A Comparison of Developed Single Phase Turbulent Flow in Two and Three Dimensions

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1 INTRODUCTION

The law of the wall, which provides a means of reducing data to standard nondimensional forms, or allows the prediction of the average velocity profiles for turbulent shear flows, remains the major discovery about turbulent shear flows. Strictly it is not a fundamental law, derived from first principles, but its existence has been demonstrated empirically by a very large number of experimental studies in both boundary layer flows, and flows in symmetrical ducts. Most experiments have been conducted in the commonest boundary layer flow situation which is substantially two-dimensional, where the velocity distribution is a function of both the wall distance and axial length in the direction of the mean velocity. Two-dimensional turbulent shear flow also includes flow through a symmetrical duct (pipe, annulus), where the circumferential distribution of the wall shear stress in a plane normal to the duct axis is uniform.

In this paper an investigation is described of the law of the wall in a non-symmetrical duct flow, in which there is an appreciable variation in the wall shear stress distribution in a plane normal to the duct axis. Such a flow is three-dimensional, although the magnitude of the secondary flow components, located in a plane normal to the axial velocity, is typically less than 1% of the mean axial velocity. The turbulent intensities, in regions of the duct where the secondary flow components are normal to the wall, appear to approximate closely the values reported for symmetrical pipe flow. The average radial velocity profiles, scaled using local values of the wall friction velocity, do conform to the law of the wall distribution, even for regions where the turbulence intensities and hence flow structure are significantly different from two-dimensional pipe flow.

NOTATION

p/d	Rod pitch-to-diameter ratio
Re	Reynolds number
u,v,w	Instantaneous values of velocity fluctuations in z, r and θ
u',v',w'	r.m.s. values of velocity fluctuations or turbulence intensities
$U(r,\theta)$	Axial mean velocity component
$W(r,\theta)$	θ component of secondary flow
$U^*(\theta)$	Friction velocity
$\tau_w(\theta)$	Wall shear stress
κ	Von Karman constant
C	Constant in law of wall

y	Radial wall distance
$\hat{y}$	Wall distance to duct centre line
ρ	Fluid density
ν	Fluid kinematic viscosity
y^+	Dimensionless wall distance (yU^*/ν)
$U^+(r,\theta)$	Dimensionless mean velocity $(U(r,\theta)/U^*(\theta))$

2 FORMULATION OF THE LAW OF THE WALL

For an isothermal flow, in which compressibility effects are negligible, the mean velocity distribution normal to the wall may be described for a large range of the parameters by

$$\frac{U}{U^*} = f\left(\frac{yU^*}{\nu}\right) . \tag{1}$$

The earliest formulation of the above form was based on pipe flow, in which (1) becomes, for yU^*/ν greater than 30,

$$\frac{U}{U^*} = \frac{1}{\kappa} \ln\left(y\frac{U^*}{\nu}\right) + C \tag{2}$$

where the constants κ and C are determined experimentally. Relationship (2), however, may be taken as both a unique and universal similarity law for turbulent flows past a smooth surface (Coles, 1956).

The predominance of laminar shear very near the wall requires U/U* to approach yU*/ν as y approaches zero. The numerical value of the Von Karman constant κ varies between 0.40 and 0.41 (Coles 1956, Clauser 1954, Lindgren and Chao 1969, Dean 1976), and the constant C varies over the greater range of 4.9 (Clauser 1954) to 5.5 (Lindgren and Chao 1969). The values used in this report for both symmetrical pipe flow and flow in a rod cluster where κ = 0.40 and C = 5.5.

The area of interest was the application of (2) to a duct flow (a typical engineering situation) in which a significant variation in the wall shear stress was present. The selected geometry represented the interior of a large array of closely spaced rods with the flow parallel to the rod bundle. The rod bundle geometry is common to most nuclear power reactor cores, in which the fuel bundles are cooled by an axially flowing turbulent fluid.

3 EXPERIMENTAL RIG

3.1 Six-Rod Cluster

The six-rod cluster air rig was designed to represent the typical flow passage for in-line flow through a large array of closely spaced heat generating rods, which is typical of the core of a

power reactor. The scale of the rig was approximately ten times larger than a light water reactor, or thirty times larger than a fast reactor fuel array. The rod dia. was 14.0 cm. The rods were arranged in a square pitch lattice (Fig.1(a)), as the circumferential variation in the wall shear stress is significantly greater than in a triangular lattice at the same rod spacing (p/d) and Reynolds number (Hooper 1977).
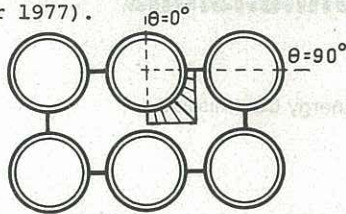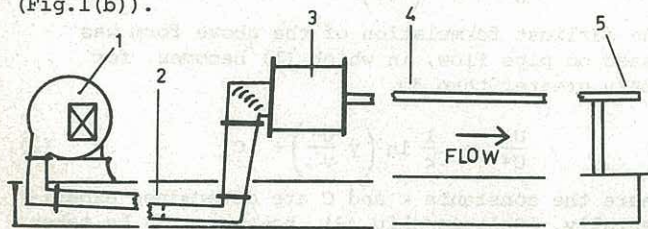

Cross-section of 6 Rod Cluster

Figure 1(a)

The p/d used was 1.193. Earlier numerical studies had shown that, for this p/d spacing, the influence on the flow pattern in the central flow region of the wall in the subchannel gap is negligible for laminar flow conditions (Hooper 1975). In turbulent flow which is of most interest, the influence of the gap wall would be further reduced, because of the significantly higher velocity gradients near the duct walls.

The test section length was 9.14 m, giving a flow development length of 95 hydraulic diameters. Air was the working fluid, and the total air flow from the 45 kW two-stage centrifugal blower was measured by orifice plate before entry to the settling tank (Fig.1(b)).

1. 45 kW Centrifugal Blower   2. Orifice Plate
3. Settling Drum   4. Test Section
5. Measurement Location

Schematic Drawing of 6 Rod Cluster

Figure 1 (b)

The circumferential variation in the wall shear stress was measured by a Preston tube of 0.40 cm dia. at the rig exit, using the correlations of Patel (1965). The mean velocity profiles were measured radially by pitot and boundary layer probes for the segment of the subchannel shown in Fig.1(a). The symmetry of flow conditions around the top and bottom central rods of the array had been shown by earlier measurements of both the wall shear stress and mean velocity (Hooper 1977).

The turbulent velocity components u',v' and w', and the Reynolds stress $\rho \overline{u \, v}$ were measured by a constant temperature hot wire anemometer, using a circuit developed by the I.S.V.R. at Southampton University. The probe response correlations were also developed by workers at Southampton (Bruun 1976) and, as the turbulence intensities were relatively low, the non-linear bridge signal was used as the signal output. The probe arrangement was a single straight wire and a single inclined wire. The axial turbulence velocity component, u', was determined by the straight wire.

Rotation of the inclined wire in four quadrants was used to determine the Reynolds stress $\rho \overline{u \, v}$ and the non-axial components v' and w'. A detailed error analysis showed that the determination of the effective flow angle of the inclined wire represented a major source of error in the measurement of the non-axial turbulence components. A probe yaw assembly was developed to calculate this flow angle at a flow velocity comparable to the test section. This calculation is described in the appendix.

4   EXPERIMENTAL RESULTS

4.1 Round Duct Average Velocity Profile

In Fig.2 is shown the radial velocity profile for the round duct of 9.50 cm dia., at a flow Reynolds number of 6.0×10^4, and a mean velocity of 10.0 m/s. The profile is typical of developed flow through a symmetrical duct with a favourable pressure gradient $(\frac{dp}{dz})$ negative and, for the turbulent core region $(y^+ > 30)$, it is well predicted by the law of the wall, almost to the centre of the duct. The near wall region is also very close to results reported for flow in the sublayer and transition region (Lindgren and Chao 1969).
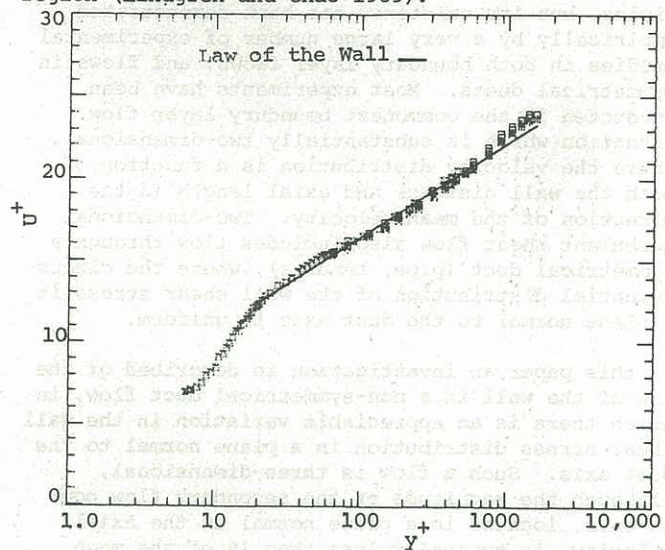

Velocity Profile in a Round Duct

Figure 2

5   SIX ROD CLUSTER

5.1 Wall Shear Stress Distribution

The variation of the wall shear stress for the top central rod of the cluster is shown by Fig.3, for an Re of 8.59×10^4, and a corresponding mean velocity of 15.5 m/s.
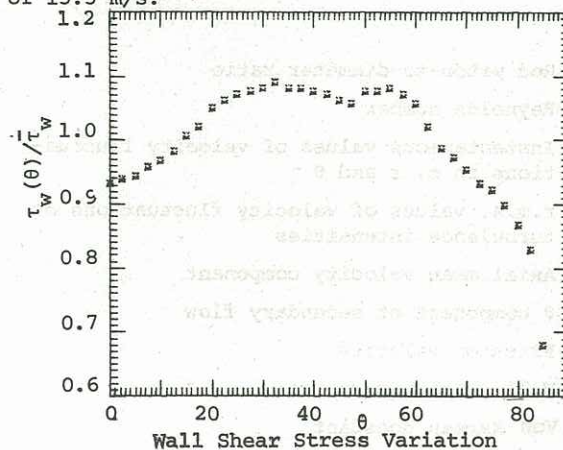

Wall Shear Stress Variation

Figure 3

The polar angle θ is 0° in the interconnection gap region for the top central rod, and increases in an anti-clockwise sense to 90° at the top right hand gap wall (Fig.1(a)). The region between θ = 0,45° is thought to approximate closely the circumferentially repeated wall shear stress distribution in a large square pitch rod array at a corresponding p/d and Re. The influence of the wall in the gap region (θ = 90°) appears to cause a progressive decrease of the local wall shear stress until it is approximately 0.70 of the mean shear stress at an angle of 85°.

5.2 Radial Average Velocity Profiles

The average velocity profiles, measured radially from the top rod for θ = 0, 15, 30, 45, 60 and 75° are shown by Fig.4.
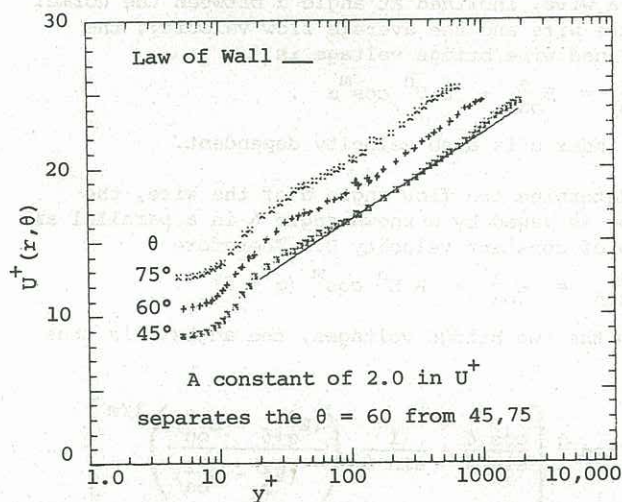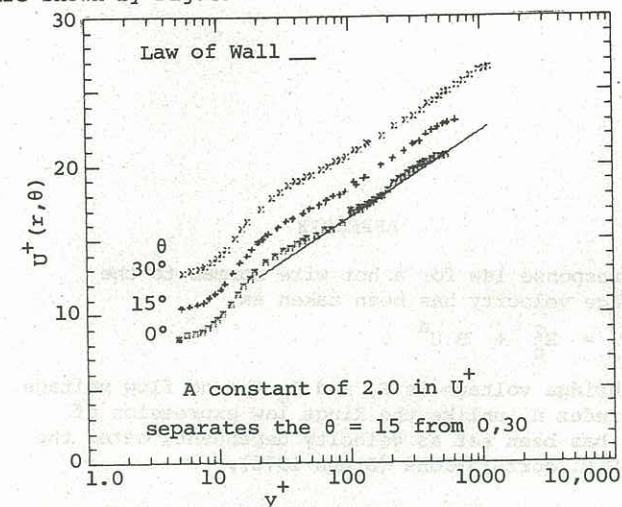
Velocity Profiles in Six-Rod Cluster

Figure 4

The logarithmic law of the wall for the turbulent core region (y$^+$ > 30) is also shown, and the similarity between the measured velocity profiles at these angles and the profile in the round duct is clear.

5.3 Radial Turbulence Intensities

The variation of the turbulence intensities u', v' and w', scaled by the local friction velocity $U_\tau(\theta)$, are shown by data points in Fig.5 for θ = 45 and 15°.

Also shown (as continuous lines) are the corresponding distributions of Laufer (1954) for a round duct. The results for the subchannel line of symmetry (θ = 45°) show close agreement to the corresponding results for flow in a round duct. For the 15° distribution, a significant difference is noted between the round duct distribution and the rod cluster, particularly for the w' distribution. This
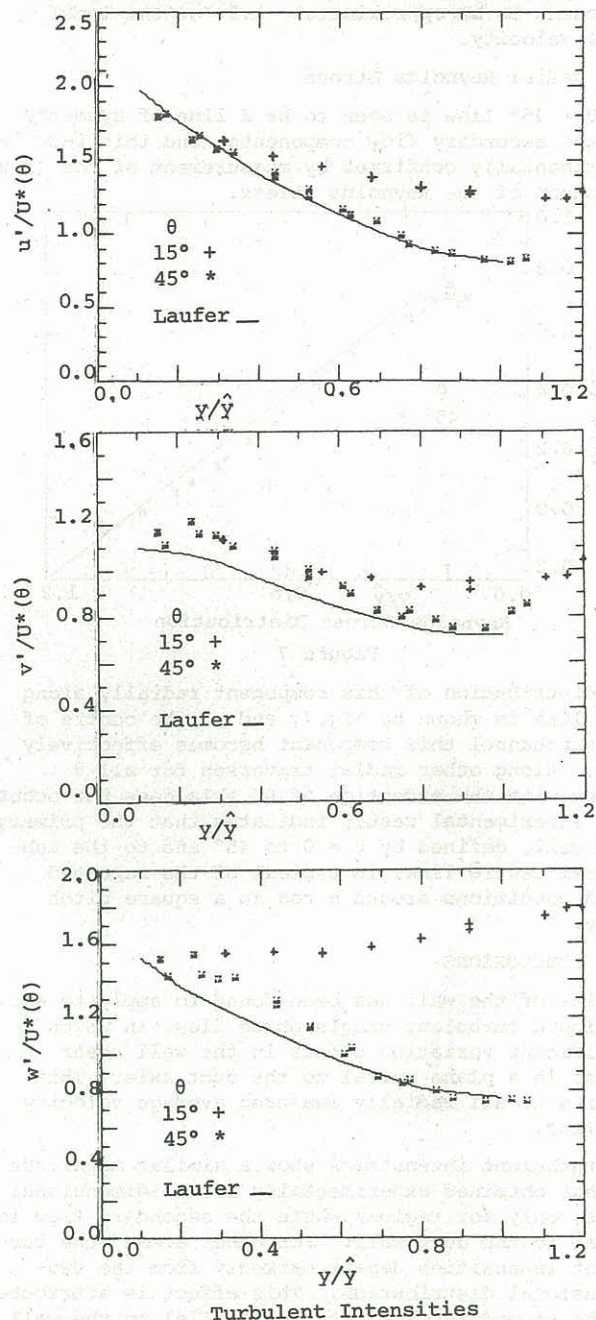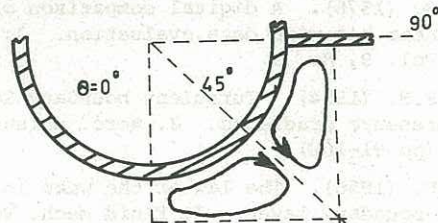
Turbulent Intensities

Figure 5

departure from the intensity distribution typical of two-dimensional flow situations is considered to be caused by secondary flows in the cross section which are circulating in the direction shown by Fig.6.

Proposed Secondary Flow Distribution

Figure 6

The magnitude of the secondary flow in the θ = 0 to 45° segment is not measurable by either pitot probes or inclined hot wires, being less than 1% of the local axial velocity. The θ = 45 to 90° segment has a higher magnitude of secondary flow; an estimate of the average secondary flow velocity close to the

wall, by inclined hot wire (see Appendix) shows this component to be approximately 1.5% of the local axial velocity.

5.4 Radial Reynolds Stress

The $\theta = 45°$ line is seen to be a line of symmetry for the secondary flow components, and this fact is experimentally confirmed by measurement of the (r-z) component of the Reynolds stress.
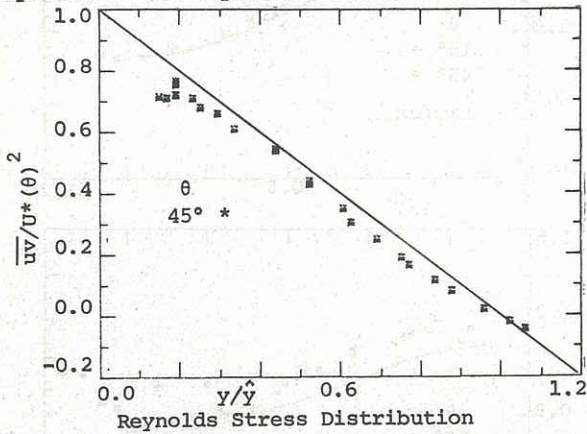
Reynolds Stress Distribution

Figure 7

The distribution of this component radially along this line is shown by Fig.7, and at the centre of the subchannel this component becomes effectively zero. Along other radial traverses for all θ angles with the exception of 0° this does not occur. This experimental result indicates that the primary flow cell, defined by $\theta = 0$ to 45° and to the subchannel centre line, is typical of the repeated fluid conditions around a rod in a square pitch array.

6 CONCLUSIONS

The law of the wall has been found to apply to a developed turbulent single phase flow, in which a significant variation occurs in the wall shear stress in a plane normal to the duct axis. This applies to all radially measured average velocity profiles.

The turbulent intensities show a similar magnitude to that obtained experimentally in two-dimensional flows, only for regions where the secondary flow is normal to the duct wall. For other areas, the turbulent intensities depart markedly from the two-dimensional distribution. This effect is attributed to the secondary flow pattern parallel to the wall.

7 ACKNOWLEDGEMENT

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APPENDIX

The response law for a hot wire normal to the average velocity has been taken as

$$E^2 = E_o^2 + B U^n .$$

The bridge voltage is E, and E_o the no flow voltage. The index n, unlike the Kings law expression of 0.5, has been set as velocity dependent, after the I.S.V.R. correlations (Bruun 1976).

For a wire, inclined at angle α between the normal to the wire and the average flow velocity, the inclined wire bridge voltage is

$$E_\alpha^2 = E_{o\alpha}^2 + B U^n \cos^m \alpha .$$

The index m is also velocity dependent.

To determine the flow angle α of the wire, the probe is yawed by a known angle δ in a parallel air flow of constant velocity U. Therefore

$$E_{\alpha+\delta}^2 = E_{o\alpha}^2 + B U^n \cos^m (\alpha + \delta) .$$

From the two bridge voltages, the angle α is thus

$$\alpha = \tan^{-1} \left[\frac{\cos \delta}{\sin \delta} - \frac{1}{\sin \delta} \left(\frac{(E_{\alpha+\delta}^2 - E_{o\alpha}^2)}{(E_\alpha^2 - E_{o\alpha}^2)} \right)^{1/m} \right] .$$

An inclined hot wire placed in a region of secondary flow will exhibit a change in the bridge voltage if rotated through 180°, because of the change in the average velocity angle with respect to the wire. This can be used to estimate the magnitude of the secondary flow component

$$(W/U) = \frac{1}{\tan \alpha} \left[\frac{1 - K}{1 + K} \right] ,$$

where

$$K = \left[\frac{E_{\alpha_1}^2 - E_{o\alpha}^2}{(E_{\alpha_2}^2 - E_{o\alpha}^2)} \right]^{1/m}$$

and E_{α_1}, E_{α_2} are the bridge voltages at 0 and 180°.