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The Evaluation of Tide Well Constants

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SUMMARY The role of viscous dissipation of energy in the operation of the common tide well is considered in detail, and a method of determining the viscous constants is described. Only measurements of the water level are required; this avoids the difficulties in obtaining accurate measurements of the fluid velocity. The results obtained demonstrate marked differences between the inflow and outflow regimes of an operating tide well. The friction coefficient is sensitive to variations in the orifice geometry.

1 INTRODUCTION

Historically tides have been measured by means of a stilling well (see Figure 1).

This consists of a large diameter pipe fixed in a vertical position and closed at its lower end. An orifice is located near the bottom and permits an exchange of sea water between the ocean and the well in response to head differences. A float senses the height of the water surface inside the well and transmits this information to a suitable recorder. The well acts as a crude filtering system by damping out the short period waves and transmitting signals with tidal frequencies. It is important to understand the nature of this damping for, once understood, it can be utilised in the design of suitable gauges for specific purposes.

Noye [1974, a, b, and c] and Braddock [1976, 1980], analysed the action of the tide well and its influence on the recording of tidal information including tides, tsunamis and long waves. The basic equations are discussed more fully in Section 2 of this paper. In his analysis, Noye [1974a] derived an approximate energy balance for the flow in the stilling well; his equation is highly non-linear. Noye also developed the drainage test to estimate the orifice parameter for the stilling well. A drainage test consists of filling the well and permitting it to empty while the external head is held constant. Noye did not include viscous effects in his model although some attempts were made to estimate the magnitude of these forces.

Braddock [1976, 1980] reanalysed the operation of the stilling well and included both laminar and turbulent dissipative effects. That analysis follows the methods developed by Noye and the major differences are the inclusion of head loss terms due to viscous forces in the energy equation. Laminar dissipation is extremely important since it corresponds to a linear term which becomes dominant for long period oscillations i.e., for tidal periods.

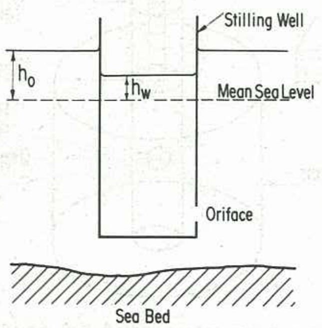

Figure 1 - The Tide Well System.

Under drainage conditions and by ignoring acceleration effects, the energy equations of both Noye and Braddock can be integrated to yield drainage history curves. In this paper, we will briefly describe the basic theory behind these equations and then show how they can be used to measure the dissipation parameters.

2 BASIC THEORY

Since the energy balance equation for the tide well has been derived elsewhere (Noye [1974a], Braddock [1976]), only a brief description will be given so as to facilitate the latter discussion. Consider a stilling well as illustrated in Figure I, and let $h_o(t)$ and $h_w(t)$ be the external and internal water levels with respect to Mean Sea Level as datum. In applying Bernoulli's Theorem to the water column, the dissipation or head losses, H_2, can be modelled in the form

$$H_2 = \psi \frac{dhw}{dt} + \xi \left(\frac{dhw}{dt}\right)^2 \qquad (1)$$

where ψ and ξ are proportionality constants. Dissipative head losses are of two forms (see Fox and MacDonald, [1973]); i.e.,

Major: these being due to viscous effects in the laminar flow,

Minor: these being due to entry and bend effects leading to turbulence.

Using this model for the head losses, Braddock [1976] used Noye's technique to derive the basic equation,

Inertia term / Orifice term

$$(\Upsilon + h_w) \frac{d^2h_w}{dt^2} + (F + \xi)\left(\frac{dh_w}{dt}\right)^2 \sigma \left(\frac{dh_w}{dt}\right) +$$

Linear dissipation / Pressure term

$$\psi \frac{dh_w}{dt} + g(h_w - h_o) = 0, \qquad (2)$$

where $F = \frac{1}{2}((A_w/A_e)^2 - 3)$

A_w is the cross sectional area of the stilling well, diameter D_w.

$A_e = C_c A_o$ is the effective cross sectional area of the orifice, actual diameter D_o and actual area A_o.

C_c is the contraction coefficient for the orifice.

$\sigma = \sigma(x)$ is the "sign of x".

$\gamma = y + D_w^3/(6\, D_o^2)$, and

g = gravitational acceleration.

The importance of the linear friction term is readily apparent on considering equation (2) with a periodic forcing term of period τ. Note that a tidal period of one day $\doteq 8.6 \times 10$ seconds. Non-dimensionalising equation (2) with respect to τ, indicates that the ratio of orifice term/linear dissipation is $F/\tau\psi$. The nature of the tide well then depends on the magnitude of the linear friction term.

Generally the geometric term F is large and the turbulent dissipation term ξ can be omitted. Noye [1974] obtained a balance between the kinetic effects (orifice term) and the external head, i.e.,

$$F \left(\frac{dh_w}{dt}\right)^2 \sigma \left(\frac{dh_w}{dt}\right) + g(h_w - h_o) = 0 \qquad (3)$$

In a drainage test, $h_o = o$ for all t, while initially $h_w = \pm a$, and $\frac{dh_w}{dt} = o$ at $t = o$. Once the water column is released from its initial configuration, the inertia term can be neglected and the hydraulic equation

$$F \left(\frac{dh_w}{dt}\right)^2 \sigma \left(\frac{dh_w}{dt}\right) + \psi \frac{dh_w}{dt} + gh_w = o, \qquad (4)$$

describes the motion of the water level. Introduce the nondimensional variables $X = h_w/a$, and $\tau = t/t_d$ where t_d is defined as the time at which $X(t_d) = 0.05$. Equation (4) reduces to

$$\sigma(\dot{X})\ a\ F(\dot{X})^2/t_d + \psi \dot{X} + g t_d X = o, \qquad (5)$$

subject to $X(o) = \pm 1$. Integrating yields

$$\{\ln[((1+C)^{\frac{1}{2}} - 1)/((1-CX)^{\frac{1}{2}} - 1)] + (1+C)^{\frac{1}{2}} - (1-CX)^{\frac{1}{2}}\}/\psi g t_d = \tau, \qquad (6)$$

with $c = \sigma(\dot{X}) 4agF/\psi^2$. When the effects of turbulent dissipation are included, F is replaced by $F + \xi$ in the expression for C.

Noye [1974a], Braddock [1976] and Braddock and Noye [1980] used drainage tests to measure $h_w(t)$ and X(t), and used these time history curves to analyse the forces operating. The velocity terms were estimated by numerical differentiation of the observed data and the coefficients estimated by least square fitting. Noye [1974a] used only Equation (3) to estimate the geometric parameter while Braddock [1976] and Braddock and Noye [1980] used Equation (4) to investigate the effects of dissipation. These latter results suggested a very strong dependence of the dissipation parameters on the orifice geometry. In fact, the geometries used in the experiments more closely approximated a pipe connection rather than a true orifice. The values of ψ which were obtained corresponded to lamina dissipation for a pipe connection, and generally were very high; so much so that the tide well seemed to be a linear device.

The need to numerically differentiate observed data is a serious criticism of this method and a likely source of error. We propose a new method based on Equation (6), an implicit algebraic relation satisfied by the time history curve, and using only directly observed data. Further experiments were conducted using precisely constructed openings which more closely approximate the mathematical orifice.

3 EXPERIMENTAL DETAILS

A glass cylinder, with a perspex plug inserted near the lower end, was used to model the tide well (see Figure II). The perspex plug was cut out using a lather to provide a thin membrane through which an orifice was drilled. The fluid meniscus was clearly visible through the glass walls, and the water level readily determined by means of a vertical scale. A digital timer measuring to 0.01 seconds was located near the apparatus and recordings taken by photographing the experiment at regular intervals. Once developed, the photographs were read to obtain readings of the time, τ_i, and height, X_i.

Outflow experiments were conducted by placing the "tide well" inside a large glass reservoir fitted with a constant head overflow pipe (see Figure II).

For inflow experiments, the "tide well" was deeply immersed in the reservoir and the external head held constant by an excess supply of water and the overflow mechanism (see Figure III). The submerged scale and the meniscus were still clearly visible and readily photographer. The experiments were conducted for both influx and efflux for a variety of orifice geometries; the various orifice diameters and wall thicknesses are shown in Figure IV. Reference will be made to these orifii in terms of the labels a, b, c, d, e and f used in Figure IV.

Data on the height above datum, X_i, and the time τ_i, $i = 1, \ldots N$, were collected from the photographic record of each experiment. The tide well constants are then obtained by minimising the least squares function

$$S(F,\psi) = \sum_{i=1}^{N} \left[\frac{\tau(X_i)}{\tau_i} - 1\right]^2 \qquad (7)$$

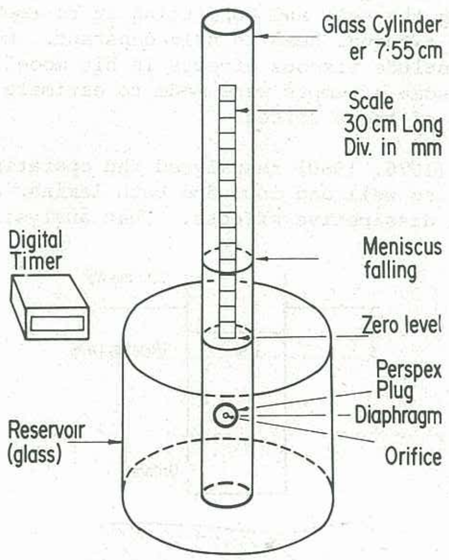

Figure 2 - The Apparatus used to study the Influx Regime.

where $\tau(X_i)$ is given by (6); the minimisation being with respect to F and ψ. This numerical minimisation was done using the Fletcher-Powell conjugate gradient algorithm (see Dixon [1972]). This is a very efficient iterative scheme using the analytic partial derivatives of (6) with respect to F and ψ. Despite the complicated dependence of τ on F and ψ, the algorithm performed splendidly, producing convergent approximations within a few iterations.

4 RESULTS

Equation (6) indicates that the drainage time for the tide well is infinite, and that the final stages of drainage occur slowly; hence the introduction of t_d in (5) and (6). The quantity t_d was readily measured from the experiments and the results are summarised in Table I.

The calculated values of F and ψ obtained from (7) are also included in this table. The normalised least squares error for the curve fitting varied from $O(3 \times 10^{-3})$ down to $O(7 \times 10^{-5})$. The observed values of F include the effects of turbulent dissipation; see (6) and the theoretical values of F assuming a true orifice, are also included in Table I.

The theoretical and observed values differ by up to a factor of two. Since F depends on the fourth power of the diameter of the orifice, estimating the orifice diameter is the most likely source of any error. Note however, that the experimental orifii are not true mathematical orifii, and that the accuracy is greatest for orifice (c) which is most like a perfect orifice. The differences between the theoretical and observed values of F are attributable to difficulties in accurately estimating the diameter of the orifice, and to the departure from an ideal opening. This accuracy far exceeds that obtained by Braddock [1976] and Braddock and Noye [1980]; the improvement is directly attributable to the use of equation (6), instead of using numerical differentiation.

The transition from a perfect orifice towards a pipe connection produces marked changes in the flow characteristics in and near the opening. The different flow geometries both inside and outside the orifice also produce different flow patterns and characeeristics. Some of these differences are

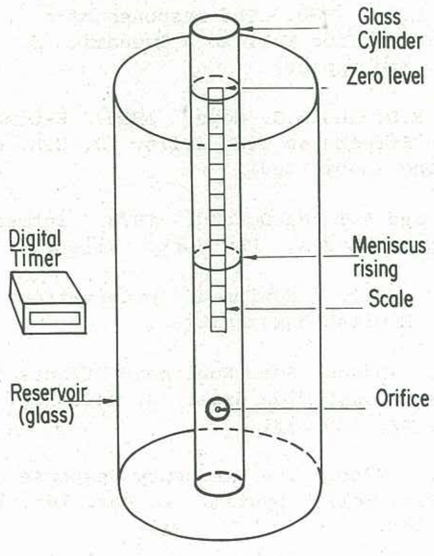

Figure 3 - The Apparatus used to study the Efflux Regime.

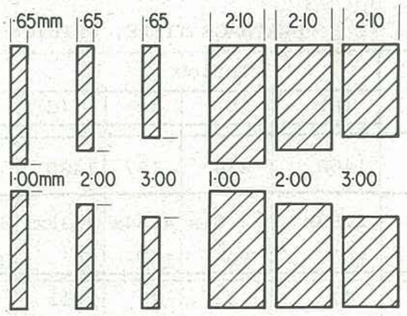

Figure 4 - Dimensions of the 6 orifii used in the Experiments.

revealed in the values of t_d in Table I; i.e. for each orifice, the inflow persists longer than for the corresponding out-flow. The percentage difference between inflow and outflow times ranges from 0.7% for orifice (a), to 23% for orifice (e); this difference also increases as the wall thickness increases. During inflow, the flow is obstructed by the opposite wall of the tide well, thus offering additional flow resistance. For efflux, there is no such obstruction to the streamlines. Obviously the effect is more pronounced for the larger orifii since the flow is faster, and the orifice "jet" penetrates more deeply into the tide well.

The increase in turbulent resistance is also reflected in the observed values of F, when compared with the theoretical values. Certainly there are errors in the estimation of F theoretical, due to errors in estimating the orifice diameter. However, the observed and theoretical F values show a marked increase in F as the orifice diameter increases. The difference between inflow and outflow are marked and clearly illustrate the additional resistance to the flow due to the orifice "jet" bending to pass from the orifice and up the tide well. The additional resistance will also depend on the velocity of the water, and hence on the magnitude of the external head. Here, the obvious inference is that the frictional resistance will vary with the velocity dh_w/dt, so that the values in Table I are averages over the full drainage curve. This inference is illustrated by the different F-values observed for inflow/outflow, for orifii (a) and (b). Both orifii have almost the same aspect ratio in that the ratio of wall thickness to orifice diameter is 0.65 for orifice (a) and 0.7 for orifice (d).

However, the ratio (F(influx)-F(efflux)/F theoretical) yields values of 0.08 for orifice (a) and 0.29 for orifice (e). This large difference is directly attributable to the increased speed of flow for orifice (e).

Linear friction in the tide well is represented by the parameter ψ, and the measured values are given in Table I. Again the differences between influx and efflux are clearly discernable, but so also is the effect of the aspect ratio and the smoothing of the flow by the "pipe like" orifice (d). For orifice (d), the aspect ratio is 2.1, there being little difference between F for inflow and outflow, and relatively large values of ψ. This orifice has produced almost perfect laminar flow, where the turbulent effects between influx and efflux have been almost eliminated. There are major differences between the values of ψ for inflow and outflow, and all the energy dissipation is now occuring as lamina friction: the large difference is due, of course, to the contorted flow lines on inflow, Obviously such differences will decrease as the aspect ratio is increased.

TABLE I.
DRAINAGE TIMES, ORIFICE AND FRICTION PARAMETER FOR INFLOW AND OUTFLOW.

Orifice	Inflow						Outflow					
	(a)	(b)	(c)	(d)	(e)	(f)	(a)	(b)	(c)	(d)	(e)	(f)
t_d (seconds)	1468	434	187	1480	372	184	1458	400	178	1426	302	157
F (observed)	2.89×10^7	2.61×10^6	4.84×10^5	2.35×10^7	1.82×10^6	4.49×10^5	2.58×10^7	2.26×10^6	4.43×10^5	2.34×10^7	1.15×10^6	2.95×10^5
ψ (observed)	77	32	11	1013	39	12	73	16	9	727	90	78
F (theoretical)	4.37×10^7	2.73×10^6	5.39×10^5	4.37×10^7	2.73×10^6	5.39×10^5	4.37×10^7	2.73×10^6	5.39×10^5	4.37×10^7	2.73×10^6	5.39×10^5

Orifice (c), with an aspect ratio of 0.225, represents the other end of the scale, in that there is very little change in the value of ψ for inflow and outflow. Here, most of the dissipation of energy is by turbulent means, and is occuring in the jet and the orifice.

5 DISCUSSION

We have described an alternate method of determining tide well parameters using the traditional drainage test. This method fits data directly to the implicit time history curve and avoids the numerical errors inherent in the numerical differentiation methods used to estimate the velocities. The method gives much more accurate estimates of the geometric parameter F. Some of the data collected by Braddock [1976], was reanalysed using the method outlined in this paper. In this previous experiment, the wall thickness of the tide well was 3/8th inches, while the orifice diameters were 1/4", 7/32", 3/16", 5/32", 1/8", 3/32", 1/16" and 1/32", so that the aspect ratio was relatively large. Using the direct fitting technique, the error between F observed and F theoretical was reduced by a factor ranging from 1.9 up to an extreme of 8.7. Those experiments cannot be compared directly with those discussed here, because of the differing well geometries. However, the advantages of the implicit method are well illustrated.

The values of ψ obtained in this paper are much smaller, by a factor of 10^2, than those obtained by Braddock [1976] and Braddock and Noye [1980]. Some of this difference could be attributed to the superior method used but the change must be attributed to the different well geometries. Braddock [1976] and Braddock and Noye [1980] performed experiments where the orifice was really a short pipe connection, while here a much better approximation to a "true orifice" was used. The values obtained for the lamina dissipation are much smaller since the orifice does not favour the establishment of lamina flow.

The results obtained in this paper indicate a delicate interplay between tide well geometry, pressure head and the dissipation mechanism. Both the fluid velocity through the orifice and hence the Reynolds number, depend on the external head and this varies during drainage. In the orifice region, the Reynolds number can be as high as $O(10^4)$ a figure obtained by estimating dh_w/dt from the time history curve and then using the mass conservation equation to calculate the velocity in the orifice. Criteria for turbulent pipe flow suggest that the flow in this region is turbulent, but observation of small particles and bubbles indicates that the turbulence is not well developed. There are also significant differences between influx and efflux. For influx, the strength of the increasing jet and proximity of the opposite wall govern the nature of the frictional resistances. The experiments suggest that perhaps the dissipation should be modelled by a head loss term of the form (see equation (2)):

$$H_2 = K(\dot{h}_w)^n \qquad (8)$$

where K is a constant and $n = n(\dot{h}_w)$ varies with the flow velocity. Here, n = 1 for lamina flow, n = 2 for fully developed turbulent flow, and the function $n(\dot{h}_w)$ takes intermediate values for mixed flows. Obviously the function n also depends on the well geometry.

The results obtained serve to illustrate the basic nonlinear character of the stilling well tide gauge as a basic measuring device. The very small values of ψ which were obtained, indicate that the tide well with a mathematical orifice is more non-linear than when a short pipe connection is used.

As shown earlier, the particular tidal component (period) in question also plays a part in determining the response of the tide gauge. However, any connector between the internal and external water levels, that removes the turbulent structure of the flow, will make the gauge response more linear. Herein lies the importance of Noye's [1974c] work on the long pipe connection for a tide well; although the fluid dynamics is some what different.

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