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Why is it that Entrainment is not related to Internal Wave Energy?

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ABSTRACT

Within both natural and man made fluid bodies, interfaces of high stability, due to buoyancy gradients, can be observed to separate well mixed homogeneous layers. Mass is diffused across such interfaces despite their static stability. The laboratory experiments reported here were performed in a small tank, with turbulence being generated by an oscillating grid of the same design as used by Turner. Both the layer above and below the interface were of constant but different density and a Richardson number, based on this difference and the velocity and lengthscales of the grid, was varied between 0.1 and 10. It was found that fluctuations of density within the interface appear to be dominated by internal waves with their period of oscillation being dependent on the tank geometry and fluid layer densities. The observed dominant frequencies were less than the Brunt Vaisala frequency of the interface. The amplitude of the internal waves, and hence a measure of the energy associated with them, was not found to be related to the rate of mixing across the interface. Therefore it would appear that the breaking of internal waves is not the dominant mixing process across the interface in the tank for the range of bulk Richardson numbers investigated.

INTRODUCTION

By using small scale laboratory experiments numerous authors have investigated various features of the mixing of mass and momentum across strongly stratified interfaces. An extensive bibliography of those experiments that involve "shear-free" mixing can be found in Fernando and Long (1985).

The experiments of concern here were typically performed in a closed tank with liquids of two densities separated by an interface at about mid depth in the tank. Agitation of one of these layers was provided by an oscillating grid.

Whether the mixing process that causes entrainment across the interface can best be characterised by a slightly modified version of that occurring at the edge of a turbulent jet or whether "breaking internal waves" are of significance is an unanswered question. Linden (1975) examined the question of whether radiation of energy away from the interface by internal waves was of importance in the efficiency of mixing and concluded that it was.

The present study is concerned with determining more directly whether the energetic internal waves that can be observed on the interface are important in the mixing process. The level of these waves is changed by varying the external conditions of the experiment and the rate of entrainment compared with the magnitude of wave produced.

EQUIPMENT

The experiments were carried out in a plexiglass tank, similar to that used by Turner (1968), of 254 x 254 mm cross-section and a depth of 465 mm (as shown in Fig.1). A single grid was used for the generation of turbulence within the upper layer of a two layer fluid system. It was made up of 10 mm square plexiglass bars arranged in a 5 x 5 array, with a bar separation of 50 mm and an overhang of 25.0 mm. This resulted in a grid solidity of 35%.

The grid was mounted on a centrally located stainless steel spindle which was free to oscillate in the vertical only. Rotation was inhibited by the use of metal guide posts. The sinusoidal motion of the stirring assembly was provided by a variable speed motor and an eccentric drive. Variations in the stroke and frequency of the grid oscillation could be made, with the latter being monitored by the use of a magnetic microswitch and electronic counter. The errors associated with the frequency and stroke of the grid were found to be ± 0.05 Hz and 0.06 cm respectively.

A single saltwater-freshwater interface was formed within the tank by initially filling it with a set amount of fresh water. Then a denser sodium chloride solution was slowly added through the bottom of the tank, so as to reduce the chance of premature mixing. In most cases the tank was completely filled with the upper water surface being in contact with the tank lid, thus giving a rigid lid approximation. The density interface was always initially 100 mm below the mean vertical grid position.

Before filling the tank, the densities of the freshwater φ_o (upper layer) and the saltwater φ_i (lower layer) were determined by the use of a hydrometer. To determine the density of the fluid at a particular point within the tank, in real time, a 4 electrode conductivity probe was used. It operated by maintaining a constant A.C. voltage across the voltage electrodes (outer pair) and measuring the current (or voltage)

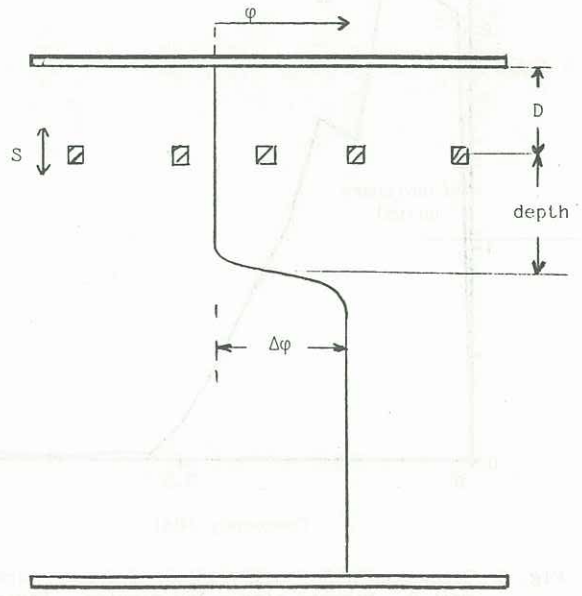

Fig. 1 The experimental arrangement.

across the inner electrodes. This gives a measure of the conductivity of the solution at the probe tip. Head (1983) states that the probe has a time response of 3dB at 800Hz and a spatial response of 3dB at 4cycles/cm. Also calibration stability is better than 1% over 8 hours, and since each experimental run took less than 6 hours a high consistency and accuracy was achieved.

The output signal from the conductivity probe was recorded by a Commodore 64 computer and thence onto magnetic disk. To convert the probe's output voltage into a signal which the computer could read an analog to digital converter was constructed.

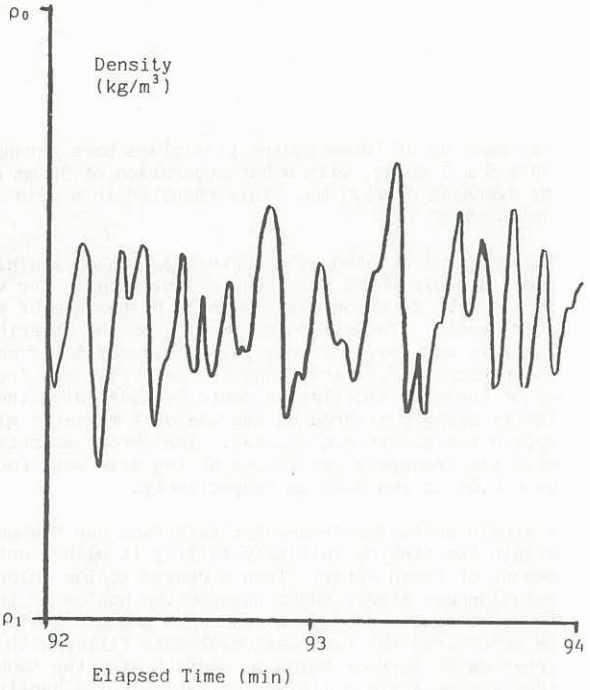

Figure 2 An example of the observed density fluctuations near the centre of the interface during experiment 6.

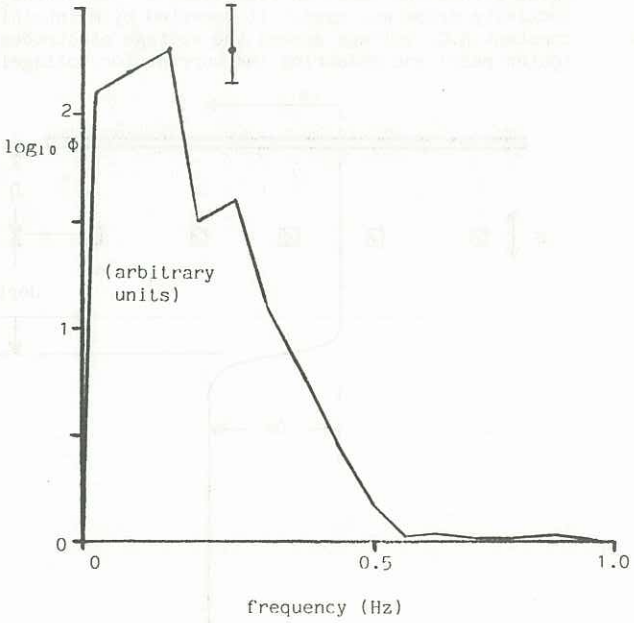

Fig. 3 Power spectral density obtained during experiment 2. Elapsed time = 160 minutes. Error bar indicates 95% confidence limits.

During the running of an experiment the probe was used in the determination of the location, thickness and time fluctuations of the density interface. The central position of the interface was found by locating the point of mean density between the upper and lower layers. It was found necessary to take at least 3 readings (at approximately the same time) to obtain a reasonable estimate of the mean interface position, because of the fluctuations.

Interface internal wave fluctuations were examined by positioning the probe as close as possible to the centre of the interface, and then sampling the conductivity at greater than 10 Hz. The data was then stored and later analysed for fluctuation periods and energy.

TABLE 1

Experimental Conditions

Exp.	S (cm)	F (Hz)	D (cm)	B (cm/s^2)	t (min)	u_e (cm/s)	X (m^{2}s)	f_p (Hz)
1	1.3	4.5	15.0	3.7	30	8.0	9.6	
				3.5	60	4.8	4.2	
2	1.3	4.5	8.0+	3.1	100	6.8	12.3	
				2.8	160	6.0	8.6	
3	1.3	4.5	9.0+	3.5	39	9.2	6.8	
				3.2	65	8.3	10.8	
				2.8	120	4.8	11.4	
4	1.5	3.0	10.0	3.8	11	6.5	6.9	7.2
				3.8	22	4.8	4.7	7.2
				3.7	32	4.3	5.8	7.2
				3.7	40	3.2	7.6	16.
				3.6	52	3.1	7.0	16.
				3.6	61	3.0	7.5	16.
				3.5	80	2.7	7.1	16.
				3.5	95	2.2	3.3	16.
				3.5	120	1.8	3.9	16.
				3.4	180	1.5	7.6	16.
5	1.6	4.5	10.0	3.5	30	8.8	17.3	
				3.5	41	7.9	16.0	
				3.4	51	6.3	16.7	
				3.4	60	5.0	14.4	
				3.3	90	3.2	16.7	
				3.2	120	2.6	13.5	
				3.1	165	2.2	13.4	
				3.1	180	1.6	5.8	
6	1.6	4.5	10.0+	3.8	11	8.3	8.3	
				3.7	21	7.7	8.5	
				3.6	40	6.8	8.2	
				3.5	60	5.1	4.9	
				3.4	70	4.8	5.4	
				3.4	90	3.1	5.3	
				3.3	120	2.9	4.3	
				3.2	180	1.9	8.2	
				3.1	240	1.4	8.4	
				3.0	290	1.2	9.1	
7	1.6	4.5	10.0+	3.8	10	8.9	9.1	6.1
				3.7	20	8.7	6.8	7.2
				3.6	30	7.7	8.4	16.
				3.6	40	7.0	12.4	16.
8	1.0	6.0	10.0	3.4	30	18.0	4.2	
				2.9	60	16.5	7.1	
				2.5	120	11.6	13.7	
				2.3	180	10.1	18.5	
9	1.1	6.0	10.0	3.4	30	15.0	6.2	
				3.1	60	12.6	15.3	
				2.9	120	9.6	26.1	
10	1.1	6.0	10.0	3.6	20	17.0	5.2	7.2
				3.5	30	16.0	9.6	7.2
				3.2	40	15.0	11.9	7.2
				3.1	50	14.1	18.6	7.2
				3.0	60	12.3	23.0	7.2
				3.0	70	11.9	22.3	7.2
				2.8	90	11.1	17.4	7.2
11	1.3	4.5	10.0	11.9	15			3.6
				11.8	22			3.6
				11.6	48			3.6
12	1.3	4.4	10.0	7.8	10			4.2
				7.6	60			5.0

$X = \frac{L^2\phi}{(\Delta\varphi)^2}$ $B = g\Delta\varphi/\varphi_o$ S = stroke

+ free surface

RESULTS

When the density fluctuations of the interface were sensed by the conductivity probe, it was obvious that there was a dominant frequency component. The recorded time series of such fluctuations (Fig. 2 shows a typical example) were then transformed into power spectra such as shown in Fig. 3. These spectra had the property

$$\overline{\varphi'^2} = \int_0^\infty \Phi(f)\, df$$

From this the energies associated with each frequency band could be examined and compared in relation to their magnitude and the rate of deepening of the interface. The frequency bands used had greater than 16 degrees of freedom and were smoothed by hanning.

The dispersion relation for an internal wave on an interface between two layers of uniform density is given by, Pond and Pickard (1983) as

$$\omega^2 = kg\,\Delta\varphi(\varphi_1 \coth kh_1 + \varphi_2 \coth kh_2)^{-1}$$

where ω is the angular frequency of the internal waves and h_1, h_2 are the depths of the two fluid layers. For k large compared with the layer depth coth (kh) $\approx$ 1 and so

$$\omega^2 \approx \frac{kg\,\Delta\varphi}{\varphi_1 + \varphi_2}$$

Consider standing waves such that the wave length λ is a multiple of the tank width (W = 0.254m), that is

$$\lambda = 2\pi/k = j\,W, \text{ where } j = 1, 2$$

$$\text{or} \quad j = 2/3,\ 1/2,\ 2/5,\ 1/3,\ \ldots\ldots$$

Given that $\varphi_o \approx 1000$ kg/m^3 then the expected periods of the standing internal waves of wavelength one half, once and twice the tank width can be calculated for varying $\Delta\varphi$ values. Comparing in Fig. 4 the predicted values to those observed experimentally, f_p, as listed in Table 1, it was observed that the fluctuations of the interface appear to be a result of standing internal waves.

A better model would assume an interface of thickness L between the two layers. A suitable profile of density gradient might be

$$\frac{\Delta\varphi}{L} \exp - (z/L)^2$$

which gives a maximum density gradient of $\Delta\varphi/L$ and a corresponding Brunt Vaisala frequency of $(g\Delta\varphi/L\varphi_o)^{\frac{1}{2}}$. In this model internal waves cannot have frequencies about the Brunt Vaisala frequency and so this limit has been drawn in Fig. 4 on the assumption that L=1cm.

In a similar tank Crapper and Linden (1974) found that the root mean square of the density fluctuations decreased rapidly away from the centre of the interface. Figure 5 shows a plot of the variation in power spectral density for the most energetic frequency band (f_p = 0.16 Hz) for a series of experiments where the interface was sampled quickly at a number of depths. The mean density $\bar{\varphi}$ is a measure of the position through the interface.

If the density fluctuations are modelled as displacements w/ω of the mean density by an internal wave with vertical velocity w and a frequency ω then the density fluctuations become

$$\varphi' \sim \frac{w\partial\bar{\varphi}}{\omega\partial z}$$

Using the diagram in Roberts (1975, p.108) for an estimate of the vertical internal wave velocity and assuming a density gradient of the form above, a model curve, shown in Figure 5, can be produced. The magnitude of the density fluctuations square, $\overline{\varphi'^2}$, in this figure is adjusted to be in reasonable agreement with the measured values of $\phi(f)$.

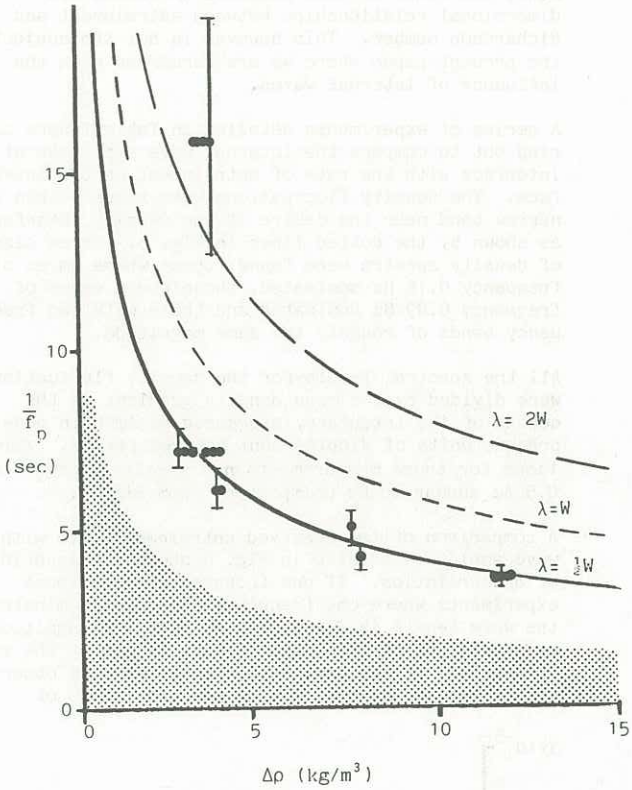

Fig. 4 Comparison between measured and predicted period of the most energetic component of density fluctuation. Bars indicate bandwidths used. Shaded area indicates maximum Brunt Vaisala frequency for interface thickness of 1 cm.

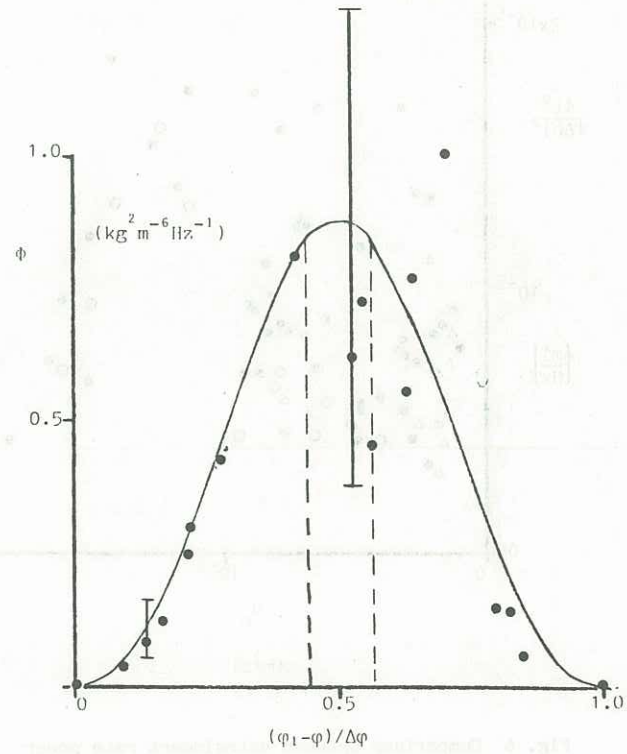

Fig. 5 Power spectral density of frequency band f = 0.16 Hz. Density difference 3.9 kg/m^3. Stroke 1.3 cm. Grid frequency 6 Hz. Error bar indicates 90% confidence limits.

The entrainment rate, u_e, was shown by Turner (1968) for salt stratified experiments to be a function of the Richardson number. While our experiment 8, with the same stroke as Turner, replicated his rate of entrainment, the other strokes produced different non-dimensional relationships between entrainment and Richardson number. This however is not the subject of the present paper where we are concerned with the influence of internal waves.

A series of experiments detailed in Table 1 were carried out to compare the internal wave amplitude at the interface with the rate of entrainment of the interface. The density fluctuations were taken within a narrow band near the centre of the density interface as shown by the dotted lines in Fig. 5. Three classes of density spectra were found; those where waves of frequency 0.16 Hz dominated, those where waves of frequency 0.09 Hz dominated and those with two frequency bands of roughly the same magnitude.

All the spectral levels for the density fluctuations were divided by the mean density gradient in the centre of the interface, expressed as $\Delta\varphi/L$ in order to produce units of displacement squared per Hz. Corrections for those measurements not precisely at $\varphi - \varphi_1 = 0.5\,\Delta\varphi$ appear to be unimportant from Fig. 5.

A comparison of the observed entrainment rate with the wave amplitude squared in Fig. 6 shows little evidence of a correlation. If one focusses on just those experiments where one frequency component dominates, the wave length is fixed, and so both the magnitude and the slope of the internal wave is set by the value of $\sqrt{\Phi}\,L/\Delta\varphi$. If the dominant internal wave we observe controlled the rate of mixing over the range of

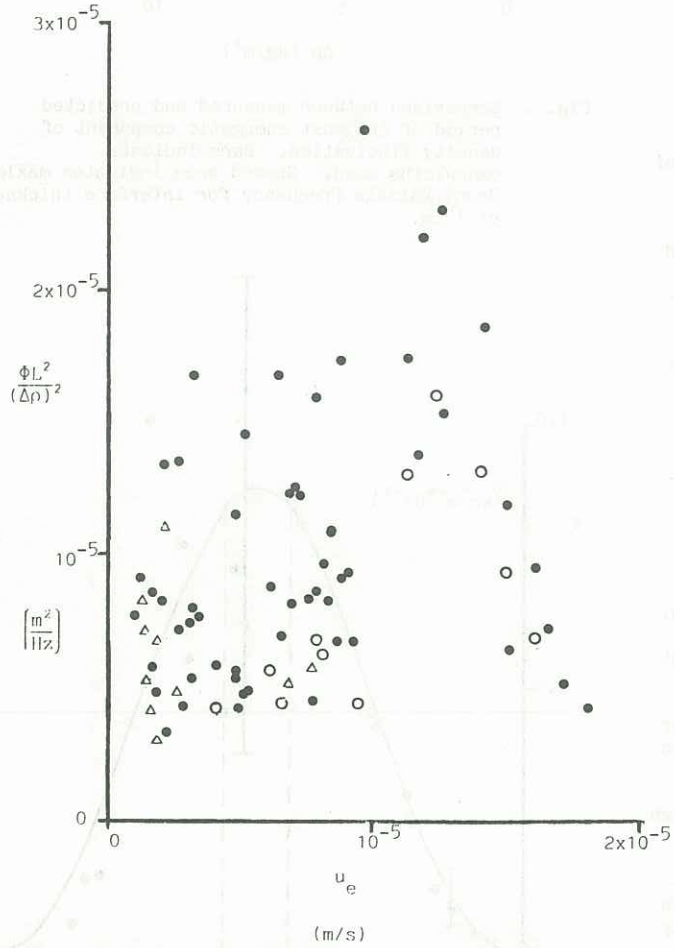

Fig. 6 Comparison between entrainment rate power spectral density of situations where frequency bands 0.16 Hz, ○, and 0.012 Hz, △, dominate. Both bands comparable, ●.

Richardson numbers studied, one would expect a correlation in Fig. 6. In the ocean Jones (1985) presented some evidence that shear instability of internal waves could occur and so it is possible that only waves of a certain magnitude can contribute to the mixing.

CONCLUSION

The most energetic density fluctuations within the interface between the two uniform layers of our grid stirred experiment appear to be standing internal waves. The characteristic frequency is a function of the density difference across the interface and occurs at frequencies below the maximum Brunt Vaisala frequency of the interface. Energy loss from the interface region due to radiation by internal waves is not a factor in the present experiment which was carried out with uniform layers each side of the interface.

When the magnitude of the internal wave displacement estimated from the spectral level of the density fluctuations divided by the mean density gradient is compared with the rate of entrainment, little correlation is observed. This suggests that the internal wave activity, while conspicuous, is not important in the mixing process.

Further measurements and modelling are needed to clarify the conditions under which internal waves might induce significant mixing. One can speculate that the lack of correlation between internal wave energy and entrainment is due to the local Richardson number of the internal waves, generated unintentionally in the present experiment, not falling below a critical value.

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