9th AUSTRALASIAN
FLUID MECHANICS CONFERENCE,
AUCKLAND, 8-12 DECEMBER 1986

Horseshoe Vortices and Bridge Pier Erosion

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ABSTRACT

The classical explanation for erosion of bed material around a bridge pier depends on the supposed formation of a strong U-shaped *horseshoe vortex* from boundary layer vorticity trapped upstream of the pier. However, boundary layer vorticity is advected continuously towards the pier and would progressively build a vortex of unbounded strength on this explanation. More careful observation shows that the vortices around obstacles in a stream are generally weak and often multiply nested with alternating sense of rotation. They involve both incident boundary layer vorticity and vorticity with a cross-stream component of opposite sense generated at the lower boundary in the adverse pressure gradient produced inertially as flow divides about the pier. The resulting net circulation per unit length calculated through the full depth of the boundary layer in the upstream plane of symmetry tends to zero at the leading edge of the pier. Erosion has been reported as starting *downstream* of the broadest transverse pier section; it depends on surface stress and hence on momentum flux from fluid to boundary, and not on horseshoe vortices.

INTRODUCTION

It has long been accepted that horseshoe vortices, which may be observed in river flow around the bases of bridge piers and other obstacles, produce a scouring that has been held responsible for the erosion of loose bed material upstream and to the sides of piers and its deposition dowstream. Thus, to take a recent example, Qadar (1981) noted that "because of the strong pressure field induced by the pier, the flow in front of a pier separates from the bed and rolls up to form the scouring vortex, which has been identified as the basic mechanism in such a scouring situation." He carried out two sets of experiments on flow past shaped wooden blocks mounted in an inclined flume. In the first series, which was designed to determine the behaviour and structure of the scouring vortex, a set of six blocks were mounted in turn on the floor of the flume. The diameter of the scouring vortex in the upstream plane of symmetry was shown to relate linearly to the block width (even though this varied from 1/12 to 1/2 of the width of the flume) and to depend on nothing else. In the second series, the blocks were bedded in sand to determine the depth of scour, which was shown to have limited dependence on grain size, no dependence on the depth of flow provided that this exceeded the diameter of the scouring vortex, and a complicated power dependence on the circulation of that vortex. In a moment of doubt, Qadar notes that the "velocity of flow ahead of the pier, where the localised scour hole develops, is small because of the proximity of the stagnation point. The velocity of flow alone, therefore, may not be considered a sufficient criterion for the development of a localised scour hole. At other sections sufficiently upstream, where the flow velocity is much greater, no such scour hole develops." Qadar, accepting conventional views, chose to relate the observed depth of scour to an estimate for the circulation of the horseshoe vortex, but otherwise took no account of vorticity, and in particular, no account of the boundary layer in the approaching stream. He considered that the scouring vortex is maintained by the drag of the flow over its top.

Many authors (including Baker, 1979; Melville, 1975; Utami, 1975) have identified the horseshoe vortex as providing the basic mechanism for scour, but few have developed the full advantages of vorticity in studying the problem. While the idea of a vortex stretched tightly around the pier and producing rapid reversed flow near the bed is attractive, both it and the classical conception of the horseshoe vortex prove to be erroneous.

THE CLASSICAL HORSESHOE VORTEX

Horseshoe vortices are believed to form when filaments of boundary layer vorticity are advected towards a pier projecting through the boundary layer and accumulate upstream of the obstacle, but are carried past on either side (Bradshaw, 1983). These filaments cannot be severed, and therefore collect into a U-shaped vortex wrapped around the upstream side of the obstacle with arms stretching away downstream. A strong, steady-state vortex then results from the supposed balance between longitudinal stretching downstream and lateral diffusion of the vortex core. This is an inadequate model, however, as boundary layer vorticity is continuously advected towards and trapped by the pier. Thus a measure for the vorticity content of the boundary layer underlying a stream U is provided by the circulation per unit streamwise length through the full depth of the layer, with magnitude U; and a measure for the mean speed of vorticity advection in the boundary layer is U/2 (exact for the Blasius boundary layer). It follows that the flux of vorticity towards the pier is approximately $U^2/2$, and hence that the trapped circulation after time t is of order $U^2t/2$ *and is unbounded.* There appears to be no limit to the accumulation of boundary layer vorticity ahead of a bridge pier! It is however, obvious that the leading edge of a pier is not a singularity of the flow. Three possible resolutions of the paradox are:

(i) that vortex filaments are drawn over the top of the obstacle;
(ii) that viscosity is responsible for steady loss of vorticity; and
(iii) that there is steady generation of vorticity at the boundaries with sense opposing that of the boundary layer.

EXPERIMENTAL OBSERVATIONS

Because of the difficulty of reproducing photographs, the main experimental detail will be presented in a series of slides. Vortices are certainly observed around obstacles in a stream, but they are generally relatively weak where they intersect the up-

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stream plane of symmetry and the circulation in each trailing arm tends to decrease fairly rapidly downstream. Indeed, it has been shown recently (Mason and Morton, 1986) that the trailing arms of horseshoe vortices contribute little to the patterns of strong trailing vortices observed in the wakes of obstacles and that strong wake vortices are generally associated with separation in the lee of the obstacle. Further study of flow detail in the upstream plane of symmetry shows Reynolds number dependence as sketched in Fig. 1. It should be noted

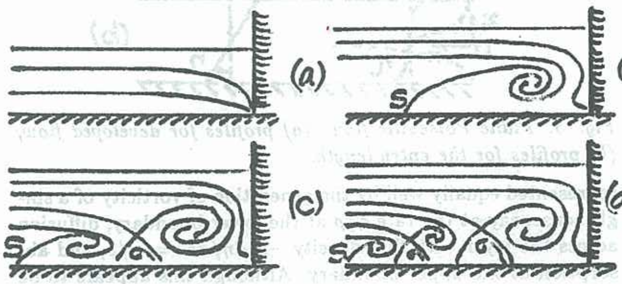

Fig. 1. Flow in the upstream plane of symmetry for a circular pier

that flow depends on the breadth of the pier, its radius of curvature at the leading edge, the thickness of the approaching boundary layer and the free stream speed: there are thus three (and, for a general obstacle, five) Reynolds numbers, which explains the complexity of flows observed (*e.g.*, plate 32, Van Dyke, 1982). There is no horseshoe vortex at low Reynolds numbers (a), separation followed by a single vortex with the sense of the boundary layer vorticity at higher Reynolds numbers (b), and with further increases (c and d) additional pairs of counter-rotating vortices appear. At still higher Reynolds numbers, the flow becomes turbulent upstream of the pier, but the patterns of vortices persist in the mean motion (Baker, 1980).

The two features of principal interest in the present context are the weakness of the observed vortices and the nesting of vortices with opposite sense of rotation. The slides show very clearly that the return velocity near the boundary is very much less than that in the outer part of the vortex as a consequence of the highly three-dimensional nature of the flow in which fluid diverges from the plane of symmetry along the core of the vortex. Even more striking is the appearance of a further pair of counter-rotating vortices in (c) corresponding with a Reynolds number (based on cylinder diameter and stream speed at cylinder top level) of 2700, in the case to be illustrated for a squat cylinder of height/diameter ratio 0.36 and height/boundary layer thickness ratio about 2. In this case the two upper clockwise vortices (of boundary layer vorticity) are separated by a lower anticlockwise vortex of triangular section, and are preceded by a wedge of slow flow behind the point of separation. As the Reynolds number is raised, an additional counter-rotating pair of vortices develops, as part of a sequence in which the number of upstream vortices is always odd with "like" upper vortices separated by lower triangular "unlike" vortices. The configuration, however, is symmetric about the upstream plane of symmetry with flow diverging to either side, and there is no way in which vorticity of sense unlike that of the boundary layer can be produced by turning boundary layer vorticity. The latter of these features, in particular, is quite incompatible with the classical model for horseshoe vortices.

There is one further piece of observational evidence against the classical horseshoe vortex model in which the trailing arms necessarily form a vortex pair with circulation producing downwash in the centre of the wake. Actual trailing vortex pairs observed in the wakes of obstacles are found to have either upwash or downwash according to the shape of the obstacle and its depth relative to the boundary layer. Where the incident flow is diverted laterally around the sides of the obstacle there is downwash in the wake, but where it is lifted over the crest of the obstacle there is upwash (Mason and Morton, 1986). This latter behaviour includes flow over broad, low obstacles such as low buildings and hills, and is both relatively common and totally inconsistent with the classical model.

THE GENERATION AND DECAY OF VORTICITY

Before we can take up possible resolutions of the paradox presented by the classical model of the horseshoe vortex, we must discuss the generation and decay of vorticity in the neighborhood of the pier or other obstacles, and in this we shall follow an earlier survey (Morton, 1984). The Helmholtz vorticity equation for an incompressible homogeneous fluid,

$$\frac{\partial \boldsymbol{\omega}}{\partial t} + (\mathbf{v} \cdot \nabla)\boldsymbol{\omega} = (\boldsymbol{\omega} \cdot \nabla)\mathbf{v} + \nu \nabla^2 \boldsymbol{\omega} ,$$

includes the processing term $(\boldsymbol{\omega} \cdot \nabla)\mathbf{v}$ describing the effects of local concentration of vorticity by stretching and of local turning of vorticity, and the term $\nu \nabla^2 \boldsymbol{\omega}$ representing the spread of vorticity by diffusion, where $\boldsymbol{\omega} = (\xi, \eta, \varsigma)$ is vorticity. However, it contains no true generation term corresponding with the creation of fresh vorticity where none existed before, and it has long been recognised that all sources of vorticity in homogeneous fluids must be on boundaries of the fluid region. Thus the source of vorticity in a homogeneous flow is clear, but the mechanisms of its generation and decay are a good deal less clear; and, in particular, it is not clear whether vorticity can be generated by wall stress, nor whether it can be lost by diffusion to solid boundaries.

An important contribution was made by Lighthill (1963), who noted that at almost all points of flow boundaries there exists a non-zero gradient of vorticity along the normal. For motion over a plane boundary, $z = 0$, the flux density of vorticity away from the boundary is

$$-\nu \left\{\frac{\partial \xi}{\partial z}, \frac{\partial \eta}{\partial z}, 0\right\}_{z=0} = \frac{1}{\rho}\left\{\frac{\partial p}{\partial y}, -\frac{\partial p}{\partial x}, 0\right\}_{z=0} ,$$

where the latter expression has been obtained by applying the Navier Stokes equation at the boundary, assumed stationary. Lighthill took this as the local strength of a distribution of vorticity sources spread over the solid boundary, and it follows that tangential vorticity must be created at the boundary in the direction of the surface isobars at a rate proportional to the tangential pressure gradient. This does not wholly resolve the situation, however, as the relation between vorticity flux density and tangential pressure gradient does not distinguish between the outward diffusion of positive vorticity and the inward diffusion of negative vorticity; nor is it clear whether vorticity can be lost by diffusion to boundaries. In a particular example (of flow separation) where the pressure first falls and then rises, we cannot say whether vorticity of a particular sense is first generated at and subsequently lost to the boundary, or whether there is continuous generation of vorticity first of one sense and then of the other as the fluid moves along the boundary. Indeed, it will not be clear whether there is any meaningful distinction between these two until we identify the mechanism or mechanisms of generation.

It will increase our insight into the generation process if we review the solution to the Rayleigh problem of a plate set impulsively into steady motion in its own plane. Motion in the semi-infinite region of fluid $z > 0$ initiated from rest when the plane boundary $z = 0$ is set impulsively into tangential motion with speed $u = UH(t)$ at time $t = 0$ is represented by the equation

$$\frac{\partial u}{\partial t} = \nu \frac{\partial^2 u}{\partial z^2} ,$$

with similarity solution

$$u = U\{1 - \frac{1}{\pi^{1/2}} \int_0^s e^{-\phi^2} d\phi\} ,$$

where $s = z/(2\nu t)^{1/2}$ is the similarity variable and the initial and boundary conditions are:

$$t < 0 , \quad u = 0 \quad 0 \leq z;$$

$$t \geq 0 , \quad u = U \; on \; z = 0 , u \to 0 \; as \; z \to \infty .$$

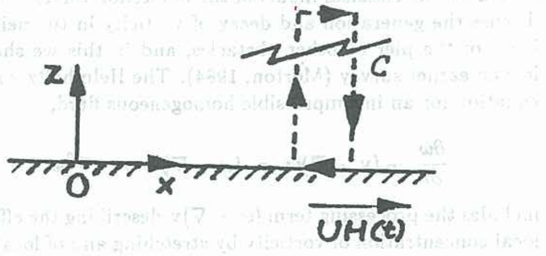

Fig. 2. The integration circuit C for the impulsively started plate.

The corresponding vorticity is

$$\omega = (0,\eta,0) = \{0, -(2\pi\nu t)^{-1/2} \, Ue^{-s^2}, 0\}$$

and has maximum magnitude $U/(2\pi\nu t)^{1/2}$ at the plate for all $t \geq 0$. The wall stress at the boundary,

$$\tau = \{\mu \left(\frac{\partial u}{\partial z}\right)_0 , 0, 0\} = \{-\mu U \, (2\pi\nu t)^{-1/2}, \, 0, \, 0\}$$

is unbounded at the initial instant and thereafter decreases in proportion to $t^{-1/2}$; and the circulation around circuit C of unit x-width and unbounded z-height (Fig. 2),

$$\oint_C \mathbf{v} \cdot d\mathbf{r} = -U,$$

increases impulsively from zero to $-U$ at the instant $t = 0$ and is thereafter constant for all time. In this case all the vorticity is generated at the initial instant as the plate is impulsively accelerated, and it is generated wholly at the boundary surface s (or z) = 0. Thereafter vorticity diffuses away from the boundary but its gross amount, which is the circulation in the contour C per unit width, remains constant, and in particular it is neither lost to nor gained from the boundary despite the continuing wall stress and the fact that the boundary remains the point of greatest vorticity concentration. The flux density of η normal to the boundary per unit area of section is

$$-\nu \frac{\partial \eta}{\partial z} = -Uz(2\pi\nu)^{-1/2} \, t^{-3/2} \, e^{-z^2/2\nu t} ,$$

and the flux density at the boundary is zero except only at $t = 0$; but that at $s = 0$, $t = 0$ it is infinite.

The solution for plane Poisenille flow adds further insight. Steady flow between fixed parallel planes, $z = \pm h$, under a uniform pressure gradient $dp/dn = -\gamma$ has velocity

$$u = \frac{\gamma h^2}{2\mu}\left(1 - \frac{z^2}{h^2}\right)$$

and vorticity, $\eta = -\gamma z/\mu$. Vorticity is generated continuously at the lower boundary at rate $+ \gamma/\rho$, and at the upper boundary at rate $-\gamma/\rho$ (sense of normal reversed) using the results given by Lighthill, and each diffuses towards the centre plane where the positive and negative counterfluxes suffer mutual annihilation. The total circulation per unit length of channel is zero. We may ask whether Poiseuille flow could be

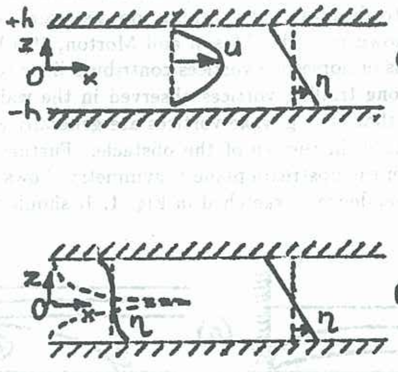

Fig. 3. Plane Poiseuille flow: (a) profiles for developed flow; (b) profiles for the entry length.

represented equally well by the generation of vorticity of a single sense, say at the rate γ/ρ at the lower boundary, diffusion across the layer with flux density $-\nu \, \partial\eta/\partial z = \gamma/\rho$, and absorption at the upper boundary. Although this appears to be a viable alternative in the region of fully developed flow, it cannot explain the entry length with upper/lower boundary layers wholly of negative/positive vorticity, generated along the boundaries and diffusing out to fill a progressively larger proportion of the channel until the two boundary layers meet at the mid-plane and the upward/downward fluxes of positive/negative vorticity are mutually annihilated by cross diffusion. There is nothing special about the positiveness or negativeness of vorticity as such, because the sign depends on our choice of axes; it is, however, essential that there exist vorticity of opposite senses or signs.

From these and other examples we note: (i) that vorticity generation results from the tangential initiation of boundary motion (or tangential acceleration of a boundary) and from tangential pressure gradients acting along a boundary; (ii) that generation is instantaneous; (iii) that vorticity once generated is not subsequently lost by diffusion to boundaries; (iv) that wall stress relates to the presence of vorticity but is not a cause of its generation; (v) that generation is unaffected by the prior presence of vorticity; (vi) that both senses of vorticity are needed to explain observations; (vii) that walls play no direct role in the decay of vorticity; and (viii) that vorticity decay results solely from cross-diffusion of two fluxes of opposite sense and takes place in the fluid interior.

The generation of vorticity is generally instantaneous and the vorticity generated is instantaneously unbounded in magnitude but in an infinitely thin sheet at the boundary. It is appropriate, therefore, to work in terms of the circulation in a small circuit C_1, with two areas of length δx parallel to the boundary and just in fluid or solid, respectively, and two short closing arms δz normal to the boundary. The circulation is then

$$\delta k = \oint_{C_1} \mathbf{v} \cdot d\mathbf{r} \approx (u - U)\delta x,$$

since the contribution from the closing arms normal to the boundary may be shown to be $0 \, (\delta z)^3$ and is negligible for $\delta z/\delta x \ll 1$. In the limit for small δx the circulation per unit length of boundary in the contour C_1 normal to Oy is

$$k_\eta = u - U,$$

where u and U are velocities parallel to the boundary in fluid and wall, respectively. Note tht the circuit is fixed, but the boundary free to move tangentially; and that normal motion of the wall does not generate vorticity. It follows that the rate of generation of y-vorticity is

$$\frac{dk_\eta}{dt} = \frac{du}{dt} - \frac{dU}{dt}$$

$$= \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x}$$

$$= -\frac{1}{\rho}\frac{\partial p}{\partial x} - \frac{dU}{dt} + \nu\nabla^2_u,$$

when we have substituted from the Navier Stokes equation. We note a difficulty: we require $\delta z/\delta x \ll 1$ for neglect of the side arm contributions, and indeed we seek the contribution from the interface between fluid and solid where genertion is concentrated. However, we have seen in the Rayleigh problem that at the instant of generation the vorticity is of infinite magnitude at the boundary surface but zero everywhere within the fluid; and its gradient is infinite at the boundary and zero elsewhere. An any later time $t > 0$ the vorticity is finite and its gradient zero at the boundary $\geq = 0$. Thus *the actual generation of vorticity is an inertial process in which viscosity plays no role.* The diffusion of vorticity does not begin until after its generation, but its effect is then exceedingly rapid close to the wall. It follows that we cannot take the limit $\delta z \rightarrow 0$ above without having the viscous term affect u and invalidate the purpose of the limit, which is to uncover the inertial generation at $z = 0$. Diffusion is a consequence of generation and does not itself affect generation, however, and if the diffusive term is neglected, we obtain the true rate of generation,

$$\frac{dk\eta}{dt} = \frac{1}{\rho}\frac{\partial p}{\partial x} - \frac{dU}{dt}.$$

This result holds also in Poiseuille flow where vorticity is generated continuously and the effects of generation and diffusion are difficult to distinguish.

The gross vorticity per unit length of a thin layer is the difference in tangential velocity across the layer, and the rate of generation of vorticity in the layer is the relative tangential acceleration across the layer. In the absence of viscosity, only pressure gradients can produce acceleration within the fluid and in a homogeneous, they produce homogeneous acceleration. Thus all relative tangential acceleration produced by the pressure field in a homogeneous fluid is necessarily at the boundaries. Viscosity plays no role in the generation precisely because the generation is an instantaneous response of fluid to inertial forces. The generation of vorticity at boundaries in homogeneous fluid is therefore an inviscid process, as it surely must be since the free slip at boundaries in inviscid flows gives an accurate estimate of the boundary layer vorticity at reasonably high Reynolds numbers for the corresponding viscous flows.

HORSESHOE VORTICES AND BED SCOUR

There is always an inertial rise in pressure whenever a stream is divided or deflected by an obstacle, with a corresponding adverse pressure gradient over the lower boundary and generation of vorticity with a component opposite in sense to that of the boundary layer. In the upstream plane of symmetry of a symmetric cylinder extending into the outer stream through the boundary layer, there will be a streamline near the top of the boundry layer with total head $p_o + \frac{1}{2}\rho U^2$ meeting the cylinder at a stagnation point with stagnation pressure $p_s = p_0 + \frac{1}{2}\rho U^2$. There is little change in pressure across the boundary layer except close to the cylinder, and the total rate of generation of opposed vorticity in the plane of symmetry is

$$\int_{-\infty}^{s} -\frac{1}{\rho}\frac{\partial p}{\partial x}\,dx = \left[-\frac{p}{\rho}\right]_{-\infty}^{s} = -\frac{p_s - p_0}{\rho} = -\frac{1}{2}U^2,$$

and this is equal and opposite to the flux of vorticity $\frac{1}{2}U^2$ in the boundary layer upstream. It follows that the net circulation per unit length through the boundary layer will decrease progressively as the cylinder is approached and will tend to zero at its leading edge. This explains both the weakness of the observed vortices and the nested patterns observed upstream of piers, because the new vorticity is generated solely at the boundary whereas the advected vorticity is distributed through the boundary layer. Well upstream of the cylinder, the new vorticity is annihilated at the instant of generation by cross diffusion into the boundary layer; but as the cylinder is approached the pressure gradient may rise until the rate of generation can no longer be matched by diffusion, at which point the flow separates from the lower boundary. Thereafter, there will be a pool of freshly generated (− ve) vorticity overlain by boundary layer (+ ve) vorticity, with net positive but decreasing circulation. Although the circulation continues to decrease, diffusion cannot directly keep pace, and as the boundary layer is stretched right and left around the cylinder, we should expect to find larger upper positive and smaller lower negative concentrated vortices which serve the role of overturning the fluid so as to enhance vorticity gradients, thereby enhancing diffusion.

Bed erosion depends on surface stress, that is momentum flux from stream to bed and the associated saltation, and has little to do with vorticity which is a gradient of velocity. At stream velocities below the critical at which the bed begins to move, the reduced velocity under the nested vortices upstream of the pier will not start erosion. However, the pressure over the surface of a pier decreases from the stagnation pressure $p_0 + \frac{1}{2}\rho U^2$ to approximmately $p_0 - 3\,\rho U^2/2$ at the sides of a circular cylinder at the level of the stagnation point, and to a pressure somewhat larger than p_0 at the upstream foot of the cylinder. The flow is therefore accelerated outwards and downwards. There is some enhancement of speed at the foot of the cylinder, but the highest speeds are close to the bed at the sides of the pier. It is scarcely surprising, therefore, that erosion has been reported (Melville, 1975) as starting downstream of the broadest cross-stream pier section. The scour holes will then extend upstream as their upstream slopes are especially liable to erosion, until they meet in front of the pier and a roller vortex is established in the hole.

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