terms of number of antennas and slow fading gains in which the instantaneous flat-fading effect diminishes. Since this model has been developed specifically to analyze the sum-rate [8], [9], it does not help analyze the outage probability of multiple-user downlink with flat-fading. An analytical method is thus proposed to study the outage probability by preserving the flat-fading randomness in [10], in which only the user-interference is treated as random.

In this letter, we consider a single-cell MU-MIMO system over independent Rayleigh channels with MF precoding. In particular, we characterize the system outage by deriving an exact expression for the cumulative distribution function (CDF) of the SINR. This new expression has the flexibility to accommodate an arbitrary number of BS antennas as well as mobile users. Moreover, insightful asymptotic expressions are derived for high transmit power and massive MIMO scenarios.

II. Multiple-User MIMO (MU-MIMO) System

We consider a single-cell MU-MIMO system, which has a BS with L antennas and K single-antenna users. The BS-to-user k channel is $\mathbf{h}_k = [h_{k,1}, \cdots, h_{k,L}]$, $k = 1, \ldots, K$. All entries are of $\mathbf{h}_k$'s are independent and identically distributed (iid) $\mathcal{CN}(0, 1)$, circularly symmetric complex Gaussian with zero-mean and unit-variance. The BS has perfect channel state information (CSI) in order to implement the MF precoding. Moreover, independent data symbols x_k, $(k = 1, \ldots, K)$ are energy normalized (e.g., $\mathbb{E}|x_k|^2 = 1$) for the K users. With MF precoding, the received signal at user k is given by [10] $y_k = \sqrt{\frac{P}{KL}}\mathbf{h}_k\mathbf{h}_k^\dagger x_k + \sqrt{\frac{P}{KL}}\sum_{j=1, j\neq k}^{K}\mathbf{h}_k\mathbf{h}_j^\dagger x_j + n_k$, where P is BS transmit power and n_k is the additive noise at user k, which is $\mathcal{CN}(0, 1)$ and is iid across different users. Now the SINR of user k may be expressed as

$$\gamma_k = \frac{\omega|\mathbf{h}_k\mathbf{h}_k^\dagger|^2}{1 + \omega\sum_{j=1, j\neq k}^{K}|\mathbf{h}_k\mathbf{h}_j^\dagger|^2} \tag{1}$$

where $\omega = \frac{P}{KL}$. Since further analysis of the SINR seems an arduous task, here we focus on obtaining an equivalent expression for γ_k. To this end, noting that $\mathbf{h}_k\mathbf{h}_j^\dagger|\mathbf{h}_k \sim \mathcal{CN}\left(\mathbf{0}, ||\mathbf{h}_k||^2\right)$,

Manuscript received July 10, 2017; revised August 24, 2017; accepted August 26, 2017. Date of publication September 6, 2017; date of current version December 8, 2017. This work is supported in part by the Australian Research Council (ARC) through the Discovery Early Career Researcher (DECRA) Award DE160100020 and the Discovery Project DP140101050. The associate editor coordinating the review of this letter and approving it for publication was X. Chen. *(Corresponding author: Saman Atapattu.)*

S. Atapattu and J. Evans are with the Department of Electrical and Electronic Engineering, University of Melbourne, Parkville, VIC 3010, Australia (e-mail: saman.atapattu@unimelb.edu.au; jse@unimelb.edu.au).

P. Dharmawansa is with the Department of Electronic and Telecommunication Engineering, University of Moratuwa, Moratuwa 10400, Sri Lanka (e-mail: prathapa@uom.lk).

C. Tellambura is with the Department of Electrical and Computer Engineering, University of Alberta, Edmonton, AB T6G 2R3, Canada (e-mail: ct4@ualberta.ca).

Digital Object Identifier 10.1109/LCOMM.2017.2749302

we can write an equivalent representation of SINR as (i.e., γ_k and Z have the same distribution)

$$Z = \frac{\omega X^2}{1 + \omega XY} \tag{2}$$

where $X = ||\mathbf{h}_k||^2$ and $Y = \sum_{j=1}^{K-1} |y_j|^2$ are independent random variables (rv's). Moreover, $y_j \sim \mathcal{CN}(0, 1)$, $j = 1, \ldots, K-1$, are also independent. To the best of our knowledge, (2) is a new representation for the SINR of user k.

III. Outage Probability

The outage probability is the probability that the SINR of user k falls below a certain predetermined threshold γ. Therefore, we write $P_o = \Pr[\gamma_k \le \gamma]$. Now we have

$$\begin{aligned} P_o = 1 &- \sum_{n=0}^{L-1} \frac{\left(\frac{\gamma}{\omega}\right)^{\frac{n}{2}} e^{-\sqrt{\frac{\gamma}{\omega}}}}{n!} + \frac{e^{-\sqrt{\frac{\gamma}{\omega}}}}{(L-1)!} \\ &\times \sum_{n=0}^{K-2} \sum_{i=0}^{n} \sum_{j=0}^{\infty} \frac{(-1)^j \binom{n}{i} (n+j)!}{\gamma^{j+n} n! j!} \left(\frac{\gamma}{\omega}\right)^{\frac{L+n+j}{2}} \\ &\times \Psi\left(j+n+1; L+n+1-i; \left(\frac{1}{\gamma}+1\right)\sqrt{\frac{\gamma}{\omega}}\right) \end{aligned} \tag{3}$$

where $\omega = \frac{P}{KL}$ and $\Psi(\cdot;\cdot;\cdot)$ is the Tricomi confluent hypergeometric function [11, Eq. (9.211.4)]. The proof is in Appendix A. While this result holds for any MIMO system, it is especially suitable for massive MIMO as the indices of the last three summations (i.e., n, i and j) do not depend on L.

A. Truncation Error

Since (3) contains an infinite sum, truncation to a finite number of terms T, i.e., $\sum_{j=0}^{\infty} \approx \sum_{j=0}^{T}$, is necessary for numerical calculations. To determine T to ensure the numerical accuracy requirements, we bound the truncation error as

$$\begin{aligned} |\epsilon_T| \le & \frac{e^{-\sqrt{\frac{\gamma}{\omega}}} \left(\frac{\gamma}{\omega}\right)^{\frac{L}{2}} {}_1F_1\left(1; T+2; \frac{1}{\sqrt{\gamma\omega}}\right)}{(L-1)!(T+1)!(\gamma\omega)^{\frac{T+1}{2}}} \\ &\times \sum_{n=0}^{K-2} \sum_{i=0}^{n} \frac{\binom{n}{i}(n+T+1)!}{n!(\gamma\omega)^{\frac{n}{2}}} \\ &\times \Psi\left(n+T+2; L+n+1-i; \left(1+\frac{1}{\gamma}\right)\sqrt{\frac{\gamma}{\omega}}\right) \end{aligned} \tag{4}$$

where ${}_1F_1(\cdot;\cdot;\cdot)$ is the Kummer confluent hypergeometric function [11]. The proof is in Appendix B.

B. High Transmit Power

Due to the interference, increasing the BS transmit power may not have the intended effect of reducing outage. To further investigate, we approximate outage of user k for large P as

$$\begin{aligned} P_o \approx 1 &- \sum_{n=0}^{L-1} \frac{e^{-\sqrt{\frac{\gamma}{\omega}}} \left(\frac{\gamma}{\omega}\right)^{\frac{n}{2}}}{n!} + \frac{e^{-\left(\frac{1}{\gamma}+1\right)\sqrt{\frac{\gamma}{\omega}}}}{(L-1)!} \\ &\times \sum_{n=0}^{K-2} \sum_{i=0}^{L+n-1} \frac{(L+n-1)! \left(\frac{\gamma}{\omega}\right)^{\frac{i}{2}}}{n! i! \gamma^n \left(\frac{1}{\gamma}+1\right)^{L+n-i}} \end{aligned} \tag{5}$$

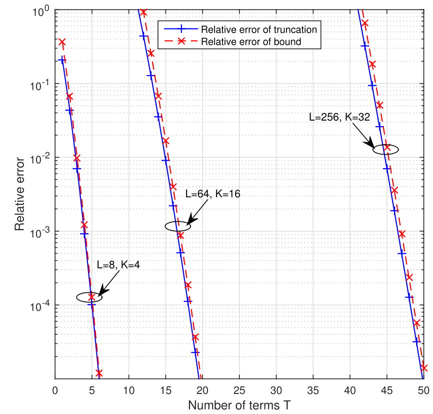

Fig. 1. Relative error vs number of terms when $P = 10$ dBm and $\gamma = 5$ dB.

$$\approx 1 - \sum_{n=0}^{L-1} \frac{(K+n-2)!}{(K-2)!n!} \frac{\gamma^n}{(1+\gamma)^{K+n-1}}. \tag{6}$$

The proof is in Appendix C. While (5) is a function of P, (6) is independent of P, thus it gives the outage-floor value.

C. Massive MIMO

In the massive MIMO setting at the BS (i.e., $L \gg K$), we can approximate (3) as

$$P_o \approx 1 - \sum_{n=0}^{L-1} \sum_{i=0}^{n} \frac{(K+i-2)! \binom{n}{i} \gamma^n e^{-\frac{\gamma}{\frac{P}{K}}}}{(K-2)! n! (1+\gamma)^{K+i-1} \left(\frac{P}{K}\right)^{n-i}}. \tag{7}$$

The proof is in Appendix C. The other important massive MIMO setting occurs when $L, K \to \infty$ such that $L/K \to \eta > 0$. To gain further insights, we focus on analyzing Z in light of stochastic convergence of rv's. As such, we write $Z = \eta P \frac{X}{L} \frac{X}{L} / \left(1 + P \frac{X}{L} \frac{Y}{K}\right)$ where $K = L/\eta$. Noting $X/L \xrightarrow{\text{a.s.}} 1$, $Y/K \xrightarrow{\text{a.s.}} 1$, and continuous mapping theorem [12], we have

$$Z \xrightarrow{\text{a.s.}} \frac{\eta P}{1+P} \tag{8}$$

where a.s. denotes almost sure convergence. This surprisingly simple result shows that, asymptotically, the SINR becomes a deterministic quantity. Therefore, extra attention is needed in setting the outage threshold of a massive MIMO system.

Remark: While the channels from the BS antennas to user k are statistically identical, the channels of different users may be non-identical. In this case, the rv Y in (2) becomes a sum of independent and non-identical exponential rvs. Moreover, for independent and non-identical $|y_k|^2 \sim \text{Exp}(\lambda_k)$ with $\lambda_k \ge 0$, $\lambda_k \ne \lambda_j$ $\forall k \ne j$, the probability density function (PDF) of $Y = \sum_{k=1}^{K-1} |y_k|^2$ takes the form $\sum_{k=1}^{K-1} a_k(\lambda_1, \ldots, \lambda_{K-1}) e^{-y}$, where $a_k(\cdot)$ is the Lagrange basis polynomial. Since this PDF has a similar structure to that in (9), the above analytical framework can readily be extended to the non-identical case as well.

IV. Numerical and Simulation Results

We consider $L = 8, 64, 256$ and $K = 4, 16, 32$, which may represent small MIMO, moderate MIMO or massive MIMO.

We calculate the relative error with truncation as $\frac{|\text{exact -truncated}|}{\text{exact}}$ and the relative error with bound

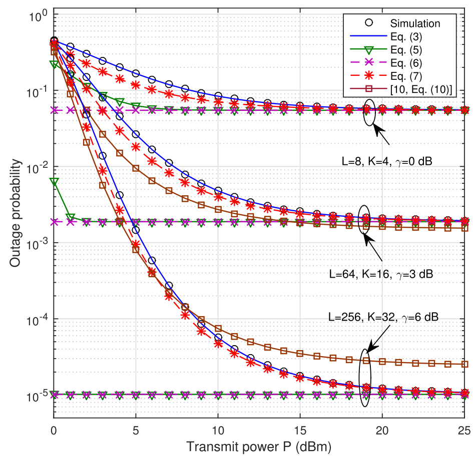

Fig. 2. Outage probability vs transmit power P for three MIMO systems.

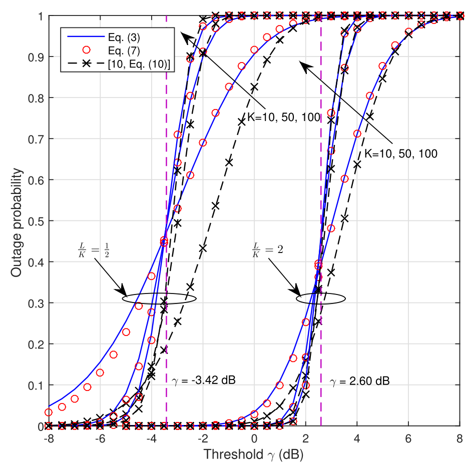

Fig. 3. Outage probability vs threshold for $P = 10\,\text{dBm}$.

as $\frac{\text{error bound in (4)}}{\text{exact value}}$. The exact value is calculated with numerical integration, e.g., Gaussian quadrature rule, by using (10) and (11). The truncated value for different T is calculated by using (3). Fig. 1 plots the relative errors with number of terms T for three MIMO systems when $P = 10\,\text{dBm}$ and $\gamma = 5\,\text{dB}$. We have the relative error less than 10^{-2}, when $T \geq 4,\ 17,\ 47$ from either plot for three systems, which confirm the tightness of the bound in (4).

For the remaining numerical examples, we use (4) to determine T to satisfy $|\epsilon_T| \leq 10^{-4}$. Fig. 2 shows outage with transmit power P for three MIMO systems with different thresholds γ. As this figure shows, the analysis (3) matches perfectly with the simulation. Also the approximations in (5), (6) and (7) approach the exact values asymptotically in high P region. Moreover, the massive MIMO approximation in (7) closely matches with the exact result over the entire P interval for $L \gg K$, e.g., $L = 256,\ K = 32$. This confirms the validity of the analysis. For $L = 64,\ 256$ and $K = 16,\ 32$, we compare the approximate outage expression from [10, Eq. (10)] with (7). The numerical results clearly show that (7) follows more closely the exact trend than [10, Eq. (10)]. Fig. 3 depicts the system outage behavior as a function of threshold γ for $\frac{L}{K} = \frac{1}{2}$ and $\frac{L}{K} = 2$ with different (L, K) pairs when $P = 10\,\text{dBm}$. As can be seen from the figure, (7) serves as a good approximation to the exact system behavior even when L and K increase simultaneously. Moreover, as (L, K) increase such that their ratio is constant, the outage curves tend to achieve more sharp transitions (e.g., at around $-3.42\,\text{dB}$ and $2.60\,\text{dB}$ for $\frac{L}{K} = \frac{1}{2}$ and $\frac{L}{K} = 2$, respectively), thereby confirming the deterministic nature of the SINR. These transitional thresholds are consistent with the theoretical results given by (8) (i.e., $10\log_{10}(5/11)$ and $10\log_{10}(20/11)$). Further, [10, Eq. (10)] works for large L and K because it is based on asymptotic deterministic equivalence for the desired signal power. However, our approximation (7) achieves higher accuracy because it utilizes both the law of large numbers and the exact distribution for both desired and interference powers.

V. Conclusion

Exact outage analysis of a single-cell MU-MIMO downlink with MF precoding has not previously been available. We thus rigorously derived the exact outage for an arbitrary number of the BS antennas and of mobile users. We used the derived expressions to characterize simple outage-floor in the large P regime and outage behavior of massive MIMO. Our analysis reveals that massive MIMO outage has deterministic behavior.

Appendix

A. Proof of (3)

The sum of $N \geq 1$ iid exponential rvs: $V = \sum_{i=1}^{N} \upsilon_i$ with $f_{\upsilon_i}(x) = e^{-x}, x \geq 0,\ \forall \upsilon_i$, is Gamma distributed. The PDF, $f_V(\upsilon)$, and the complementary cumulative distribution function (CCDF), i.e., $\bar{F}_V(\upsilon) = 1 - F_V(\upsilon)$, are

$$f_V(\upsilon) = \frac{\upsilon^{N-1} e^{-\upsilon}}{(N-1)!} \text{ and } \bar{F}_V(\upsilon) = \sum_{n=0}^{N-1} \frac{\upsilon^n e^{-\upsilon}}{n!}, \tag{9}$$

respectively. Since X and Y in (2) are sums of L and $K-1$ exponential rv's, their distributions are special cases of (9). The CDF of Z in (2), $F_Z(z) = \Pr\left[\frac{\omega X^2}{1+\omega XY} \leq z\right]$ is

$$F_Z(z) = \int_0^{\sqrt{\frac{z}{\omega}}} \underbrace{\Pr\left[Y \geq \frac{\omega x^2 - z}{\omega z x}\right]}_{=1} f_X(x)\mathrm{d}x + \int_{\sqrt{\frac{z}{\omega}}}^{\infty} \underbrace{\Pr\left[Y \geq \frac{\omega x^2 - z}{\omega z x}\right]}_{=\bar{F}_Y\left(\frac{\omega x^2 - z}{\omega z x}\right)} f_X(x)\mathrm{d}x. \tag{10}$$

Following (9), the first integral gives $F_X\left(\sqrt{\frac{z}{\omega}}\right)$. The second term can be written using (9) as

$$I(z) = \sum_{n=0}^{K-2} \frac{\int_{\sqrt{\frac{z}{\omega}}}^{\infty} x^{L-1} e^{-x} \left(\frac{\omega x^2 - z}{\omega x z}\right)^n e^{-\frac{\omega x^2 - z}{\omega x z}} \mathrm{d}x}{(L-1)!n!}. \tag{11}$$

To facilitate further analysis, the integral in (11) is written as

$$J(z) \overset{(a)}{=} \frac{e^{-\sqrt{\frac{z}{\omega}}}}{z^n}\left(\frac{z}{\omega}\right)^{\frac{L-1}{2}} \int_0^{\infty} u^n \left(u\sqrt{\frac{\omega}{z}} + 1\right)^{L-1} \times e^{-\left(1+\frac{1}{z}\right)u} \left(1 + \frac{1}{u\sqrt{\frac{\omega}{z}} + 1}\right)^n e^{-\frac{\frac{u}{z}}{\left(1+u\sqrt{\frac{\omega}{z}}\right)}} \mathrm{d}u$$

$$\stackrel{(b)}{=} \frac{e^{-\sqrt{\frac{z}{\omega}}}}{z^n}\left(\frac{z}{\omega}\right)^{\frac{L-1}{2}} \sum_{i=0}^{n}\sum_{j=0}^{\infty} \frac{(-1)^j \binom{n}{i}}{z^j j!} \times \int_0^\infty u^{j+n}\left(1+u\sqrt{\frac{\omega}{z}}\right)^{L-1-i-j} e^{-\left(1+\frac{1}{z}\right)u} \mathrm{d}u$$

$$\stackrel{(c)}{=} \frac{e^{-\sqrt{\frac{z}{\omega}}}}{z^n} \sum_{i=0}^{n}\sum_{j=0}^{\infty} \frac{(-1)^j \binom{n}{i}}{z^j j!}\left(\frac{z}{\omega}\right)^{\frac{L+j+n}{2}} \times \int_0^\infty v^{j+n}(1+v)^{L-1-i-j} e^{-\left(\frac{1}{z}+1\right)\sqrt{\frac{z}{\omega}}v}\mathrm{d}v$$

where (a) follows as $u = x - \sqrt{\frac{z}{\omega}}$, (b) follows with the binomial and power series expansions, and (c) follows as $v = \sqrt{\frac{\omega}{z}}u$. Finally, we obtain (3) using [11, Eq. (9.211.4)].

B. Proof of (4)

For T terms, the truncation error of (3) is expressed as

$$|\epsilon_T| = \left| \frac{e^{-\sqrt{\frac{\gamma}{\omega}}}}{(L-1)!} \sum_{n=0}^{K-2}\sum_{i=0}^{n} \frac{\binom{n}{i}\left(\frac{\gamma}{\omega}\right)^{\frac{L+n}{2}}}{\gamma^n n!} \sum_{j=T+1}^{\infty} \frac{(n+j)!\left(\frac{\gamma}{\omega}\right)^{\frac{j}{2}}}{(-1)^j \gamma^j j!} \times\Psi\left(j+n+1; L+n+1-i;\beta\right)\right|$$

where $\beta = \left(\frac{1}{\gamma}+1\right)\sqrt{\frac{\gamma}{\omega}}$. Let us now focus on simplifying the inner infinite sum. To this end, we write

$$\begin{aligned}\hat{\epsilon}_T &\stackrel{(a)}{=} \sum_{j=T+1}^{\infty} \frac{(-1)^j\left(\frac{\gamma}{\omega}\right)^{\frac{j}{2}}}{\gamma^j j!}\int_0^\infty \frac{e^{-\beta x}x^{j+n}}{(x+1)^{i+j-L+1}}\mathrm{d}x \\ &\stackrel{(b)}{=} \sum_{\ell=0}^{\infty} \frac{(-1)^{\ell+T+1}\left(\frac{\gamma}{\omega}\right)^{\frac{\ell+T+1}{2}} \int_0^\infty \frac{e^{-\beta x}x^{\ell+T+1+n}}{(x+1)^{i+\ell+T+2-L}}\mathrm{d}x}{\gamma^{\ell+T+1}(\ell+T+1)!} \\ &\stackrel{(c)}{=} \frac{(-1)^{T+1}\left(\frac{\gamma}{\omega}\right)^{\frac{T+1}{2}}}{\gamma^{T+1}(T+1)!}\int_0^\infty \frac{e^{-\beta x}x^{T+1+n}}{(x+1)^{i+T+2-L}} \times\sum_{\ell=0}^{\infty}\frac{(-1)^\ell(1)_\ell\left(\frac{x}{1+x}\frac{1}{\sqrt{\gamma\omega}}\right)^\ell}{\ell!(T+2)_\ell}\mathrm{d}x\end{aligned} \tag{12}$$

where (a) follows by using the integral representation of $\Psi(\cdot;\cdot;\cdot)$ function, (b) follows by changing the summation index as $\ell = j-T-1$, and (c) follows by using identities $(\ell+T+1)! = (T+1)!(T+2)_\ell$ and $(1)_\ell = \ell!$. Nothing that $|(-1)^{T+1}| = |(-1)^\ell| = 1$ and $x/(x+1) < 1$, we use the definition ${}_1F_1(a;b;z) = \sum_{k=0}^{\infty}\frac{(a)_k z^k}{(b)_k k!}$, $b \neq 0,-1,-2,\ldots$, and (12) to obtain the upper bound for $|\hat{\epsilon}_T|$ as

$$|\hat{\epsilon}_T| \leq \frac{{}_1F_1\left(1; T+2; \frac{1}{\sqrt{\gamma\omega}}\right)}{\left(\frac{\gamma}{\omega}\right)^{-\frac{T+1}{2}}\gamma^{T+1}(T+1)!}\underbrace{\int_0^\infty \frac{e^{-\beta x}x^{T+1+n}}{(x+1)^{i+T+2-L}}\mathrm{d}x}_{\Psi(n+r+2;L+n+1-i;\beta)}$$

which completes the proof of (4).

C. Proof of (5), (6), and (7)

For large P, we may write the CCDF term in (10) as $\bar{F}_Y\left(\frac{\omega x^2 - z}{\omega z x}\right) = \bar{F}_Y\left(\frac{\frac{P}{KL}\frac{x^2}{z}-1}{\frac{P}{KL}x}\right) \approx \bar{F}_Y\left(\frac{x}{z}\right); z \neq 0$. Then, we can approximate $I(z)$ in (11) as

$$I(z) \approx \sum_{n=0}^{K-2}\frac{\int_{\sqrt{\frac{z}{\omega}}}^{\infty} e^{-\left(1+\frac{1}{z}\right)x}x^{L+n-1}\mathrm{d}x}{(L-1)!n!z^n} \tag{13}$$

in which the integral can be solved using [11, Eq. (3.351.2)]. Now with the aid of (9), (10), and (13), we obtain (5).

For large P, we approximate $1+\omega XY \approx \omega XY$ and obtain $Z \approx \tilde{Z} = \frac{X}{Y}$. Following (9), the CDF of $\tilde{Z}$, $\int_0^\infty F_X(zy)f_Y(y)\mathrm{d}y$, can be written as

$$F_{\tilde{Z}}(z) = 1 - \sum_{n=0}^{L-1}\frac{z^n\int_0^\infty y^{K+n-2}e^{-(1+z)y}\mathrm{d}y}{(K-2)!n!}. \tag{14}$$

Now we Use [11, Eq. (3.351.3)] to obtain (6).

For large L, noting that $\frac{X}{L} \to 1$ (the law of large numbers), we obtain the approximation $Z = \frac{\frac{P}{K}\frac{X}{L}X}{1+\frac{P}{K}\frac{X}{L}Y} \approx \frac{\frac{P}{K}X}{1+\frac{P}{K}Y}$. We thus write an approximate CDF of Z as

$$\begin{aligned}F_Z(z) &\approx \int_0^\infty F_X\left(\frac{Kz}{P}\left(1+\frac{Py}{K}\right)\right)f_Y(y)\mathrm{d}y \\ &\approx 1 - \sum_{n=0}^{L-1}\frac{\int_0^\infty y^{K-2}\left(1+\frac{Py}{K}\right)^n e^{-(1+z)y}\mathrm{d}y}{(K-2)!n!\left(\frac{K}{Pz}\right)^n e^{\frac{Kz}{P}}}\end{aligned} \tag{15}$$

where we have used (9). Now we apply the binomial expansion followed by [11, Eq. (3.351.3)] to obtain (7).

References

[1] L. Zheng and D. N. C. Tse, "Diversity and multiplexing: A fundamental tradeoff in multiple-antenna channels," *IEEE Trans. Inf. Theory*, vol. 49, no. 5, pp. 1073–1096, May 2003.

[2] J. G. Andrews *et al.*, "What will 5G be?" *IEEE J. Sel. Areas Commun.*, vol. 32, no. 6, pp. 1065–1082, Jun. 2014.

[3] M. Joham, W. Utschick, and J. A. Nossek, "Linear transmit processing in MIMO communications systems," *IEEE Trans. Signal Process.*, vol. 53, no. 8, pp. 2700–2712, Aug. 2005.

[4] F. Rusek *et al.*, "Scaling up MIMO: Opportunities and challenges with very large arrays," *IEEE Signal Process. Mag.*, vol. 30, no. 1, pp. 40–60, Jan. 2013.

[5] K. R. Kumar, G. Caire, and A. L. Moustakas, "Asymptotic performance of linear receivers in MIMO fading channels," *IEEE Trans. Inf. Theory*, vol. 55, no. 10, pp. 4398–4418, Oct. 2009.

[6] A. H. Mehana and A. Nosratinia, "Diversity of MIMO linear precoding," *IEEE Trans. Inf. Theory*, vol. 60, no. 2, pp. 1019–1038, Feb. 2014.

[7] H. Tataria, P. J. Smith, L. J. Greenstein, and P. A. Dmochowski, "Zero-forcing precoding performance in multiuser MIMO systems with heterogeneous Ricean fading," *IEEE Commun. Lett.*, vol. 6, no. 1, pp. 74–77, Feb. 2017.

[8] H. Deng, H.-M. Wang, C. Liu, and W. Wang, "Performance analysis of linear precoding for secure multiuser MIMO systems with a multiple-antenna eavesdropper," in *Proc. IEEE Global Telecommn. Conf. (GLOBECOM)*, Dec. 2015, pp. 1–6.

[9] J. Zhu, R. Schober, and V. K. Bhargava, "Linear precoding of data and artificial noise in secure massive MIMO systems," *IEEE Trans. Wireless Commun.*, vol. 15, no. 3, pp. 2245–2261, Mar. 2016.

[10] C. Feng, Y. Jing, and S. Jin, "Interference and outage probability analysis for massive MIMO downlink with MF precoding," *IEEE Signal Processing Lett.*, vol. 23, no. 3, pp. 366–370, Mar. 2016.

[11] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series and Products*, 7th ed. San Francisco, CA, USA: Academic, 2007.

[12] A. W. van der Vaart, *Asymptotic Statistics*. Cambridge, U.K.: Cambridge Univ. Press, 1998.