Periodicity of large-scale coherence in turbulent boundary layers

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A B S T R A C T

This study examines the pronounced periodicity of large-scale coherent structures in turbulent boundary layers, which are of the order of the boundary layer thickness (δ) and reside in the logarithmic and wake regions. To this end, a series of multi-camera planar particle image velocimetry (PIV) measurements are conducted in a streamwise/spanwise and streamwise/wall-normal planes at a friction Reynolds number of \( Re_\tau = 2500 \). The experiments are configured to capture a large field-of-view with velocity fields that cover a streamwise extent in excess of 15δ. The resulting vector fields reveal large-scale streamwise and spanwise organisation instantaneously, which is often lost when only examining mean statistics. By extracting the dominant streamwise and spanwise Fourier modes of the large-scale motions, a clearer picture of these structural organisations and coherence is presented. A targeted inspection of these dominant modes reveal that these features remain coherent over a significant fraction of the boundary layer thickness in the wall-normal direction, but only a fraction of them have coherence that extends to the wall (wall-coherent). Further, the spatial extents and the population density of these wall-coherent and wall-incoherent modes are characterised, with the former conforming to the attached eddy arguments of Townsend (1976) and the subsequent attached eddy models. Collectively, through the evidence gathered here, we provide a conceptual picture of the representative large-scale structures in turbulent boundary layers, which are likely to have implications on the type of representative structures to be used in structure-based models for these flows.

1. Introduction

The structural composition of turbulent boundary layers has been the subject of many investigations over the last half a century (see Jiménez (2018) for a recent review). A large number of these studies have shown that the flow is populated by recurrent turbulent structures, commonly referred to as ‘coherent structures’. Of particular interest in the present study is to capture instantaneous snapshots of large-scale structures that are of the order of the boundary layer thickness (δ), which are known to inhabit the logarithmic and outer regions of boundary layers at moderate to high Reynolds number (Smits et al., 2011). Prior evidence of these structures has largely been provided by inferring spatial information from temporally resolved single-component point measurements using Taylor’s frozen turbulence hypothesis (Hutchins and Marusic, 2007). However, this technique is limited, particularly over large spatial extents (Dennis and Nickels, 2008). More recently, direct numerical simulations with large spatial domains have provided insight into these structures (Sillero et al., 2014). Measurements using particle image velocimetry (PIV) with large spatial domains, such as those presented herein, offer a promising approach to examine these large-scale motions. However, the approach is not without its own difficulties; these structures are known to commonly persist for streamwise distances exceeding 10δ (Hutchins and Marusic (2007) report streamwise coherent structures with lengths > 20δ), requiring the use of multiple imaging sensors to obtain sufficient spatial resolution over such a large streamwise extent.

One distinct feature of the large-scale coherence reported for turbulent boundary layers is the appearance of a pronounced spanwise periodicity (Hutchins et al., 2004; 2005a). In particular, the near-wall region is thought to be composed of streaky patterns of positive and negative streamwise fluctuations (u) with a characteristic spanwise spacing of approximately 100 viscous wall units (Kline et al., 1967). Past works using PIV measurements have confirmed that such periodic patterns are also present further away from the wall and appear to scale with wall-distance (Tomkins and Adrian, 2003; Hutchins et al., 2005b; Hambleton et al., 2006). However, due to the limited field of view (typically less than 3δ in the streamwise direction) of these works the
periodicity exhibited by the $\delta$-scaled structures remains yet to be fully characterised.

The behaviour is best described with reference to Fig. 1, which shows a representative snapshot of the streamwise velocity fluctuations, $u$, from the present PIV experiments on a streamwise wall-normal plane. Here, the boundary layer thickness, $\delta$, corresponds to the wall distance where the mean streamwise velocity is 99% of the free-stream velocity, $U_{\infty}$. From this velocity field, it is clear that the positive and negative $u$ coherence appears to have some pronounced degree of periodicity at a particular wavelength (regions encapsulated by the ellipses), which is often lost when only examining mean statistics. These regions of large-scale coherence are known to account for a large proportion of the turbulent kinetic energy production at high Reynolds numbers (Smits et al., 2011). If this pronounced periodicity observed implies some orderliness of these energetic flow features, their understanding would allow a compact low-order representation of turbulent boundary layers to be developed.

Accordingly, in the present work, a series of multi-camera particle image velocimetry measurements are conducted in a planar arrangement, which are configured to capture velocity fields on streamwise-wall-normal and streamwise/spanwise planes that cover a streamwise extent in excess of $15\delta$, allowing us to capture the full extent of the large periodic motions that are present in the boundary layer.

Throughout this work, the coordinate system $x$, $y$ and $z$ refer to the streamwise, spanwise and wall-normal directions, respectively. Corresponding instantaneous streamwise, spanwise and wall-normal velocity fluctuations are represented by $u$, $v$ and $w$. Overbars denote average quantities and the superscript $+$ refers to normalisation by velocity fluctuations. For example, we use $\overline{u} = U_{\tau} / \nu$ for velocity and $l^+ = lU_{\tau} / \nu$ for length, where $U_{\tau}$ is the friction velocity and $\nu$ is the kinematic viscosity of the fluid.

2. Experimental databases

The experiments described in this paper are performed in the High Reynolds Number Boundary Layer Wind Tunnel (HRNBLWT) at the University of Melbourne. The tunnel consists of a large development length of approximately 27 m, offering the capability of achieving high Reynolds numbers at relatively low freestream velocities (Nickels et al., 2005). The present campaign is tailored to obtain snapshots of very large scale streamwise motions with sufficient fidelity. Hence, experiments are conducted near the upstream end of the test section ($x \approx 4$ m) where the boundary layer thickness is $\delta \approx 90$ mm. This enables us to capture well-resolved instantaneous snapshots at moderate Reynolds numbers with a streamwise extent in excess of $15\delta$. To achieve this, the field of view (FOV) is constructed by stitching the imaged region from eight high-resolution 14 bit PCO 4000 PIV cameras with a sensor resolution of 4096 × 2672 pixels each.

Two camera arrangements are employed (see Fig. 2): One to quantify velocity fields on a large streamwise/wall-normal plane (taken at the centre of the wind tunnel) and the other on a streamwise/spanwise plane, hereafter referred to as SW and WP, respectively. The red solid lines in Figs. 2(a) and (b) show the combined field of view for each arrangement. Table 1 details the key experimental parameters for both arrangements. Measurements for both planes are conducted at a freestream velocity of $10 \text{ m/s}$, with a corresponding friction based Reynolds number of $Re_{\tau} \approx 2400 - 2900 \ (Re_{\tau} = \delta U_{\tau} / \nu)$, across the FOV. The WP database is acquired at wall-normal locations of $z/\delta \approx 0.1$ and 0.4. Particle illumination was provided during measurements by a Big Sky Nd-YAG double pulse laser that delivers 120 mJ/pulse.

The image pairs are processed using an in-house PIV package developed at the University of Melbourne (de Silva et al., 2014). First, a calibration procedure is used to account for distortions within the image plane and to stitch the velocity fields from each camera. Velocity vector evaluation is performed based on a cross-correlation algorithm using multi-grid with window deformation applied at each pass. Two multigrid passes are performed in all measurements, with the final window sizes for each dataset and other processing parameters detailed in Table 1. 50% overlap is employed for all images at the final interrogation window size. Further details on the measurements can also be found in de Silva et al. (2015).

3. Detection of dominant energetic modes

The large spatial extent of the present databases allows us to capture the periodicity in the large-scales present in the flow. However, these features, that are generally readily visible on the instantaneous velocity fields (see Fig. 1), are not so evident in the time-averaged statistics (see Sillero et al. (2014) for recent results). This is likely the result of the statistical averaging that is caused by the superposition of the wide range of dominant energetic scales present in turbulent boundary layers once time-averaged, which masks the periodic patterns of the large-scale streamwise or spanwise coherence that appear over a region of $O(\delta)$. Therefore, to sort the energetic modes or scales, here we employ the pre-multiplied energy spectrum of each instantaneous velocity field as a tool to detect the dominant modes (the peak in the pre-multiplied energy spectrum has been shown to capture the wavelength of the energetic structures in turbulent boundary layers well (Hutchins et al., 2005a; Smits et al., 2011)). We note, even though not adopted in the present study, alternative techniques such as proper orthogonal decomposition (POD) may also be formulated to highlight the wavelengths of the dominant modes. However, due to its simplicity, here we have employed a Fourier based decomposition, which has been shown to work well for detecting periodic features (Hutchins et al., 2004; 2005a). Moreover, it would be not as easy to categorise the dominant POD modes with a single wavelength, which is an objective of the present study.

Accordingly, for the SW database, a Fourier decomposition is performed in $x$ at each $z$ location of the FOV to determine the energetic modes which are of order $\delta$. Towards this, a streamwise trace of the $u$

<table>
<thead>
<tr>
<th>Plane</th>
<th>$U_{\infty}$ (m/s)</th>
<th>$Re_{\tau}$</th>
<th>$z/\delta$</th>
<th>$\nu U_{\tau}$ (µm)</th>
<th>Window size</th>
<th>$j^*$</th>
<th>pixels</th>
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<tbody>
<tr>
<td>SW</td>
<td>10</td>
<td>2500</td>
<td>-</td>
<td>42</td>
<td>26</td>
<td>32×32</td>
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<td>WP</td>
<td>10</td>
<td>2500</td>
<td>0.1, 0.4</td>
<td>42</td>
<td>50</td>
<td>32×32</td>
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fluctuations is extracted from each frame at a prescribed reference height $z_{ref}$. As an example, the red line in Fig. 3(a) shows the signal extracted from the instantaneous velocity field shown previously in Fig. 1 at $z_{ref}/\delta \approx 0.4$, while the blue line corresponds to the most dominant streamwise Fourier mode (or wavelength $\lambda_x = 2\pi/k_x$, where $k_x$ is the wave-number) in the pre-multiplied energy spectrum $(k_x \phi_{uu})$ obtained from the signal.

To validate that the dominant periodicity has been extracted from the aforementioned approach from the trace shown in Fig. 3(a) the corresponding velocity field for $u$ for the dominant streamwise mode is presented in Fig. 3(b), where the phase is determined by performing a Fourier decomposition in $x$ at each $z$ location. It should be noted that even though the mode wavelength is prescribed here, the wall-normal coherence and inclination is not (and comes about only due to the phase relationship between these modes). The PIV frames are then sorted or ‘binned’ based on the streamwise mode $\lambda_x$ that carries the highest energy for that particular frame and reference wall height (see arrow in Fig. 3). The resulting probability distribution of dominant streamwise Fourier modes from all PIV frames at $z_{ref}/\delta = 0.4$ from the SW database at $Re_{\tau} \approx 2500$ is shown in Fig. 3(c). In a similar fashion, the WP databases, a Fourier decomposition is performed in both $x$ and $y$, which allows us to extract the dominant modes in both the streamwise ($\lambda_x$) and spanwise ($\lambda_y$) directions.

4. Characterisation of the periodicity in the large-scale coherence

To illustrate the persistent periodicity in the flow, the normalised two-point correlation functions for the streamwise velocity fluctuations, $R_{uu}$, are computed from the PIV frames corresponding to the four most dominant streamwise modes at $z_{ref}/\delta = 0.4$ from the SW database following the methodology outlined in §3. The results are shown in Figs. 4(a-d) which reveal that when the PIV frames are sorted in this manner.

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Fig. 1. Instantaneous velocity field from the PIV datasets at $Re_{\tau} = 2500$. Results are presented on a streamwise/wall normal plane and the colour contours represent the instantaneous streamwise velocity fluctuations $u$. Spatial extents are normalised by the boundary layer thickness $\delta$. Regions encapsulated by the solid ellipses highlights coherent regions of alternating positive (red) and negative (blue) $u$ that exhibit periodicity at a particular wavelength.

Fig. 2. Experimental setup used to conduct large field of view planar PIV experiments in the HRNBLWT. (a) and (b) shows the configuration used to capture a streamwise/wall-normal plane and streamwise/spanwise plane, respectively. The red solid line corresponds to the combined field of view captured from the multi-camera imaging system.

Fig. 3. (a) Streamwise trace of the $u$ fluctuations extracted at $z_{ref}/\delta = 0.4$ from the example shown in Fig. 1. The solid blue line shows the dominant streamwise Fourier mode. (b) The reconstructed $u$ using only the dominant Fourier mode from the signal shown in (a) across all wall-normal locations. (c) Probability distribution of dominant streamwise Fourier modes from each PIV frame at $z_{ref}/\delta = 0.4$ from the SW database at $Re_{\tau} = 2500$. The C.M. de Silva, et al. International Journal of Heat and Fluid Flow 83 (2020) 108575
fashion, the two-point correlations clearly exhibit an underlying periodicity as denoted by the alternating positive and negative \( R_{uu} \). It is worth noting that the conditional \( R_{uu} \) is computed from an unfiltered subset of the original PIV frames that includes all the energetic scales/wavelengths present in those velocity fields. Therefore it is interesting to note that despite the broad range of scales existing in the velocity fields, the periodic patterns in \( R_{uu} \), that are associated with the strongest dominant modes detected at \( z_{ref} \), are discernible. Further, our results reveal that these modes have coherence that extends a considerable distance in the wall-normal direction in excess of 0.5 \( \delta \) (to be discussed further in §5.2) and appears to exhibit a characteristic inclination angle and also maintain the dominant wavelength detected at \( z_{ref} \) throughout its wall-normal extent (which is not prescribed). Moreover, despite these repeating patterns describing most of the data (~80% of the total frames when combined), the superposition of these modes leads to an \( R_{uu} \) that exhibits no sign of periodicity (see Fig. 4e) and is similar to the unconditioned \( R_{uu} \) (see Fig. 4f). Therefore, care must be taken when interpreting statistically averaged measures, such as \( R_{uu} \) where instantaneous features might be masked. Furthermore, we note that even though our results reveal periodic patterns in the u coherence with streamwise extents that span several \( \delta \), the true extent of some of these structures is likely to be larger due to their meandering nature (Hutchins and Marusic, 2007). Figs. 4(g-l) presents \( R_{uu} \) for the four most dominant spanwise modes \( \lambda_y \) at a reference wall-normal location of \( z_{ref} = 0.1 \) within the logarithmic region. The results exhibit an underlying periodicity for the coherent u-motions that appears to extend for several \( \delta \) in the streamwise direction. Further, our results confirm that these four modes account for over 80% of the PIV frames and once combined appear to recover the behaviour of the unconditioned \( R_{uu} \). Moreover, their spatial extents are observed to be longer in the logarithmic region, which is likely due to the presence of superstructures in this region (Hutchins and Marusic, 2007). This observation is also consistent with the reported shorter streamwise extent of the u coherence as the reference wall height, \( z_{ref} \), moves away from the wall in a turbulent boundary layer (Ganapathisubramani et al., 2005; Hutchins et al., 2005b; Monty et al., 2009). We note, although inaccessible from the present experimental databases due to the lack of near-wall information, it would be interesting to examine whether the large-scale periodic patterns observed in the present study also has a signature in the wall-shear stress signals. This, in turn, would have implications to flow control strategies of large-scale motions, where it may be possible to exploit these periodicities.

![Fig. 4. (a-f) Contours of the normalised two-point correlation function \( R_{uu} \) computed from PIV frames with strongest dominant modes, \( \lambda_x \), in the range (a) \( 1 < \lambda_x / \delta < 3 \), (b) \( 3 < \lambda_x / \delta < 5 \), (c) \( 5 < \lambda_x / \delta < 7 \) and (d) \( 7 < \lambda_x / \delta < 9 \) at a reference location \( z_{ref} / \delta = 0.4 \). The percentage shown in (a-d) corresponds to the fraction of PIV frames included from the full dataset. (e) \( R_{uu} \) for the sum of the modes in (a-d), while (f) corresponds to the unconditioned \( R_{uu} \) from all PIV frames. (g-l) corresponds to the same results computed at a reference location of \( z_{ref} / \delta = 0.1 \).

To elucidate the three-dimensional periodicity of the coherent u-motions, the same technique can be applied to the WP database to obtain the dominant Fourier modes in the spanwise direction (\( \lambda_y \)). To this end, similar to the method applied to SW database (see Fig. 3) a trace of u fluctuations is extracted from PIV frame at \( \Delta x / \delta = 0 \) and thereafter a Fourier decomposition is applied to this signal to determine the most energetic spanwise modes. This process is then repeated for each PIV frame from SW databases. The results are presented in Fig. 5, which shows contours of \( R_{uu} \) for the four most dominant spanwise modes \( \lambda_y \) at a reference wall-normal location of \( \lambda_x / \delta = 0.4 \) (see Figs. 5a-f) and \( z_{ref} = 0.1 \) (see Figs. 5g-l). The results exhibit an underlying periodicity for the coherent u-motions that appears to extend for several \( \delta \) in the streamwise direction. Further, our results confirm that these four modes account for over 80% of the PIV frames and once combined appear to recover the behaviour of the unconditioned \( R_{uu} \). Moreover, their spatial extents are observed to be longer in the logarithmic region, which is likely due to the presence of superstructures in this region (Hutchins and Marusic, 2007). This observation is also consistent with the reported shorter streamwise extent of the u coherence as the reference wall height, \( z_{ref} \), moves away from the wall in a turbulent boundary layer (Ganapathisubramani et al., 2005; Hutchins et al., 2005b; Monty et al., 2009). We note, although inaccessible from the present experimental databases due to the lack of near-wall information, it would be interesting to examine whether the large-scale periodic patterns observed in the present study also has a signature in the wall-shear stress signals. This, in turn, would have implications to flow control strategies of large-scale motions, where it may be possible to exploit these periodicities.
A closer look at the wall-normal coherence of periodic structures

Several past works have highlighted that a turbulent boundary layer is composed of a collection of representative/recurrent structures that carry a significant proportion of the turbulent kinetic energy. In particular, models based on hairpin-like structures have gained significant interest (see Adrian et al. (2000); de Silva et al. (2016); Baidya et al. (2017); Hwang and Sung (2018) and others). These works report that the representative structures in a turbulent boundary layer appear to scale with distance from the wall. In particular, models based on studies by Perry and co-workers (see Perry and Chong (1982); Marusic and Monty (2019)) on the attached-eddy hypothesis have been shown to reasonably reproduce flow statistics of wall-bounded turbulence. These models propose that the boundary layer is a collection of structures that scale with wall-distance and are largely wall-coherent or have coherence that extends to the wall. The SW database in the present work allows us to examine whether the large-scale periodic structures have coherence which extends to the wall. This is illustrated in Fig. 6, where two instantaneous velocity fields (see Figs. 6a, d) and the corresponding reconstructed $u$ for the dominant Fourier modes (see Figs. 6b, e) at a wall-normal height of $z_{ref}/\delta \approx 0.4$ are shown. The results reveal that the dominant Fourier mode for the first field (see Fig. 6b) appears to exhibit $u$ coherence that extends to the wall (wall-coherent), conversely the dominant Fourier mode in Fig. 6(e), computed at the same $z_{ref}/\delta$, appears to show coherence that does not extend to the wall (wall-incoherent), on this $xz$ slice.

In order to quantify this behaviour further and attempt to compute the proportion of dominant wall-coherent and wall-incoherent Fourier modes, it is imperative to estimate the wall-normal extents of these modes. To this end, a threshold is applied to the turbulence intensity $u^2$ of the dominant Fourier modes, (see figures 6c). For the present case, the threshold is set to be equal to the freestream turbulence intensity (vertical dashed line in Figs. 6c). Based on this threshold, the wall-normal extents, $z_e$, of the dominant Fourier modes from the respective flow fields, are highlighted by the light grey shaded region in Figs. 6(c) and 6(f). Further, for the wall-incoherent mode in Fig. 6(e) where $z_e$ does not extend to the wall, its separation from the wall is denoted by

![Fig. 5. (a-f) Contours of the two-point correlation function $R_{uu}$ for strongest dominant spanwise modes, $\lambda_y$, from the WP database at $z_{ref}/\delta \approx 0.4$. (a) $0.5 < \lambda_y/\delta < 0.75$, (b) $0.75 < \lambda_y/\delta < 1$, (c) $1 < \lambda_y/\delta < 1.25$ and (d) $1.25 < \lambda_y/\delta < 1.5$. The percentage shown in (a-d) corresponds to the fraction of PIV frames included from the full dataset. (e) $R_{uu}$ for the sum of the modes in (a-d), while (f) corresponds to the unconditioned $R_{uu}$. (g-i) Corresponds to the same results computed at a reference location of $z_{ref}/\delta \approx 0.1$.](image-url)
these parameters will be characterised further in §5.2. Fig. 7 presents results for the fraction of wall-coherent dominant modes as a function of wall-normal height once the aforementioned technique is employed to all the PIV frames of the SW database at $Re_\tau \approx 2500$. The results show that the proportion of wall-coherent dominant streamwise Fourier modes decreases with wall-normal height, which is expected. However, even within the first 15% of $\delta$, in the logarithmic region of the flow (bounded by the vertical dashed lines), approximately 20% of the dominant $\delta$-scaled periodic features appear to be wall-incoherent. To further examine this trend Fig. 7(b) decomposes the percentage of wall-coherent and wall-incoherent features based on $\lambda_x$ ($\sim$ streamwise extent) of the detected modes. The results reveal that dominant modes with smaller streamwise extents are categorised as wall-coherent only when $z_{ref}/\delta \lesssim 0.3$, while the larger streamwise modes ($\lambda_x > 5$) retains a relatively higher proportion of wall-coherent features further away from the wall. These observations are likely to have implications on the type of representative eddies to be used in structure-based models for turbulent boundary layers, which are unlikely to be only purely a set of wall-coherent structures such as those described in the original attached eddy model (Type A eddies) (Perry et al., 1986), but rather an increasingly larger proportion of wall-incoherent structures particularly in the wake region ($z/\delta > 0.15$) (Perry and Marusic, 1995). Specifically, the wall-incoherent structures are associated with the Type B and C eddies discussed (see also a recent review by Marusic and Monty (2019)). It should be noted that the results presented here are computed from two-dimensional $xz$ slices, while the structures examined are three-dimensional, and therefore some wall-incoherent modes could have coherence extending to the wall on different $xz$ slice, nevertheless, we do not expect this to affect the general trends reported. It is also worth noting that recent studies based on direct numerical simulations with access to three-dimensional flow-fields have also shown the presence of both wall-coherent and wall-incoherent flow features (Hwang and Sung, 2018).
5.1. Streamwise extent of wall-coherent / wall-incoherent features

Fig. 8 (a,d) presents the distribution of all detected dominant streamwise Fourier modes at $z_{ref}/\delta \approx 0.4$ from the SW database at $Re_\tau \approx 2500$, where (a), (b) and (c) correspond to results computed from all, wall-coherent and wall-incoherent modes, respectively. (d-f) reproduces the same statistics computed at a reference location of $z_{ref}/\delta \approx 0.1$ (logarithmic region).

In order to exclusively examine structures/modesthat have coherence which extends to the wall (wall-coherent), Figs. 8(b,c) and Figs. 8(e,f) show the probability of $\lambda_x$ for wall-coherent or wall-incoherent modes exclusively. The results reveal that the wall-coherent modes are associated with larger $\lambda_x$ on average, while the wall-incoherent structures appear to be associated with a smaller $\lambda_x$ mostly around $6\delta$ or less. These results are in agreement with findings by Chandran (2019), who reported by examining the two dimensional energy spectrum of the streamwise velocity component that regions of coherence that do not extend to the wall have smaller streamwise extents.

5.2. Wall-normal extent and separation of wall-coherent / wall-incoherent features

As depicted in Fig. 6, the SW database enables us to investigate the wall-normal extent ($z_e$) of all the dominant Fourier modes examined so far in Fig. 8. The results are shown in Fig. 9, which shows a population distribution of $z_e$ at various wall-normal heights (different symbols). Here, (a), (b) and (c) correspond to statistics computed from all, wall-coherent and wall-incoherent modes, respectively, and the distributions are normalised by $P_T$, which corresponds to the total number of detections. The results from all modes (see Fig. 9a) reveals a larger population of energetic modes closer to the wall, particularly at smaller $z_e$ magnitudes. This behaviour diminishes with increasing $z_e$, which is in agreement with the notion that a larger range of scales is present closer to the wall (Adrian, 2007; Jiménez, 2012; Smits et al., 2011).

Fig. 9(b) reproduces the distribution of $z_e$ from the wall-coherent dominant modes exclusively, which reveals a pattern for the population of different length-scales ($\sim z_e$) as a function of $z_{ref}$. Specifically, the results reveal that the population of $z_e$ appear to follow a $1/z_e$ distribution (black dashed line in Fig. 9b), with a clear ‘peel-off’ at different wall-heights along the distribution. This is in agreement with the structural composition described in the attached eddy model, where the boundary layer is conceived as a collection of wall-coherentstructures that scale with wall-height and have a population distribution which is inversely proportional to the wall height (Perry and Chong, 1982). We note, purely considering all modes (see Fig. 9a) does not reveal evidence of such scaling for the population of structures as a function of wall-height since $z_e$ of the wall-incoherent modes does not appear to
To further quantify this behaviour, figure 11(a) presents the average wall-normal extent $\bar{\eta}$ of the dominant Fourier modes as a function of wall-normal height, $z$. Here, $\bar{\eta}$ is computed at a particular $z$ as the average of the wall-normal extents, $z_e$, of all detected modes at $z$. The results reveal that $\bar{\eta}$ computed exclusively from the wall-coherent modes (■ symbols) appear to increase with $z$. To unravel the underlying structural composition we overlay the expected mean behaviour for $\bar{\eta}$ based on the hierarchical self-similar structural framework described in the attached eddy model. The model assumes discrete hierarchies of eddies whose geometric sizes scale with $z$ and their probability densities to be inversely proportional to $z$. Hence, for a geometric progression of self-similar eddies with the wall-normal extent of the largest eddy being $\delta$, at a particular wall-height $z = \delta/2^n$,

$$\frac{\bar{\eta}}{\delta} = \frac{1 + n}{\sum_{k=0}^{\infty} 2^k},$$

(1)

where the denominator is a geometric series with a common ratio of 2. $\bar{\eta}/\delta$ computed using the above expression for various $z/\delta$ is represented using the dashed line in Fig. 11. The results in Fig. 11(a) suggest that the dominant wall-coherent modes detected from the experimental data conform to the framework assumed in the attached eddy model. In a similar fashion, Fig. 11(b) presents the average wall-normal separation of the wall-incoherent modes $\bar{\zeta}$, which also exhibits that $z_e$ scales with wall-distance confirming our observations from the p.d.f of $z_e$ (see Fig. 10b).

Based on the preceding discussion, Fig. 11(c) presents results for the average wall-normal extent of the wall-incoherent modes inclusive of its separation to the wall, i.e. $z_e + \zeta_e$. Based on this metric, the results reveal that the behaviour of the wall-incoherent modes closely agrees to the wall-coherent modes revealing similar scaling behaviour. Therefore, despite the wall-normal coherence of the wall-incoherent modes not extending to the wall, their sizing appears to scale with $z$, which is in agreement with the physical models described in (Marusic and Perry, 1995; Adrian et al., 2000; Chandran, 2019). Although inaccessible through the databases in this study, it is plausible that the dominant wall-incoherent modes detected are ‘older’ structures that were once wall-coherent or originated from the wall (see Baars et al. (2017); Marusic and Monty (2019)).

Inspired by our findings on the streamwise ($\lambda_x$) and wall-normal $\bar{\eta}$ extent ($z_e$) of the dominant wall-coherent and wall-incoherent modes, figure 12 presents a joint p.d.f of $\lambda_x$ and $z_e$ at a reference wall-height of $z_{ref} = 0.1\delta$ from the SW database at $Re_e \approx 2500$. Here, Figs. 12(a) and 12(b) correspond to results computed exclusively from wall-coherent and wall-incoherent modes, respectively. The results reveal that the aspect ratio of the wall-coherent modes appears to be larger than the wall-incoherent modes (i.e. larger streamwise wavelength, $\lambda_x$, on average at a fixed wall-normal extent, $z_e$). Further, a subtle linear relationship between $\lambda_x$ and $z_e$ is evident for both cases at small $z_e$.

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**Fig. 10.** (a) Probability distribution of the separation distance $z_s$ of the dominant wall-incoherent streamwise Fourier modes from the SW database at $Re_e = 2500$. (b) Probability distribution of the separation distance $z_s$ normalised by the reference wall-normal height $z_{ref}$. Symbols refer to different wall-normal heights.

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**Fig. 11.** (a) The average wall-normal extent, $\bar{\eta}$, of the dominant $\delta$-scaled Fourier modes as a function of reference wall-normal height $z_{ref}$. The ■ and ▲ symbols correspond to results computed from all detected $\delta$-scaled dominant Fourier modes, only wall-coherent modes and only wall-incoherent modes, respectively. The dashed line corresponds to the expected $\bar{\eta}$ if the modes conform to the framework assumed in the attached eddy model following eq. 1. (b) The average wall-normal separation, $\bar{\zeta}$ of the dominant $\delta$-scaled Fourier modes as a function of wall-normal height for wall-incoherent modes only (△ symbols). (c) The average wall-normal extent in (a) reproduced, now inclusive of the wall-normal separation, $z_e$ which is non-zero for the wall-incoherent modes.
highlighting some degree of self-similarity. At large $z_e$, there is a departure from this behaviour which is expected as the structures are limited by the outer limit $\delta$, while we expect the largest $\lambda_x$ to be associated with superstructures in the boundary layer which are known to not scale in a self-similar manner (Baars et al., 2017).

5.3. An idealised conceptual model

Inspired by our observations so far, here we aim to present an idealised conceptual model of the representative large-scale eddies in a turbulent boundary layer. To this end, the schematic in Fig. 13 illustrates trains of representative eddies viewed on a $xz$ plane organised in a manner to reproduce the periodic coherence observed in the present work. We note the general appearance of each coloured region adheres to the widely accepted representative/average shape of structures in a turbulent boundary layer which is conceived as being populated by streamwise-elongated coherent structures with a shallow forward inclination angle (Adrian et al., 2000). Beyond this, we have distilled the structures to wall-coherent and wall-incoherent features that adhere to the scaling observed in the preceding discussions. Specifically, the wall-coherent modes are modelled with structure sizing that scales with $z$ with an increasingly larger population of small-scales near the wall as prescribed in the attached eddy model (Perry and Chong, 1982), while the wall-incoherent structures are pictured to be larger in size away from the wall with a correspondingly larger separation distance, $z_e$, on average. Collectively, the spatial organisation of these structures, in turn, lead to the large-scale period patterns observed in this work over a spatial extent of order $\delta$. These refinements are also in line with preceding works by Adrian et al. (2000); Hwang and Sung (2018); Chandran (2019).

6. Summary and conclusions

This paper examines the large-scale periodicity of the streamwise coherence in turbulent boundary layers. To this end, a set of multi-camera PIV measurements are employed to capture a sufficiently large spatial domain in excess of ten times the boundary layer thickness, $\delta$. By extracting the dominant streamwise and spanwise Fourier modes of the large-scale motions we observe that instantaneously the $u$ coherence exhibits periodic patterns that extend a considerable distance in the wall-normal direction and have inclination angles to the direction of the mean flow similar to those reported in previous works. However, these periodic patterns are not easily discernible directly through time-averaged multi-point statistics.

Through targeted inspection of these dominant energetic modes, we observe that only a fraction of these modes appears to have coherence that extends to the wall. Further, we report that once these modes are categorised to either wall-coherent or wall-incoherent modes, the sizing and population distribution of the wall-coherent modes closely resembles those reported in the attached eddy model (Perry and Chong, 1982). Further, we report that once these modes are categorised to either wall-coherent or wall-incoherent modes, the sizing and population distribution of the wall-coherent modes closely resembles those reported in the attached eddy model (Perry and Chong, 1982). Moreover, despite the coherence of the wall-incoherent modes not extending to the wall, our results show that their sizing still scales with distance from the wall. Collectively, these observations provide insight towards improved modelling of the large-scale structures of turbulent boundary layers as the dominant modes examined and characterised in this work carry a large proportion of the turbulent energy and are hence prime candidates for the representative structures that could be employed in structure-based models. Further, in future works, these findings may also have implications to flow control strategies of large-scale motions, where it may be possible to exploit these periodicities.
Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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CRediT authorship contribution statement

C.M. de Silva: Conceptualization, Software, Formal analysis, Investigation, Writing - original draft, Writing - review & editing. D. Chandran: Conceptualization, Investigation, Writing - review & editing. R. Baidya: Conceptualization, Investigation, Software, Writing - review & editing. N. Hutchins: Supervision, Writing - review & editing. I. Marusic: Supervision, Writing - review & editing.

References