Expansions and Matrix Expressions of 3-D Compressible Turbulent Flow Eqs. in Non-Orthogonal Curvilinear Coordinates and the Method of Numerical Calcuation

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ABSTRACT

The time-averaged turbulent flow equations in vector form are established in this paper. They can be expressed by matrices and expanded in non-orthogonal curvilinear coordinate system as well. In the end, the method of solving the 3-D turbulent flow field by numerical difference is discussed briefly.

NOMENCLUTURE

E-Inner energy.
f-Viscous force.
g-Metric tensor and its components.
Π-Stress tensor.
p-Pressure.

T-Absolute temperature
V-Absolute velocity or its components.
x-Curvilinear coordinate curves.
Γ-Christoffel symbol.
ρ-Gas density.

I. INTRODUCTION

In recent years because of the rapid development of the computer technology & the continuous improvement of the calculation methods, it is now possible to solve the 3-D viscous flow equations numerically. However it is very difficult to solve the viscous flow field with a boundary of complex geometry by finite difference method because one should deal with the "broken" mesh on the boundary.

In order to solve the 3-D viscous flow field with the boundary of complex geometry in non-orthogonal curvilinear coordinate, the matrix expression of viscous terms in the basic aerodynamic equations are given in the references [1] [2]. However these matrix expressions could not be used for calculation of turbulent flow directly. Therefore the basic aerodynamic equations with viscous terms should be handled with the theory of time-averaging. For this purpose the studies represented in this paper can be considered as continuation of the works carried out in above-mentioned references.

II. TIME-AVERAGED BASIC AERODYNAMIC EQUATIONS IN VECTOR FORM

In order to make the procedure of time-averaging of the turbulent flow basic equations clear and easy, the time-averaged equations in vector form are first derived.

1. Equation of continuity

Time-averaged equation of continuity can be written in following form

$$\frac{\partial \bar{\rho}}{\partial t} + \nabla \cdot (\bar{\rho}\,\bar{V} + \overline{\rho'V'}) = 0 \qquad (2\text{-}1)$$

In order to simplify the above equation, we analyse the condition of $M_r \ll 1$[4] then $\rho'/\rho \ll 1$. Therefore $\bar{\rho} \approx \rho$. Under this condition, Eq. (2-1) can be simplified as

$$\frac{\partial \bar{\rho}}{\partial t} + \nabla \cdot (\bar{\rho}\,\bar{V}) = 0 \qquad (2\text{-}2)$$

2. Momentum equation

After being time-averaged, momentum equation

$$\frac{\rho(\rho V)}{\partial t} + \nabla \cdot (\rho V V) + pI) = \rho m + \rho f \qquad (2\text{-}3)$$

becomes

$$\frac{\rho(\bar{V})}{\partial t} + \cdot \nabla (\bar{\rho}\,\bar{V}\,\bar{V}) + pI) = \overline{\rho m} + \bar{\rho}f - \frac{\partial(\overline{\rho'V'})}{\partial t} \qquad (2\text{-}4)$$
$$- \nabla \cdot (\tilde{\rho}\,\overline{V'V'} + \overline{\rho'V'}\bar{V}' + 2\bar{V}\,\overline{\rho'V'})$$

The time-averaged momentum equation in vector form is given by (because $M_r \ll 1$)

$$\frac{\partial(\bar{\rho}\bar{V})}{\partial t} + \nabla \cdot (\bar{V}\,\bar{V} + \overline{pI}) = \overline{\rho m} + \nabla \cdot (\overline{\Pi} - \rho\overline{V'V'}) \qquad (2\text{-}5)$$

where

$$\nabla \cdot \overline{\Pi} = \overline{\rho f} \qquad (2\text{-}6)$$

3. Energy equation

Under the condition of adiabatic flow energy equation is

$$\frac{\partial(\rho E)}{\partial t} + \nabla \cdot (\rho EV + pV) = \rho m \cdot V + \nabla \cdot (k\nabla T) + \nabla \cdot (\Pi \cdot V) \qquad (2\text{-}7)$$

After being time-averaged, Eq. (2-7) becomes

$$\frac{\partial(\overline{\rho E})}{\partial t} + \nabla \cdot (\bar{\rho}\overline{E}\bar{V} + \bar{p}\,\bar{V}) = \overline{\rho m \cdot V} + \nabla \cdot (k\nabla\overline{T}) + \nabla \cdot (\overline{\Pi \cdot V})$$
$$- \frac{\partial(\overline{\rho'E'})}{\partial t} - \nabla \cdot (\bar{\rho}\,\overline{E'V'} + \overline{E}\overline{\rho'V'} + \overline{V}\overline{\rho'E'} + \overline{\rho'E'V'} + \overline{p'V'}) \qquad (2\text{-}8)$$

Since $M_r \ll 1$, the equation is simplified into

$$\frac{\partial(\rho E)}{\partial t} + \nabla \cdot (\rho\,\overline{E}\bar{V} + \overline{p}\bar{V}) = \rho\,\overline{m \cdot V} + \nabla \cdot (k\nabla\overline{T})$$
$$+ \nabla \cdot (\overline{\Pi} \cdot \bar{V}) + \nabla \cdot (\overline{\Pi'V'}) - \nabla \cdot (\rho\,\overline{E'V'} + \overline{p'V'}) \qquad (2\text{-}9)$$

The above-mentioned time-averaged equations can be easily expanded in various coordinate system

III. EXPANSIONS OF THE TIME-AVERAGED BASIC AERODYNAMIC EQUATIONS IN VECTOR FORM

Because non-orthogonal curvilinear coordinate system is used for solving the flow fields with complex boundary, we will expand the time-averaged equations in this coordinate system, (the bar over all fluctuated terms in equations is dropped for ease of prescription).

1. Equations of continuity

Eq. (2-2) can be written as

$$\frac{\partial \rho}{\partial t} + \frac{1}{\sqrt{g}}\frac{\partial(\sqrt{g}\,\rho v^{\alpha})}{\partial x^{\alpha}} = 0 \qquad (3\text{-}1)$$

2. Momentum equation

The second term $\nabla \cdot (\rho VV)$ on the left-hand side of Eq. (2-3) can be expanded as

$$\nabla \cdot (\rho VV) = \left[\frac{1}{\sqrt{g}}\frac{\partial(\sqrt{g}\,g^{\alpha_k}\rho v_k v_{\beta})}{\partial x^{\alpha}} + \frac{1}{2}\frac{\partial g^{\lambda\gamma}}{\partial x^{\beta}}\rho v_{\lambda}v_{\gamma}\right]e^{\beta} \quad (3\text{-}2)$$

Using this equation and neglecting the mass force, Eq.(2-5) can be expanded into

$$\frac{\partial(\rho v_{\beta})}{\partial t}e^{\beta} + \left[\frac{1}{\sqrt{g}}\frac{\partial(\sqrt{g}\,g^{\alpha_k}\rho v_k v_{\beta})}{\partial x^{\alpha}} + \frac{1}{2}\frac{\partial g^{\lambda\gamma}}{\partial x^{\beta}}\rho v_{\lambda}v_{\gamma}\right]e^{\beta}$$

$$= -\frac{\partial p}{\partial x^{\beta}}e^{\beta} + \left[\frac{1}{\sqrt{g}}\frac{\partial}{\partial x^{\alpha}}(\sqrt{g}\,g^{\alpha_k}\Pi_{k\beta} - \sqrt{g}\,g^{\alpha_k}\rho\,\overline{v_k'v_{\beta}'})\right.$$

$$\left.+ \frac{1}{2}\frac{\partial g^{\lambda\gamma}}{\partial x^{\beta}}(\Pi_{\lambda\gamma} - \overline{\rho v_{\lambda}'v_{\gamma}'})\right]e^{\beta} \qquad (3\text{-}3)$$

This is the time-averaged expansion of momentum equation. where $\Pi_{k\beta}$ and $\Pi_{\lambda\gamma}$ are expressed by $\Pi_{\gamma\beta} = \mu\left(\frac{\partial v_{\beta}}{\partial x_{\gamma}} + \frac{\partial v_{\gamma}}{\partial x^{\beta}} - 2v_{\alpha}\Gamma_{\beta\gamma}^{\alpha}\right) - \frac{2}{3}\mu\frac{1}{\sqrt{g}}\frac{\partial(\sqrt{g}\,v^{\alpha})}{\partial x^{\alpha}}$ (3-4)

3. Energy equation

Neglecting the mass force the main difficulty to expand the energy equation lies in expanding the term of work done by viscous stress, i.e. $\nabla \cdot (\Pi \cdot V)$. Its expansion is

$$\nabla \cdot (\Pi \cdot V) = \frac{1}{\sqrt{g}}\frac{\partial(\sqrt{g}\,\Pi_{\gamma\beta}v^{\beta}g^{\gamma_k})}{\partial x^{k}} \qquad (3\text{-}5)$$

Thus, the expansion of energy equation in non-orthogonal curvilinear coordinates is

$$\frac{\partial(\rho E)}{\partial t} + \frac{1}{\sqrt{g}}\left\{\frac{\partial}{\partial x^{\alpha}}[\sqrt{g}\,(\rho E v^{\alpha} + p v^{\alpha})]\right\}$$

$$= \frac{1}{\sqrt{g}}\frac{\partial}{\partial x^{\beta}}\left(\sqrt{g}\,g^{\alpha\beta}\frac{\partial T}{\partial x^{\alpha}}\right) + \frac{1}{\sqrt{g}}\left\{\frac{\partial}{\partial x^{k}}[\sqrt{g}\,g^{\gamma_k}(\Pi_{\gamma\beta}v^{\beta}\right.$$

$$\left.+ \overline{\Pi_{\gamma\beta}'v^{\beta}})]\right\} - \frac{1}{\sqrt{g}}\left\{\frac{\partial}{\partial x^{\alpha}}[\sqrt{g}\,(\rho\overline{E'v'^{\alpha}} + \overline{p'v'^{\alpha}})]\right\} \quad (3\text{-}6)$$

Eqs. (3-1), (3-3) and (3-6) constitute the time-averaged basic equations of turbulent flow. With the addition of an equation of state the turbulent flow fields can be solved by using these five equations connected with "zero model" of turbulence.

IV. MATRIX EXPRESSION OF THE TIME-AVERAGED BASIC AERODYNAMIC EQUATIONS

From Eqs. (3-3) and (3-6) we can see that the number of the terms in these equations are too large. This makes it difficult to design the calculation program. overcome this problem, a number of matrix operator are introduced and defined. Thus, the basic equations are expressed in matrix form, so that the calculation program library of computer can be used for simplifying the computation

$$D = \left(\frac{\partial}{\partial x^1},\ \frac{\partial}{\partial x^2},\ \frac{\partial}{\partial x^3}\right)^T \qquad (4\text{-}1)$$

$$\widetilde{V}(v^1,\ v^2,\ v^3)^T, \qquad V = (v_1,\ v_2,\ v_3)^T$$

$$\widetilde{G} = \begin{pmatrix} g^{11} & g^{12} & g^{13} \\ g^{21} & g^{22} & g^{23} \\ g^{31} & g^{32} & g^{33} \end{pmatrix}, \quad G = \begin{pmatrix} g_{11} & g_{12} & g_{13} \\ g_{21} & g_{22} & g_{23} \\ g_{31} & g_{32} & g_{33} \end{pmatrix} \qquad (4\text{-}2)$$

$$G_{j\beta} = \frac{\partial}{\partial x^{\beta}}\,\widetilde{G} \qquad (4\text{-}3)$$

$$Q_{\pi} = (C_{\alpha\alpha\pi;1},\ C_{\alpha\alpha\pi;2},\ C_{\alpha\alpha\pi;3})^T \qquad (4\text{-}4)$$

$$\text{(sum for } \alpha\text{)} \qquad (4\text{-}5)$$

$$Q_{\nu} = (C_{\alpha\alpha\nu;1},\ C_{\alpha\alpha\nu;2},\ C_{\alpha\alpha\nu;3})^T \qquad (4\text{-}6)$$

$$\text{(sum for } \alpha\text{)}$$

$C_{\alpha\alpha\pi;\beta}(\beta=1,2,3)$ are the elements laid on the diagonal of matrix $C_{\pi;\beta}$. where $C_{\pi;\beta}$ is represented by

$$C_{\pi;\beta} = G_{j\beta}\,\Pi \qquad (4\text{-}7)$$

where

$$\Pi = \begin{pmatrix} \Pi_{11} & \Pi_{12} & \Pi_{13} \\ \Pi_{21} & \Pi_{22} & \Pi_{23} \\ \Pi_{31} & \Pi_{32} & \Pi_{33} \end{pmatrix} \qquad (4\text{-}8)$$

$C_{\alpha\alpha\nu;\beta}(\beta=1,2,3)$ are the elements laid on the diagonal of matrix $C_{\nu;\beta}$, which is given by

$$C_{\nu;\beta} = G_{j\beta}\,V\,V^T \qquad (4\text{-}9)$$

Using Eqs. (4-1) ~ (4-9), the equation of continuity, the momentum equation and the energy equation can be expressed in matrix form respectively by

$$\frac{\partial \rho}{\partial t} + \frac{1}{\sqrt{g}}D^T\sqrt{g}\,\rho\,\widetilde{V} = 0 \qquad (4\text{-}10)$$

$$\frac{\partial(\rho V)}{\partial t} + \frac{1}{\sqrt{g}}(D^T\sqrt{g}\,\rho\widetilde{G}\,V\,V^T)^T + \frac{\rho}{2}Q_{\nu} = -Dp$$

$$+ \frac{1}{\sqrt{g}}\left\{D^T\,[\sqrt{g}\,\widetilde{G}\,(\Pi^T - \rho\,\overline{V'V'^T})]\right\}^T + \frac{1}{2}(Q_{\pi} - \rho Q_{\nu}) \quad (4\text{-}11)$$

$$\frac{\partial(\rho E)}{\partial t} + \frac{1}{\sqrt{g}}D^T\,[\sqrt{g}\,(\rho E\widetilde{V} + p\widetilde{V}] = \frac{1}{\sqrt{g}}D^T(\sqrt{g}\,\widetilde{G}kDT)$$

$$+ \frac{1}{\sqrt{g}}D^T\,[\sqrt{g}\,\widetilde{G}(\Pi\,\widetilde{V} + \Pi'\widetilde{V'})]$$

$$- \frac{1}{\sqrt{g}}D^T\,[\sqrt{g}\,(\rho E'\overline{\widetilde{V'}} + \overline{p'\widetilde{V'}})] \qquad (4\text{-}12)$$

The components of Π in Eqs. (4-11) and (4-12) are expressed by Eq. (3-4).

In order to use the implicit difference scheme for computing, it is best that does not appear in the matrix expressions. For this reason, let

$$B_{\beta\gamma} = \frac{\partial v_{\beta}}{\partial x^{\gamma}} + \frac{\partial v_{\gamma}}{\partial x^{\beta}} - 2v_{\lambda}\Gamma_{s\gamma}^{\lambda} \qquad (4\text{-}13)$$

then

$$B = (DV^T)^T + DV^T - [(\widetilde{V}D^T)^T\,G]^T - (\widetilde{V}D^T)^T G^T + (\widetilde{V}^TD)\,G \quad (4\text{-}14)$$

and let

$$Q_F = (C_{\alpha\alpha F;1},\ C_{\alpha\alpha F;2},\ C_{\alpha\alpha F;3})^T \qquad (4\text{-}15)$$

$$\text{(sum for } \alpha\text{)}$$

where $C_{\alpha\alpha F;\beta}(\beta=1=1,2,3)$ are the elements located at the diagonal of matrix $C_{F;\beta}$, which is expressed by $C_{F;\beta} = G_{j\beta}\,B$

Using Eqs. (4-1) and (4-15) the second term on the right-hand side of Eq. (2-5) can be expressed in matrix form by

$$\nabla \cdot \Pi = \frac{1}{\sqrt{g}}[D^T\mu\sqrt{g}\,\widetilde{G}\,B^T] - \frac{2}{3}D\mu\frac{1}{\sqrt{g}}D^T\sqrt{g}\,\widetilde{V} + \frac{1}{2}\mu Q_F \quad (4\text{-}16)$$

and $\nabla \cdot (\Pi \cdot V)$ in energy equation (2-9) can be expressed in matrix form by

$$\nabla \cdot (\Pi \cdot V) = \frac{1}{\sqrt{g}} D^T \left\{ \mu \sqrt{g} \, \widetilde{G} \, [\widetilde{V}^T D \, \underset{\sim}{V}^T]^T + \mu \sqrt{g} \, \widetilde{G} \, D \widetilde{V}^T \cdot \underset{\sim}{V} \right\}$$

$$- \frac{2}{3} \mu \widetilde{V}^T D \left[\frac{1}{\sqrt{g}} D^T \sqrt{g} \, \widetilde{V} \right] - \frac{2}{3} \frac{1}{g} D^T \sqrt{g} \, \widetilde{V} \cdot D^T \mu \sqrt{g} \, \widetilde{V}$$

$$(4-17)$$

Eqs. (4-10) - (4-12) represent the expansions of the time-averaged turbulent flow equations in matrix form in non-orthogonal curvilinear coordinate system which we expected.

V. SIMPLIFICATION OF THE EXPANSIONS OF THE TIME-AVERAGED BASIC EQUATIONS

The method of iteratively calculating boundary layer-inviscid flow is often used for solving viscous flow field with high Reynolds number. Therefore, under this condition the time-averaged equations mentioned above should be simplified.

The simplified conditions given in Refs.[1] and [2] are used in this paper.

In non-orthogonal curvilinear coordinate system x^1 takes the direction of a quasi-streamline and x^2, x^3 take the other directions The order of magnitude of x^1, v^1 and metric tensor are considered to be $O(1)$, that of x^2, x^3, v^2, v^3 to be $O(\delta)$, and that of viscous coefficient μ to be δ^2. Besides, the unsteady and fluctuation terms in the inertial term of basic equations are considered not to be neglected. Therefore they have the same order of magnitude as the pressure term, convection term and viscous term of laminar flow. According to the simplifying condition mentioned above, the order of magnitude of matrix operator D and velocity matrix $\widetilde{V}$ are given respectively by

$$D = \begin{vmatrix} \frac{\partial}{\partial x^1} \\ \frac{\partial}{\partial x^2} \\ \frac{\partial}{\partial x^3} \end{vmatrix} \sim \begin{vmatrix} 0 \\ \frac{1}{\delta} \\ \frac{1}{\delta} \end{vmatrix}, \quad \widetilde{V} = \begin{vmatrix} v^1 \\ v^2 \\ v^3 \end{vmatrix} \sim \begin{vmatrix} 1 \\ \delta \\ \delta \end{vmatrix}$$

If we let

$$D_{(1/\delta)} = \begin{vmatrix} 0 \\ \frac{\partial}{\partial x^2} \\ \frac{\partial}{\partial x^3} \end{vmatrix} \sim \begin{vmatrix} 0 \\ \frac{1}{\delta} \\ \frac{1}{\delta} \end{vmatrix}, \quad D_{(1)} = \begin{vmatrix} \frac{\partial}{\partial x^1} \\ 0 \\ 0 \end{vmatrix} \sim \begin{vmatrix} 1 \\ 0 \\ 0 \end{vmatrix}$$

$$\widetilde{V}_{(1)} = \begin{vmatrix} v^1 \\ 0 \\ 0 \end{vmatrix} \sim \begin{vmatrix} 1 \\ 0 \\ 0 \end{vmatrix}, \quad \widetilde{V}_{(\delta)} = \begin{vmatrix} 0 \\ v^2 \\ v^3 \end{vmatrix} \sim \begin{vmatrix} 0 \\ \delta \\ \delta \end{vmatrix}$$

then we obtain

$$D = D_{(1)} + D_{(1/\delta)}, \quad \widetilde{V} + \widetilde{V}_{(1)} + \widetilde{V}_{(\delta)} \quad (5-1)$$

Using Eq. (5-1) the basic aerodynamic equations can be expanded into the expressions with more terms which have the different order of magnitude. The simplified equations will be obtained by analysing the order of magnitude

of each term and neglecting the terms have less order of magnitude than the others

1. Equation of continuity

Using Eq. (5-1), Eq. (4-10) becomes

$$\frac{\partial \rho}{\partial t} + \frac{1}{\sqrt{g}} D_{(1)} \sqrt{g} \rho \widetilde{V}_{(1)} + \frac{1}{\sqrt{g}} D_{(1/\delta)} \sqrt{g} \rho \widetilde{V}_{(\delta)} = 0 \quad (5-2)$$

It will be shown that the expansion of Eq. (5-2) is the same as that of Eq. (4-10). From this result it can be concluded that even if the non-orthogonal coordinates are adopted, the equation of continuity used for boundary layes can not be simplified.

2. Momentum equation

In order to get a simplified momentum equation, we analyse the order of magnitude of each term, that is, unsteady term, convection term, pressure term, viscous term and fluctuation term, respectively, in Eq. (4-11)

1) Unsteady term

Because $V = G \widetilde{V}$, and using Eq. (5-1) the unsteady term becomes

$$\frac{\partial (\rho V)}{\partial t} \approx \frac{\partial}{\partial t} (\rho G \widetilde{V}_{(1)}) \qquad \sim 1 \quad (5-3)$$

2) First convection term

Using eq. (5-1) and $V = GV$, first convection term of Eq. (4-11) can be written as

$$\left(D^T \sqrt{g} \rho \widetilde{G} \, \underset{\sim}{V} \, \underset{\sim}{V}^T \right)^T = \left[D_{(1)}^T \sqrt{g} \rho \widetilde{V}_{(1)} \, \widetilde{V}_{(1)}^T \, G \right]^T \quad (5-4)$$

$$+ \left[D_{(1/\delta)}^T \sqrt{g} \widetilde{V}_{(\delta)} \, V_{(1)}^T \, G \right]^T$$

3) Second convection term

We have

$$Q_v = (Q_{1v}, Q_{2v}, Q_{3v})^T = (C_{\alpha \alpha v; 1}, C_{\alpha \alpha v; 2}, C_{\alpha \alpha v; 3})^T$$

and

$$G_{;\beta} = \begin{vmatrix} G_{;1} \\ G_{;2} \\ G_{;3} \end{vmatrix} \sim \begin{vmatrix} 1 \\ 1/\delta \\ 1/\delta \end{vmatrix}$$

After neglecting the terms which have the order of magnitude of $O(\delta)$ or $O(\delta^2)$ one obtains

$$Q_v = \begin{vmatrix} C_{\alpha \alpha v; 1} \\ C_{\alpha \alpha v; 2} \\ C_{\alpha \alpha v; 3} \end{vmatrix} \sim \begin{vmatrix} 1/\delta \\ 1/\delta^2 \\ 1/\delta^2 \end{vmatrix}$$

4) Pressare gradient term

It is well known from Eq. (5-13) that the order of magnitude of Dp is $O(1)$ in direction x^1, and $1/\delta$ in directions x^2 and x^3

5) Laminar viscous terms

From Ref. [3] we can know $B \sim 1/\delta$ thus $\mu Q_F \sim (\delta, 1, 1)^T$. Hence, Eq. (4-16) becomes

$$\nabla \cdot \Pi = \frac{1}{\sqrt{g}} \left[D_{(1/\delta)}^T \mu \sqrt{g} \, \widetilde{G} B^T \right]^T$$

$$- \frac{2}{3} D_{(1/\delta)} \mu \frac{1}{\sqrt{g}} D_{(1/\delta)} \sqrt{g} \, \widetilde{V}_{(1)} \sim 1$$

6) Fluctuation terms

In consideration that the order of magnitude of each component of V' is same, we will only analyse the order of magnitude of operator D.

It follows from the analyses mentioned above that under the simplified condition given in this paper the order of magnitude of simplified momentum equation is $O(1)$ in drection

x^1 and $O(1/\delta)$ in direction x^2 and x^3. Since the components of the metric tensor are variable in non-orthogonal curvilinear coordinates, the terms of pressure gradient in the directions x^2 and x^3 can not be neglected even if the boundary layer is thin.

3. Energy equation

The procedure of analysing the order of magnitude of energy equation (4-12) is similar to that of momentum equation. The order of magnitude of each term in energy equation is also maitained in $O(1)$. The terms $\overline{\widetilde{\Pi'}\widetilde{V'}}$ and $\overline{p'\widetilde{V'}}$ represented the rate of work done by fluctuation stress and static pressure respectively are neglected but the term $D_{(1/\delta)}^T \overline{\rho E'\widetilde{V'}}$ in the expression of $D^T \overline{\rho E'\widetilde{V'}}$ of energy equation is maitained.

VI. METHOD OF NUMERICAL DIFFERENCE

In order to perform the numerical calculation effectively for the expressions with matrix form, an difference operator matrix is set up in this paper by using the method of numerical differentiation.

Let q denote a scalar function. Utilizing the formula of three-point differentiation, the difference expressions of the first and second derivations of q at the arbitrary point β are given by

$$\left(\frac{\partial q}{\partial x}\right)_\beta = b_{\beta-1}q_{\beta-1} + b_\beta q_\beta + b_{\beta+1}q_{\beta+1} \qquad (6-1)$$

$$\left(\frac{\partial^2 q}{\partial x^2}\right)_\beta = C_{\beta-1}q_{\beta-1} + C_\beta q_\beta + C_{\beta+1}q_{\beta+1} \qquad (6-2)$$

If A is a 3-D array, then its differentiation can be represented by DA^T. The difference operator matrix at point p (i, j, k.) can be defined by Δ_p as $\Delta_p = \Delta_{-1} + \Delta_o + \Delta_{+1}$

where $\Delta_{-1} = \begin{bmatrix} b_{i-1} \\ b_{j-1} \\ b_{k-1} \end{bmatrix}$, $\Delta_o = \begin{bmatrix} b_i \\ b_j \\ b_k \end{bmatrix}$, $\Delta_{+1} = \begin{bmatrix} b_{i+1} \\ b_{j+1} \\ b_{k+1} \end{bmatrix}$

The expression of DA^T is $\Delta_p A^T = (\Delta_{-1} + \Delta_o + \Delta_{+1})A^T$ When one wants to calculate the basic equations expressed by matrix form using a kind of numerical difference scheme, it is only required to substitute Δ_p for D. If C_β in Eq. (6-2) are substituted for b_β, One obtains a difference operator matrix of second derivatives.

SUMMARY

1. The time-averaged turbulent flow equations in tensor form are established. They are expressed by matrices, which are expanded in non-orthogonal curvilinear coordinates.

2. The equations mentioned above are simplified under the condition of larger Reynolds number.

3. In order to design the calculation program easly by using the matrix form, a numerical difference operator of matrix is introduced.

4. The method of solving the 3-D turbulent flow field by numerical defference is discussed briefly.

REFERENCES

[1].Chen, N.: Some Problems in Heat-transfer Terms, And Method for Solution of Basic Equations (China P.R.). Scientia Sinica (Series A), Vol. 27,No.6,June 1984.

[2].Wang, Z., And Kang Shun: The Expansions of Viscous Terms in the Basic Aerodynamic Equations in Non-orthogonal Curvilinear Coordinate and Method of Their numerical Differentiation (China P.R.). J. of Eng. Thermophysics, Vol.6,No.2,May 1985.

[3].Wang, Z.: Three-Dimensional Flow Solution in Turbomachinery and its Mathematical and Aerodynamic Foundation (China P.R.). 1st ed, Engineering Industry Press, Beijing, 1983.

[4].Hinge, J. O.: Turbulence. McGraw-Hill, New York, 1959.