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Flow Resistance in Open Channels with Standardised Triangular Roughness Elements

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ABSTRACT

The analysis of results of experiments conducted in a rectangular channel with two-dimensional triangular roughness elements representing a standard pattern of bed undulations has shown that the bed friction factor can be expressed by a logarithmic relationship between the friction factor and the relative roughness involving the bed hydraulic radius and geometric parameters of the roughness elements together with a correction factor which takes into account the wake length and height below the crest to the point of reattachment of flow with the upstream face of the roughness element.

1 INTRODUCTION

In cases of flow in alluvial channels, canals taken through rock cuttings and flow below ice covers wavy surfaces are obtained on the boundary giving rise to bed undulations. The pattern of these undulations is irregular and three-dimensional. Such bed undulations exert considerable drag and contribute significantly to the hydraulic resistance.

The resistance coefficients of the commonly used formulas of Chezy and Manning are not non-dimensional and the effect of bed undulations is not integrally considered in these coefficients. The friction factor, on the other hand, is non-dimensional and varies with R_e as such the fluid and flow properties as well as the geometric characteristics of the channel bed - which can be expressed non-dimensionally in the form of relative roughness - can be related to friction factor.

To gain insight into the flow over undulated bed, experiments were conducted in a rectangular channel with rigid boundaries having smooth painted aluminium surfaces on the sides and bed covered with two-dimensional wooden triangular roughness elements to represent a standard pattern of undulations. The shape, size, concentration λ (defined as the ratio of height to base length of the elements) and the scheme of experiments are presented in Table 1.

2 THEORETICAL CONSIDERATIONS

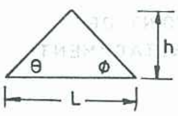
The bed of the experimental channel, being covered with wooden triangular elements was rougher than the painted aluminium surface of the sides. So the bed friction factor (f_b) was differentiated from wall friction factor (f_w) by applying side-wall correction procedure, outlined by Johnson (3). For steady uniform flow in rectangular channels of width B, and depth of flow D, this procedure yields the relationship

$$f_b = f + \frac{2D}{B}(f - f_w) \qquad .. (1)$$

where f is the total friction factor given by Darcy-Weisbach formulae

$$f = \frac{8g\ RS}{V^2} \qquad .. (2)$$

in which g is the acceleration due to gravity,

Sketch of roughness elements (Not to scale)	Element Type	Details of Roughness elements					Total no. of elements used in the entire length.	Bed Slope	Run Numbers
		L(cm)	h (cm)	λ	θ	ϕ			
	1	3·8	1·9	0·5	45°	45°	256	0·001	111 – 161
								0·002	112 – 162
								0·0029	113 – 163
	2	5·7	1·9	0·33	26·57°	45°	171	0·001	211 – 261
								0·002	212 – 262
								0·0029	213 – 263
	3	7·6	1·608	0·21	18·42°	30°	128	0·001	311 – 361
								0·002	312 – 362
								0·0029	313 – 363

SHAPE OF CHANNEL :- RECTANGULAR ; BED WIDTH = 29·05 cms.
ROUGHNESS ON SIDES :- SMOOTH PAINTED ALUMINIUM SURFACES. LENGTH OF FLUME = 9·75m

TABLE 1. SCHEME OF EXPERIMENTS

R is the hydraulic mean depth, S is the bed slope of the channel and V is the mean velocity of flow. Following the application of Nikuradse roughness standards to open channels by Keulegan (4) the friction factor for the smooth side walls can be written as

$$\frac{1}{\sqrt{f_w}} = a_w \cdot \log\left[(R_e)_w \cdot \sqrt{f_w}\right] + b_w \quad .. (3)$$

where $(R_e)_w$ is the Reynold's number of flow pertaining to wall; the values of the constants a_w and b_w were obtained from a series of experiments where both the sides and the bed of the rectangular channel were of smooth painted aluminium surface as 2.03 and -0.47 respectively.

When the bed is covered with roughness elements, the friction factors f and f_w were calculated from Eqs.(2) and (3) respectively. Substituting these values in Eq.1 the bed friction factor was calculated.

The sking friction of the bed, f_b', can be represented by relationship, similar to that given by equation (3), and the corresponding constants were found to be 2.0 and -0.8 respectively from a series of experiments where the sides were of smooth painted aluminium, but the bed was of plane painted wooden surface.

The form friction f_b'' can then be obtained by deducting from the total friction factor f_b, the contribution of the skin friction factor f_b'. Pillai (5) has shown that for two-dimensional triangular roughness elements the form friction factor of the bed f_b'' can be expressed by the equation of the form

$$\frac{1}{\sqrt{f_b''}} = A_o \log\left(\frac{R_b}{h\lambda}\right) + B_o \quad .. (4)$$

The value of the constant A_o is a measure of the Karman constant K for mixing length, while the additive constant B_o is a function of shape, size and spacing of the roughness elements. By introducing a parameter T to take care of such effects Eq.(4) can be modified as

$$\frac{1}{\sqrt{f_b''}} = A_o \log\left(\frac{R_b}{T\lambda h}\right) \quad .. (5)$$
$$-(B_o/A_o)$$

where T = 10 .. (6)

The values of A_o and T are to be determined experimentally. The effect of variation due to size and spacing of the elements is taken into account by modifying the characteristic roughness height λh to a new value of $T \lambda h$.

As the flow passes over triangular roughness elements on the bed, separation occurs at the crest and the flow gets reattached on the upstream face of the next element (Fig.1). It is seen that with the change in the concentration of the elements, the wake length L_w, as well as h_e which is the difference in level between the crest and the point of reattachment, also change. As a result the characteristic roughness height λh gets affected because, (i) the actual height of obstruction offered to the flowing water by the bed is h_e and not h; (ii) the horizontal distance between the points of separation and reattachment is L_w, whereas L is the length of the elements.

As the effect of variation in the geometric characteristics of the bed undulations is represented by T, it is conceived that as the bed roughness elements change, T modifies the characteristic roughness height, h, by modifying h in accordance with h_e, and the length for measuring the concentration i.e. L in accordance with L_w. The following functional relationship is assumed for T.

$$T = \phi\left(\frac{h_e}{h}, \frac{L_w}{L}\right) = \frac{(h_e/h)^b}{(L_w/L)^a} \quad .. (7)$$

Since $\lambda h = h \times (h/L) = h^2/L$, b was assumed to be 2, and a was to be found from experimentally observed values of h, h_e, L and L_w.

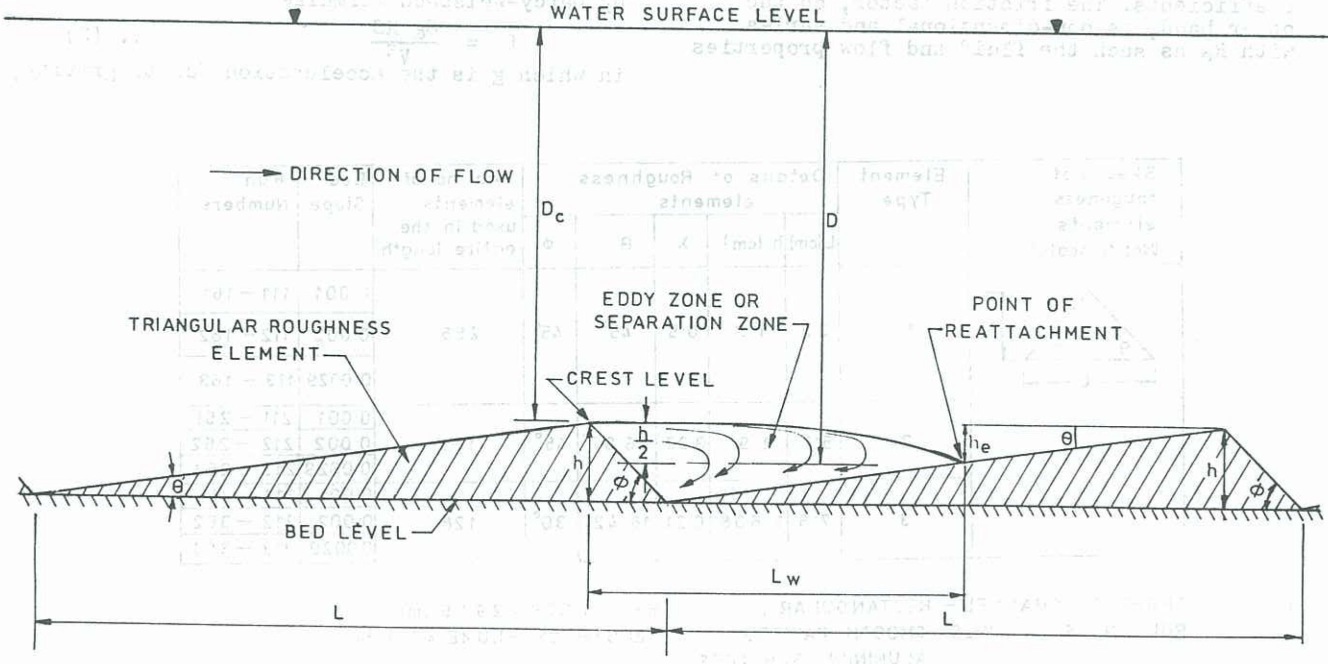


FIG.I. DEFINITION SKETCH AND SCHEMATIC DIAGRAM OF EDDY ZONE

3 EXPERIMENTS : GENERAL ARRANGEMENT

The experiments were conducted in a rectangular test channel (9.75 m long, 29.05 cm wide and 38.1 cm deep) built from 6.4 mm thick aluminium alloy sheets. A tail bay with a tail gate at the downstream end and a smooth transition with proper stilling arrangements at the upstream end enabled the establishment of uniform flow for each run. A calibrated V-notch at the overhead tank near the upstream end of the test channel was used for measurement of discharge. The triangular roughness elements were made of solid wood. They extended the entire width of the test channel and were screwed to wooden planks secured by fixing to the bottom-sheet of the aluminium test channel. Three patterns of wooden elements and three different bed slopes were used. For each set, six experiments with six different depths ranging from 8.3 cm to 25.89 cm were conducted (Vide Table-1).

Measurement of Length of Wake

For measuring the length of the wake, a very thin layer of plastic clay was applied on the bed. Tiny crystals of potassium permanganate were placed - one at a time - at a particular point on the plastic clay layer. During flow of water a thin streak of pink colour emerged from the tiny crystal, which after travelling a certain distance dispersed and disappeared into the flowing water. The average location of the point, at which the change in direction of the streak from upstream to downstream took place, could be ascertained with a number of trials. From the experimentally observed data, the different friction factors were calculated and are shown in Table-2. More details are given elsewhere (1). It was observed that the contribution of skin friction in the bed resistance was negligible and as such the bed resistance was taken to be all due to form resistance alone.

4 DISCUSSION OF RESULTS

Investigators like Vanoni and Hwang (7), Pillai (5) have fitted their experimental points, to a straight line of the form given by Eq.4. When the results of all 54 runs of the present experiments were taken and a relationship between $1/\sqrt{f_b}$ and $(R_b/\lambda h)$ was tried upon, a large amount of scatter about the best fit line was noticed. Following Sayre and Albertson (6) each particular set of points were fitted by the method of least squares to a straight line and the constants evaluated. For the nine sets of experiments the values of A_o varied from 1.6166 to 1.8947, whereas the values of B_o varied from 0.016 to 0.832.

The experimental points of the nine different sets are represented graphically by a series of nine parallel lines in Fig.2 the slope of the parallel lines being equal to the average of the A_o values which is 1.8162. Eq.4 can then be written as

$$\frac{1}{\sqrt{f_b}} = 1.8162 \log \left(\frac{R_b}{\lambda h} \right) + B_1 \quad .. (8)$$

and the additive constants will assume new values, designated as B_1 which are slightly different from the calculated values of B_o. The values of the changed constants for each set are computed by taking the average value of six differences given by $1/\sqrt{f_b} - 1.8162 *$ $\log (R_b/\lambda h)$. With $A_o = 1.8162$ and the nine values of B_1, a set of nine values of T were calculated using Eq.(6). It is seen that by using the average of the different values of the multiplying constant A_o, all the experimental points can be represented very satisfactorily by a single straight line, by introducing the parameter T. Combining the present experimental data and the published data of Pillai, 18 sets comprising 108 test runs covering the values of λ ranging from .0625 to 0.5 were studied together. The results are presented in Table-3.

TABLE-2 : Experimental Data and Results

Run No.	D_{av} cms	Q lit/sec	Viscosity $m^2/sec \times 10^{-7}$	Mannings n	f	f_w	R_b cms	f_b
111	8.48	4.8	8.46	.0230	.1098	.0261	7.74	.1587
131	15.40	13.0	8.38	.0193	.0690	.0207	13.01	.1203
151	22.44	24.1	8.42	.0170	.0506	.0182	17.56	.1008
122	11.86	11.6	9.31	.0216	.0910	.0222	10.58	.1472
142	18.87	25.1	9.28	.0184	.0612	.0187	15.61	.1164
162	25.89	42.8	9.17	.0162	.0451	.0167	19.75	.0956
113	8.45	8.5	9.85	.0221	.1013	.0236	7.71	.1465
133	15.37	22.4	9.99	.0191	.0675	.0193	13.11	.1186
153	22.35	41.8	9.95	.0166	.0483	.0169	17.62	.0966
221	11.93	8.5	8.72	.0210	.0853	.0230	10.49	.1364
241	18.97	18.1	8.61	.0182	.0595	.0194	15.45	.1119
261	25.95	29.9	8.65	.0164	.0462	.0176	19.60	.0974
212	8.39	6.2	8.46	.0247	.1274	.0255	7.77	.1863
232	15.37	17.6	8.65	.0201	.0756	.0200	13.29	.1345
252	22.20	31.7	8.76	.0180	.0568	.0178	17.98	.1165
223	11.83	14.4	9.44	.0208	.0842	.0210	10.52	.1357
243	18.91	30.4	9.44	.0184	.0607	.0181	15.73	.1162
263	25.89	50.7	9.26	.0164	.0464	.0163	20.06	.1002
311	8.33	5.0	8.57	.0214	.0956	.0252	7.53	.1360
331	15.25	13.4	8.59	.0185	.0637	.0204	12.74	.1092
351	22.17	24.6	8.54	.0163	.0470	.0179	17.07	.0913
322	11.65	12.0	8.28	.0203	.0801	.0210	10.30	.1276
342	18.73	26.2	8.36	.0175	.0553	.0179	15.30	.1034
362	25.77	44.6	8.31	.0154	.0410	.0160	19.32	.0853
313	8.30	8.6	9.66	.0212	.0935	.0231	7.56	.1338
333	15.16	22.7	9.85	.0185	.0637	.0190	12.86	.1103
353	22.20	43.0	9.85	.0159	.0448	.0165	17.25	.0880

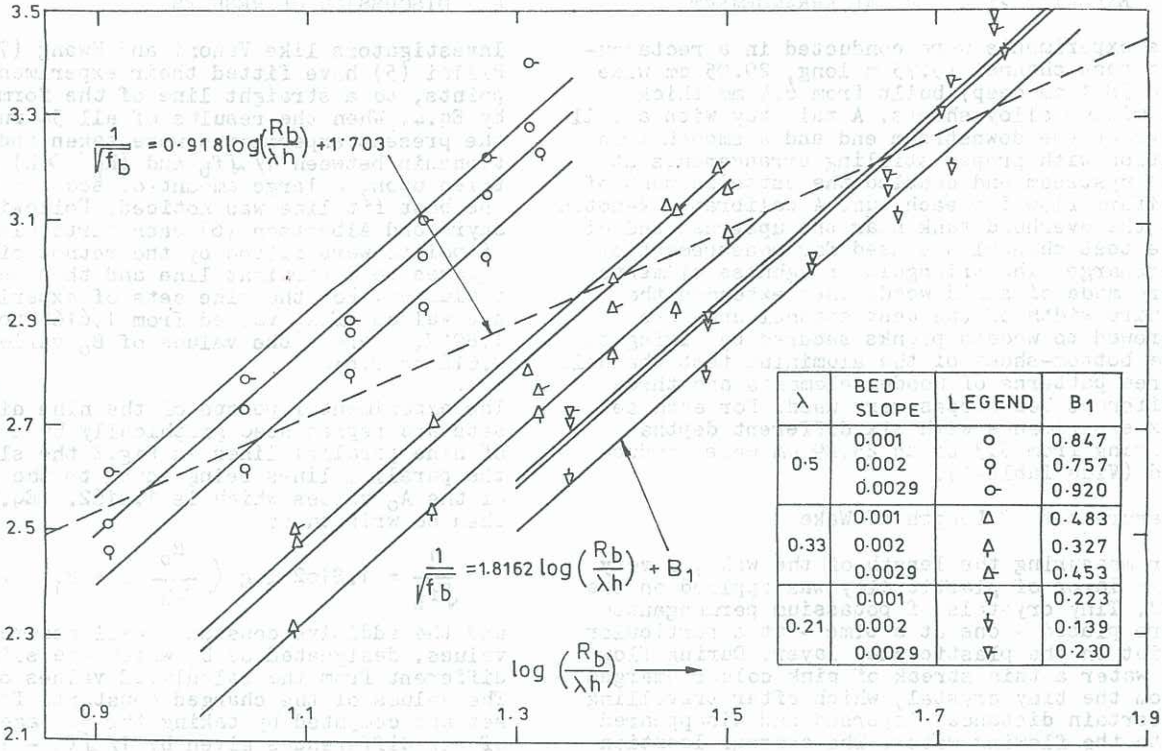


FIG. 2. BED FRICTION FACTOR VS. RELATIVE ROUGHNESS WITH ROUGHNESS HEIGHT λh (----FOR ALL DATA, ——— FOR INDIVIDUAL SET)

Table-3: Published Data of Pillai

λ	Bed slope x 10^{-2}	D_{av} cms	Q lit/sec	R_b cms	f_b
.125	.1	11.43	8.64	9.91	.1147
		20.19	22.54	15.37	.0815
		27.78	36.65	19.35	.0735
	.1933	12.35	14.19	10.58	.1024
		18.78	28.38	14.75	.0825
		25.37	44.83	18.50	.0756
	.2733	8.18	7.84	7.50	.1469
		13.72	20.08	11.61	.0979
		20.00	37.38	15.54	.0804
.0833	.1	14.06	13.88	11.34	.0769
		23.42	30.22	16.67	.0662
		30.30	44.83	19.60	.0592
	.1933	8.94	9.18	7.86	.0953
		15.22	21.30	12.31	.0804
		21.53	36.87	16.00	.0697
	.2733	11.19	15.69	9.57	.0881
		16.83	30.30	13.32	.0742
		21.59	44.69	16.06	.0677
.0625	.1	8.51	6.80	7.31	.0757
		17.69	21.92	12.92	.0556
		24.51	35.88	16.03	.0495
		10.58	13.71	8.81	.0672
		17.53	30.22	13.08	.0562
		23.39	46.78	15.91	.0508
	.2733	6.62	7.53	5.94	.0828
		12.57	21.44	10.18	.0633
		17.96	36.99	13.50	.0575

The average value of the multiplying constant for 18 sets worked out to be 2.06675. The values of B_1 for each of the 18 sets and then the value of T were calculated. The slope of the best fit line through the points given by $1/\sqrt{f_b}$ and log ($R_b/T\lambda h$) was determined. It was found that the slope was 2.06.

Since T is an implicit function of the multiplying constant, its values will change if A_o is assigned different values. In this analysis the value of A_o, was achieved through successive iterations, till the difference in the assigned value of the multiplying constant and the obtained value of the slope of the best fit line was limited to 0.0001. The final relationship worked out to be

$$\frac{1}{\sqrt{f_b}} = 2.0432 \log \left(\frac{R_b}{T\lambda h} \right) \quad .. \quad (9)$$

The corresponding value of Von-Karman constant comes out to be 0.39799, which is very close to the most widely accepted value of 0.4.

A plot of $1/\sqrt{f_b}$ versus log ($\frac{R_b}{T\lambda h}$) for all the 108 experimental points is shown in Fig.3, which shows a very good agreement of the experimental points about the line given by Eq.(9).

It was found for all the experiments the Froude number was less than 0.5. It was also observed that for a particular type of element, when the depth of flow was changed, the length of wake L_w remained unaltered. So it is taken that the length of wake is independent of Froude number which is in agreement with the findings of Engel (2).

The values of h_e/h and L_w/L were computed for all the types of elements. A log plot of $(h_e/h)^2/T$ against (L_w/L) for all the elements has shown that a straight line having a slope of about 0.5 can be best fitted to the points giving thereby a value of 0.5 for 'a' in Eq.(7).

5 CONCLUSIONS

The early empirical formulas involving flow resistance in channels, such as those of Chezy and Manning do not consider in particular the effects of bed forms and its variations on flow resistance. The logarithmic formulas of friction factor developed for pipe flow by Prandlt-Karman-Nikuradse are extended to determine the resistance to flow in open channels. The undulations of the bed forms can be expressed by the relative roughness ($R_b/\lambda h$), which takes into account the geometry and the distribution of the roughness elements on the bed. For the undulated bed represented by two-dimensional triangular roughness elements, it is found that for values of λ ranging from .0625 to .5, the bed friction factor can be very satisfactorily expressed by Eq.9, where T is a factor that takes into account the variation in the height, length and spacing of the roughness elements.

Experimental determination of the length of separation zone, (L_w) with Froude Number less than 0.5 has shown that L_w is independent of depth of flow. From the observed values of L_w and h_e for various values of λ, it is found that T can also be expressed very satisfactorily by Eq.7.

The parameter T introduced as a correction factor will be helpful in the further studies on the resistance to flow in channels with undulated beds.

REFERENCES

1. Datta, T K (1983): Studies on resistance in open channels with bed undulations simulated by closely spaced triangular roughness elements - Ph.D. Thesis presented to I.I.T., Kharagpur, India.

2. Engel, P (1981): Length of flow separation over Dunes. Journal ASCE, Hyd. Div. Vol.107, No.HY10, 1133-1143.

3. Johnson, J W (1942): The importance of side wall friction in bed load investigation. Civil Engineering, ASCE, Vol.112, 329-331.

4. Keulegan, G H (1938): Laws of turbulent flow in open channels. Jour.National Bureau of Standards, Research paper 1151, Vol.21, 704-741.

5. Pillai, C R S (1976): Effects of undulations of bed on friction factor in open channels, Ph.D.Thesis. presented to I.I.T., Kharagpur, India.

6. Sayre, W W and Albertson, M L (1963): Roughness spacing in rigid open channels. Trans. A.S.C.E., Vol.128, Part I, 343-427.

7. Vanoni, V A and Hwang, L (1967): Relations between bed forms and friction in streams. Journal, A.S.C.E.,Hyd.Div., Vo.93, No.HY3, 121.144.

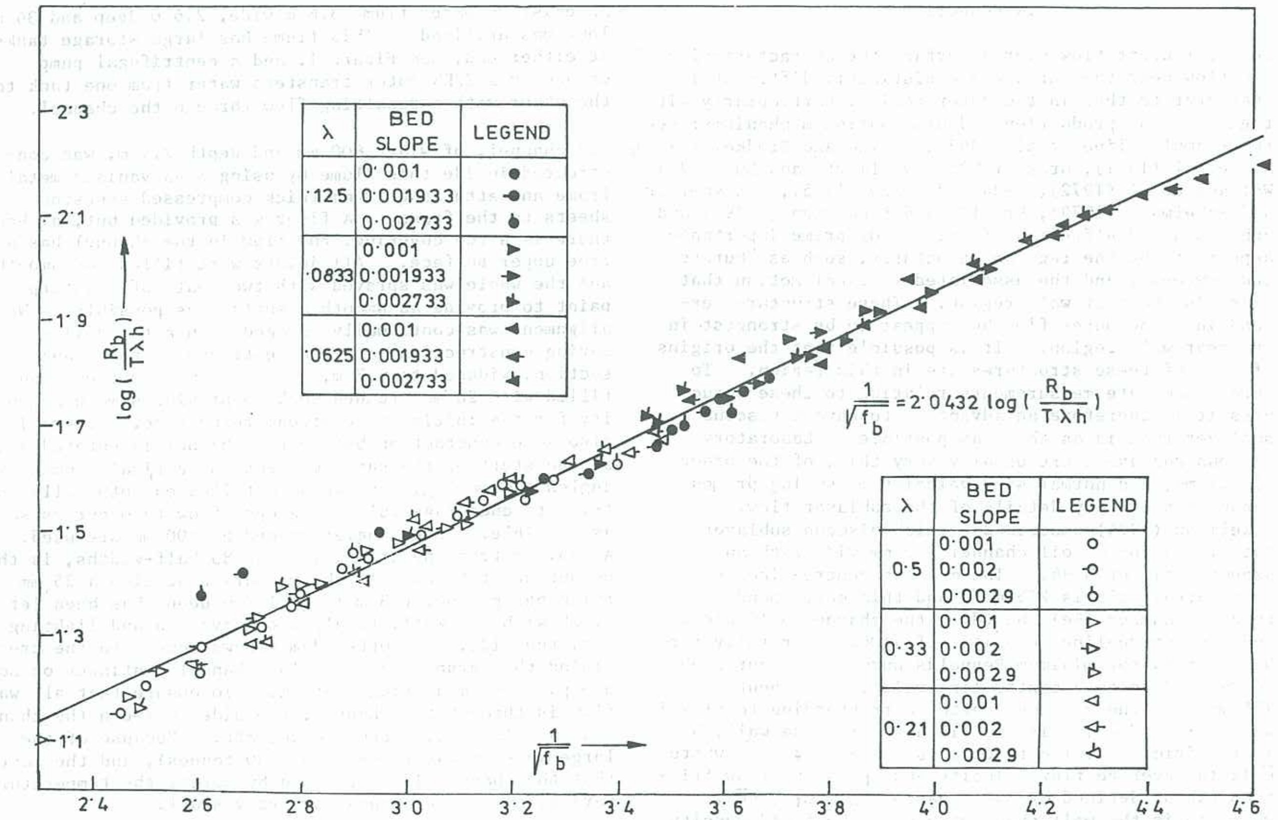


FIG. 3. BED FRICTION FACTOR VS. RELATIVE ROUGHNESS WITH ROUGHNESS HEIGHT Tλh.