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Pre-Dilution Devices with Swirl Control

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SUMMARY A model for axisymmetric swirling turbulent core flows, embedded in a uniform co-directional ambient stream, is developed to investigate utilization of swirl in enhancing and controlling the mixing associated with a suggested pre-dilution device design. The model is based on an approximate integral-method solution technique, which requires an entrainment assumption to be postulated from dimensional and physical arguments. The results obtained suggest both a use for swirl control in this pre-mixing device design; and an alternative design which would more effectively take advantage of both pre-mixing and swirl control of mixing beyond the device.

1 INTRODUCTION

The modern-day submarine outfall for the sea disposal of sewage effluent consists of a long pipe to convey the waste to the point of discharge and a multiple-port diffuser, situated on the bottom of the receiving water, to discharge the waste. The design of the outfall depends upon many factors, including the volume and characteristics of the effluent to be discharged, receiving water quality standards, and oceanographic conditions. The major design objective for the diffuser, however, is to maximize the initial dilution of the effluent in order to reduce its buoyancy. This seeks to utilize any natural stable ocean stratification as a barrier to prevent surfacing of the effluent, as well as to considerably reduce surface concentrations when the stratification is insufficient to provide total submergence.

In an effort to more satisfactorily achieve this design objective a number of experimental and theoretical studies have been made [e.g. Silvester (1967), Agg and White (1974)] of pre-mixing devices to maximize immediate dilution of the effluent within the outfall, or some extension of it. Agg and White (1974) used small scale models to investigate six possible designs for premixing devices. One of the designs utilizes a reversing tidal flow and emits the effluent horizontally to the environment with azimuthal as well as axial velocity. The resulting flow initially takes the form of a swirling wake, which suggest we investigate the effect of swirl on mixing for swirling core flows in a following stream, in an effort to determine the effect of the device on mixing beyond the outlet as well as within the device. Agg and White were primarily concerned with internal pre-mixing and only measured flow characteristics just outside the outlet.

The model developed below is based on approximate forms of the integrated equations for conservation of mass, axial momentum and azimuthal momentum, with closure of the system obtained by postulating an assumption which relates the unknown entrainment to mean flow variables. Chervinsky and Lorenz (1967) gave semi-empirical solutions of approximate forms of the integrated axial momentum and azimuthal momentum equations which are valid for weak swirl only. Narain (1974) gave a theoretical treatment based on approximate forms of the integrated equations for conservation of axial momentum, azimuthal momentum and mechanical kinetic energy. Narain assumed similarity of the flow profiles and related the Reynolds stress variable in the latter equation to mean flow quantities by an assumed self-similarity relation. These features limit the applicability of his results to weak swirl.

2 THE EQUATIONS OF MOTION

Consider an axisymmetric swirling core flow embedded in a parallel following stream moving with a uniform velocity W_0. In all cases the flow is considered to have originated from a steadily-maintained circular source of fluid and to have developed fully its appropriate turbulent character before the region under consideration. The flow in the 'entry' or 'development' region is outside the scope of the present treatment.

Relative to cylindrical polar co-ordinates (r,ϕ,z) with origin at the centre of the source and Oz directed horizontally, the velocity field has components $(u + u', v + v', w + w')$ where primes signify fluctuations and unprimed symbols signify ensemble mean values.

The equations of motion, averaged over realizations for steady-in-the-mean, incompressible, axisymmetric-in-the-mean turbulent flow at large Reynolds number, such that viscous stresses are negligible relative to Reynolds stresses, are taken as the governing equations for investigating these flows. An order of magnitude analysis of these equations provides details for their reduction to approximate forms suitable for narrow swirling core flows in a parallel stream. This proceeds like the analysis presented by Morton (1968) for swirling jets in a still environment, except that in a parallel stream there are two different scales for the axial component of velocity w; as a convective velocity $w \sim W_0$, and under differentiation $w \sim W_1$, the appropriate disturbance velocity scale.

The ensemble mean representation of the turbulent core will be assumed to remain confined within a region $0 < r < R(z)$. Outside this region the external flow is taken to be turbulence free (laminar) with uniform freestream velocity W_0 and uniform pressure p_∞. In what follows p is taken as the absolute pressure and ρ as the constant density.

Integration of the reduced equations for conservation of mass, azimuthal momentum and axial momentum over a core section z = constant to radius $r = R(z)$, with the boundary conditions:

$$r = 0;\ u = v = \frac{\partial w}{\partial r} = \frac{\partial p}{\partial r} = 0,\ w,\ p \text{ finite, Reynolds stress bounded,}$$

$$r = R(z);\ w = W_o,\ v = 0,\ ru \text{ finite but unknown,}$$

$$\overline{u'^2} = \overline{w'^2} = \overline{u'w'} = \overline{u'v'} = 0,$$

and some further reduction leads to the following set of flux equations:

$$\frac{d}{dz}\int_0^R \rho rw dr = \rho RW_o \frac{dR}{dz} - (\rho ru)_{r=R}, \quad (1)$$

$$\frac{d}{dz}\int_0^R \rho r^2 vw dr = 0, \quad (2)$$

$$\frac{d}{dz}\int_0^R \{\rho w(w - W_o) + \rho\overline{w'^2} + (p - p_\infty)\} r dr = 0 \quad (3)$$

The equation for conservation of radial momentum is more conveniently left in the reduced form

$$\frac{\partial p}{\partial r} = \rho\left(\frac{v^2}{r} - \frac{\partial \overline{u'^2}}{\partial r}\right). \quad (4)$$

Equation (1) indicates that the axial rate of increase of absolute mass flux in the core is equal to the rate of entrainment of ambient fluid. The form of this equation assumes that all entrainment is due to turbulent mixing outward from within the column into the laminar ambient stream. Fluid flowing in the main stream cannot of itself enter the turbulent core, but will merely convect the core boundary until reached by lateral turbulent diffusion of relative momentum. The terms on the right hand side of equation (1) may therefore be interpreted as longitudinal and lateral contributions to the entrainment flux, respectively, and are in the ratio

$$\frac{R^2 W_o}{Z} \cdot \frac{1}{RU} \sim \frac{W_o}{W_1},$$

where Z is taken as a representative axial distance. Thus, for jets or wakes in a following stream with $W_o >> |W_1|$ the supply of fluid which enters the core comes largely from upstream, rather than by lateral inflow of ambient fluid as in the case of jets in a still or low velocity environment with $W_o << W_1$. In any transition region with $W_o \sim |W_1|$ this ratio provides a measure of the angle at which entrained fluid enters the core.

After a further integration equation (2) becomes

$$\int_0^R \rho r^2 vw dr = \frac{G}{2\pi}, \quad (5)$$

where the constant G represents the invariant flux of angular momentum transmitted by the core, which is equal to the torque transmitted from the source to the fluid.

After a further integration equation (3) becomes

$$\int_0^R \{\rho w(w - W_o) + \rho\overline{w'^2} + (p - p_\infty)\} r dr = \frac{F}{2\pi}, \quad (6)$$

where F is the invariant 'flow force' or net axial force, transmitted by the core. The perturbation pressure $(p - p_\infty)$ provides a measure of the dynamical significance of the azimuthal velocity, and in 'strong' swirling cores provides a coupling between axial and azimuthal flow fields.

After some reduction equation (4) can be used to eliminate the pressure term from equation (6) to obtain

$$\int_0^R \rho\{w(w - W_o) - \frac{1}{2}v^2\} r dr = \frac{F}{2\pi}, \quad (7)$$

where $(\overline{w'^2} - \overline{u'^2})$ has been neglected relative to $(w^2 - \frac{1}{2}v^2)$ with modest accuracy, in accordance with the approximations of Chigier and Chervinsky (1967). Equation (7) serves to indicate that a swirling core undergoing progressive lateral spread of angular momentum by turbulent diffusion can produce a quite large axial deceleration and a consequent increase in the core rate of spread.

2.1 The Choice of Profile Shapes

Equations (1), (5) and (7) comprise a set of flux equations in the dependent mean flow variables (w,v,r). They exhibit one advantage over the original field equations in their reduced dependence on the Reynolds stress variables, although we still have an implicit dependence on the Reynolds stresses in the unknown entrainment. Most models using this approach 'close' the system of equations by assuming a relation between the unknown entrainment and mean flow variables. This is considered in section 2.2.

The 'closed' set of integrated equations cannot be solved for lateral profiles of velocity, as this information has been suppressed by integration. However, solutions for axial variations in behaviour of mean flow quantities can be obtained using the Pohlhausen technique of assumed r-profile shapes.

Due to lack of suitable experimental results for swirling core flows in a following stream we are forced to base our choice of r-profile shapes on available experimental results for swirling jets in a still environment. The results of Chigier and Chervinsky (1967) indicate that a similarity assumption is valid up to moderate degrees of swirl, with axial and azimuthal velocities adequately described by

$$w(r,z) = W_o + (w_m(z) - W_o)e^{-k\eta^2},$$

$$v(r,z) = v_m(z)\sqrt{k}\,\frac{\eta}{\lambda}\,e^{-\frac{1}{2}\left(\frac{k\eta^2}{\lambda^2} - 1\right)}, \quad (8)$$

where $\eta = r/r_m(z)$ is a similarity variable, $R(z) = r_m(z)$, and the suffix m indicates maximum for the mean velocities. The lateral spread parameter $\lambda = 0.85$ with modest accuracy and the constant k should be chosen so that the lateral length scale r_m provides a measure of the actual core radius. For the present investigation we will take the radius as where $(p_\infty - p)/\rho p_m = 1/100$ (where p_m is the value of the pressure perturbation on the core axis), which yields $k = 3.83$.

2.2 Closure - The Entrainment Assumption

In simple free turbulent shear flows the eddies which effect the turbulent entrainment of exterior fluid are characterized by the relative velocities of the two fluid streams. It would be expected, therefore, that entrainment of ambient fluid into a non-swirling turbulent core in a following stream, with the assumption of approximate self-similarity of the turbulent structure, could be fully represented by an inflow speed at the core edge which is proportional to the magnitude of the axial disturbance velocity. These arguments plus dimensional considerations led Morton (1961) to postulate an entrainment assumption for turbulent non-swirling core flows in a following stream as

$$E_R = r_m W_o \frac{dr_m}{dz} - (ru)_{r=r_m} = Er_m |w_m - W_o| ,$$

where E is the entrainment constant, defined as the ratio of the inflow speed at the core radius r_m to the speed difference between the core and ambient flows, and whose value must be determined from experimental observations. It is important to realise that the form of this entrainment assumption makes no distinction between lateral and longitudinal contributions to the entrainment flux, but merely characterizes entrainment by a bulk inflow speed at the core edge.

Emmons (1967) and Rubel (1973) have suggested that in a rotating core flow entrainment depends also on the radial stability, according to circulation, that occurs in rotating flows. In a swirling core the circulation K(r) in circular paths steadily increases with r in a rotationally stable inner region , extending from zero at the axis to a maximum value $K_M(r_M)$. This maximum is reached within the core, and at greater radial distances the circulation falls to zero at the core edge through an unstable outer sheath.

The overall radial stability of the core must be a result of the combined properties of the two possible types of stability region. If we assume that the overall level of turbulence (and hence entrainment) is enhanced by an unstable outer annulus, even though the stable inner region may act as a sink of the energy transferred to fluctuations, it appears reasonable to base an azimuthal velocity scale for entrainment on the magnitude of the gradient of the unstable section of the circulation curve. In the context of a model in which lateral structure is suppressed by integration and quasi-similarity is assumed, this gradient can only be represented by the azimuthal scale v_m.

An appropriate entrainment assumption for a swirling core in a following stream, based on the physical arguments above and appropriate scaling appears to be,

$$E_R = Er_m \{(w_m - W_o)^2 + \beta^2 v_m^2\}^{\frac{1}{2}} , \qquad (9)$$

where E is taken as an entrainment constant to be determined experimentally, and β is an additional constant to be determined experimentally and which allows different mixing roles for the axial and azimuthal flows. In the case of a still environment assumption (9) reduces to the form postulated by Morton (1968) for a model of a swirling jet and that used by Ross (1974) in modelling a swirling forced plume.

3 A MODEL FOR SWIRLING CORE FLOWS

A model for the swirling core flows being investigated can now be developed by adopting the similarity profile forms (8) and closing equations (1), (5) and (7) using entrainment assumption (9). Transformations to non-dimensional variables will be based on core radius $(r_m)_o$ and axial velocity on the core axis $(w_m)_o$ at the physical source, z = 0. This transformation base yields the non-dimensional variables:

$$\bar{r} = \frac{r_m}{(r_m)_o} , \quad \bar{\phi} = \frac{(w_m - W_o)}{(w_m)_o} , \quad \bar{v} = \frac{v_m}{(w_m)_o} , \quad \bar{z} = \frac{2Ez}{(r_m)_o} ,$$

and the working equations:

$$\frac{d}{d\bar{z}}\{\bar{r}^2(I_1\Lambda + I_2\bar{\phi})\} = \bar{r}(\bar{\phi}^2 + \beta^2\bar{v}^2)^{\frac{1}{2}} ,$$

$$\bar{r}^3\bar{v}(I_3\Lambda + I_4\bar{\phi}) = \frac{G}{2\rho\pi(r_m)_o^3(w_m)_o^2} , \qquad (10)$$

$$\bar{r}^2(I_2\Lambda\bar{\phi} + I_5\bar{\phi}^2 - I_6\bar{v}^2) = \frac{F}{\rho\pi(r_m)_o^2(w_m)_o^2} ,$$

where

$$I_1 = 2\int_0^1 \eta d\eta , \quad I_2 = 2\int_0^1 \eta f d\eta , \quad I_3 = \int_0^1 \eta^2 h d\eta ,$$

$$I_4 = \int_0^1 \eta^2 h f d\eta , \quad I_5 = 2\int_0^1 \eta f^2 d\eta , \quad I_6 = \int_0^1 \eta h^2 d\eta ,$$

with

$$f(\eta) = e^{-k\eta^2} \text{ and } h(\eta) = \sqrt{k}\,\frac{\eta}{\lambda}\, e^{-\frac{1}{2}(k\eta^2/\lambda^2 - 1)} .$$

The transformations are based on the source conditions $r_m = (r_m)_o$, $w_m = (w_m)_o$, $v_m = (v_m)_o$ at z = 0, or alternatively $\bar{r} = 1$, $\bar{\phi} = 1 - \Lambda$, $\bar{v} = S$ at $\bar{z} = 0$, where

$$\Lambda = \frac{W_o}{(w_m)_o} \text{ and } S = \frac{v_m}{w_m\,o} .$$

Thus, the solution of equations (10) with these initial conditions can be used to describe the family of swirling core flows from a series of sources of specified $[(r_m)_o, (w_m)_o]$, with the parameters $\Lambda \propto W_o$, $S \propto (v_m)_o$ incorporating changes in the relative magnitude of the uniform freestream velocity and the source azimuthal velocity, respectively.

The parameter Λ is the ratio of freestream velocity to axial velocity on the core axis at z = 0. The value Λ = 0 corresponds with a jet in a still environment, values 0 < Λ < 1 correspond with jets in a following stream, and Λ > 1 correspond with forced wakes; the limiting value for a simple wake is about 1.25 [Morton (1961)], although larger values may be possible in the presence of swirl. The non-dimensional source swirl parameter S is necessarily positive and provides a measure of the dynamic role of the centrifugal pressure perturbation in the core flow.

Although the analysis is based on simplified equations of motion, the resulting equations are too complicated to be solved analytically. Numerical solutions were found using Runge-Kutta integration for selected values of Λ and a range of values of S.

4 RESULTS

This paper only presents results which illustrate the effect of swirl on the particular case of swirling wakes with $\Lambda = 2$, although the results described are typical of the entire wake range $\Lambda > 1$. These results are contained in figure (1) which illustrates the effect of swirl on the axial variation of wake radius, azimuthal velocity, axial velocity and absolute mass flux.

The model predicts that swirl increases a wakes rate of spread, axial decay of azimuthal velocity and mixing at all downstream axial stations. Figure (1) indicates that swirl increases the axial velocity deficiency and makes the core more 'wake-like', as a result of the swirl induced axial deceleration. Above a sufficiently large degree of swirl the axial velocity on the core axis first decelerates to a minimum, in a swirl controlled region, and then accelerates with increasing downstream distance as swirl decays and the role of entrained axial momentum excess becomes more significant.

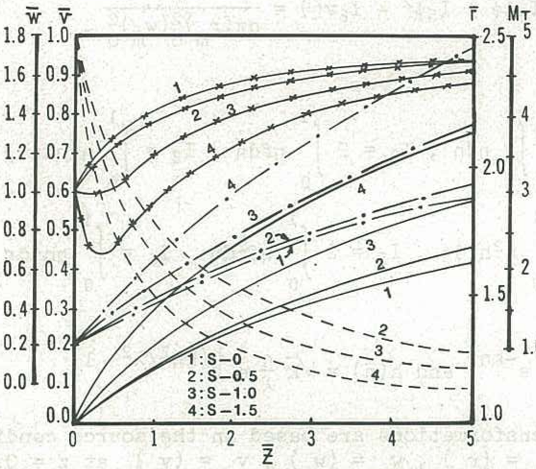

Figure 1 Non-dimensional curves showing axial variations of wake radius, $\bar{r}$ (——), azimuthal velocity, $\bar{v}$ (– – –), axial velocity, $\bar{w}$ (-x-x-) and absolute mass flux, M_T (—·—) of swirling wakes with $\Lambda = 2.0$ ($\beta^2 = 1$), for various values of the swirl parameter S. The non-dimensional mass flux $M_T = \bar{r}^2 (I_1\Lambda + I_2\bar{\phi}) / [I_2 + \Lambda(I_1 - I_2)]$

5 APPLICATION TO PRE-DILUTION DEVICES

One of the small scale pre-mixing device models investigated by Agg and White (1974) consists of a horizontal cylinder which would be aligned with the direction of a reversing tidal current flow while effluent is introduced horizontally and tangentially. Thus, the effluent will be emitted from the outlet horizontally as a swirling wake which, in view of the results obtained in the previous section, suggests that increasing the degree of swirl at the outlet, by say increasing the tangential effluent flow rate, will increase mixing beyond the outlet as well as within it. It must be remembered, however, that the pre-diluted sewage-sea water mixture is still positively buoyant and can only be considered as a horizontal swirling wake in an initial region, where buoyancy has not produced significant upward bending of the flow. The axial length of such a region increases with the level of pre-mixing.

The results presented in the previous section indicate that swirl increases mixing and hence dilution in wakes. It should be noted, however, that with Λ fixed and S varied we examined the effect of a source azimuthal velocity on wakes from a series of sources of fixed radius and axial velocity. Such a set of initial conditions would require a different pre-mixing device design, in order to maintain a constant axial disturbance velocity (Λ constant) for varying source azimuthal velocities. This could be designed with a combination of axial and tangential flows, similar to those in the swirl generator used by Chigier and Chervinsky (1967), but would require pumping of the effluent input to provide the necessary axial flow, with the ambient flow providing at least part of the controlled tangential flow. It should be noted, however, that for such a device, the emitted sewage-sea water mixture may also take the form of a swirling jet for some levels of the ambient flow. The effect of swirl on mixing in these flows, will be considered elsewhere.

Increasing the degree of swirl at the outlet in the Agg and White model being considered, by increasing the tangential effluent flow rate, for a certain ambient stream velocity, appears likely to produce both an increase in source azimuthal velocity and a decrease in source axial velocity. Consequently, we would expect swirl to increase mixing beyond the outlet, as well as inside it, since the level of turbulence, and hence entrainment, in the core will be increased by the larger mean axial shear across the core as well as by the direct effects of swirl.

6 CONCLUSIONS

The results presented indicate that increasing the level of source swirl in a forced wake will increase mixing, and hence dilution of effluent, at all downstream axial stations. This suggests that swirl control associated with the pre-dilution device considered could be utilized to take advantage of both pre-mixing within the device and swirl control of mixing beyond the device.

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