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Diffuser-Augmented Wind Turbine Analysis

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SUMMARY The performance of diffuser-augmented wind turbines has been established by matching the forces acting on a blade element to overall momentum and energy balances. This permits the radial variation, wake rotation and Reynolds number effects on the turbine blades to be taken into account. Good agreement with experimental data is obtained for both turbines and turbine-simulating screens. Optimising the pitch setting to take account of the total drag as well as the power output indicates that a substantial drag reduction with only a small loss of power output is possible; this is relevant to jet-stream electricity generation.

1 INTRODUCTION

Electricity generation from jet-stream winds (Fletcher and Roberts, 1979) requires the deployment of tethered aerodynamic platforms containing wind turbines. Because of their light weight and high performance, diffuser-augmented wind turbines are well-suited to this application.

Diffuser-augmented wind turbines have received considerable attention recently (Igra, 1977 and Gilbert et al, 1978 and 1979). At the present time the power output from a diffuser-augmented wind turbine is about four times that of an unshrouded wind turbine of the same area. Results obtained with the present analysis are compared with the experimental results of Igra, 1977 and Gilbert et al, 1978 and 1979.

A high tip speed ratio ($r_t.\Omega/U_o$) produces a large power output by minimising the rotational energy left in the wake. At jet-stream altitudes the resultant speed at the tip is limited to about 200 m/s by the onset of sonic conditions. With a typical rated speed (Fletcher et al, 1979) of 45 m/s, the maximum power output is required at a tip speed ratio of about four. In contrast ground-based, unshrouded wind turbines produce maximum power at a tip speed ratio of about eight to ten.

An economic analysis of jet-stream wind energy (Fletcher et al, 1979) indicates that the drag associated with the production of power plays an important role in determining the size of the lifting surfaces of the aerodynamic platform. Therefore it would appear feasible to operate the wind turbines at less than the maximum power coefficient if, by so doing, the drag due to power could be significantly reduced. The present analysis considers this aspect.

2 ANALYSIS

The analysis is based on the concept of non-interacting circular stream tubes which connect the undisturbed freestream, far upstream (station 0, Figure 1), through the turbine disc to the freestream far downstream (station 5, Figure 1). At the plane of the turbine disc the local forces on the blade elements are given by classical aerofoil theory when allowance is made for the relative motion (Figure 2).

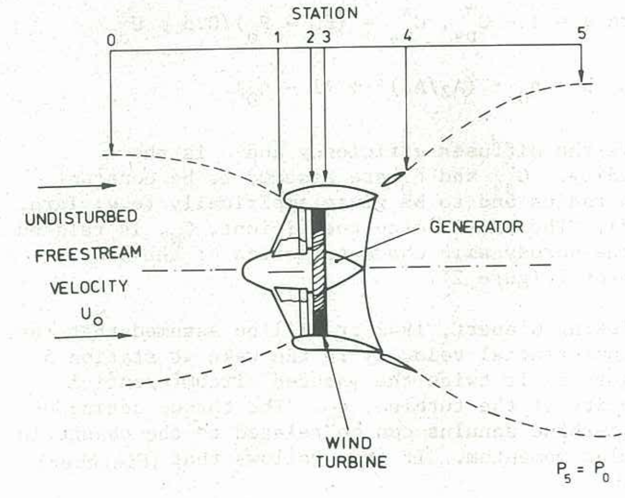

Figure 1 Cross-section through wind turbine reference stations

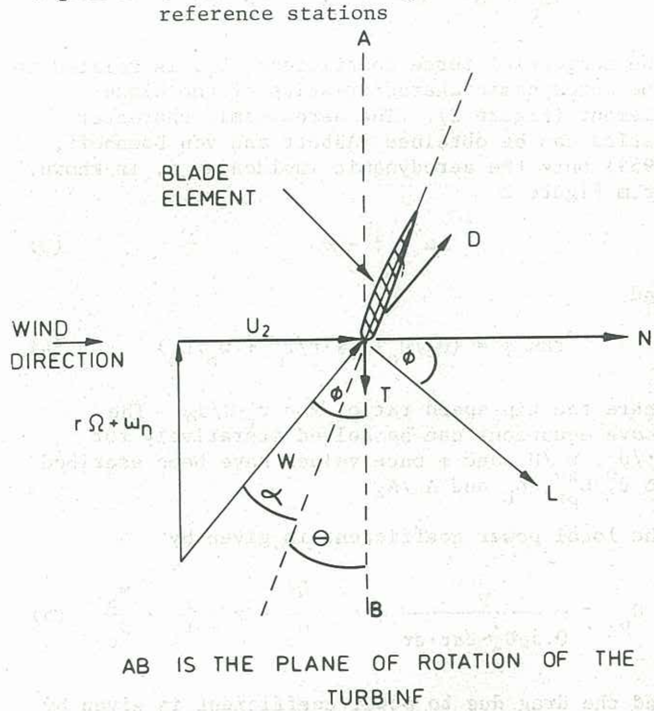

Figure 2 Velocity diagram for blade element

The axial force acting on the turbine is calculated from blade element theory and equated to the force acting on the turbine calculated from an energy balance between stations 0 and 4. The torque acting on the blade elements can be related to the angular velocity in the wake at station 5. In turn this allows the induced angular velocity in the plane of the turbine and the power extracted by the turbine to be calculated.

It may be noted that the present analysis permits the radial variation of most of the major parameters, retains the correct overall continuity, momentum and energy balances and permits the Reynolds number influence on the local blade element properties to be correctly accounted for.

The equations of motion will be established by considering a circular stream tube which has a radius r and thickness dr in the plane of the turbine disc. Equating the axial force on the blade sections with the axial force calculated from an overall energy balance gives (Fletcher, 1979)

$$\frac{\sigma}{2} \cdot \frac{r_t}{r} \cdot \frac{(U_2/U_o)^2}{\sin^2\phi} \cdot C_N = a - b\,(U_2/U_o)^2 \qquad (1)$$

where $a = 1 - C^*_{p4}$, $C^*_{p4} = (P_4 - P_o)/0.5\,\rho\,U_o^2$

and $b = \eta_D \cdot (A_2/A_4)^2 + (1 - \eta_D)$.

η_D is the diffuser efficiency and σ is the solidity. C^*_{p4} and η_D are assumed to be constant with radius and to be given empirically (e.g. Igra, 1977). The axial force coefficient, C_N, is related to the aerodynamic characteristics of the blade element (Figure 2).

Following Glauert, 1942 it will be assumed that the circumferential velocity in the wake at station 5 (Figure 1) is twice the induced circumferential velocity at the turbine, w_n. The torque acting on the turbine annulus can be related to the change in angular momentum. It then follows that (Fletcher, 1979)

$$\frac{w_n}{U_o} = \frac{\sigma}{8} \cdot \frac{r_t}{r} \cdot \frac{C_T}{\sin^2\phi} \cdot \frac{U_2}{U_o} \qquad (2)$$

The tangential force coefficient, C_T, is related to the aerodynamic characteristics of the blade element (Figure 2). The aerodynamic characteristics can be obtained (Abbott and von Doenhoff, 1959) once the aerodynamic incidence, α, is known. From Figure 2

$$\alpha = \phi - \theta \qquad (3)$$

and

$$\tan\phi = (U_2/U_o)/(x \cdot r/r_t + w_n/U_o) \qquad (4)$$

where the tip speed ratio, $x = r_t \cdot \Omega/U_o$. The above equations can be solved iteratively for U_2/U_o, w_n/U_o and ϕ once values have been ascribed to θ, C^*_{p4}, η_D and A_4/A_2.

The local power coefficient is given by

$$C_{P\ell} = \frac{P}{0.5\rho U_o^3 \cdot 2\pi r \cdot dr} = 4 \cdot \frac{U_2}{U_o} \cdot x \cdot \frac{r}{r_t} \cdot \frac{w_n}{U_o} \qquad (5)$$

and the drag due to power coefficient is given by

$$C_{DP\ell} = \frac{D_P}{0.5\,U_o^2 \cdot 2\pi r \cdot dr} = 2 \cdot \frac{U_2}{U_o} \cdot \left(1 - \frac{U_5}{U_o}\right) \qquad (6)$$

For each radius, θ has been chosen to maximise the power generated by the aerodynamic platform. The total power from all speed bands is given by

$$P_{tot,\ell} = \sum_{i=1}^{j} p_i \cdot \frac{1}{2}\rho V_{mpi}^3 \cdot C_{P\ell i} + C_{P\ell k} \cdot \frac{1}{2}\rho V_R^3 \sum_{i=k}^{n} p_i \qquad (7)$$

where p_i is the probability that the wind speed is in the i^{th} speed band (data from Maher and McRae, 1964). V_{mpi} is the mean power speed for the i^{th} speed band.

The various parameters, e.g. $C_{P\ell}$, have been defined based on a local area. The corresponding integral parameters, e.g. C_P, are based on the turbine area, A_2. C_P can be obtained from the local parameter, $C_{P\ell}$, in the following manner,

$$C_P = \frac{1}{A_2}\int_{r_{hub}}^{r_t} C_{P\ell} \cdot 2\pi r \cdot dr \qquad (8)$$

From Figure 2 it is apparent that the local velocity over a blade element is given by

$$\frac{W}{U_o} = \left[\left\{x \cdot \frac{r}{r_t} + \frac{w_n}{U_o}\right\}^2 + \left\{\frac{U_2}{U_o}\right\}^2\right]^{\frac{1}{2}} \qquad (9)$$

Clearly W varies significantly with radius and tip speed ratio, x. This, combined with the variation in chord, causes a significant variation in Reynolds number and, as a result, the aerodynamic characteristics C_L and C_D. The average chord for the diffuser-augmented turbine has been based on typical dimensions for the 1 MW prototype (Fletcher and Roberts, 1979).

For a NACA 4418 aerofoil the aerodynamic characteristics, C_L vs. α and C_D vs. α, at Reynolds numbers of 0.5×10^6 and 2.0×10^6 have been deduced from experimental data (Abbott and von Doenhoff, 1959). The data at $Re = 0.5 \times 10^6$ and 2×10^6 have been interpolated to give the aerodynamic characteristics corresponding to the Reynolds number of the local blade element. A more detailed development of the analysis may be found in Fletcher, 1979.

3 RESULTS AND DISCUSSION

In a series of experiments, Gilbert, 1978 and 1979, and Igra, 1977, the behaviour of diffuser-augmented wind turbines has been investigated including the use of screens to simulate the turbine. Results for a number of cases that have been subjected to the present analysis are indicated in Figures 3 and 4. The close agreement between the present analysis and the experimental data, shown in Figures 3 and 4, supports the validity of the present approach.

In obtaining the power coefficient with the screen simulation method the following expression is used

$$C_P = C_{DT} \cdot U_2/U_o \qquad (10)$$

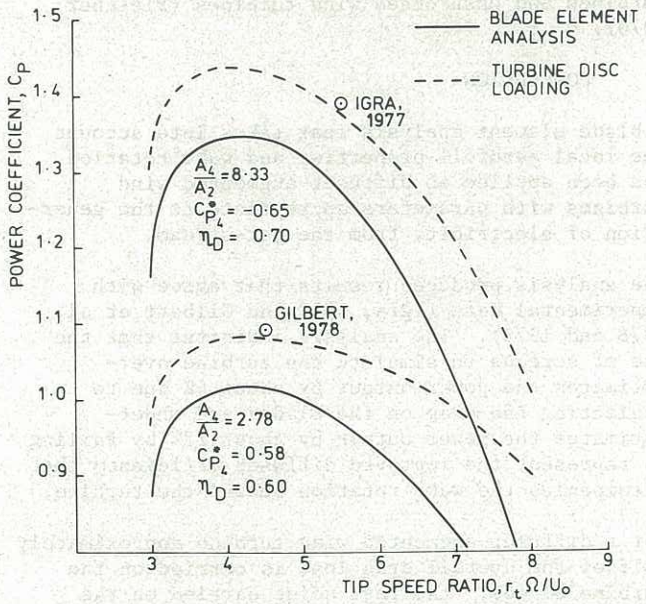

Figure 3 Power output for the Igra, 1977 and Gilbert, 1978 test cases

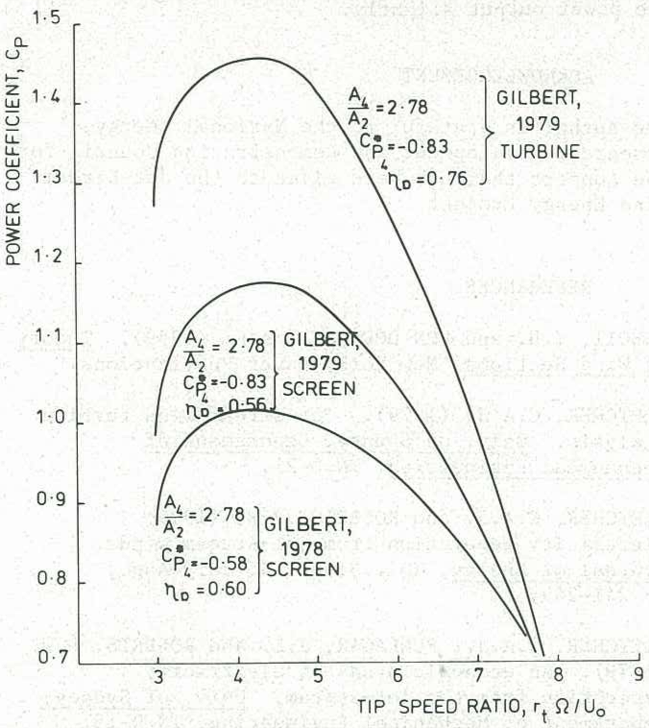

Figure 4 Power output for different Gilbert, 1978 and 1979 test cases

This approach neglects both the drag on the turbine blades and the small amount of energy associated with the wake rotation, $2\,w_n$. In the present analysis C_p has been calculated from (10) and presented along with the value of C_p evaluated from the integral form of (5). The present analysis using (10) includes the wake rotation effect and consequently produces slightly lower values of C_p than given by the results from Gilbert et al, 1978 and Igra, 1977. The difference between the full and dashed lines in Figure 3 arises from the neglect of the drag on the individual turbine blades and is seen to be about 6% at maximum power. Away from the maximum power condition the difference is greater.

Gilbert et al, 1979 have also undertaken experiments in which a three bladed, fixed pitch, constant chord turbine has replaced the screen. Although they were unable to achieve sufficient pitch adjustment or rotational speed to produce the optimum conditions, the power output was still significantly above that produced by a screen. It was found that the rotation of the wake assisted in the re-energising of the boundary layer on the inside of the diffuser and consequently produced a higher diffuser efficiency.

The present analysis has also been applied to these conditions and the results are shown in Figure 4. It can be seen that a substantial improvement in performance is obtained when an actual turbine is used instead of a screen. This improvement is about four times as large as the loss in performance due to not accounting for the drag of the turbine blades and wake rotation.

The variation of generated power with tip speed ratio for various values of solidity is shown in Figure 5. The results shown in Figure 5 have been obtained with a rated speed of 56.6 m/s and at an area ratio, $A_4/A_2 = 4$. An approximate correlation between $C^*_{p_4}$, η_D and area ratio, A_4/A_2 (Igra, 1977) has been used.

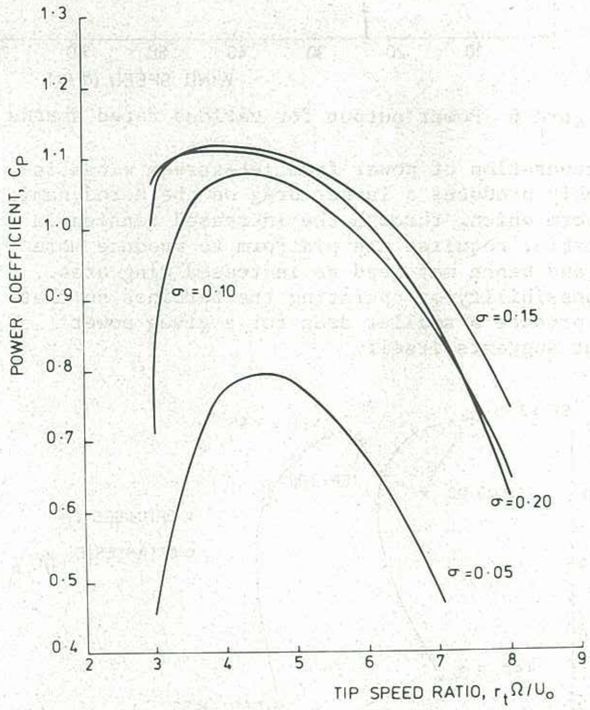

Figure 5 Power output for various solidity ratios

It is apparent from Figure 5 that the highest power output over the tip speed ratio range occurs with a solidity, $\sigma = 0.15$. A large value of tip speed ratio corresponds to a small ϕ and a small aerodynamic incidence, α, at which the lift/drag ratio is small and a low power output is obtained. Reducing the tip speed ratio increases ϕ and α and hence C_L and C_D. The ratio C_L/C_D reduces significantly if the turbine blades stall, with a consequent reduction of power output. This has occurred at small values of tip speed ratio for σ = 0.05 and 0.10 but not for σ = 0.15 and 0.20.

The proper choice of different rated speeds is an economic one, Fletcher et al, 1979. The results shown in Figure 6 indicate that if the pitch angle distribution, $\theta(r)$, is chosen to optimise a larger speed range, then the power coefficient will tend to fall off at the limits of the required speed range. Because of the V^3 effect on delivered power the fall off in C_p close to the rated speed as the rated speed is increased is considered to be the more important effect.

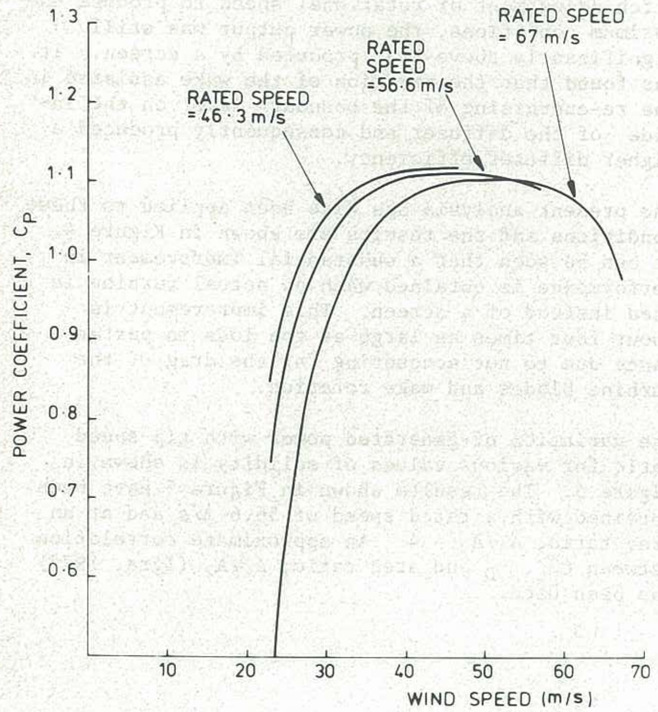

Figure 6 Power output for various rated speeds

The generation of power from jet-stream winds inevitably produces a larger drag on the aerodynamic platform which, through the increased tension in the cable, requires the platform to produce more lift and hence may need an increased wing area. The possibility of operating the turbines so that they produce a smaller drag for a given power output suggests itself.

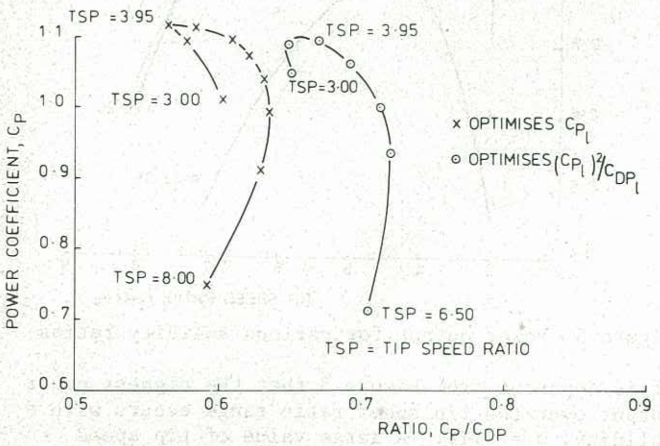

Figure 7 Power output and total drag for different pitch angle optimisation criteria

To illustrate this C_p has been plotted against C_p/C_{DP} in Figure 7. Results have also been obtained for which $P_{tot,\ell}$ in equation (7) is multiplied by $C_{p\ell}/C_{DP\ell}$ and $\theta(r)$ is chosen to optimise the product. This particular choice, $C^2_{p\ell}/C_{DP\ell}$, was made merely to give the notion of reducing drag some weight in the optimisation. It is clear that a substantial reduction in drag can be achieved with only a small reduction in generated power. The precise balance between maximising the power output and minimising the drag would appear to require an economic analysis, as in Fletcher et al, 1979. The present analysis has been used to compare diffuser-augmented wind turbines and unshrouded wind turbines (Fletcher, 1979).

4 CONCLUSION

A blade element analysis that takes into account the local aerofoil properties and wake rotation has been applied to diffuser-augmented wind turbines with parameters appropriate to the generation of electricity from the jet-stream.

The analysis produces results that agree with experimental data (Igra, 1977 and Gilbert et al, 1978 and 1979). The analysis indicates that the use of screens to simulate the turbine overestimates the power output by about 6% due to neglecting the drag on the blades and underestimates the power output by about 25% by failing to represent the improved diffuser efficiency that accompanies the wake rotation behind the turbine.

For a diffuser-augmented wind turbine approximately half of the overall drag load is carried on the turbine blades; the rest being carried on the diffuser. An optimum pitch distribution produces a large power coefficient over the complete speed range. For diffuser-augmented wind turbines it is possible, with a different pitch setting, to reduce the overall drag significantly while only reducing the power output slightly.

5 ACKNOWLEDGEMENT

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6 REFERENCES

ABBOTT, I.H. and VON DOENHOFF, A.E. (1959). Theory of Wing Sections, New York, Dover Publications.

FLETCHER, C.A.J. (1979). Jet-stream wind turbine analysis. Univ. of Sydney, Department of Mechanical Engineering, TN D-21.

FLETCHER, C.A.J. and ROBERTS, B.W. (1979). Electricity generation from jet-stream winds. Journal of Energy, Vol. 3, No. 4, July-Aug., pp 241-249.

FLETCHER, C.A.J., FULLAGAR, J.L. and ROBERTS, B.W. (1979). An economic study of electricity generation from the jet-stream. Univ. of Sydney, Department of Mechanical Engineering, TN D-15.

GILBERT, B.L., OMAN, R.A. and FOREMAN, K.M. (1978). Fluid dynamics of diffuser-augmented wind turbines. Journal of Energy, Vol. 2, No. 6, Nov.-Dec., pp 368-374.

GILBERT, B.L. and FOREMAN, K.M. (1979). Experimental demonstration of the diffuser-augmented wind turbine concept. Journal of Energy, Vol. 3, No. 4, July-Aug., pp 235-240.

GLAUERT, H. (1942). The elements of aerofoil and airscrew theory, Cambridge, C.U.P.

IGRA, O. (1977). Compact shrouds for wind turbines. Energy Conversion, Vol. 16, pp 149-157.

MAHER, J.V. and McRAE, J.N. (1964). Upper wind statistics, Australia. Bureau of Meteorology, Australia.