Conditionally averaged flow topology about a critical point pair in the skin friction field of pipe flows using direct numerical simulations

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It has recently been shown that critical points in the skin friction field of wall turbulence exist and negative streamwise skin friction events occur. Further analysis of the critical points at the wall using direct numerical simulation data of turbulent pipe flow is presented here. After identifying and characterizing features of the “no-slip” critical points in a turbulent pipe flow, conditional averages of the flow are presented to reveal the magnitude and extent of the fluctuations in the flow around these extreme events. It is found that the velocity and skin friction fields near critical points resemble the three-dimensional U separation proposed by Perry and Chong [J. Fluid Mech. 173, 207 (1986)].

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I. INTRODUCTION

In a recent paper, Chong et al. [1] have shown that there exist critical points in the skin-friction field in turbulent flows over walls. They adopt the terminology of Chong et al. [2] who define a “no-slip” critical point as a point where the skin friction in the streamwise and spanwise directions is simultaneously zero. As such, a no-slip critical point represents a point of local separation in the flow field. If a no-slip critical point exists, there must also be a point of negative streamwise skin friction in the vicinity of the critical point, which also means there are negative streamwise velocity events in wall turbulence. The existence of such extreme streamwise velocity events near a wall was shown by Lenaers et al. [3]. This result contradicts the findings by Eckelmann [4] that in favorably and zero pressure gradient flows “there are no negative velocities near the wall.” A recent high magnification particle image velocimetry experiment by Willert et al. [5] provided a direct observation of back flow events. They reported observing between 8 and 12 back flow events in the 64 000 sample images taken. Other experimental techniques such as molecular tagging velocimetry and laser doppler velocimetry are methods that can be utilized to observe back flow events.

The instantaneous skin-friction field in wall turbulence has not received much attention in literature owing to the difficulties of measuring the quantity. In bluff-body flows, particle image velocimetry (Depardon et al. [6]) with oil or various types of paint applied to a surface (Gregory et al. [7], Woodiga and Liu [8]) have been used to visualize and quantify the mean skin-friction field. The results from these techniques are often extrapolated to infer information about the flow field above the wall, which is used to understand the turbulent processes that give rise to extreme skin-friction events. With the advent of high Reynolds number direct numerical simulations (DNS) of wall turbulence, we now have access to simultaneous skin friction and highly resolved velocity fields. Although Chong et al. [1] found critical points in the DNS of a turbulent channel flow, they did not investigate the flow field above those points. Lenaers et al. [3], however, investigated the flow above locations of negative streamwise skin friction. Using conditional averaging, they

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showed that a strong spanwise vortex is responsible for strong negative skin-friction events and the negative velocities in the flow above them. The spanwise vortex was centered approximately 20 viscous units above the wall and slightly upstream of the negative skin-friction event. Cardesa et al. [9] investigated these critical points for a turbulent channel flow looking at statistics such as lifetime, average distance, velocity, and area density. Recently, Chin et al. [10] investigated the flow topologies of critical points between toroidal pipes and channels. They reported that critical points exist in both wall-bounded flows, however, the occurrence in a toroidal pipe is less than a channel. Their findings allude to the possibility that critical points appear to be universal in wall-bounded flows. Their work was performed at a single curvature for the toroidal pipe and therefore more research will be required to understand how curvature influences critical point topology.

Analytically, Perry and Chong [11] have explored flow patterns that could produce certain arrangements of critical points on the surface. Perry and Chong [11] applied high order series expansion solutions to the Navier-Stokes and continuity equations and showed various examples of local separations that are possible above a no-slip critical point. Two examples of flow patterns above a saddle-node combination of no-slip critical points are shown in Fig. 1. These flow patterns are referred to as three-dimensional U separations.

In this paper, a previous study on critical point flow topology on channel flows by Chong et al. [1], Cardesa et al. [9], and Chin et al. [10] is extended to include no-slip critical points using numerical simulations of turbulent pipe flow. Attempts will be made to classify and relate these extreme events to the flow field above.

**II. NUMERICAL SIMULATION DETAILS**

In recent times, DNS data have provided unprecedented access to flow fields. Here, we employ direct numerical simulation data of turbulent pipe flow [12,13]. The employed DNS pipe data has been previously compared and validated with other pipe flow DNS [14] and also experimental data [15]. The DNS is performed with a periodic boundary condition, which is the accepted norm for such a computation. It is important to note that no physical experiment could impose such boundary condition. The details of the simulation are provided in Table I. In this table, the friction Reynolds number is defined as \( \text{Re}_\tau = \frac{\delta U_\tau}{\nu} \approx 1000 \), where \( \delta \) is the pipe radius, \( U_\tau \) is the friction velocity, and \( \nu \) is the kinematic viscosity. Note that a superscript “+,” as in \( \langle u'_1 \rangle_{\text{CP}}^+ \), indicates normalization.
with inner variables, $U_\tau$ and $\nu/U_\tau$. For the pipe flow data, many independent flow fields were available, allowing for converged statistics and conditional averages to be calculated.

III. CRITICAL POINTS IN THE SKIN-FRICTION FIELD

Following the introduction of invariants of the velocity gradient tensor by Chong et al. [2], many researchers have applied it successfully to study turbulent structures [17,18]. The velocity gradient tensor ($A_{ij}$) is defined by the first-order Taylor series expansion at any point in a flow field. Therefore, the velocity field $u_i$ can be written as

$$u_i = \dot{x}_i = A_{ij}x_j,$$

where $\dot{x}_i = dx_i/dt$ is the velocity and $x_i$ is the local coordinate ($t$ is time). The characteristic equation of $A_{ij}$ is

$$\lambda_i^3 + P\lambda_i^2 + Q\lambda_i + R = 0,$$

where $\lambda_i$ are the eigenvalues and $P$, $Q$, and $R$ are the tensor invariants. For incompressible turbulent flows, the first invariant $P$ is zero and the topology of the flow structures can be investigated in terms of the second and third invariants, $Q$ and $R$. At the surface of the wall, invariants $Q$ and $R$ are both zero, which makes Eq. (2) invalid.

At the wall, since it is a no-slip condition, i.e., $u_i = 0$, when $x_3 = 0$, in order to classify the topology of critical points in the skin-friction field (and surface vorticity field), we define a no-slip tensor. This is derived from a Taylor-series expansion at the wall with higher-order terms leading to

$$\left[\begin{array}{c} u_1 \\ u_2 \\ u_3 \end{array}\right] = \left[\begin{array}{ccc} 0 & 0 & A_{13} \\ 0 & 0 & A_{23} \\ 0 & 0 & 0 \end{array}\right]\left[\begin{array}{c} x_1 \\ x_2 \\ x_3 \end{array}\right] + \left[\begin{array}{ccc} A_{11}x_3 & A_{12}x_3 & A_{13}x_3 \\ A_{21}x_3 & A_{22}x_3 & A_{23}x_3 \\ A_{31}x_3 & A_{32}x_3 & A_{33}x_3 \end{array}\right]\left[\begin{array}{c} x_1 \\ x_2 \\ x_3 \end{array}\right],$$

with rescaling by $u_3$ and defining $\dot{x}_i = dx_i/(x_3dt)$

$$\left[\begin{array}{c} u_1/x_3 \\ u_2/x_3 \\ u_3/x_3 \end{array}\right] = \left[\begin{array}{ccc} d\dot{x}_1/(x_3dt) \\ d\dot{x}_2/(x_3dt) \\ d\dot{x}_3/(x_3dt) \end{array}\right] = \left[\begin{array}{c} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{array}\right] = \left[\begin{array}{ccc} A_{13} \\ A_{23} \\ 0 \end{array}\right] + \left[\begin{array}{ccc} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{array}\right]\left[\begin{array}{c} x_1 \\ x_2 \\ x_3 \end{array}\right],$$

where $A_{ij} = \partial \dot{x}_i/\partial x_j$ is defined as the no-slip tensor. More details and derivations on the no-slip tensor can be found in Chong et al. [1] and Cardesa et al. [9].

Similar to the invariants of the velocity gradient tensor, the no-slip tensor has three invariants $P$, $Q$, and $R$. They are related by the expansion

$$2P^3 + PQ + R = 0.$$

Integration of Eq. (4) yields the surface skin-friction field and hence a critical point in the skin-friction field occurs when $\dot{x}_1 = \dot{x}_2 = \dot{x}_3 = 0$. It can be shown that the no-slip tensor $A_{11}$, $A_{12}$, $A_{21}$, and $A_{22}$ are related to vorticity gradients, and $A_{13}$ and $A_{23}$ are related to pressure gradients. The topology of the critical points can be studied using critical point concepts described in Perry and
Chong [11]. It can also be shown that at a critical point in the skin-friction field, the no-slip tensor $A_{ij}$ is reduced to

$$A_{ij} = \begin{bmatrix}
    A_{11} & A_{12} & A_{13} \\
    A_{21} & A_{22} & A_{23} \\
    0 & 0 & -\frac{1}{2}(A_{11} + A_{22})
\end{bmatrix}. \tag{6}$$

The invariants of this no-slip tensor $P$, $Q$, and $R$ are not necessarily zero. At the surface of the wall

$$\begin{bmatrix}
    \hat{x}_1 \\
    \hat{x}_2
\end{bmatrix} = \begin{bmatrix}
    A_{13} \\
    A_{23}
\end{bmatrix} + \begin{bmatrix}
    A_{11} & A_{12} \\
    A_{21} & A_{22}
\end{bmatrix} \begin{bmatrix}
    \hat{x}_1 \\
    \hat{x}_2
\end{bmatrix}. \tag{7}$$

This means only two-dimensional information very near the wall is needed to classify the critical points of the skin-friction field as might be expected. The classification of these critical points can be determined by the $p$-$q$ chart as shown in Fig. 2. Here $p = -(A_{11} + A_{22})$ and $q = (A_{11}A_{22} - A_{12}A_{21})$.

Given that by definition a critical point occurs at $\hat{x}_1 = \hat{x}_2 = 0$, locating it on a presubscribed grid is extremely unlikely. The critical points are more likely to fall within the grid spacing $\Delta x_1 \times \Delta x_2$. The algorithm employed to identify critical points is described as follows. The left-hand side of Eq. (7) is set to $\hat{x}_1 = \hat{x}_2 = 0$, hence it becomes a straightforward approach to solving Eq. (7) as follows:

$$\begin{bmatrix}
    x_1 \\
    x_2
\end{bmatrix} = \begin{bmatrix}
    A_{11} & A_{12} \\
    A_{21} & A_{22}
\end{bmatrix}^{-1} \begin{bmatrix}
    A_{13} \\
    A_{23}
\end{bmatrix}. \tag{8}$$

This operation is performed at every grid point and produces a corresponding solution for $x_1$ and $x_2$. As illustrated in Fig. 3(a), only solutions within a filter $\Delta F_1$ of size $\pm \Delta x_1/2$ by $\pm \Delta x_2/2$ are considered. For example, the grid point $P_1$ (blue dot) has a critical point (CP) solution (denoted by the “×” symbol) that lies within the defined filter size ($\pm \Delta x_1/2$, $\pm \Delta x_2/2$; blue box), hence it
FIG. 3. Critical points identification and filtering process. (a) A single critical point is within the filter $\Delta F_1 (\pm \Delta x_1/2, \pm \Delta x_2/2)$ shown as blue and red rectangles. Gray dashed lines represent the computational grid and the critical points are denoted by symbol “×.” (b) Critical points $CP_1$ and $CP_2$ are within the same filter $\Delta F_1 (\pm \Delta x_1/2, \pm \Delta x_2/2$; gray box). The red, blue, green, and black dashed line boxes are of a bigger filter $\Delta F_2 (\pm \Delta x_1, \pm \Delta x_2$).

is accepted. If the CP solution lies outside the defined filter $\Delta F_1$, it is discarded, due to the fact that the CP will be a valid solution for another grid point. Figure 3(a) also shows the instance when two CPs are within the computational grid size shown as gray dashed lines. As discussed, with the chosen filter size $\Delta F_1$, the grid points ($P_1$ and $P_2$) will only pick up solutions closest to itself. The second filtering operation is performed with a bigger filter $\Delta F_2$ of size $\pm \Delta x_1$ by $\pm \Delta x_2$ denoted by red, blue, black, and green dashed line boxes shown in Fig. 3(b). This particular filtering operation considers the possibility when two CPs are within the initial filter $\Delta F_1$ defined in the first operation. Using Fig. 3(b) as an illustration, the grid point $P_2$ picks up the location of the critical point $CP_1$, however, there exists another critical point, $CP_2$, that does not get picked up by the other grid points ($P_1$, $P_3$, and $P_4$) surrounding it. Hence this second operation will not enable grid point $P_2$ to identify the critical point $CP_2$ but will instead allow grid points $P_1$, $P_3$, and $P_4$ to locate it. These two operations will ensure that no critical points are ignored. The final operation is a simple check that involves removing duplicated critical points due to the above operations performed. In the ideal case for identifying the exact locations of critical points, the grid resolution of the DNS should be fine enough so that all critical points lie on the grid. However, this is not feasible for the current study due to availability of computational resources necessary for the required resolution. The current technique identified approximately two critical points per $10^6$ viscous unit area, which is similar to what is reported for a channel flow [9]. These critical points are usually located within the grid as discussed above. Upon determining the exact location of the critical point, the pipe flow data is interpolated unto a new mesh (of identical grid resolution to the initial mesh), whereby the critical point will now lie on the new grid as illustrated in Fig. 4. On closer inspection, it was found that CP regions consist of one or more pairs for critical points and each pair consists of a node and a saddle (see Chong et al. [1] or Foss [20]). It was conjectured in Chong et al. [1] that this was expected by mapping the skin-friction field in the DNS of fully developed pipe flow (with streamwise periodicity) onto the surface of a torus (see Fig. 5). The periodic pipe flow can also be wrapped from the outlet to inlet similar to a sphere with two holes. For the surface of a torus, the Poincaré-Hopf theorem states that the number of nodes must equal the number of saddles.

A sample skin-friction field is shown in Fig. 6. This is the typical field where skin-friction lines (black lines) are predominantly oriented in the streamwise direction and the streamwise velocity is positive. The vorticity lines by definition, which are orthogonal to the skin-friction lines, are shown as red lines. There are no critical points and very little variation in the skin friction (which has a positive streamwise velocity value at all points in the field in Fig. 6).
However, at locations such as those shown in Fig. 7, the skin-friction fields (black lines) exhibit a more complex flow pattern. Critical points are observed and these cause significant disruption to the skin-friction field, resulting in back flow above the wall and bifurcation lines upstream of and downstream from the critical points. Very strong fluctuations (negative) in $\tau_{+1}$ are observed in the vicinity of the critical points; indeed, these fluctuations can be seen as extreme given that the mean value of wall shear stress is $\tau_{+1} = 1$. Figure 7(a) shows a saddle-node combination, with the node downstream from the saddle. This combination sets up a skin-friction field very much like that below the U separation shown in Fig. 1. With this arrangement of saddle and node, the U separation is stable. This appears to be the most common arrangement of no-slip critical points, yet certainly not the only type. It can also be shown that vorticity lines (found by integrating the vorticity field) are orthogonal to skin-friction lines. Figure 7(a) displays the vorticity lines (red lines) around the same critical point. A saddle in the skin-friction field corresponds to a saddle in the vorticity field and a node in the skin-friction field corresponds to a focus in the vorticity field. Note that away from the critical point, the vorticity lines are generally oriented in the spanwise direction.

In Fig. 7(a), a critical point pair appears in isolation; however, there are many instances where multiple pairs of critical points occur together, forming more complex flow patterns. An example is shown in Fig. 7(b). In this instance, there are multiple pairs of critical points (two nodes and two saddles) in very close proximity. The vorticity lines are quite different [compared with Fig. 7(a)] owing to the presence of the extra foci; although, the saddle and node combination with strongly positive and negative $\tau_{+1}$ still dominates the vicinity of the CP. Figure 7(b) shows a case where the top critical point pair is a saddle point located upstream of a node and the lower critical point pair is the reverse. Here the skin-friction field resembles that below an unstable U separation. This suggests a different flow field locally above the critical point, although all critical points shown would be expected to be associated with a pair of streamwise vortices in the wider flow field (if

![FIG. 4. Interpolation of data unto a new mesh grid in the $x_1-x_2$ plane. Gray dashed lines show the initial mesh grid and red dashed lines show the new mesh grid where the pipe flow data is interpolated unto.](image)

![FIG. 5. Illustration of how the planar surface of a turbulent, wall-bounded flow simulation with periodic boundary conditions (left) can be mapped onto a torus (right). Colors represent contours of the streamwise component of skin friction.](image)
the U-separation analogy holds). Figure 7 also illustrates that critical points exist in pairs in close proximity. Although only two instances are shown, the authors have viewed the field around many more critical points and found that the figures shown are indeed representative of all critical points in the field. The critical points are always of the saddle-node combination (the nodes can be stable or unstable), which ensures the Poincaré-Hopf theorem is satisfied.

IV. RESULTS

The Reynolds number and large computational volume of the DNS dataset means that critical points appear in sufficient quantities for statistical convergence of a conditional average. Here we calculate the conditional average of a fluctuating quantity, \( u'_i(x_1, x_2, x_3) \), given a critical point event:

\[
\langle u'_i \rangle_{CP} = \left( u'_i(x_1, x_2, x_3) \bigg| \frac{\partial u_1}{\partial x_3} = \frac{\partial u_2}{\partial x_3} = 0 \right),
\]

where \( \langle u'_i \rangle_{CP} = \langle u'_i \rangle_{CP}(\Delta x, \Delta r\theta, r) \) and \( \frac{\partial u_1}{\partial x_3}, \frac{\partial u_2}{\partial x_3} \) are evaluated at \((x_1 - \Delta x, x_2 - \Delta r\theta, 0)\). The superscript “+” denotes viscous scaling as defined earlier. The fluctuation quantity is a temporal variation to the complete population mean value.

The conditional average of all components of velocity was calculated in this way for a rectangular volume defined by the vertices \((-120, -120, 0), (750, 120, 250)\) with a no-slip critical point pair nominally centered at \((0,0,0)\). This is a limitation of the grid resolution to locate the precise locations of critical points as discussed previously. The volume size is chosen to enable the conditional

![Conditional average of fluctuating quantities](image)
averaging to sufficiently capture the turbulence physics important to the critical points. Conditional streamwise and spanwise skin-friction data are shown in Figs. 8(a) and 8(b), respectively. Firstly, it is statistically confirmed that strong fluctuations are associated with critical points. The unconditional mean viscous-scaled skin friction is unity and the conditional skin friction has fluctuations of that order in both the streamwise and spanwise components. Positive streamwise fluctuations of skin friction around a critical point pair are much less in magnitude than the negative fluctuations such that a strongly negative skewness of the skin friction is observed in these regions [see Fig. 8(a)].

Secondly, in Fig. 8(b), the spanwise wall shear stress also exhibits strong fluctuations upstream of and downstream from the critical point pair. These conditional average results are similar to those presented in a turbulent channel flow [9].

Turning our attention to the flow away from the wall, we now attempt to identify any coherent structure around the critical point. Figure 9(a) displays conditional averages of streamwise velocity fluctuation in the $x_1-x_3$ plane. The streamwise velocity above the critical point pair exhibits strong, compact velocity changes in the vicinity of the critical point, with average fluctuations from the mean of over $2\bar{U}_t$. Moreover, the negative velocity fluctuation seen near the wall (at $\Delta x^+ \approx 0$) switches sign above the wall, with high (positive) fluctuations extending right through the logarithmic region ($100 \lesssim x_3^+ \lesssim 0.15\delta^+$). The change in sign indicates the presence of a shear layer in the buffer region ($5 \lesssim x_3^+ \lesssim 60$) above a critical point. The magnitude of the observed velocity fluctuation is high with a maximum of over $+10\%$ observed in the logarithmic region, while the minimum descends below $-30\%$ close to the wall. In the streamwise direction, the velocity field shows significant correlation with the critical point event even up to 700 wall units ($\sim 0.7\delta$) downstream. The scale of these flow variations was somewhat surprising given that the critical point pair is a local, small-scale event confined to the wall. The wall-normal extent would suggest a connection between the large-scale outer flow and the critical point pair. Indeed, the angle of the shear layer emanating from the critical point pair follows the large-scale structure inclination angle.
FIG. 9. Conditional averages above a critical point centered at $x_1 = 0$ in the $x_1$-$x_3$ plane for (a) streamwise velocity fluctuation, (b) spanwise velocity fluctuation, and (c) Reynolds stress. The black dashed lines are at an incline angle of 14°.

of 14° (see Marusic and Heuer [21] and Brown and Thomas [22]) as indicated in the figure as a black dashed line. It is interesting to note that the region of high (positive) fluctuations also exhibits a similar inclination angle of 14°. This is possibly due to hierarchies of large-scale motions in the flow above the critical point pair. The streamwise extent of the low speed region and its confinement below $x_3^+ = 50$ implies a strong connection with the near-wall cycle (see Schoppa and Hussain [23] and Jimenez and Pinelli [24], for example). However, the moderate Reynolds number of the available data precludes a conclusive analysis of the scaling of structure associated with the critical point pair.

Figure 9(b) displays the conditional average of the wall-normal velocity in the streamwise–wall-normal plane. It appears that flow is ejected from the wall upstream of and downstream from a critical point pair, with a strong downward flow directly above it. Again, the magnitude of the fluctuations is very high, reaching 5% of the mean streamwise velocity in the logarithmic region (negative fluctuations) and up to 50% near the wall (positive fluctuations). Also, the scale of the associated field is again demonstrated as the downward velocity fluctuations are still of notable magnitude in the logarithmic region. There is a strong spanwise vortical motion slightly upstream of the critical point pair, preceded by an inclined shear layer in the buffer layer extending over 100 viscous units in the streamwise direction. A similarly located spanwise vortex was shown by Lenaers et al. [3] in presenting the conditional average of the flow above negative flow regions near the wall. However, they performed the analysis on a small field of view so there was no mention of an extended shear layer downstream. Figure 9(c) shows the conditional average of Reynolds stress
above the critical point pair. One would immediately notice two strong regions of Reynolds stress: one above the critical point pair and the other at a downstream distance of $\Delta x^+ \approx 100$. These regions are directly connected with the ejection event and it is likely that there exist individual turbulence structures associated with each region.

The three-dimensional nature of the conditionally averaged flow is shown by an isocontour plot of $\lambda^+$ in Fig. 10. According to Jeong and Hussain [25], this quantity can be used to identify regions of vortical activity. It is clear that a vortical structure exists above the critical point pair resembling a hairpinlike shape. Downstream from the critical point, there is a pair of counter-rotating quasistreamwise vortices, which could be the legs of another, larger hairpin vortex or the streaks in the near-wall cycle. Another interpretation is the counter-rotating quasistreamwise vortices closely resemble the U separation shown in Fig. 1(a). The contour plots shown in Fig. 9

FIG. 10. Isocontour of $\lambda^+$ calculated from the conditionally averaged velocity gradient tensor. The contour level is $\lambda^+ = -0.0013$.
indicate a downward force on these vortical motions imparted by the flow above the buffer layer. It is possible that the critical point pairs are not necessarily the cause of the activity above the wall, but a result of the pressing down of a hairpin-shaped (or at least a spanwise-oriented) vortex by the larger-scale, high-speed flow above the wall. These near-wall vortical structures potentially play an important role in the vorticity transport [26]. Finally, Fig. 11 shows the percentage of the type of critical point that dominates the pipe flow in a normalized $p-q$ plane. As expected, the number of saddles is equal to the number of nodes (sum of stable and unstable). Interestingly, the occurrence of stable and unstable U separation is similar, which would explain the mean turbulence structure shown in Fig. 1.

A concise analysis of the temporal evolution of critical points has been conducted on the pipe flow simulations. It should be noted that it is difficult to conduct this analysis with statistical convergence owing to the enormous amount of data required to do so at the current Reynolds number. Firstly, a critical point pair has been found in the pipe flow numerical simulations as shown in Fig. 12. The figure shows the same critical point pair at two time instances separated by $11v/U_2^+ s$. While many critical points are observed to appear and disappear on timescales of $T^+ = O(1)$, there is evidence that some critical points are long-lived, such as that shown here. The total lifetime of this critical point pair is approximately 20 wall time units. A convection velocity of the critical point pair can also be calculated from this figure, since the critical point pair appears to have shifted downstream by $\Delta x^+ \approx 130$ in $T^+ \approx 11$. The streamwise convection velocity of this particular critical point pair is therefore estimated at $U_c^+ \approx 12$. This is considered very high speed for the near-wall region; in fact, it is characteristic of a velocity of the near-wall cycle (i.e., the mean velocity at $x_3^+ \approx 20$), suggesting the critical point pair may be related to the sublayer streaks. The time evolution of the turbulence structures above a critical point pair is illustrated in Fig. 13. The critical point pair is located at wall location $x_1^+, x_2^+, x_3^+ \approx (500, 600, 0)$ in Fig. 13(a) and the turbulence structures are shown using the $\lambda_2$ criterion colored by vorticity magnitude.

FIG. 12. Skin friction in the vicinity of a critical point at time (a) $T^+$ and (b) $T^+ + 11$. Colored contours represent the skin friction.

FIG. 13. Time evolution of turbulence structures ($\lambda_2^+$ colored by vorticity magnitude) above a critical point at time instances (a) $T^+$, (b) $T^+ + 6$, and (c) $T^+ + 12$. The critical point in (a) is at $(x_1^+, x_2^+) \approx (500, 600)$. 

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magnitude. These turbulence structures appear to convect downstream with the critical point pair as shown in Figs. 13(b) and 13(c). Above this critical point pair, a vortical structure (resembling a hairpinlike shape) is present with a pair of streamwise vortices flanking both sides. This is similar to the conditionally averaged $\lambda^+$ structures observed in Fig. 10. These flanking streamwise vortices could potentially be part of a larger-scale hairpinlike structure; however, further research is required to validate this.

V. CONCLUSION

From an analysis of a turbulent pipe flow DNS at $Re_\tau \approx 1000$, further evidence for the existence of no-slip critical points has been found. Critical points always appear in saddle-node pairs; in many cases multiple pairs of critical points interact in close proximity. Applying a two-dimensional classification of critical points in the $p$-$q$ plane shows that 50% of critical points are in a saddle in front of the node configuration and 50% vice versa. These result in skin-friction patterns around critical points most often resembling either an unstable or a stable three-dimensional U separation [11], due to the presence of the saddle-node pair and the mean flow. This finding suggests the U separation could form the basis of a simple analytical model for the turbulent skin-friction field. The conditional average of the velocity field above a critical point pair (or collection of critical points in close proximity) reveals a strong spanwise vortex above, with further vortical activity downstream. Violent fluctuations above and around the critical point pairs were observed in all components of velocity and skin friction. The extent of the flow related to the critical point pair was also quite surprising, with activity in the logarithmic region clearly associated with it. It is not yet clear whether these critical points are a result of the larger-scale flow field or a source of the large-scale vortical activity. It is possible that the large-scale motions act to push spanwise vortical structures toward the wall thereby creating a critical point event. However, further work is required to understand such mechanisms; particularly to determine causality of the critical point with respect to the flow field.

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