Forcing frequency effects on turbulence dynamics in pulsatile pipe flow


Department of Mechanical Engineering, The University of Melbourne, Melbourne 3010, VIC, Australia

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ABSTRACT

The turbulence dynamics of pulsatile pipe flow are investigated using direct numerical simulation (DNS) at a mean friction Reynolds number of 180. Results are shown for a range of forcing frequencies at a fixed amplitude, which, based on existing classifications, corresponds to the current-dominated regime. This work directs attention towards the phase-variations of single and two-point turbulence statistics, with a particular emphasis on the response of the Reynolds shear stress to systematic changes in the applied forcing frequency. The work yields two key outcomes. (i) A new frequency classification procedure for pulsatile turbulent flows (at low-to-moderate friction Reynolds numbers), informed by the Reynolds shear stress frequency co-spectra and the value of the applied forcing frequency. (ii) A detailed account of single- and two-point Reynolds shear stress statistics, in response to high, very-high and ultra-high forcing frequencies in order to study turbulence dynamics in the physical and Fourier domains. Furthermore, the oscillatory velocity field obtained from the DNS data is compared against the laminar Womersley solution in order to assess the interaction (or lack thereof) between the oscillatory velocity field and phase-averaged Reynolds shear stress fluctuations. For the highest frequencies considered in this work, single- and two-point Reynolds shear statistics all enter the so-called “frozen” regime — which occurs as the forcing time-scale becomes smaller than that of the highest-frequency, energy-containing motions in the Reynolds shear stress co-spectra.

1. Introduction

Pulsatile flows are encountered in a wide range of engineering applications and physical systems. Examples include biological flows, e.g. pulmonary ventilation and haemodynamics (Varghese et al., 2007; Xiao and Zhang, 2009; Huang et al., 2010), environmental flows, e.g. flow over ocean beds and sediment transport in coastal flows (Van Rijn et al., 1990; Yan, 2011) and reciprocating flow in internal combustion engines (Semlitsch et al., 2014). Furthermore, the challenges associated with predicting unsteady turbulent flows using commercial computational fluid dynamics (CFD) software, e.g. computer codes that solve the Reynolds-averaged Navier-Stokes (RANS) have also been highlighted in a past review by Scotti and Piomelli (2002). Ultimately, improved modelling capabilities can only be achieved through improved physical understanding. Therefore, obtaining a first-principles understanding of the physics that governs pulsatile turbulent flows is of great practical interest. However, as noted in past work by Akhavan et al. (1991) and several others, the mean-squared effect of unsteadiness greatly complicates the statistical analysis of the instantaneous flow, relative to a traditional Reynolds-averaged approach, e.g. see Reynolds and Hussain (1972) for details. Hence, the fluid dynamic properties of pulsatile turbulent flows continue to be an active area of experimental and numerical research.

The instantaneous fluid motion in a pulsatile turbulent flow can be split into three separate parts: (i) a mean component; (ii) an oscillatory component and (iii) a turbulent fluctuation. In the absence of (i) and (ii), the flow reduces to an oscillatory laminar flow (with zero mass flux) which forms the basis for Stokes’ First and Second Problems (Stokes, 1851). Particularly relevant to this study are the past works of Womersley (1955) and Uchida (1956), who derived the theoretical solution for pulsatile laminar flow in a straight circular pipe. A basic theoretical outcome of these past studies is that the thickness of the oscillatory shear layer, ℓ, and the applied forcing frequency, ω, are related through the formula ℓ = ½ω, where ν is the kinematic viscosity of the fluid. The impact of varying the forcing frequency, and, hence, the forcing length-scale, upon pulsatile turbulent pipe flows has been considered in several past experimental (Ramaprian and Tu, 1980; Tu and Ramaprian, 1983; Shemer and Kit, 1984; Shemer et al., 1985; Mao and Hanratty, 1986; Brereton et al., 1990; Brereton and Reynolds, 1991; Brereton and Hwang, 1994; Lodahl et al., 1998; He and Jackson, 2009) and computational (Manna and Vacca, 2008; Manna et al., 2012; Papadopoulos and Vouros, 2016) studies. In addition, the impact of unsteadiness upon turbulent channel flow has been examined in detail by Tardu et al. (1994), Binder et al. (1995), Scotti and Piomelli (2001)...

⁎ Corresponding author.
Tu and Ramaprian (1983) conducted experiments in a fully-developed turbulent pipe flow, focusing on how pulsation affects the time-averaged turbulence intensity. Their work considered two separate forcing frequencies: one was comparable to the bursting frequency of near-wall cycle, whereas the other was far lower. Considerable differences between the mean turbulence intensity acquired under pulsatile and non-pulsatile conditions were observed for low frequency forcing, while negligible differences were observed for the higher forcing frequency. A later experimental study by Brereton and Hwang (1994) investigated how the instantaneous turbulence structures responded to pulsation by computing phase-averaged streak spacing over a range of forcing frequencies, ultimately yielding a scaling law based using rapid-distortion theory. He and Jackson (2009) experimentally studied phase-averaged turbulence intensities in a pulsatile turbulent pipe flow, noting that turbulence structure in the core region was not affected by the higher frequency forcing conditions, which they referred to as the “frozen” state. Manna et al. (2012) used direct numerical simulation (DNS) to investigate the near-wall region of pulsatile turbulent pipe flow, focusing on penetration depth of disturbances from the wall in the context of time- and space-averaged statistics of the first- and second-order moments, including vorticity fluctuations and Reynolds stress budgets. More recently, Papadopoulos and Vouros (2016) conducted a DNS study covering a wide range of high and very high frequencies. Following an analysis of the mean velocity and turbulence intensities, they also noted that phase-averaged turbulent quantities exhibit independence with respect to pulsation once the forcing frequency exceeds a certain level. In addition, Papadopoulos and Vouros (2016) also noted that the upper level of high frequency forcing remains somewhat ambiguous.

In addition to studying how pulsation affects turbulence dynamics and flow structures, past studies have proposed several classification schemes for forcing conditions encountered in pulsatile turbulent flows. Early classifications were based on the relative magnitude of the applied forcing frequency and the bursting frequency measured in the near-wall region under non-pulsatile conditions. Detailed descriptions of the bursting process and associated flow events can be found in the work of Kline et al. (1967) and later studies by Corino and Brodkey (1969) and Bogard and Tiederman (1986). One of the first classifications was developed by Mizushima et al. (1974), where the forcing conditions in pulsatile turbulent pipe flow were categorised based on the relative magnitude between the applied forcing frequency and the maximum value of the bursting period. Later work by Ramaprian and Tu (1983) introduced a classification scheme based on a turbulent Stokes number, representing the ratio of the pipe radius to the turbulent diffusion in one oscillation period. Additional classifications based on variants of the bursting frequency (Tardu and Binder, 1993) and the so-called turbulent Stokes layer thickness — which accounts for the summed effect of molecular and eddy diffusivities (Scotti and Piomelli, 2001) — have also been put proposed in past research.

As was previously mentioned, a recent DNS study by Papadopoulos and Vouros (2016) noted that the upper limit of high frequency forcing in a pulsatile turbulent pipe flow remains somewhat ambiguous. In that work, attention was directed towards the phase-averaged response of single-point turbulence statistics, namely, profiles of mean velocity and axial turbulence intensity, across a systematic range of forcing frequencies. The current study extends the past work of Papadopoulos and Vouros (2016) by performing DNS of pulsatile turbulent pipe flow across a wide range of forcing frequencies — including high, very-high and a new “ultra-high” regime. In addition, whilst the majority of past work regarding pulsatile turbulent pipe flow studies have focussed on single-point data, we extend our analysis to two-point statistics in both the frequency, wave-number and physical domains, with a particular emphasis on the Reynolds shear stress (RSS). Finally, this work introduces a new classification procedure to categorise forcing frequency conditions in pulsatile turbulent pipe flow based on the relative magnitude of the applied forcing frequency and the frequency co-spectra of the instantaneous RSS fluctuations.

This document is organised into four sections. The computational aspects of this study are described in Section 2, which includes details of the numerical method, averaging procedures and the validation tests. A list of the simulations considered in this study is also provided in Section 2, along with the classification of each case using the RSS co-spectra approach. Section 3 contains the key results of this study, where a detailed analysis of phase-averaged single- and two-point RSS statistics is performed. Finally, in Section 4, the conclusions of this work are given and recommendations for future research are made.

2. Computational details

This section describes the computational details and is divided into three parts. First, the governing equations and details of the simulation setup are provided. Second, the statistical averaging procedures are described and a validation of the current numerical algorithm is presented. Finally, a classification of the forcing conditions is given and a list of the cases considered in this study is provided.

2.1. Governing equations and simulation set up

In this study, DNS of incompressible pulsatile turbulent pipe flows were performed. The mass conservation and Navier-Stokes equations for an incompressible fluid can be written as

\[ \nabla \cdot \mathbf{u} = 0 \]  
(1)

\[ \frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla p + \frac{1}{Re_l} \nabla^2 \mathbf{u} + \mathbf{f} \]  
(2)

where \( u_r \), \( u_\theta \) and \( u_z \) are the instantaneous velocity components in the radial (r), azimuthal (\( \theta \)) and axial (z) directions and \( p \) is the fluctuating pressure. The Reynolds number based on the pipe centre-line velocity, \( u_\theta \), and pipe radius, \( R \), is defined here as \( Re_\theta = u_\theta R/\nu \), where \( \nu \) is the kinematic viscosity of the fluid. The friction Reynolds number is defined here as \( Re_c = u_r R/\nu \), where \( u_r \) denotes the mean friction velocity. The flow is driven using a time-varying axial pressure gradient, \( f \), which can be written as

\[ f = [0, 0, \Psi(t)] = [0, 0, \Psi_0(1 + A \sin(\omega t))] \]  
(3)

where \( \Psi_0 \) is the constant (negative) axial pressure gradient and \( A \) and \( \omega \) are the forcing amplitude and frequency, respectively.

The Navier-Stokes equations were solved numerically using the OpenFOAM library (Greenshields, 2015) based on the PISO and PIMPLE methods (Ferziger and Peric, 2012), which return a divergence-free velocity field at each time-step. Temporal integration was achieved using a second-order accurate backward difference scheme. Spatial discretisation was based on a second-order-accurate finite volume method. The computational domain, as shown in Fig. 1 (a), is a cylindrical volume of \( V = \pi R^2 L \), where \( L \) is the length of the pipe. The “unwrapped” cylindrical volume is shown in Fig. 1(b) and corresponds to the transformed coordinate system, \( x' = (y, r_0, x) \), where \( y = R - r \) denotes the wall-normal distance from the wall. At the pipe walls (\( r/R = 1 \)), impermeable, no-slip boundary conditions were enforced on velocity, whereas a Neumann boundary condition was applied on pressure. A periodic boundary conditions was applied at the pipe inlet-outlet plane.

A hybrid Cartesian ‘O-grid’ mesh was used for all simulations considered in this study. This meshing approach is based on the past work of Chan et al. (2015). A uniform grid spacing was used in the axial direction and a linear expansion was prescribed in the radial direction in order to attain sufficient resolution at the wall of the pipe. At the pipe centre-line, the cells are approximately cubic (\( \Delta r \approx \Delta \theta \approx \Delta z \)). A cross-section of the computational mesh is shown in Fig. 1 (c). Before
statistical quantities were post-processed, the instantaneous DNS data on the hybrid mesh was interpolated to a cylindrical polar coordinate system.

2.2. Statistical averaging procedures and validation

Throughout this work, the instantaneous DNS data was triple-decomposed using the statistical averaging procedures introduced by Reynolds and Hussain (1972). For example, the instantaneous velocity can be decomposed as

\[ \mathbf{u}(x, t) = \bar{\mathbf{u}}(y) + \mathbf{u}\!(y, \phi(t)) + \mathbf{u}'(x, t) \]

where \( \bar{\mathbf{u}} \) is the global mean velocity, \( \mathbf{u} \) is the periodic component and \( \mathbf{u}' \) is the stochastic fluctuation. The phase-averaged velocity, \( \langle \mathbf{u} \rangle \), is the sum of the global mean and periodic components and is obtained by averaging points that share a common temporal phase by applying the phase-averaging operator

\[ \langle \mathbf{u}(y, \phi) \rangle = \frac{1}{2\pi N} \sum_{n=0}^{N-1} \int_0^{2\pi} \mathbf{u}(x, t + n\tau) dx d\phi \]

where \( \tau \) is the period of the forcing cycle and \( N \) is the number of completed cycles. All phase-averaged quantities were computed using 20 cycles, i.e. \( N = 20 \), resolved using thirty-two equally-spaced points per cycle. Temporal phase is defined in as \( \phi = 2\pi (t/\tau \mod 1) \), where \( \mod \) is the modulo operator. The global mean velocity can be recovered by integrating the phase-average (Eq 5) with respect to temporal phases

\[ \bar{\mathbf{u}}(y) = \frac{1}{\tau} \int_0^{2\pi} \langle \mathbf{u}(y, \phi) \rangle d\phi \]

Note that the periodic component, \( \mathbf{u} \), can be recovered by phase-averaging the triple decomposition Eq 4 and rearranging the result to obtain \( \mathbf{u} = \langle \mathbf{u} \rangle - \bar{\mathbf{u}} \). Finally, considering the phase-averaging operator 5 and global-average operator 6, the phase-averaged RSS can be expressed as

\[ \langle u_i'u_i' \rangle(y, \phi) = \langle u_i \rangle - \langle u_i \rangle \langle u_i \rangle \]

In order to validate the accuracy of the current DNS algorithm, past results relevant to this work were reproduced (see Table 1 for details). As a first step, past results for non-pulsatile turbulent pipe flow were reproduced. With reference to Fig. 2, the mean axial velocity profile and RSS profiles acquired at a friction Reynolds number of 180 are compared against a number of past results. Overall, excellent levels of agreement are observed on both first- and second-order velocity statistics. To verify the accuracy and reliability of the current DNS algorithm in simulating pulsatile turbulent pipe flow, results from the past study by Papadopoulos and Vouros (2016) were reproduced. Phase-averaged mean velocity and axial turbulence intensity profiles corresponding to “case 1d” in their work are computed using Eq 5 and compared at four phases in Fig. 3. Overall, good agreement between the two data sets is observed at all wall-normal locations for each phase.

In order to verify that the computational mesh was fine enough to resolve the smallest eddies, premultiplied one-dimensional wavenumber spectra were computed and compared on two separate grids. Details of the coarse and fine meshes are given in Table 1. Streamwise wavenumber spectra were computed in phase-averaged form using the formula

\[ \langle \Phi \rangle_k = \int_{-\infty}^{\infty} \langle R(e) \rangle e^{-ikx} e^{ikx} dx \]

where \( \langle R(e) \rangle = \langle u_i(x, t)u_i(x + \Delta x, t) \rangle \) is the phase-averaged two-point velocity correlation tensor, \( \Delta x = (0, 0, \Delta z) \) denotes the streamwise separation distance, \( k_x = 2\pi/L, \) is the streamwise wavenumber and \( i \) is the unit complex number. Premultiplied wavenumber spectra of streamwise velocity fluctuations computed using the coarse and fine mesh listed in Table 1 are compared in Fig. 4. The two spectra show very similar results across all wavenumbers and wall-normal positions at each phase. Hence, all results presented herein correspond to the “coarse” mesh as listed in Table 1.

2.3. Classification and description of forcing conditions

It is possible to classify the forcing conditions in a pulsatile turbulent flow using two length scales: (i) the inner-scaled thickness of the oscillatory shear layer, which, for a laminar flow, is defined as \( \delta^* = \sqrt{2/\omega^*} \), and (ii) the inner-scaled pipe radius, \( R^* \). As was previously mentioned, Scotti and Piomelli (2001) argued that the turbulent Stokes length, defined here as \( \ell^* = \ell^*(\delta^* + \sqrt{1 + (\delta^*)^2}) \), where \( \delta^* \) is von Kármán coefficient, is a better measure of the oscillatory shear layer thickness as it accounts for the combined effect of molecular and turbulent diffusivities. The laminar and turbulent Stokes lengths become commensurate

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Fig. 1. Schematic of pipe flow configuration. (a) Pipe geometry in cylindrical coordinate system \((r, \theta, x)\); (b) Unwrapped pipe in Cartesian coordinate system \((y, r\theta, x)\) and (c) Cross-section hybrid computational mesh.
at very high forcing frequencies. Scotti and Piomelli (2001) classified pulsatile turbulent pipe flow into five separate categories based on the value of $\omega^*$. (I) Quasi-steady regime ($\omega^* \approx 0$). The flow in this regime can be considered a series of steady states. (II) Low-frequency regime ($0.005 < \omega^* < 0.02$). The oscillatory shear layer encompasses the entire flow, from the wall to the pipe centre-line, and, hence, $l_t^*$ is on the order of $R^*$. (III) Intermediate frequency ($0.02 < \omega^* < 0.04$). The thickness of the oscillatory shear layer is commensurate with the viscous sublayer. (IV) High frequency regime ($\omega^* > 0.04$). The forcing frequency is comparable to the bursting frequency of the steady turbulent flow. In this frequency range, $l_t^* < < R^*$. The same classification was used in the recent DNS study of Papadopoulos and Vouros (2016).

To complement the past classifications based on $\omega^*$, here we compare the value of the applied forcing frequency against the frequency spectra of turbulence statistics acquired under non-pulsatile, or “equilibrium”, conditions. Frequency spectra were recovered using the formula

$$X_i(\omega^*, y^+) = \int_{-\infty}^{\infty} R_{ij} e^{-i\omega \Delta t} d\Delta t$$

where $R_{ij} = \bar{u}_i(x, t)u_j(x, t + \Delta t)$ denotes the temporal correlation function and $\Delta t$ denotes the time separation between two consecutive samples. Throughout this work, all frequency co-spectra were computed using an inner-scaled sampling period of $T^* \equiv Tu^*/v = 10800$, or $Tu^*/R = 30$ in outer-scaling, for both the non-pulsatile and pulsatile cases. The pre-multiplied RSS co-spectra acquired under non-pulsatile conditions is shown in Fig. 6a, where the vertical dashed lines demarcate the values of $\omega^*$ corresponding to Type (II), (III), (IV) and (V) forcing conditions introduced in past work by Scotti and Piomelli (2001) and Papadopoulos and Vouros (2016). In addition to RSS co-spectra, the frequency spectra of streamwise velocity fluctuations (with the same annotations) is shown in Fig. 6b for comparison.

As anticipated, the frequency co-spectra of RSS shown in Fig. 6a reveals negligible levels of energy in the near-wall region (across all frequencies), where the turbulent fluctuations are damped out by viscosity. Type II forcing frequencies ($\omega^* < 0.005$) represent a “deadzone” where negligible levels of energy are observed from the wall all
the way to the pipe centre-line. The region bounded by Type II and IV forcing frequencies (0.005 < $\omega^+ < 0.04$) represents the “inner-shelf” of the RSS co-spectra where non-negligible energy levels are found in the range $10 < \gamma^+ < 100$. The peak energetic content of the RSS co-spectra is bounded between Type IV and V forcing conditions (0.04 < $\omega^+ < 0.35$). Type VI forcing frequencies ($\omega^+ > 0.35$) define the “outer-shelf” of the RSS co-spectra, where the energy distribution begins to roll off towards zero. For frequencies greater than $\omega^+ > 1$, the applied forcing frequency becomes smaller than the viscous time-scale, and, as a result, the effects of pulsation are anticipated to become negligible. The pre-multiplied streamwise energy spectra shown in Fig. 6b show the same overall trends. However, we prefer to use the RSS co-spectra since it is a more physically meaningful quantity in the context of mean (and phase-averaged) momentum transport.

A list of the simulations considered in this study is provided in Table 2, where details of the forcing frequency, forcing amplitude, laminar Stokes lengths and other key parameters are provided. In addition, the corresponding flow properties including Reynolds numbers, bulk velocity and centreline velocity are also listed. As was previously mentioned, the current study focuses on the frequency-response of the flow field, and to be consistent with the work by Papadopoulos and Vouros (2016), the amplitude $a_u$ is 0.64 throughout this paper. In total, ten different simulations were performed including nine pulsatile cases and a reference non-pulsatile case. Finally, in order to put the cases listed in Table 2 into the context of past work, each case is plotted in the $Re_u - Re_b$ plane in Fig. 7 along with a number of previous studies. Here, the oscillatory and bulk Reynolds number are defined as $Re_u = \max(\delta_x^* U^+)/\nu^+$ and $Re_b = U_c D/\nu$, respectively. The “laminar to turbulent transition” boundary was obtained from the experimental work of Lodahl et al. (1998). For the bulk Reynolds number considered in this study, relaminarisation is anticipated to occur at an oscillatory Reynolds number of $Re_u = 1.4 \times 10^4$, corresponding to a viscous-scaled forcing frequency of $\omega^+ \approx 0.01$, i.e. just within the laminar region. Therefore, under Type III forcing conditions (0.005 < $\omega^+ < 0.02$), corresponding to case “64_010” in Table 2, we observed relaminarisation — consistent with the behaviour of the “WD1” case in past work by Manna et al. (2012). In order to avoid any effects related to turbulence-to-laminar transition (and vice versa), herein we focus on forcing frequencies in excess of $\omega^+ > 0.02$.

3. Results

This section includes the key results of this work and is divided into four parts. First, the influence of pulsation upon first-order velocity statistics is presented in the context of phase-averaged and oscillatory velocity profiles. Second, the phase-averaged response of the RSS and its frequency spectra are analysed. Third, a quadrant analysis of the phase-averaged RSS and its premultiplied wave-number spectra are presented and discussed.

3.1. Analysis of phase-averaged axial velocity profiles

To aid the analysis of phase-averaged axial velocity profiles, the data was expanded in a Fourier series of the form

$$\langle u_r(y, \phi) \rangle = \hat{u}_r(y) + \hat{u}(y) \cos(\omega t + \Theta(y)) + \sum_{n=2}^{\infty} \hat{a}_n(y) \cos[n(\omega t + \Theta(y))]$$

(10)

Table 1

<table>
<thead>
<tr>
<th>Validation Case</th>
<th>$Re_c$</th>
<th>$\Delta^*$</th>
<th>$\Delta_{y,b}^+$</th>
<th>$\Delta^+$</th>
<th>$L^*$</th>
<th>$L/R$</th>
<th>Designation</th>
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<tbody>
<tr>
<td>Wu and Moin (2008)</td>
<td>180.0</td>
<td>0.17</td>
<td>2.20</td>
<td>5.30</td>
<td>2700</td>
<td>15</td>
<td></td>
</tr>
<tr>
<td>Eggels et al. (1994)</td>
<td>180.0</td>
<td>1.88</td>
<td>[0.05, 8.84]</td>
<td>7.03</td>
<td>1800</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>Chan et al. (2015)</td>
<td>180.0</td>
<td>0.33</td>
<td>6.50</td>
<td>6.10</td>
<td>2262</td>
<td>4x</td>
<td></td>
</tr>
<tr>
<td>Fukagata and Kasagi (2002)</td>
<td>180.0</td>
<td>0.46</td>
<td>8.84</td>
<td>7.03</td>
<td>1800</td>
<td>10</td>
<td></td>
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<tr>
<td>Present study (non-pulsatile)</td>
<td>180.0</td>
<td>0.20</td>
<td>5.89</td>
<td>5.89</td>
<td>2262</td>
<td>4x</td>
<td></td>
</tr>
<tr>
<td>Present study (pulsatile)</td>
<td>180.0</td>
<td>0.50</td>
<td>4.32</td>
<td>5.65</td>
<td>2160</td>
<td>12</td>
<td></td>
</tr>
<tr>
<td>Present study (coarse mesh)</td>
<td>180.0</td>
<td>0.20</td>
<td>4.71</td>
<td>5.89</td>
<td>2262</td>
<td>4x</td>
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</tr>
<tr>
<td>Present study (fine mesh)</td>
<td>180.0</td>
<td>0.13</td>
<td>3.14</td>
<td>3.92</td>
<td>2262</td>
<td>4x</td>
<td></td>
</tr>
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</table>

Key simulation parameters for validation cases including: viscous-scaled grid spacings in radial $\Delta_r$, azimuthal $\Delta_{y,b}$ and axial $\Delta_{x}$ directions, the viscous-scaled pipe length $L^*$ and the ratio of the pipe length to the pipe radius, $L/R$. All validation cases in this work correspond to a friction Reynolds number of $Re_u = 180$.  

Fig. 4. Grid refinement study showing premultiplied wave-number spectra of streamwise velocity fluctuations. Data is compared on a coarse mesh, —, and a fine mesh, - - - , as listed in Table 1. The spectra are compared at two separate phases, namely, (a) $\phi = 0$ and (b) $\phi = \pi$ with contour levels $k(\xi) = 0.5, 1.0, 1.5, 2.0, 2.5$ and 0.5, 1.0, 1.5, 2.0, respectively. All data has been normalised using the mean friction velocity, $u_c$.  

Fig. 6. Case “64_010” comparison of instantaneous streamwise velocity profiles; (a) $\phi = 0$ and (b) $\phi = \pi$. Data is compared at two different phases, namely, (a) $\phi = 0$ and (b) $\phi = \pi$. All data has been normalised using the mean friction velocity, $u_c$.
Table 2

Key simulation parameters for the pulsatile pipe flow cases including: friction Reynolds number ($R_{\text{fc}}$), forcing amplitude ($a_{\text{osc}}$), inner-scaled forcing frequency ($\omega^*$), laminar Stokes length ($l^*_l$), turbulent Stokes length ($l^*_t$), ratio of laminar and turbulent Stokes lengths ($l^*_l/l^*_t$) and grid spacings in radial $\Delta^r$, azimuthal $\Delta^\theta$ and axial $\Delta^z$ directions. Bulk flow properties are also listed including: bulk Reynolds number ($R_{\text{bc}}$); centre-line Reynolds number ($R_{\text{cl}}$) and the ratio of centre-line velocity to bulk velocity ($\pi_{\text{bc}}/\pi_{\text{cl}}$). The forcing classification for each case (III, IV, V, or VI) is included, along with non-pulsatile data for reference. All data has been normalised using the mean friction velocity, $u_*$ and pipe radius, $R$.

<table>
<thead>
<tr>
<th>Case</th>
<th>$R_{\text{fc}}$</th>
<th>$a_{\text{osc}}$</th>
<th>$\omega^*$</th>
<th>$l^<em>_l/l^</em>_t$</th>
<th>$\Delta^r$</th>
<th>$\Delta^\theta$</th>
<th>$\Delta^z$</th>
<th>$R_{\text{bc}}$</th>
<th>$R_{\text{cl}}$</th>
<th>$\pi_{\text{bc}}/\pi_{\text{cl}}$</th>
<th>Type Designation</th>
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<td>-</td>
<td>-</td>
<td>0.20</td>
<td>4.71</td>
<td>5.89</td>
<td>5332</td>
<td>7016</td>
<td>1.316</td>
<td>-</td>
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<td>64_010</td>
<td>179.6</td>
<td>0.64</td>
<td>0.022</td>
<td>9.535</td>
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<td>4.71</td>
<td>5.89</td>
<td>5679</td>
<td>7410</td>
<td>1.299</td>
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<tr>
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<td>0.64</td>
<td>0.030</td>
<td>8.165</td>
<td>0.315</td>
<td>4.71</td>
<td>5.89</td>
<td>5576</td>
<td>7294</td>
<td>1.316</td>
<td>IV</td>
</tr>
<tr>
<td>64_087</td>
<td>182.7</td>
<td>0.64</td>
<td>0.043</td>
<td>6.820</td>
<td>0.364</td>
<td>4.71</td>
<td>5.89</td>
<td>5697</td>
<td>7343</td>
<td>1.316</td>
<td>V</td>
</tr>
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<td>64_123</td>
<td>180.0</td>
<td>0.64</td>
<td>0.123</td>
<td>4.032</td>
<td>0.518</td>
<td>4.71</td>
<td>5.89</td>
<td>5444</td>
<td>7048</td>
<td>1.316</td>
<td>V</td>
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<td>64_172</td>
<td>179.8</td>
<td>0.64</td>
<td>0.172</td>
<td>3.410</td>
<td>0.568</td>
<td>4.71</td>
<td>5.89</td>
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<tr>
<td>64_346</td>
<td>179.8</td>
<td>0.64</td>
<td>0.346</td>
<td>2.404</td>
<td>0.664</td>
<td>4.71</td>
<td>5.89</td>
<td>5366</td>
<td>7057</td>
<td>1.316</td>
<td>VI</td>
</tr>
<tr>
<td>64_698</td>
<td>179.4</td>
<td>0.64</td>
<td>0.698</td>
<td>1.693</td>
<td>0.747</td>
<td>4.71</td>
<td>5.89</td>
<td>5371</td>
<td>7065</td>
<td>1.316</td>
<td>VI</td>
</tr>
</tbody>
</table>

where $\hat{A}$ and $\Theta$ are the amplitude and phase of the fundamental Fourier mode, respectively. After evaluating Eq 10 at the pipe centre-line, accelerating and decelerating phases were defined using the oscillatory component of axial velocity, which we approximate here as the fundamental Fourier mode, i.e. $\tilde{u}_a = \hat{A}_a \cos(\omega t + \Theta_a)$. With reference to Fig. 8, the four phases of interest are the stationary-deceleration phase, $\Theta_a = 0$, the mid-acceleration phase, $\Theta_a = \pi/2$, the stationary acceleration phase, $\Theta_a = \pi$, and mid-deceleration phase, $\Theta_a = 3\pi/2$. These four phases are shown on Fig. 8, along with the normalised profile of the oscillatory pressure gradient, $\Pi (\Pi - \Pi_0)$. Note that the pressure gradient and centre-line velocity exhibit a phase difference of $\pi/2$. This result can be readily obtained from the force balance at the pipe centre-line, e.g. see theoretical analysis by Mao and Hanratty (1986). Furthermore, whilst it is clear that the centre-line quantities are well approximated by the fundamental Fourier mode (see Fig. 8), it is instructive to check how this approximation holds at other wall-normal positions. The phase-lag of the fundamental Fourier mode referenced against its centre-line value, $\Theta(y) - \Theta_{cl}$, is shown in Fig. 9(a). The amplitude of the fundamental Fourier mode normalised by its centre-line amplitude, $\hat{A}(y)/\hat{A}_{cl}$, is shown for the same data in Fig. 9(b). In the outer region, the phase lag under Type VI forcing conditions shows no difference compared to its centre-line value. On the other hand, a phase dependence of approximately $\pm$ 2 degrees is evident on the range

![Graph](image-url)
for Type IV forcing. The amplitude ratio in the outer region shows similar trends whereby only Type IV shows a deviation from its centre-line value. Further data-processing revealed that the summed amplitude of all higher harmonics normalised by the amplitude of the fundamental Fourier mode was less than 3% at all wall-normal positions for both forcing conditions. As a result, the summed effect of all higher harmonics is herein considered negligible and the phase-averaged axial velocity profiles were approximated as the sum of the global mean profile and fundamental Fourier mode, i.e.

\[
\langle u_x \rangle (y, \phi) \approx \hat{u}_x + \hat{\Delta U}^0(\phi) \cos(\omega t + \Theta).
\]

Phase-averaged axial velocity profiles are compared at type IV ($\omega^+ = 0.022$) and type VI ($\omega^+ = 0.346$) forcing conditions in Fig. 10(a). Data is shown at the four phases annotated in Fig. 8. Both type IV and VI appear to preserve a nominal log-law profile across different phases. The similarity (or lack thereof) in the outer region of the flow becomes more obvious when the data is shown in defect form — as shown in the inset panels on Fig. 10. Under type IV forcing conditions, the defect profiles of $\langle u_x \rangle$ retain a discernible phase-dependence at all wall-normal positions, and, as a result, strict outer similarity is not observed for this case. In contrast, under type VI forcing conditions, the profiles collapse onto a single line beyond a height of 10 wall units and remain invariant with respect to phase in the outer region. Therefore, in the limit of very high forcing frequencies, the phase-averaged axial velocity profiles exhibit a logarithmic dependence of the form

\[ u_x = \kappa \log y + B + \Delta \hat{U}(\phi) \]

where $\kappa$ is the von Karman coefficient, $B$ is the intercept and $\Delta \hat{U}(\phi)$ is the oscillatory component of axial velocity (which achieves a constant value in the outer region, e.g. see Fig. 10 insets). As expected, the velocity profiles at the stationary acceleration phase ($\phi_1 = \pi$), exhibits a momentum surplus, relative to value averaged across all phases, and, as a result, $\Delta \hat{U} > 0$. The opposite behaviour is observed at the stationary deceleration phase ($\phi_6 = 0$). In addition, a phase-averaged reverse flow occurs below $y^+ < 10$ and $y^+ < 20$ for type IV and VI forcing condition, respectively. This is in line with past observations by Tardu et al. (1994).

To further characterise the unsteady effects in the axial velocity field, the behaviour of the oscillatory component of the phase-averaged axial velocity can be investigated. Profiles of the oscillatory axial velocity, $\hat{u}_x$, are plotted in Fig. 11 at the same forcing conditions shown previously in Fig. 10. Velocity profiles corresponding to Womersley's laminar solution at matched forcing conditions are included for reference. An initial observation based on Fig. 11 is that the phase-averaged DNS data and laminar profiles match very closely within the viscous sublayer, i.e. below a height of ten wall units, for both type IV and VI forcing conditions. Moreover, in the latter regime, the DNS data and Womersley’s solution are practically indistinguishable at all wall-normal positions. This observation agrees well with past work by Manna et al. (2012); Papadopoulos and Vouros (2016); Scotti and Piomelli (2001); Lodahl et al. (1998); He and Jackson (2009); Tardu et al. (1994); Shemer and Kit (1984); Lodahl et al. (1998); He and Jackson (2009); Tardu and Binder (1993); Scotti and Piomelli (2001); Lodahl et al. (1998); He and Jackson (2009); Tardu et al. (1994); Shemer and Kit (1984); Lodahl et al. (1998); He and Jackson (2009); Tardu and Binder (1993); Scotti and Piomelli (2001); Lodahl et al. (1998); He and Jackson (2009); Tardu and Binder (1993); Scotti and Piomelli (2001).
Piomelli (2001), which also found that the oscillatory velocity component collapses on top of Womersley’s solution in the limit of very high forcing frequencies. On the other hand, clear differences between the DNS data and Womersley’s solution are evident above 20 wall units under type IV forcing conditions and highlighted in Fig. 11—consistent with the behaviour of the phase-averaged axial velocity profiles shown previously in Fig. 10.

Deviations from Womersley’s solution can be explained by considering the dynamical equation that governs the time-evolution of the oscillatory velocity field, $u\tilde{\epsilon}_i$. For the current flow configuration, the oscillatory velocity field is strictly one-dimensional, i.e. $\tilde{u}_i = (0, 0, \tilde{u}_i)$, and, as a result, its governing equation can be written as

$$\frac{\partial \tilde{u}_i}{\partial t} = -\frac{\partial \tilde{p}}{\partial x} + \frac{1}{Re} \frac{\partial}{\partial r} \left( \frac{\partial \tilde{u}_i}{\partial r} \right) - \frac{\partial}{\partial r} \left( \tilde{u}_i \tilde{u}_i \right)$$

(12)

A full derivation of Eq 12 can be found in past work of Reynolds and Hussain (1972). The last term on the right-hand side of Eq 12 represents the action of the phase-averaged RSS against the axial velocity oscillations. More specifically, this term represents the wall-normal gradient of the difference between the local phase-averaged value RSS and its mean value and is herein referred to as the RSS modulation, defined here as $\tilde{u}_i \tilde{u}_i \equiv \langle \tilde{u}_i \tilde{u}_i \rangle - \tilde{u}_i \tilde{u}_i$. Note that in the limit of zero RSS modulation, the dynamical Eq 12 reduces to the starting point to derive the analytical Womersley (1955) solution. As a result, the RSS modulation is the only physical mechanism capable of producing the difference between the phase-averaged oscillatory velocity profiles and Womersley’s solution previously observed in Fig. 11.

3.2. Analysis of phase-averaged RSS

The preceding analysis of oscillatory axial velocity profiles highlighted a difference between the DNS data and Womersley’s solution for type IV forcing conditions (see Fig. 11(a)). In contrast, no such difference was observed in the type VI regime (see Fig. 11(b)). As was previously mentioned, any deviation from Womersley’s solution can be explained by the RSS modulation, $\tilde{u}_i \tilde{u}_i$, whose gradient appears on the right-hand side of the Eq 12. In this subsection, the phase-averaged response of RSS under type IV, V and VI forcing conditions is analysed in detail along with a complementary analysis of the time-averaged RSS frequency spectra. Note that the frequency spectra were computed using Eq 9, and, hence, do not contain any phase-averaged information. In order to obtain the phase-averaged frequency spectra for the pulsatile pipe flow data, special statistical treatments related to the theory of cyclostationary processes must be applied — see Tardu and Vezin (2004) for details.

Phase-averaged profiles of RSS accumulated under type IV, V and VI
forcing conditions are shown in Fig. 12, where data is plotted for the four phases annotated in Fig. 8, along with the non-pulsatile RSS profile at the same friction Reynolds number for reference. The pre-multiplied frequency spectrum of the RSS is also included on the right-hand column of Fig. 12. Despite the small difference between the Womersley profile and DNS profiles under type IV forcing conditions (see Fig. 11(a)), the phase-averaged RSS profiles plotted in Fig. 12(a) show a strong phase dependence. As expected, no phase-variations occur in the near-wall region and the largest differences between the phase-averaged and mean RSS values occurs on the range $10 < y^+ < 100$. This
observation is consistent with the deviation of the DNS data from the laminar Womersley profile shown previously in Fig. 11(a). Moving from type IV to V forcing conditions, it is clear that the RSS modulation weakens and that the phase-averaged profiles begin to approach the so-called “frozen” state. At type VI forcing, the phase-averaged RSS profiles becomes indistinguishable from the non-pulsatile data, and, in addition, the two RSS frequency spectra match very closely for all frequencies and wall-normal positions. These observations are in-line with the past analysis of Papadopoulos and Vouros (2016), who observed a similar response in the phase-averaged streamwise turbulence intensities.

In order to associate particular flow events to the phase-averaged
RSS (Fig. 12), the co-variance integrand of $\langle u'_i u'_j \rangle$ can be examined. Following Ong and Wallace (1998), we compute the co-variance integrand of the RSS, which, in phase-averaged form, can be expressed as

$$\langle u'_i u'_j \rangle = \int_{-\infty}^{\infty} u'_i u'_j \langle P(u'_i, u'_j, \phi) \rangle du'_i du'_j$$  \hspace{1cm} (13)

where $u'_i u'_j \langle P(u'_i, u'_j, \phi) \rangle$ is the weighted joint probability density function (WJPDF) of the instantaneous RSS fluctuations evaluated at a particular phase. Each JPDF was computed using 40 × 40 equally-spaced bins on the range $(u'_i/a\omega, u'_j/a\omega) \in [-5, 5]$, where $a\omega$ and $a\omega$ denote the standard deviation of the axial and radial velocity fluctuations acquired under non-pulsatile conditions, respectively. The WJPDF of phase-averaged RSS (i.e. the integrand of Eq 13) are plotted in Fig. 13. From left to right, the data corresponds to type IV, V and VI forcing conditions. All data is shown at a wall-normal height of 30 wall units in order to coincide with the peak value of RSS for each forcing frequency (see Fig. 12). The WJPDF contours are classified using the quadrant classification of Wallace et al. (1972) with Fig. 8 in the same forcing cycle. In addition, the contours are colour-coded using the quadrant classification of Wallace et al. (1972) with Willmarth and Lu (1972), which can be used to decompose the phase-averaged RSS as follows

$$\langle u'_i u'_j \rangle = \sum_{Q} Q_n$$  \hspace{1cm} (14)

where $Q_1(u'_i > 0, u'_j < 0)$ and $Q_2(u'_i > 0, u'_j > 0)$ represent ejection and sweep events, respectively. The remaining $Q_3(u'_i < 0, u'_j > 0)$ and $Q_4(u'_i < 0, u'_j < 0)$ represent so-called outward and inward interaction events, respectively. A detailed overview of quadrant analysis is provided in the review article by Wallace (2016).

Under type IV forcing conditions, the joint-PDF of RSS exhibits a clear phase-dependence in all four quadrants. The largest phase-variations occur in the second ($Q_2$) and fourth ($Q_4$) quadrants — indicating that the Reynolds-stress-producing events (i.e. ejections and sweeps) both react strongly to the applied forcing condition. The strongest sweeps occur at the stationary deceleration phase $\phi_4 = 0$ where the covariance is distributed across a wider range of fluctuations compared with non-pulsatile results. The most violent ejections occur at the mid-deceleration phase $\phi_4 = 3\pi/2$, where the intensification is due to a larger variance of the streamwise fluctuations. Under type V forcing conditions, pulsation does not produce significant differences, relative to type IV. However, it is worth mentioning that, in this situation, $Q_3$ is not strongly modulated. In contrast, pulsation still affects $Q_2$ activity. As anticipated, each quadrant appears frozen under type VI forcing.

To aid the analysis of the weighted-JPDFs plotted in Fig. 13, contours of the instantaneous RSS fluctuations acquired under Type IV, V and VI forcing conditions at a height of 30 wall units are shown in Fig. 14. The RSS fluctuations are shown at the four phases defined in Fig. 8 in the same forcing cycle. In addition, the contours are colour-coded using the quadrant classification of Wallace et al. (1972) with green, blue, yellow and red regions corresponding to instantaneous $Q_1$, $Q_2$, $Q_3$ and $Q_4$ events, respectively. Plotting the data in this way is effectively an extension of the “quadrant map” technique developed in past work by, for example, Pokrajac et al. (2007). Overall, the instantaneous quadrant activity is consistent with trends in the WJPDF analysis shown previously in Fig. 13. For instance, both the shape and intensity of the RSS fluctuations show strong phase variations under Type IV and V forcing conditions. More specifically, as the flow accelerates the intensity and coherence of the RSS fluctuations decreases, ultimately leading to a weaker quadrant activity, relative to the decelerating phases. On the other hand, the intensity and coherence increase as the flow decelerates, which, again, is in line with the intensification of the covariance in weighted-JPDFs. Instantaneous realisations such as Fig. 14 provide empirical evidence that the spatial distribution of RSS fluctuations can exhibit a strong phase dependence.
In order to associate particular length-scales with this behaviour, an analysis of two-point data is desirable.

3.3. Two-point correlations and wavenumber spectra

So far, our analysis has focussed on phase-averaged single-point statistics including axial velocity profiles (Fig. 10), oscillatory velocity profiles (Fig. 11), RSS profiles (Fig. 12) and WJPDF (Fig. 13). Whilst these quantities offer insight into the phase-dependence of quantities at fixed points in the flow, information related to the spatial structure of the flow is best described using two-point methods. Therefore, in this section, we examine the two-point cross-correlation of RSS (and premultiplied wavenumber spectra) with the general aim of complementing the single-point analysis. The RSS cross-correlation coefficient function is defined here as

$$R_{ij}(\Delta y, \Delta x, \phi) = \frac{\langle u'_i(x, t) u'_j(x + \Delta x, t) \rangle}{\sigma_{u_i} \sigma_{u_j}}$$

where the separation vector is $\Delta x = (\Delta y, 0, \Delta x')$ and $\sigma_{u_i}$ and $\sigma_{u_j}$ denote the standard deviation of the axial and radial velocity fluctuations acquired under non-pulsatile conditions, respectively. As noted by Siller et al. (2014), although the cross-correlation of $u'_i$ and $u'_j$ is typically associated with momentum transfer in the form of RSS at fixed points in the flow, it should be clear that two-point cross-correlations expresses the statistical dependence of $u'_i$ at point to the value of $u'_j$ at another.

The RSS cross-correlation is shown in the streamwise-wall-normal plane at zero azimuthal separation in Fig. 15. Data is shown for Type IV, V and VI forcing conditions where the reference wall-normal location at which the correlation is computed is $y^+ = 30$. Non-pulsatile data is also shown for comparison. Overall, the two-point correlations reflect the behaviour of the instantaneous RSS fluctuations shown in Fig. 14, where the “compression” and “extension” of contours in the streamwise direction reflects the phase-dependence of large-scale coherence for Type IV and V forcing conditions. On the other hand, the two-point RSS correlation under Type VI forcing shows little difference, relative to the non-pulsatile data for each phase.

Phase-averaged premultiplied one-dimensional streamwise energy spectra of RSS are shown in Fig. 16. From left to right, the data corresponds to Type IV, V and VI forcing conditions. The highest energy state is achieved at the stationary deceleration phase ($\phi = 0$), where streamwise wavelengths of $\lambda^* \approx 1000$ contain most of the energy. A lower energy state is attained under Type IV and V forcing conditions at the mid-acceleration phase ($\phi = \pi/2$), relative to the previous phase, whereas the spectra under Type VI forcing shows little difference. As shown in Fig. 14, the instantaneous RSS fluctuations also show a reduced energetic state at the mid-acceleration phase ($\phi = \pi/2$),
Premultiplied one-dimensional streamwise energy spectra of RSS. From left to right, each column corresponds to (a, d, g, j) Type IV ($\phi_A = 0$), (b, e, h, k) Type V ($\phi_B = \pi/2$), and (c, f, i, l) Type VI ($\phi_C = \pi$) forcing conditions. From top to bottom, each row corresponds to one of the four phases defined in Fig. 8. All data has been normalised using the mean friction velocity, $u_\tau$, and pipe radius, $R$. 

Fig. 16. Premultiplied one-dimensional streamwise energy spectra of RSS. From left to right, each column corresponds to (a, d, g, j) Type IV ($\phi_A = 0.022$); (b, e, h, k) Type V ($\phi_B = 0.043$) and (c, f, i, l) Type VI ($\phi_C = 0.346$) forcing conditions. From top to bottom, each row corresponds to one of the four phases defined in Fig. 8. All data has been normalised using the mean friction velocity, $u_\tau$, and pipe radius, $R$. 

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particularly under Type IV forcing where weaker instantaneous RSS and smaller streamwise length-scales are clearly visible. A further reduction of energy is observed at the stationary-acceleration phase ($\phi_{1} = \pi$) where the energy spectra exhibit multiple peaks under Type IV and V forcing. Finally, at the mid-deceleration phase ($\phi_{2} = 3\pi/2$), streamwise wavelengths of $\lambda_{x} \approx 1000$ once again contain most of the energy — as visible in Fig. 14.

4. Summary & conclusions

DNS of pulsatile turbulent pipe flow were performed at a mean friction Reynolds number of 180 across a range of forcing frequencies. In order to classify the forcing conditions considered in this work, a categorisation procedure based on the applied forcing frequency and the frequency co-spectra of RSS was devised (Fig. 6). The impact of unsteady forcing upon the turbulence dynamics was investigated in the context of phase-average velocity profiles (Fig. 10), oscillatory velocity profiles (Fig. 11), phase-averaged RSS profiles and associated frequency spectra (Fig. 12), quadrant analysis (Fig. 13) and two-point RSS statistics in both the physical and wave-number domain (Fig. 15 and 16). Whereas recent work has directed attention towards the phase-averaged response of mean velocity profiles and turbulence intensities (Papadopoulos and Vouros, 2016), the principal aim of this work was to investigate, in detail, the phase-averaged response of single- and two-point quantities, with a particular emphasis on statistics of the RSS. The key contributions of this study are summarised below.

(i) This study provides a new classification procedure for forcing conditions in pulsatile turbulent pipe flow. Specifically, we compare the applied forcing frequency against the direct computation of the Reynolds shear stress frequency co-spectra (see Fig. 6) — a rarely reported quantity in most experimental and DNS studies. In addition to establishing an explicit link between the inner-scaled forcing frequency, $\omega^*$, and the frequency content of the instantaneous Reynolds shear stress fluctuations, an additional benefit of this approach is that existing classifications outlined in past work by, for example, Scotti and Piomelli (2001), can be put into a clearer context.

(ii) Whereas recent work argued that there is no upper limit to the high frequency regimes in pulsatile turbulent pipe flow (Papadopoulos and Vouros, 2016), our classification procedure shows that the upper limit is determined by the “outer-shell” of the RSS co-spectra (see Fig. 6 and 12). Put in other words, if the time-scale of the applied forcing frequency becomes shorter than the highest-frequency energy-containing motions in the RSS co-spectra, then any further increase in the applied forcing frequency will have no effect. Likewise, if the time-scale of the applied forcing frequency becomes longer than the lowest-frequency energy-containing motions in the “inner-shell” of the RSS co-spectra, then a quasi-steady behaviour will be observed.

(iii) Whereas recent studies of pulsatile turbulent pipe flow have directed attention towards the phase-averaged response of mean velocity profiles and axial turbulence intensities (Papadopoulos and Vouros, 2016), our analysis focussed on single- and two-point RSS statistics. This choice was motivated by the leading role that RSS plays in determining the mean (and phase-averaged) momentum balance, e.g. see last term on right-hand side on the momentum balance Eq 12. Following an analysis of phase-averaged RSS profiles (Fig. 12) and associated quadrant activity (Fig. 13), as well as the instantaneous RSS fluctuations (Fig. 14) and two-point statistics (Fig. 15 and 16), our results show that a “frozen state” is achieved for all wall-normal positions, frequencies and wavenumbers at an inner-scaled forcing frequency of $\omega^* = 0.346$. In contrast, as the forcing frequency is lowered the phase-dependence of RSS begins to emerge. With reference to the RSS Fig. 16, the premultiplied one-dimensional streamwise energy spectra show that RSS tends to form streamwise large-scale coherence with less intensive motions during the deceleration phases ($\phi_{3}, \phi_{4} = 0, 3\pi/2$). whilst these structures become unstable and more wavy and eventually break into small-scales at acceleration ($\phi_{5}, \phi_{6} = \pi/2, \pi$). A comparable scenario has been reported in the time evolution of streamwise velocity streaks in pulsatile channel flows (Scotti and Piomelli, 2001).

Finally, whereas this work has focused on the turbulence dynamics of pulsatile turbulent pipe flow at a low friction Reynolds number, future work should extend the frequency classification procedure introduced in this work (see Fig. 6) to higher values of $Re$, where the influence of very-large scale structures in the outer region could come into effect (Marusic et al., 2010). In addition, a systematic investigation of multi-mode oscillations in pulsatile turbulent pipe flow would also be of interest and has direct applications to unsteady biological flows (e.g. see work by Chen et al. (2016)).

CRediT authorship contribution statement

Z. Cheng: Conceptualization, Software, Validation, Formal analysis, Investigation, Resources, Writing - original draft, Writing - review & editing.
T.O. Jelly: Conceptualization, Writing - original draft, Writing - review & editing, Supervision.
S.J. Illingworth: Supervision.
I. Marusic: Supervision.
A.S.H. Ooi: Supervision, Funding acquisition.

Declaration of Competing Interest

We wish to confirm that there are no known conflicts of interest associated with this publication and there has been no significant financial support for this work that could have influenced its outcome. We confirm that the manuscript has been read and approved by all named authors and that there are no other persons who satisfied the criteria for authorship but are not listed. We further confirm that the order of authors listed in the manuscript has been approved by all of us. We confirm that we have given due consideration to the protection of intellectual property associated with this work and that there are no impediments to publication, including the timing of publication, with respect to intellectual property. In so doing we confirm that we have followed the regulations of our university concerning intellectual property. We understand that the corresponding author is the sole contact for the editorial process (including the editorial manager and direct communications with the office). He is responsible for communicating with the other authors about progress, submissions of revisions and final approval of proofs. We confirm that we have provided a current, correct email address which is accessible by the corresponding author. We thank you for the further consideration of our manuscript. We hope that the revised manuscript will be considered and accepted for publication in the International Journal of Heat and Fluid Flow.

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References


