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Energy Flux of Continental Shelf Waves

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ABSTRACT

Longuet-Higgins originally recognised that the energy flux defined by pressure work from the equations of motion was not the same as the mean energy density times the group velocity for planetary waves on a beta-plane. This paper addresses a similar paradox for continental shelf waves on an arbitrary shaped (in the offshore direction) straight continental shelf. The approach is to first examine a wavetrain solution to the problem and then to use a multiple scale argument which results in a solution as a group of waves modulated about a central frequency σ and wavenumber k. The paradox is resolved in both instances by noting that a divergence free quantity J can be included in the energy conservation equation to establish an equivalence between the two definitions of mean energy flux. For the wavetrain solution

$$J(y) = \frac{-1}{8h}\,[(\sigma AA^*)_{ky} - 4\sigma\, \mathbf{re}\,(A_y A^*_k)],$$

where y is the offshore direction; h(y) is the depth; A(k,y) is the complex stream function amplitude; σ is the frequency; and k is the wavenumber. For the modulated group, the quantity J is given by $J = J(y)\, BB^*$ where $B = B(X,T)$ is part of the shelf wave complex stream function amplitude $A(k,y)B(X,T)$ and X,T are the long longshore and time variables respectively.

INTRODUCTION

Recent research into the definition of the energy density and the energy flux of edge waves on a linearly sloping continental shelf (Shillington, 1985) has revealed that there are problems as to the "best" definition of these properties. These difficulties are similar to those that were originally recognised by Longuet-Higgins (1964) for planetary waves on a beta-plane. There are three basic ways in which to calculate the mean energy flux in a wave system. The first approach is to start directly from the primitive equations; the second way is to start from the governing wave equation , and the third approach is to use a definition of the mean energy flux vector as the mean energy density multiplied by the group velocity. Whitham (1974) prefers the definition of the mean energy flux vector $\bar{F}^1$ as

$$\bar{F}^1 = \underset{\sim}{c}_g \bar{E}, \tag{1.1}$$

and notes that the two approaches are identical for linear wave systems, ($\underset{\sim}{c}_g$ is the group velocity vector, $\bar{E}$ is the mean energy density averaged over one wavelength, σ and k are the radian frequency and wave number respectively.)

Longuet-Higgins (1964) showed that a definition of the mean energy flux vector obtained directly from the pressure work for planetary waves on a beta-plane, did not equal $\bar{F}^1$ as defined in equation (1.1). He then resolved the apparent paradox by examining the behaviour of a modulated group of planetary waves, where it became clear that the two definitions of energy flux were equivalent to within the addition of a non divergent vector field. Other authors have addressed similar difficulties in topographic planetary waves (e.g., LeBlond and Mysak, 1978). Shillington (1985) discovered a similar paradox for linear, long period edge waves trapped to the coast by a linearly sloping continental shelf. Shillington and Brundrit (1986) generalized these results to include the case of arbitrary shaped offshore topography. In this paper, we extend the results to include continental shelf waves.

GENERAL EQUATIONS

The appropriate equations required to study freely propagating continental shelf waves, are the shallow water equations for a rotating fluid (e.g., Gill and Schumann, 1974; LeBlond and Mysak, 1978):

$$u_t - fv + g\eta_x = 0, \tag{2.1}$$

$$v_t + fu + g\eta_y = 0, \tag{2.2}$$

$$\eta_t + (hu)_x + (hv)_y = 0, \tag{2.3}$$

where u, v are the vertically integrated x, y components of velocity (we choose a right handed co-ordinate system with x alongshore, y offshore and f>0), η the surface elevation, h(x,y) is the depth, f is the Coriolis parameter, g is the gravitational constant and subscripts represent partial differentiation. For long period shelf waves, Gill and Schumann (1974) have shown that the term v_t is very much smaller than u_t and can be neglected so that (2.2) represents geostrophic balance. We prefer to retain it at this stage and show later in the scaling how it could be neglected for long shelf waves. We use the rigid lid approximation, (Gill, 1982), and neglect η_t in comparison to other terms in (2.1)-(2.3). Hence the system of equations can be solved via the introduction of a stream function $\psi(x,y,t)$ (e.g.,Leblond and Mysak, 1978) where

$$hu = -\psi_y; \quad hv = \psi_x,$$

and equations (2.1) and (2.2) can be combined to give the vorticity equation

$$(u_y - v_x)_t - f(v_y + u_x) = 0,$$

$$[(h^{-1}\psi_y)_y + (h^{-1}\psi_x)_x]_t - h^{-2} f h_y \psi_x = 0, \tag{2.4}$$

where we now consider $h = h(y)$ only, so that we are dealing with a straight continental shelf that has no variations in the x direction (alongshore). To obtain an energy relation, multiply (2.4) by ψ which after some re-arrangement gives

$$[(2h)^{-1}[(\psi_y)^2 + (\psi_x)^2]]_t + [-h^{-1}\psi_{xt} + \tfrac{1}{2}h^{-2} f h_y \psi^2]_x + [-h^{-1}\psi_{yt}]_y = 0. \tag{2.5}$$

In equation (2.5), it is natural to regard the expression:

$$E = (2h)^{-1}\,[(\psi_x)^2 + (\psi_y)^2], \tag{2.6}$$

as the energy density; and the terms

$$F^x = -h^{-1}\psi_{xt} + \tfrac{1}{2}h^{-2} f h_y \psi^2$$

$$F^y = -h^{-1}\psi_{yt} \qquad (2.7)$$

as the components of the energy flux vector. Thus equation (2.5) expresses the conservation of energy:

$$\frac{\partial E}{\partial t} + \nabla.F = 0. \qquad (2.8)$$

CONTINENTAL SHELF WAVES

We now consider a plane wave solution in the presence of a continental shelf which has prescribed offshore structure h(y) between the coast (y=0) and the open sea ($y\to\infty$), but is straight in the longshore (x) direction. The plane wavetrain

$$\psi = \text{re}\,[\,A(x)\exp i(ky - \sigma t)\,],$$

where A is the complex amplitude , σ is the frequency, and k is the wavenumber of the wave, is a solution of the governing wave equation for the stream function ψ in (2.4) for the prescribed topography h(y) provided that

$$(h^{-1}A_y)_y + (\frac{kf}{\sigma h^2}\frac{dh}{dy} - \frac{k^2}{h})A = 0. \qquad (3.1)$$

Equation (3.1), together with the appropriate boundary conditions poses an eigenvalue equation for the eigenvalues $\sigma^n(k)$ and eigen functions $A^n(y)$. The boundary conditions are typically

$$hu \to 0 \quad \text{as } y \to 0; \text{ and } A \to 0 \quad \text{as } y \to \infty,$$

so as to impose the trapping at the coast. Families of solutions to this eigen equation can be recognised and have been noted. (e.g., Buchwald and Adams, 1968; LeBlond and Mysak, 1978). It should be noted that the eigen equation can be interpreted as a dispersion relation once it is realised that the eigen equation (3.1) puts severe constraints on the form of A(k,y) which have been discussed for the edge wave problem by Shillington and Brundrit (1986). The dispersion relation can be differentiated with respect to wavenumber k to give the group velocity which is directed longshore. The constraints on the complex amplitude A(k,y) will ensure that the expression for the group velocity is both real and independent of y.

Following the same method as in Shillington and Brundrit (1986), we can derive alternative dispersion and group velocity relations as

$$h^{-1}(k^2AA^* + A_yA^*_y) = \tfrac{1}{2}(h^{-1}[AA^*]_y)_y + \frac{kf}{\sigma h^2}\frac{dh}{dy}AA^*,$$

$$\text{with} \qquad \text{im}\,[h^{-1}A^*A_y] = 0, \qquad (3.2)$$

$$(2\frac{k}{h} - \frac{f}{\sigma h^2}\frac{dh}{dy} + \frac{kf}{(\sigma h)^2}\frac{dh}{dy}\frac{d\sigma}{dk})AA^* = [h^{-1}[\tfrac{1}{2}(AA^*)_{ky} - 2\text{re}(A_yA^*_k)]]_y \qquad (3.3)$$

ENERGY RELATIONS

Using the plane wave solution

$$\psi = \text{re}[\,A(y)\exp\ i(kx - \sigma t)\,], \qquad (4.1)$$

we can write the mean energy density and mean energy flux (averaged over the wave period)

$$\bar{E} = (4h)^{-1}[A_yA^*_y + k^2AA^*], \qquad (4.2)$$

$$\bar{\underset{\sim}{F}} = ([(4h^2)^{-1}fh_y - (2h)^{-1}k\sigma]AA^*;\ 0) \qquad (4.3)$$

where the overbar represents the time average, and use has been made of relations (3.2) and (3.3). We can note that the averaged version of the conservation of energy (2.8) is

$$\frac{\partial\bar{E}}{\partial t} + \nabla.\bar{\underset{\sim}{F}} = 0.$$

We would anticipate that the mean energy density will move with the group velocity so that

$$\frac{\partial\bar{E}}{\partial t} + \nabla.[(\frac{d\sigma}{dk},0)\,\bar{E}\,] = 0,$$

which can also readily be confirmed.

Both offshore components of the two different forms of the mean energy flux vectors are zero, but the longshore components can be subtracted to give

$$\bar{F}^x - \frac{d\sigma}{dk}\bar{E} = \frac{-1}{8}[h^{-1}\{(\sigma AA^*)_{ky} - 4\sigma\,\textbf{re}\,(A_yA^*_k)\}]_y,$$

on using (3.2) and (3.3). We are left with the result that

$$\bar{\underset{\sim}{F}} = \bar{E}\underset{\sim}{c_g} - (\frac{\partial}{\partial y}, -\frac{\partial}{\partial x})\,J, \quad \text{where}$$

$$J = \frac{-1}{8h}[(\sigma AA^*)_{ky} - 4\sigma\,\textbf{re}\,(A_yA^*_k)],$$

but $\nabla.\bar{\underset{\sim}{F}} = \underset{\sim}{c_g}.\nabla\bar{E}.$

The mean energy flux does not equal the group velocity times the mean energy density, although the mean energy density does propagate with the group velocity.

MODULATED WAVES

It has been recognised that a proper discussion of energy propagation must touch on the propagation of a group of waves modulated about a central frequency and wavenumber. There are three reasons for this. Such a solution has an identifiable area of generation; it can be adapted to more physically realistic conditions; and it directly confirms assumptions about propagation velocities of wavetrains.

We shall adopt a multiple scale approach to the propagation of a group of shelf waves, recognising that there are fast and slow time scales and short and long longshore length scales associated with the wave and the group respectively.

We take L and T to be the short length scale and fast time scale respectively, and $\frac{1}{\varepsilon}$ L and $\frac{1}{\varepsilon}$ T as the long longshore length scale and slow time scale, where ε is a small ratio of fast to slow scales. We choose the offshore length scale as l. We then have two sets of non-dimensional variables, (x^*,y^*,t^*) scaled by L,l,T and (X^*,T^*) scaled with slow variables $\frac{1}{\varepsilon}$ L and $\frac{1}{\varepsilon}$ T. We choose horizontal velocity scales U,V for u,v; H as the depth scale; and N as a scale for the free surface displacement, with

$$(\frac{L}{T}) = fl, \quad \frac{N}{H} = \frac{Ufl}{gH}, \quad \text{and} \quad \frac{V}{U} = \frac{l}{L},$$

then the equations (2.1) (2.2) and (2.3), with the * variables all properly scaled, become

$$(\frac{\partial}{\partial t^*}+\varepsilon\frac{\partial}{\partial T^*})u^* - v^* + (\frac{\partial}{\partial x^*}+\varepsilon\frac{\partial}{\partial X^*})\eta^* = 0, \qquad (5.1)$$

$$(\frac{1}{L})^2(\frac{\partial}{\partial t^*}+\varepsilon\frac{\partial}{\partial T^*})v^* + u^* + \frac{\partial}{\partial y^*}\eta^* = 0, \qquad (5.2)$$

$$(\frac{\partial}{\partial x^*}+\varepsilon\frac{\partial}{\partial X^*})(h^* u^*) + \frac{\partial}{\partial y^*}(h^* v^*) = 0, \qquad (5.3)$$

with $\eta^* = \eta^*(x^*,y^*,t^*,X^*,T^*)$,

$$u^* = u^*(x^*,y^*,t^*,X^*,T^*), \quad v^* = v^*(x^*,y^*,t^*,X^*,T^*).$$

This notation for the derivatives has been chosen to conform with the operator approach which follows. Henceforth dropping * from the non-dimensional variables, the governing non-dimensional wave equation becomes

$$(\frac{\partial}{\partial t} + \varepsilon\frac{\partial}{\partial T})[(\frac{\partial}{\partial y}(\frac{1}{h}\frac{\partial}{\partial y}) + \delta^2(\frac{\partial}{\partial x} + \varepsilon\frac{\partial}{\partial X})(\frac{1}{h}(\frac{\partial}{\partial x} + \varepsilon\frac{\partial}{\partial X}))]\psi - \frac{1}{h^2}h_y(\frac{\partial}{\partial x} + \varepsilon\frac{\partial}{\partial X})\psi = 0, \qquad (5.4)$$

where $\delta = 1/L$. It is clear that for very long shelf waves, as discussed by Gill and Schumann (1974), that the first term in (5.2) can be neglected. (The "rigid lid" approximation has also been invoked in (5.3) at the outset.) Using the two time and length scales, we can define a non-dimensional energy density scaled by $H U^2$, and a non-dimensional energy flux scaled by $fl.HU^2$. The ratio of the scales for energy flux and energy density is fl.

A solution to the governing wave equation (5.4) in the form of a slowly modulated wave group about a central wavenumber k and frequency σ is

$$\psi = \mathbf{re}[\, C(X,y,T) \exp\ i(kx - \sigma t)\,],$$

provided

$$[(-i\sigma + \varepsilon\frac{\partial}{\partial T})[\frac{\partial}{\partial y}(h^{-1}\frac{\partial}{\partial y}) + \delta^2(ik + \varepsilon\frac{\partial}{\partial X})^2 h^{-1}] - h^{-2}\frac{dh}{dy}(ik + \varepsilon\frac{\partial}{\partial X})]\ C(X,y,T) = 0. \qquad (5.5)$$

The solution to this equation must be valid for all small ε, including $\varepsilon = 0$. Given the detail of the offshore topography and the longshore wavenumber k, we can anticipate that the solution will take the form of a separable operator-function product

$$C(X,y,T) = \mathbf{A}[\tfrac{1}{i}(ik + \varepsilon\frac{\partial}{\partial X}),\ y]\ B(X,T). \qquad (5.6)$$

With $\varepsilon = 0$, the equation (5.5) with solution (5.6) inserted becomes

$$[i\sigma\frac{\partial}{\partial y}(h^{-1}\frac{\partial}{\partial y}) + i\sigma(\frac{k}{\sigma h^2}\frac{dh}{dy} - \delta^2 k^2 h^{-1})]\ \mathbf{A}[\tfrac{1}{i}(ik), y]B = 0.$$

This is now functionally separable and, for appropriate boundary conditions, has the solution

$$\sigma = \sigma(k), \qquad \mathbf{A}[\tfrac{1}{i}(ik), x] = A(k,x),$$

as provided in the dimensionless eigen system equivalent to (3.1). Thus the detail of the operator **A** is provided by the function A. To first order in ε, the operator **A** is given by

$$\mathbf{A}[\tfrac{1}{i}(ik + \varepsilon\frac{\partial}{\partial X}),\ y] = A(k,y) + [A(k,y)]_k\,\varepsilon\,\tfrac{1}{i}\frac{\partial}{\partial X},$$

so that $C(X,y,T) = \mathbf{A}\,B(X,T)$ is the properly determined solution to (5.5). It can be shown that the zero order eigen equation reduces to the form in (3.1), while the first order equation becomes

$$[-i\sigma(\frac{\partial}{\partial y}(\frac{1}{h}\frac{\partial}{\partial y}) + \delta^2\frac{(ik)^2}{h}) - \frac{1}{h^2}h_y ik]\frac{1}{i}\frac{\partial A}{\partial k}\frac{\partial B}{\partial X}$$
$$+ [\frac{\partial}{\partial T}(\frac{\partial}{\partial y}(\frac{1}{h}\frac{\partial}{\partial y}) + \delta^2\frac{(ik)^2}{h}) - (\frac{2i^2k\sigma}{h} + \frac{1}{h^2}\frac{dh}{dy})\frac{\partial}{\partial X}]AB = 0,$$

or
$$\frac{\partial B}{\partial T} + \frac{d\sigma}{dk}\frac{\partial B}{\partial X} = 0, \qquad (5.7)$$

on using the dimensionless equivalent of the group velocity relation. Thus the B part of the solution travels at a velocity which is properly referred to as the group velocity. Indeed the solution of (5.7) is

$$B = B(X - \frac{d\sigma}{dk}T),$$

which shows that the group of waves is propagated longshore without distortion.

ENERGY RELATIONS FOR MODULATED SHELF WAVES

The complete combined zero and first order solution for modulated continental shelf waves can be shown to take the form

$$\psi = \mathbf{re}[\mathbf{A}\, B \exp\ i(kx - \sigma t)\,], \qquad (6.1)$$

$$u = \mathbf{re}[-h^{-1}(\mathbf{A}\, B)_y \exp i(kx - \sigma t)\,], \qquad (6.2)$$

$$v = \mathbf{re}[h^{-1}(ik + \varepsilon\frac{\partial}{\partial X})\,\mathbf{A}\, B \exp i(kx - \sigma t)\,], \qquad (6.3)$$

provided the extended eigen equation (5.5) is satisfied by solution (5.6).

Once again the energy density and energy flux are rapidly varying functions of the fast variables (y,t) and these quantities are more usefully averaged over the wave period to give (to first order in ε)

$$\bar{E} = [(4h)^{-1}(A_y B)(A^*_y B^*) + \delta^2 k^2 (AB)(A^* B^*)\,], \delta^2 \qquad (6.4)$$

$$\bar{\underset{\sim}{F}} = (\tfrac{1}{2}\,\mathbf{re}[-h^{-1}\,\mathbf{A}^* B^*(ik + \varepsilon\frac{\partial}{\partial X})(-io + \varepsilon\frac{\partial}{\partial T})\mathbf{A}B]\ \delta^2$$
$$+ \tfrac{1}{2}\mathbf{re}[\tfrac{1}{2}h^{-2}\frac{dh}{dy}(\mathbf{A}B)(\mathbf{A}^* B^*)\ ; \tfrac{1}{2}\mathbf{re}[\,\mathbf{A}^* B^*(-io + \varepsilon\frac{\partial}{\partial T})\mathbf{A}_y B]). \qquad (6.5)$$

The energy relation can be simplified to give

$$\frac{\partial \bar{F}_0{}^y}{\partial y} = 0,$$

at zero order in ε, and

$$\frac{\partial}{\partial T}(\bar{E}_0) + \frac{\partial}{\partial X}(\bar{F}_0{}^X) + \frac{\partial}{\partial y}(\bar{F}_1{}^y) = 0, \qquad (6.6)$$

to first order in ε. Thus we will require the following zero and first order terms from (6.4) and (6.5)

$$\bar{E}_0 = (4h)^{-1}[A_y A^*_y + \delta^2 k^2 AA^*)]\, BB^*,$$

$$\bar{F}_0{}^X = [(4h^2)^{-1}h_y - \delta^2 (2h)^{-1}k\sigma]\, AA^*\, BB^*,$$

$$\bar{F}_0{}^y = \tfrac{1}{2}\mathbf{re}[\, h^{-1}\, i\sigma\, A_y A^*]\, BB^*,$$

$$\bar{F}_1^{\,y} = -(2h)^{-1}\,\mathbf{re}[\sigma A^*{}_k A_y \frac{\partial}{\partial X}(BB^*) + A_y A^*\; B^*\frac{\partial B}{\partial T} - \sigma A^* A_{yk}\; B^*\frac{\partial B}{\partial X}], \qquad (6.7)$$

where $A = A(k,y)$ and $B = B(X,T)$. From the dimensionless form of (4.2) and (4.3), it will immediately be seen that

$$\bar{F}_o^{\,y} = 0,$$

$$\bar{F}_o^{\,x} - \frac{d\sigma}{dk}\bar{E}_o = \frac{-1}{8}\,[h^{-1}\,[(\sigma AA^*)_{ky} - 4\sigma\,\mathbf{re}\,(A_x A^*{}_k)]]_y\; BB^*. \qquad (6.8)$$

because E_o takes the separable form $E_o = D(y)\,BB^*$, and because

$$\frac{\partial}{\partial T}(BB^*) + \frac{d\sigma}{dk}\frac{\partial}{\partial X}(BB^*) = 0,$$

follows from the propagation of the group velocity (5.7), we can note that

$$\frac{\partial \bar{E}_o}{\partial T} + \frac{d\sigma}{dk}\frac{\partial \bar{E}_o}{\partial X} = 0. \qquad (6.9)$$

The mean energy density is propagated with the group velocity. Comparing (6.9) with the first order energy relation (6.6) gives

$$\frac{\partial}{\partial y}\bar{F}_1^{\,y} + \frac{\partial}{\partial X}\left(\bar{F}_o^{\,x} - \frac{d\sigma}{dk}\bar{E}_o\right) = 0.$$

Substituting from (6.8) and integrating with respect to y gives

$$\bar{F}_1^{\,y} = -\frac{1}{8}\,[h^{-1}[(\sigma AA^*)_{ky} - 4\sigma\,\mathbf{re}\,(A_y A^*{}_k)]]\frac{\partial}{\partial X}BB^*.$$

This last result can also be formally derived from the longer expression in (6.7).

Thus once more we have the mean energy flux not equal to the group velocity times the mean energy density although the mean energy density does move with the group velocity. We can establish an equivalence between the two definitions of the energy flux by noting that

$$(\bar{F}_o^{\,x}\,;\,\bar{F}_1^{\,y}) - (\frac{d\sigma}{dk}\,;\,0)\,\bar{E} = (\frac{\partial}{\partial y}\,,-\frac{\partial}{\partial X})\,\mathbf{J}(X,y,T),$$

where the divergence free

$$\mathbf{J}(X,y,T) = \frac{-1}{8h}\,[\sigma(AA^*)_{ky} - 4\sigma\,\mathbf{re}\,(A_y A^*{}_k)]\;BB^*.$$

DISCUSSION

In this paper we have analysed the propagation of non-divergent continental shelf waves, trapped to a long straight coastline. Using the method of multiple scales developed by Shillington and Brundrit (1986) for the study of long period edge waves, we have investigated the energy propagation of both a plane wave train and a modulated group of shelf waves. As with the edge waves, we found that the mean energy flux did not equal the group velocity tmes the mean energy density, either in the plane wave solution or the modulated group solution. However, it is well known that the definition of energy flux is arbitrary to within the addition of a divergence free quantity. This term, J, has been found for both the plane wave and the modulated group solution. It is interesting to note that even though the analysis was carried out with a parameter $\delta = 1/L$ (the ratio of the offshore topography scale to the longshore wavelength scale), the result for J turned out to be independent of this parameter. This means that although the definitions for the energy and energy flux inluded δ, the term cancelled exactly due to incorporation of the dispersion relation and did not result in differences in J between "long" and "short" shelf waves.

In contrast to the way in which we defined the energy and energy flux for edge waves, we found it more convenient to define the energy directly from the governing wave equation as suggested by Pedlosky. Recent work in progress suggests that a definition involving the velocities u,v and elevation h leads to a different definition of the energy density.

In answer to the question as to which definition of energy flux is preferable, we note that both Longuet-Higgins and Pedlosky indicated that a definition which led to the use of mean energy times group velocity would be preferable for an application in which the group rather than the individual waves was the essential feature. It is difficult to say categorically which definition would be preferable for shelf waves, except that one might expect that the coastline may not be long enough to contain more than a few wavelengths. However, we have shown that the group of shelf waves propagates without distortion or leakage to the open sea.

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