Measurements from flame chemiluminescence tomography of forced laminar premixed propane flames

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A R T I C L E   I N F O

Article history:
Received 24 January 2017
Revised 9 March 2017
Accepted 3 May 2017
Available online 18 May 2017

Keywords:
Tomography
Premixed combustion
Laminar flames
MART

A B S T R A C T

The unsteady, visible, chemiluminescence fields of two non-axisymmetric, forced, laminar, premixed, propane–air flames are tomographically reconstructed using between 3 and 36 equally-spaced views. Algorithms for measuring flame surface area, flame curvature, flame thickness, and the normal component of the flame propagation velocity (surface speed) are demonstrated. The sensitivity of each measurement to the number of views used in the reconstruction is then assessed. For both flames studied, the difference between flame surface area fluctuations measured using 36 views and those measured using as few as 9 views was less than 1%. For the other three quantities, a measurement sample is acquired over the entire flame surface at one phase of the forcing cycle for each flame. The sensitivity to the number of views is compared by assessing the similarity of measurement distributions obtained using the maximum number of views to those obtained using fewer views. The surface speed measurement distribution is found to converge fastest as the number of views was increased, though results for mean curvature are similar. However, the flame thickness measurement distribution was found to have significantly slower convergence and more than twice the number of views are required to measure flame thickness compared to curvature or surface speed. The demonstrated measurement algorithms are generally applicable to the chemiluminescence fields of wrinkled, premixed flames. The results suggest that for laboratory-scale, weakly turbulent, premixed, jet flames, statistical measurements of flame curvature and surface speed may be accurately obtained using as few as six views, while greater than 20 views are likely to be required to obtain useful measurements of flame thickness.

1. Introduction

There has recently been significant progress in the development of measurement methods capable of obtaining pseudo-instantaneous and time-resolved flame measurements over an entire combustion region. Two main approaches are used: laser scanning methods [1–4], and tomographic methods [5–24].

Tomographic methods involve reconstructing a three-dimensional scalar field from integral measurements. The use of tomography in combustion has been limited primarily by the difficulties and cost associated with obtaining measurements of the required number of projections. As reconstruction algorithms are optimized for combustion applications, and optical equipment continues to decrease in price, tomographic methods are anticipated to have greater applicability in both combustion research and for online diagnostics of industrial combustors [25].

Most previous high resolution flame tomography has used one of two approaches: light scattered by oil droplets or solid particles in the unburned premixture, and using the light emitted from the flame itself. The latter is known as Flame Chemiluminescence Tomography (FCT) and is the subject of this paper. The following review of light scattering methods is therefore kept concise. Upton et al. demonstrated tomographic reconstruction of oil droplet evaporation on turbulent premixed flames using 12 flame images [26]. Tomographic particle image velocimetry (PIV) is a related method that uses lower seeding densities to allow reconstruction of the scattering from individual droplets [11,27] or solid particles [12]. The resolution of the flame front position is limited by the sparse seeding density, typically to approximately 1 mm. In comparison, FCT involves imaging the light emission from the flame, and likely provides a more accurate determination of the reaction zone and the combustion processes. Chemiluminescence imaging is also non-intrusive and passive. Recently, tomographic reconstruction of laser induced fluorescence (LIF), termed ‘volumetric LIF’ has also been demonstrated [17,18]. This approach has the advantage that the entire flame need not be reconstructed, but

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in many ways is similar to FCT, and the algorithms used to obtain measurements from the reconstructed fields are likely to be similar. Several works detail the reconstruction methods typically used in FCT, e.g. [14,28]. Methods can be categorized as series expansion methods and transform methods. Series expansion methods involve representing the field as a set of basis functions and solving a system of equations for the basis function coefficients [28,29]. Series expansion methods have been favored over transform methods for FCT due to the limited number of views often available. The system of equations is typically underdetermined and sometimes inconsistent, so several methods to find an optimal reconstruction have been proposed.

The additive and multiplicative variants of the algebraic reconstruction techniques (ART/MART) are the series expansion methods which have been the most widely used in previous FCT implementations [5,13,23,30]. Other methods involve formulating the problem with additional optimization criteria, often incorporating prior knowledge of the field. Anikin et al. reconstructed the OH* emission of a turbulent bluff-body flame using a regularization parameter based on the Euclidean norm [31]. Ishino and Ohiwa used Maximum Likelihood Expectation Maximization to reconstruct the broadband chemiluminescence of a turbulent jet flame [15]. Cai et al. reported improved results over ART by using a total variation regularization method in reconstructing artificial data representing a modified McKenna burner [32]. Denisova demonstrated an improvement of the maximum entropy (MENT) method over ART in the reconstruction of two numerical examples, but did not consider examples representative of flame chemiluminescence [33]. Denisova et al. later reported reconstructions of a conical flame using MENT and showed the MENT method could be improved by local smoothing of the projections [14]. Further study was said to be required for applying this method to non-axisymmetric flame tomography.

To the authors’ knowledge, for unsteady, wrinkled, premixed flames, there are only two instances in the literature where 3D flame measurements at points on an identified “flame surface” have been made using FCT. Ishino et al. report measuring the flame propagation velocity on a small part of a turbulent flame surface reconstructed using 20 equally-spaced, coplanar views, but do not report their methods [34], and Ma et al. recently reported 3D curvature measurements on a turbulent slot flame calculated on an isosurface of the CH* chemiluminescence field, reconstructed using 6 coplanar views [16].

FCT based measurements have also been reported in several other combustion studies. Ma et al. reconstructed a combustion region in a Mach-2 combustor using 8 views [20,21]. Flame surface area and volume were computed using an isosurface and voxel thresholding respectively. Cross-sections of the combustion zone did not reveal the presence of wrinkled flames, and the computed isosurface enclosed the combustion region. FCT has also been applied to the reconstruction of time-averaged turbulent flame brushes [7,8,10,13,35]. However, chemiluminescence fields from turbulent flame brushes typically have very different characteristics to ‘instantaneous’ flame surfaces. Using volumetric LIF, Ma et al. tomographically reconstructed part of a turbulent jet flame and reported flame length measurements, measured on 2D cross-sections [17].

Whilst a significant amount of work has been dedicated to determining optimal reconstruction algorithms for FCT, the identification, demonstration, and development of robust and accurate algorithms for obtaining measurements from the reconstructed chemiluminescence field is equally important for minimizing the number of views required to obtain measurements with a desired accuracy. Furthermore, it is not currently clear which quantities can be most accurately measured when using a low number of views (≤ 20).

In this paper, algorithms for measuring the flame surface area, flame surface curvature, the normal component of the flame propagation velocity (surface speed), and the flame thickness are proposed and demonstrated on the time-resolved chemiluminescence field of two non-axisymmetric, forced, laminar, premixed, propane flames at different equivalence ratios: $\phi = 0.71$ and $\phi = 0.84$. Measurements of the flame thickness and surface speed over an ‘instantaneous’, unsteady, non-axisymmetric, premixed flame have not previously appeared in the literature. The measurement algorithms are then applied to flame chemiluminescence fields reconstructed using between 3 and 36 views and the relative sensitivity of the measurements to the number of views is assessed.

The presented results: (i) show the relative sensitivity of the measured quantities to the number of views used in the reconstruction, and (ii) provide an estimate of how many views are likely to be required to obtain statistical measurements of these flame quantities in flames of similar character, that is, laboratory-scale, weakly-turbulent, lean, premixed flames. Such measurements are likely to be useful for the validation of theoretical models and numerical simulations.

2. Experimental and tomographic methods

2.1. Forced laminar premixed non-axisymmetric burner

A flame holder with a square port was used to stabilise non-axisymmetric, laminar, premixed flames (Fig. 1). The sides of the port are 22 mm long with rounded corners of 2 mm radius. The lip has a vertical inner surface and is tapered to a knife edge at a 60° angle. The flame holder can be rotated about the vertical axis to allow any number of coplanar views to be obtained without moving the camera. A vernier angle scale is used to measure the viewing angle $\beta$ to an accuracy of ± 0.1°.

Flow rates of dry, filtered, compressed air and propane (99.95% purity) were controlled using MKS thermal flow meters, models 1559A and M100B respectively. Two lean, premixed flames of different stoichiometry are investigated in this study. These are referred to as Case 1 and Case 2 (Table 1), and were chosen given their strong flame dynamics, including annihilation events that the group has been studying as acoustic sources [36-38]. The propane and air are mixed approximately three meters upstream of the burner to ensure a well mixed mixture at the burner port [39]. This mixture passes through a 60 mm honeycomb flow straightener in the burner plenum before entering a contraction upstream of the rotatable flame holder. Layers of fine steel mesh before and after the contraction are used to dissipate turbulence. The flame geometry was found to be particularly sensitive to small irregularities in the final mesh layer. This mesh layer was therefore installed to
Table 1
Key experimental parameters for cases 1 and 2. The mean bulk velocity is estimated based on the measured mass flow rates and the component gas densities at 25 °C and 1 atm. Reynolds number is based on the length of the square port edges.

<table>
<thead>
<tr>
<th></th>
<th>Case 1</th>
<th>Case 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>φ</td>
<td>0.71</td>
<td>0.84</td>
</tr>
<tr>
<td>Air flow rate (gs⁻¹)</td>
<td>0.53</td>
<td>0.94</td>
</tr>
<tr>
<td>Propane flow rate (gs⁻¹)</td>
<td>0.024</td>
<td>0.051</td>
</tr>
<tr>
<td>Unburned gas mean bulk velocity (ms⁻¹)</td>
<td>0.87</td>
<td>1.6</td>
</tr>
<tr>
<td>Reynolds number</td>
<td>1100</td>
<td>1900</td>
</tr>
</tbody>
</table>

2.2. Flame imaging

Images from 36 coplanar views, equally spaced through 180°, were taken of both test flames. These were captured with a LaVision Flowmaster 3S camera system with a LaVision high-speed IRO image intensifier. A Sigma 150 mm f/2.8 lens was used with a Kenko MC UV low pass filter. The aperture was fully open, giving an f-number of 2.8. A gate width of 100 μs was used. This 12 bit camera has a full-frame resolution of 1280 × 1024. The camera was positioned so that the image plane was parallel to the flame holder’s axis of rotation.

Arduino microcontrollers were used to generate a 50 Hz sinusoidal input to the speaker amplifier and to control the relative phase of the camera exposure. One hundred phases of each flame cycle were imaged, which is equivalent to imaging at 5000 Hz. At each phase and for each view, three images were acquired. For 36 views, this amounts to 10,800 images acquired for each of the flame cycles. A background image, obtained without the flame, was subtracted from each image, and each image was corrected for non-uniformity in the response of the image intensifier.

Figure 2 shows three images acquired from the same view and at the same phase for both cycles. The images were taken 1785 cycles (36 s) apart. SNRs were measured using the definition used by Floyd et al. [5],

\[
SNR = \frac{\overline{P}}{\sigma_s^2}
\]

where \(\overline{P}\) is a representative mean signal power and \(\sigma_s^2\) is the signal variance. To measure the SNR, sample areas of the images were used where the imaged flame surfaces are perpendicular to the imaging axis and the true image intensity is therefore nearly uniform. For Case 1, the SNR in the three images in Fig. 2 was 30.2, 31.3, and 36.0. For Case 2, the SNR in the three images was 115.0, 115.9, and 107.5. These SNRs are conservative as the signal increases in areas where the flame surface is at a lesser angle to the imaging axis.

The images input to the reconstruction algorithm were composite images, constructed using the median intensity of each pixel in three images taken at the same phase. The composite images have an increased SNR when measured in the manner just described, but imperfect repeatability of the flame cycle may limit the temporal resolution. The images in Fig. 2 demonstrate that repeatability is high. For both cases, the flame edge was found to move by no more than five pixels at any position. It is reasonable to expect the effect of imperfect repeatability would be a thickening of the chemiluminescence region. This is likely to have little effect on measurements of curvature and velocity, which, as will be described later, are computed using the layer of peak emission, but an increase in the measured thickness is expected and is estimated to be about 3 pixel lengths, or 0.2 mm. However, the main results of this study, which are to estimate the number of views required for measurements of certain statistical quantities to converge, and to assess the relative sensitivity of the measured quantities to the number of views, is unlikely to be significantly affected.

2.3. Tomographic reconstruction

Horizontal cross-sections of the flame were reconstructed using a MART reconstruction algorithm [42,43]. The MART method approximates the chemiluminescence emission field, I, using basis functions, \(b_j\):

\[
\hat{I} = \sum_j b_j b_j
\]

where \(b_j\) are unknown basis function coefficients. Voxel basis functions were used and the coefficients \(b_j\) are voxel intensity values constrained to non-negative values. The algorithm is essentially a 2D image reconstruction algorithm, using the same row of camera pixels from each viewing angle to reconstruct the intensity field of a cross-section. This reconstruction method is possible when orthographic projections are assumed and coplanar views are used.
associated with each camera pixel. A common simplification has been implemented where voxels are treated as cylinders with an equal volume [44]. Weightings are then independent of the angle of intersection and are only dependent on the perpendicular distance between a line projected from the centre of the camera pixel and the centre of the voxel. The weighting array is the same for each cross-section and so need only be calculated for one cross-section.

For each cross-section, five iterations of the MART algorithm [44] were used to reconstruct the emission intensity field on a 400 × 400 grid, with a grid spacing of 74.1 µm. The grid spacing was chosen to be equal to the image sensor pixel length (67.3 µm) divided by the image magnification (0.0904), so that the individual cross-sections form a 3D cartesian grid when stacked. A relaxation factor of 0.3 was used.

3. Post-processing algorithms

3.1. Flame surface area

MATLAB’s isosurface function was applied to the reconstructed emission fields. The algorithm produces a set of triangular faces, each defined by their vertices $v_1$, $v_2$, $v_3$. The surface area of face $p$ is then

$$A_p = |(v_2 - v_1) \times (v_3 - v_1)|/2.$$  (3)

Two isosurfaces are created either side of the peak emission layer and the flame surface area is therefore $A_f = \frac{1}{2} \sum_p A_p$. Three dimensional mean filtering was applied to the reconstructed fields prior to computing the isosurfaces. A sensitivity analysis was undertaken to determine appropriate values for the filter kernel size and the isosurface value. A filter kernel size of 11 voxels was used and the isosurface value is 15% of the maximum emission intensity $I_{max}$ at a chosen instant in the cycle. A wide range of isosurface values, from less than 7% to greater than 20% of $I_{max}$, was found to result in computed areas within 1%, though a greater amount of filtering is required when using smaller isosurface values.

3.2. Local surface fitting

Measuring curvature, surface speed, and thickness requires identifying a surface to represent the flame location. This study defines the flame surface as the set of points that are at the peak of the emission profile along a flame surface normal. This is chosen instead of the more common isosurface method to avoid processing issues, such as sensitivity to thresholds levels in regions of high curvature, and failure of surface identification near flame holes.

The algorithm comprises two key steps:

1. identifying points near the flame surface.
The proximity of a pixel to the flame surface was determined using a two part algorithm. The first part (Algorithm A) flags a given pixel if the difference in direction of the gradient between any of the four opposing pairs of pixels is greater than 90° (Fig. 3c). Gradient vectors typically point towards the pixels that are flagged by Algorithm A. The exception is in regions of high intensity, such as near the flame tip (Fig. 3d). Here the gradient vectors can point away from the flagged pixels and towards the flagged pixels in the high intensity region. Algorithm A then fails to flag pixels near these regions, as shown in the dashed box in Fig. 3c.

The second part (Algorithm B) is then applied (Fig. 4). This algorithm only considers pixels neighboring those that have already been flagged. If the gradient vector at these pixels is directed away from previously flagged pixels then these pixels are also flagged. Algorithm B loops until no further pixels are flagged, proceeding from the ‘typical’ flamelet regions that were identified by Algorithm A to the crest of the more highly curved flame regions, following a path with the highest emission intensity.

This two part algorithm is generalized for the 3D data in this study by using the neighboring 26 voxels (13 opposing pairs) for Algorithm A. Algorithm B was unchanged.

3.2.2. Locally fitting polynomial surfaces to the identified points

The linear regression method described by Flynn and Jain [45] has been used to locally fit polynomial surfaces of the form:

\[
h(x', y') = a_{00} + a_{10}x' + a_{01}y' + a_{11}x'y' + a_{20}x'^2 + a_{02}y'^2 + a_{21}x'y'^2 + a_{12}x^2y' + a_{22}x'^2y'^2
\]

Each local surface fit is centered on one of the flagged voxels. The centre points of other flagged voxels within a specified radius, called the inclusion sphere, are included in the surface fit. Prior to the surface fit, the coordinate system is transformed to put the center of the inclusion sphere at the origin and rotated such that the support plane \((x', y')\) is perpendicular to the third principal component direction [46]. This plane is parallel to the plane for which the sum of squared perpendicular distances between the points and the plane is a minimum. Rotation of the coordinate system in this manner increases the probability of a good surface fit.

A root mean square error (RMSE) criteria of 1.4 voxel lengths and a maximum residual (MR) criteria of 4 voxel lengths were used as goodness of fit criteria. An inclusion radius of 20 voxel lengths was used in the first surface fit and the inclusion radius was decreased by a factor of 21/4 until the goodness of fit criteria were met or until an inclusion radius of 10 voxel lengths was reached. This factor was used for reducing both the computational effort and the sensitivity of the surface fit to the scatter of the points around the flame surface. Once a surface meets the goodness of fit criteria, the inclusion volume centre point, originally at \((x', y', z') = (0, 0, 0)\) is projected onto the fitted surface to obtain the flame surface point at \((x', y', z') = (0, 0, h(x', y'))\).

3.3. Calculating the curvature

The principal curvatures \((\kappa_1, \kappa_2)\) at a point on a surface \(\sigma(x', y')\) are the maximum and minimum normal curvatures. The mean curvature that is commonly referred to in the combustion literature is the mean of the principal curvatures. The principle curvatures of a surface are given by the roots of the equation:

\[
\begin{bmatrix}
L - \kappa E & M - \kappa F \\
M - \kappa F & N - \kappa G
\end{bmatrix} = 0
\]

where \(E, F, G\) and \(L, M, N\) are the coefficients of the first and second fundamental forms respectively [47]. For a surface \(\sigma(x', y') = (x', y', h(x', y'))\) these coefficients are:

\[
E = 1 + h'^2
\]
Fig. 7. Sample unfiltered cross-sections of the emission field reconstructed using 36 views (EF36). The z values refer to the height above the burner lip. Dashed boxes indicate slices in Fig. 8.

![Image](image1.png)

Fig. 8. A sample cross-section of the emission field indicated in Fig. 7 and reconstructed using different numbers of equally-spaced views.

\[
L = \frac{h_{v'}}{\sqrt{1 + h_x^2 + h_y^2}} \quad (7)
\]

\[
F = h_x h_y \quad (8)
\]

\[
M = \frac{h_{v'}}{\sqrt{1 + h_x^2 + h_y^2}} \quad (9)
\]

\[
G = 1 + h_y^2 \quad (10)
\]

\[
N = \frac{h_{v'}}{\sqrt{1 + h_x^2 + h_y^2}} \quad (11)
\]

In Level Set descriptions of the flame surface, \( G(x, y, z) = c \) (an isosurface). The mean curvature is then related to the divergence of the normal vector [48]:

\[
\kappa_m = \nabla \cdot (\nabla G/|\nabla G|)/2. \quad (12)
\]

3.4. Calculating the normal component of the flame propagation velocity

The velocity of the flame surface in its normal direction at some point \( x_0 \) is estimated from the distance between intersections of that normal with the flame surfaces that are 200 \( \mu s \) either side of the current instant. See Fig. 5 for a 2D example. The flame surface normal at \( x_0 \) is \( (h_{x'}, h_{y'}, -1) \). Starting from \( x_0 \) in the earlier (or later) instant, voxels intercepted by the surface normal are stepped through in both directions. When a flagged voxel is encountered the intersection point of the normal line and the surface tangent plane at the flame surface point associated with the intercepted voxel is calculated. If a flagged voxel is not encountered when moving along the surface normal, the speed is not calculated at that point.

The flame speed \( (s_f) \) and flame propagation velocity \( (V_f) \) are related by:

\[
V_f = v + s_f n \quad (13)
\]

where \( v \) is the unburned gas velocity. The surface speed measured in this paper is an estimate of \( V_f \cdot n \) [48,49].

3.5. Calculating the flame thickness

The Full Width at Half Maximum (FWHM) of the emission profile in the flame surface normal direction is calculated and reported as the flame thickness. Trilinear interpolation was used to determine the intensity in steps of 19 \( \mu m \), equal to one quarter of the voxel length. As can be seen from Fig. 6, filtering of the reconstructed emission field significantly affects the measured profile maximum and FWHM. Thickness measurements are therefore derived from the unfiltered emission field.

3.6. Determining the unburned and burned gas side

Flame velocity and curvature measurements are only useful if the side of the unburned gas is known. A segmentation method was applied to the entire reconstruction volume to assign voxels to the flame region, unburned gas region, or the burned gas region. The unburned gas was then determined by analysing the assigned regions intercepted by the flame surface normal in both directions.

4. Results and discussion

4.1. Reconstructed emission field

The emission field reconstructed using x views is referred to as EFX. Sample cross-sections from EFX are shown for both cases in Fig. 7. Thin, connected, high intensity areas are observed in the
cross-sections, characteristic of forced, laminar, premixed flames [50].

The accuracy of the approximate solution computed using the MART algorithm is understood to generally increase as the number of views is increased [28,51]. Figure 8 demonstrates the effects of reducing the number of views using the sample cross-sections indicated in Fig. 7 for each of the cases. Tomographic artefacts progressively appear, starting with breaks in the flame surface and followed by the appearance of lines perpendicular to the projection planes. The flame surface begins to lose coherence with fewer than nine views. A similar trend was observed for other cross-sections and at other instants in the forcing cycle, with Fig. 9 showing that 3D isosurfaces of these two flames also feature these artefacts as the number of views is reduced.

4.2. Flame surface area measurements

Figure 10 shows the flame surface area fluctuations \( \frac{A' - \bar{A}}{\bar{A}} \) calculated using 36 views for 100 equally-spaced instants through the flame cycle for both cases. The flame surface area fluctuations are less than 2% for Case 1, despite large changes in the flame geometry. For Case 2, area fluctuations reach roughly 6%. Also shown are the fluctuations obtained using different numbers of views at eight instants through the cycle. The cycle is well represented with as few as nine views for both cases.
The difference in the area fluctuation between the two cases is related not only to the amplitude of the velocity forcing but also the Strouhal number \( \text{St} = \frac{\omega R}{S_L \cos \theta_0} \) [39,52]. Here, \( \omega \) is the angular forcing frequency, \( R \) is the burner radius, \( S_L \) is the laminar flame speed, and \( \theta_0 \) is the flame half apex angle for the unforced flame. The flame speed for Case 2 is 39% higher than for Case 1. The Strouhal number is 28% lower as a consequence and the wavelength of the transverse wave along the flame surface is larger. The impact of these variations has been studied extensively in other works and so is not considered further [39,52–56].

Table 2 shows \( \bar{A} \) calculated from eight instants, using different numbers of views. The computed flame surface area usually, but not always, increases as the number of views used in the reconstruction decreases. Figure 11 compares the flame surface area fluctuations, calculated at 100 instants through the cycle using 36 views, to fluctuations in the total chemiluminescent emission. The total chemiluminescent emission was obtained by summing the pixel counts in the flame images for each instant. There is close agreement, as has been observed previously in axisymmetric, forced, laminar flames [57].

![Fig. 10. Flame surface area fluctuations measured using different numbers of views: solid line 36, ○ 18, □ 12, ◊ 9, △ 6, ▽ 4.](image1.png)

![Fig. 11. Fluctuations in the flame surface area and total chemiluminescent emission.](image2.png)

![Fig. 12. Overlapping coefficient between measurement distributions obtained from EF36 and those obtained using fewer views.](image3.png)

<table>
<thead>
<tr>
<th>Number of views</th>
<th>Case 1</th>
<th>Case 2</th>
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<tbody>
<tr>
<td>36</td>
<td>18.9</td>
<td>21.6</td>
</tr>
<tr>
<td>18</td>
<td>18.9</td>
<td>21.7</td>
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</tr>
<tr>
<td>4</td>
<td>18.9</td>
<td>22.6</td>
</tr>
</tbody>
</table>
4.3. Probability density functions of the flame curvature, speed, and thickness

Probability density functions (PDFs) are presented to assess convergence as the number of views used in the reconstruction varies. The overlapping coefficient (OVL) \[ \text{OVL} = \sum_{i} \min[P_i, Q_i] \times 100\% \] (14)

where \( P_i \) and \( Q_i \) are the \( i \)th element, corresponding to the \( i \)th bin, of two discrete probability distribution vectors \( P \) and \( Q \). An OVL of 100% denotes two identical PDFs.

The OVL reported for \( x \) views compares the distribution found using \( E_F \) to the distribution found using \( E_{F_{18}} \), which is the best available estimate of the true measurement distribution. The bin widths for these PDFs are based on estimates of the measurement resolution: 0.01\( \delta_{th} \) for the curvature measurements, 0.1\( \delta_L \) for the speed measurements and 0.05\( \delta_{th} \) for flame thickness.

The laminar flame properties used for normalization were estimated based on kinetic modelling. The AramcoMech 2.0 mechanism [59] was used in a Chemkin premixed laminar flame-speed model, using an unburned mixture at 25 \( ^\circ \)C and 100 kPa. The calculated planar laminar flame speeds were 0.23 ms\(^{-1} \) and 0.32 ms\(^{-1} \) for Case 1 and Case 2 respectively and the planar flame thicknesses were 0.53 mm and 0.43 mm respectively, based on the definition \( \delta_{th} = (T_b - T_u)/|dT/\text{dx}|_{\text{max}} \), where \( \delta_{th} \) is the flame thickness, \( T_u \) and \( T_b \) are the unburned and burned gas temperatures, and \( dT/\text{dx} \) is the temperature gradient.

To reduce the computational expense, flame measurements were taken at a sample of points. The sampling algorithm progresses down a randomly ordered list of the flame surface points, adding a point to the measurement sample if it is further than 5 voxel lengths from any flame surface point already selected for the sample.

Calculated OVLs are presented in Fig. 12 for these three quantities. The observed trends and their causes are considered in the following sections.

4.4. Normalized mean curvature

Measurements of normalized mean curvature for both cases are shown in Fig. 13 and the corresponding PDFs are given in Fig. 14. From Fig. 12, when increasing from 6 to 18 views the OVL increases from 85% to 97% for Case 1 and from 81% to 94% for Case 2. Figure 13 demonstrates how these measurements are affected as the number of views varies. For Case 1, the results obtained with 36 views and 18 views are very similar, and this is reflected in the high OVL of 97%. With fewer views, measurement omissions and errors continue to increase. A similar trend is observed for Case 2.

4.5. Normalized surface speed

Measurements of the normal component of the propagation velocity are shown in Fig. 15 and the corresponding PDFs in Fig. 16. The mean unburned bulk gas velocities, when normalized by the laminar flame speed, are 3.8 for Case 1 and 4.8 for Case 2. Figure 12 shows the convergence of the speed distribution as the number of views varies. The OVL is generally higher for the speed measurement distribution than for the curvature distribution, indicating that surface speed can be measured with fewer views than curvature. This is despite higher numbers of measurement omissions for speed measurements. Omissions occur for both curvature
and speed measurements when a poor surface fit is identified in the present instant. Speed measurements are also omitted when the surface normal does not intersect with the flame surface in the previous or next instants. Two factors are likely to be involved in the faster convergence for speed measurements: a weaker correlation between the occurrence of measurement omission and the surface speed; and a greater tolerance to reconstruction errors. Quantification of the relative contribution of each factor is beyond the scope of this study.

Figures 17 and 18 show the results for the flame thickness. The position of the PDF peak monotonically decreases from 0.89 mm ($1.7\delta_m$) with 6 views to 0.49 mm (0.92$\delta_m$) with 36 views for Case 1 and from 1.07 mm (2.5$\delta_m$) with 6 views to 0.49 mm (1.1$\delta_m$) with 36 views for Case 2. The OVLs in Fig. 12 are lower than for curvature or speed measurements, reaching only 76% with 18 views for Case 1 and 59% with 18 views for Case 2. With 36 views, it is not known whether the measured flame thickness distribution has converged. Few measurements of the visible flame thickness have been reported in the literature. The visible chemiluminescence measured by unfiltered optical systems is mostly produced by CH$^+$ and CO$_2^*$. Numerical results from Samaniego et al. [60] found that for a lean methane-air flame ($\phi = 0.55$), the CO$_2^*$ emission profile is approximately 400 μm wide (FWHM), with the peak located approximately 200 μm downstream of the peak heat release rate. The FWHM of the CH$^+$ profile is approximately 100 μm [61] and the profile peak is located within 100 μm downstream of the peak heat release rate [62]. Based on these results, the true thickness of the visible chemiluminescence profile is likely to be between 0.1 mm and 0.5 mm. And, as discussed in Section 2.2, imperfect repeatability of the flame cycle may increase the flame thickness in the reconstructions in this study. Overall, our results suggest flame thickness measurement is significantly more sensitive to the number of views than the other parameters studied in this work.

One of the reasons is that the curvature and speed measurement methods fit surfaces to the emission field within an inclusion radius that is between 10 and 20 voxel lengths, and in so incorporate information from a large volume of the reconstructed emission field, and so are relatively insensitive to changes in intensity values of any single voxel. The radius of curvature is also much larger than the flame thickness over most of the flame surface, allowing for the use of large inclusion radii. Measurement of the flame thickness may also be more sensitive to errors in the projection measurements than measurements related to the flame surface topology.

As this appears to be the first demonstration of flame thickness measurement on a non-axisymmetric wrinkled flame using any form of tomography, future studies should consider such effects further. The relatively large number of coplanar views required to obtain reasonable measurements in this work suggest that accurate reconstructions of the flame thickness will be a significant practical challenge for turbulent flames.

4.7. Implications for FCT of turbulent premixed flames

The potential study of turbulent premixed flames is a key motivation for developing FCT. Reconstruction resolution is known to be dependent on the field to be reconstructed [5,51], as demonstrated by the differences observed between the two test cases. However, the cross-sections of the chemiluminescence field for the
test cases are similar to those reported for weakly turbulent jet flames by Ishino et al. [15,34]. It is therefore reasonable to expect similar results for weakly turbulent jet flames of similar dimensions. The number of unknowns scales with the volume within the convex hull of the flame [63] whereas the number of equations scales with the area of the flame images. More views are therefore likely to be required to achieve the same accuracy when applying these methods to more highly wrinkled jet flames or to larger flames.

5. Conclusions

The relative sensitivity of several key flame measurements to the number of views used in the tomographic reconstruction of the visible chemiluminescence field has been assessed. A single, intensified camera was first used to acquire 36 equally-spaced, coplanar views of two forced, premixed, non-axisymmetric, laminar flames at 100 phases of their forcing cycle. A standard MART algorithm was then used to reconstruct the time-resolved chemiluminescence field using different numbers of equally-spaced views.

Algorithms for measuring the flame surface area, mean curvature, the normal component of the flame propagation velocity (surface speed) and the flame thickness were then demonstrated on the reconstructed chemiluminescence fields. The flame surface area was calculated using isosurfaces of the chemiluminescence field while measurements of the other three quantities were made on the layer of peak emission. Flame surface area was measured at 100 phases of each cycle using 36 views, and at 8 equally-spaced phases of the cycle using 4, 6, 9, 12, and 18 views. Fluctuations in the flame surface area calculated using as few as nine views were within 1% of those calculated using 36 views for both flames. Fluctuations in the flame surface area were found to closely track fluctuations in the total chemiluminescent emission, as has been previously observed in studies of axisymmetric, forced, laminar, premixed flames [57].

Mean curvature, surface speed and thickness were then measured at a sample of positions over the entire flame surface, reconstructed using between 6 and 36 views. The relative sensitivity of these measurements to the number of views was assessed by comparing the similarity of the measurement distributions. A defined similarity coefficient is used to quantify the similarity between two probability density functions (PDFs), computed from the measurement samples. The coefficient is 100% by definition for two identical PDFs. For both cases, the mean curvature and the surface speed measurements achieved over 80% and 90% similarity respectively, between the 6 view and 36 view reconstructions. The similarity between these two quantities obtained with a given number of views and those from the 36 view reference was also only relatively weakly dependent on the number of views. In contrast, the flame thickness measurements achieved less than 80% similarity between the 18 view and 36 view reconstructions, and the similarity with the 36 view reference fell significantly as the number of views was further reduced. Therefore, whilst this study appears to be the first that measures the flame thickness over a wrinkled flame surface using FCT, the results show a large number of views is required to obtain accurate measurements of this quantity in comparison to measurements of flame curvature and surface speed.

In general, the measurements converged slightly more slowly for the more wrinkled flame (Case 2). Though our results do not suggest a strong relationship between the degree of flame wrinkling and the number of views required, a more comprehensive study of this relationship would be valuable.
This work suggests that FCT with a practical number of intensified cameras may have acceptable accuracy when measuring certain statistical quantities on laboratory scale, turbulent flames. For flame surface area, flame curvature and flame surface speed, this includes using fewer than approximately 10 coplanar views. Since this number of views can be obtained using 5 or more high speed intensified cameras with 2 or more megapixel resolution and image doubling optics, this is thought to already be practical in some laboratories. A reduction in the required number of views is likely to be achievable through further advancements of the reconstruction methods and measurement algorithms. It is plausible that FCT could therefore generate rich data sets that feature multiple, simultaneously-measured turbulent quantities.

Acknowledgments

This study was supported by the Advanced Centre for Automotive Research and Testing (ACART, www.acart.com.au), the Australian Research Council (ARC), and the Australian Commonwealth Government through an Australian Government Research Training Program Scholarship.

Appendix A. Analysis of uncertainty

As the number of views used in FCT decreases, the system becomes increasingly underdetermined. When a sufficiently large number of views are used, the problem becomes overconstrained, and uncertainty in the reconstructed field is a result of uncertainty in the input projection measurements and in the estimated weighting functions. Additional uncertainty may also arise from measurement algorithms. In this section, these sources of uncertainty are discussed and quantified.

A.1. Projection measurements

Projection measurements are spatially weighted volume integrals of the chemiluminescence emission field:

\[ p_i(\alpha_0, \Delta \alpha) = \int_V w_i(x, y, z) \phi(x, y, z, \alpha_0, \Delta \alpha) dV \]  

(A.1)

where \( p_i \) is the \( i \)th projection measurement, \( \alpha_0 \) is the phase of the forcing cycle at which the camera exposure starts, \( \Delta \alpha \) is the camera exposure time, \( w_i \) is a weighting function associated with projection measurement \( p_i \), and \( \phi \) is a wavelength weighted integral of the chemiluminescence emission field over the exposure time:

\[ \phi(x, y, z, \alpha_0, \Delta \alpha) = \int_{\alpha_0}^{\alpha_0 + \Delta \alpha} \int_{\lambda} s(\lambda) l^I(x, y, z, \alpha, \lambda) d\lambda d\alpha \]  

(A.2)

where \( s(\lambda) \) is the wavelength dependent sensitivity of the imaging system, and \( l^I \) is the instantaneous chemiluminescence field. In Eqs. A.1 and A.2, it has been assumed that self-absorption and the wavelength dependence and time dependence of beam steering can be ignored. From the assumed weightings functions \( w_i \), weightings \( W_{ij} \), describing the contribution of basis function \( b_j \) to projection measurement \( p_i \) are then calculated:

\[ W_{ij} = \int w_i(x, y, z) b_j(x, y, z) dV \]  

(A.3)

Uncertainty in the inputs to the tomographic reconstruction algorithm therefore arises from two sources: (i) uncertainty in the measured flame images, (ii) uncertainty in the estimated weightings.

A.1.1. Flame images

Uncertainty in the images is quantified by the image resolution and the SNR. The image SNR has been defined in Section 2.2. Based on the image magnification and pixel size, the object length at the focal plane imaged on to a pixel is 74.2 µm. The optical resolution was measured using a USAF 1951 resolution target. The stated resolution is the wavelength at which the estimated modulation transfer function is 10% [64]. The resolution at the focal plane, which was positioned at the center of the flame holder, was 270 µm or approximately 4 pixels. For a moving flame, the spatial resolution is also dependent on the motion of the flame through the camera exposure time. For the flames presented here, the flame velocity relative to the laboratory does not exceed 3 m s\(^{-1}\) anywhere on the flame surface. The 100 µs exposure time used for the results presented here limits the movement of a flame surface element travelling at 3 m s\(^{-1}\) to 0.3 mm or 4 pixels.

In phase-shifted or stroboscopic measurement methods, the temporal resolution is determined not only by the time shift interval, but also the repeatability of the cycle. The repeatability of the flame cycles was quantified by the temporal shift in the occurrence of the flame annihilation events. Both annihilation events were found to occur within 50 µs intervals of the forcing signal. This is much lower than the 200 µs time shift interval and the temporal resolution is therefore estimated at 200 µs.

A.1.2. Weighting functions

Flame images formed on the image sensor were treated as orthographic projections. Using this common assumption [5,13,15,26], the weightings can be cheaply computed based on calculating the resulting intersecting volumes. Additionally, if using coplanar views, subvolumes or even individual horizontal slices of the flame can be reconstructed independently. However, coplanar views are generally not considered optimal for maximizing the tomographic reconstruction accuracy [32]. It is, nonetheless, one of the simpler arrangements for the acquisition of images, and in many applications it is the only practical arrangement.

Image blur and perspective are the two main sources of error when the assumption of orthographic projection is used. Image blur at the near and far edges of the subject is related to the depth of field and depends on the lens aperture. Flame regions are a maximum of 15.6 mm from the focal plane, occurring when the flame holder is rotated so that the optical axis is aligned with the burner diagonal. The measured optical resolution using the USAF 1951 resolution target was 0.84 mm and 0.83 mm respectively when the target was positioned 15 mm behind, and in front of, the focal plane.

Perspective is the change in magnification with distance from the image plane. For a fixed magnification, perspective is governed by the focal length of the lens and does not depend on the lens aperture. To quantify the uncertainty due to perspective, a calibration target was traversed along the imaging axis towards the camera. The change in position of the imaged points at the furthest edges of the target was less than 0.35 mm.

A.2. Tomographic reconstruction

A comparison between particular camera images and computed views from EF\(_{36}\) and EF\(_{73}\) is given in Fig. A.19. This observed high degree of similarity between these computed and measured images is a necessary but not sufficient condition for accurate reconstructions.

The spatial resolution of the reconstructed emission field is limited by the optical resolution of the input images and the flame’s repeatability. It is dependent on the number of views and the uncertainty in the inputs to the reconstruction algorithm. Estimating the propagation of uncertainty through the reconstruction process,
by numerical or theoretical means, is beyond the scope of this paper.

A.3. Method of locating the flame surface

The emission profile in the surface normal direction was used to quantify the error in the proposed method for locating points on the flame surface. As defined in Section 3.2.2, the flame surface comprises the set of points located at the peak of the emission profile along the surface normal direction. To be consistent with this definition, the flame surface points identified on the locally fit surfaces should therefore be located at the peak of the profile. Figure A.20 compares the distance from these flame surface points to the peak of the normal profiles using EF36. The PDF of these measurements is given in Fig. A.21. The length of the stepping distance used in creating the profiles was 19 μm. 7 × 7 × 7 mean filtering was applied to the emission field prior to running the surface fitting algorithm. The distance to the profile peaks in both the unfiltered and filtered emission fields are shown. Using the filtered emission field, the distance to the profile peak exceeds 50 μm in fewer than 1% of the measurements. A total of 72% of the profile peaks are located on the unburned gas side of the fitted surface, suggesting a small bias. Figure A.20a does not show any decrease in performance in the more curved regions. The effect of filtering on the location of the profile peak is assessed by considering the distance to the profile peak in the unfiltered emission field. Due to noise in the unfiltered emission field, the standard deviation of the distance distribution increases from 19 μm for the filtered emission field to 46 μm. The PDF peak for the unfiltered emission field is located approximately 20 μm towards the unburned gas side. Figure A.20 again does not reveal any decrease in performance in regions of high curvature. Importantly, both biases in Fig. A.21 are an order of magnitude smaller than the FWHM of the chemiluminescence profile.

References


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