FIFTH AUSTRALASIAN CONFERENCE

on

HYDRAULICS AND FLUID MECHANICS

at

University of Canterbury, Christchurch, New Zealand

1974 December 9 to December 13

ANALYSIS OF SOME DISCONTINUOUS INITIAL-VALUE PROBLEMS FOR FREE-MOLECULAR GAS MIXTURES

by

Russell East and Aleksander Glikson

SUMMARY

Free-molecular solutions of a certain system of the Boltzmann equations are found, assuming the absence of external forces, for some discontinuous initial-value problems, when either a mixture of highly rarefied gases is considered over an arbitrary time, or a mixture of not so rarefied gases is considered over a sufficiently short time of evolution. Initially, the distinct gases are in absolute equilibrium and completely separated by means of an impenetrable boundary. After this boundary is removed, diffusion of the gases and smoothing out of the initial, not necessarily weak, discontinuities begins. The mean field quantities of particular gases and field quantities for the whole gas-mixture are evaluated analytically and numerically. Comparison with some previous results and classical gas-dynamical solutions by means of corresponding graphs, and discussion of the results obtained, are given.

R. East, Department of Mathematics, Univ. of New England, N.S.W., Australia.
A. Glikson, Department of Mathematics, Univ. of New England, N.S.W., Australia.

I. Introduction

Keller [1] and Kornowski [2] have obtained nonstationary free-molecular* solutions of the non-linear Boltzmann equation for the problem of the filling of a particular vacuum by a neutral gas, assuming the absence of external forces and under the assumption that initially (at time t=0, say) the gas is in absolute equilibrium and completely separated from the vacuum by an impenetrable boundary. In [1], this boundary was $x_3=0$ [where x_k (k=1,2,3) are physical space X coordinates] with the vacuum in the half-space $x_3<0$. Analytical expressions for the lower thermodynamic moments of the distribution function were given, showing in particular that the pressure tensor remains diagonal for t>0 and that the macroscopic quantities not only depend on the same ratio (x_3/t) as the classical gasdynamics solutions of the Euler and Navier-Stokes equations** but also represent similar (although distinguishable) results. It seems useful to mention that this problem can also be interpreted as one of a gas confined to the "positive" half of a tube of constant rectangular cross-section with perfectly reflecting walls, thus representing an idealized shock-tube experiment. (For an interesting and simple review of various shock-tube (shock tunnel) driving techniques and their applications, especially for a description of some quite recent achievements at the Australian National University, see [7].) Kornowski [2] studied the problem of the filling of a vacuum: (i) between two parallel planes, (ii) inside a sphere, (iii) inside an infinitely long circular cylinder. Only the density profiles were found. Strictly speaking, the exact expressions for the density in the whole of X were obtained in the case of the first two configurations, but only on the centreline and boundary in the case of the third one. An approach to some problems of upper atmosphere flight on the basis of the results obtained was also described.

It should be mentioned here that "the method of instantaneous point sources", used by Molmud [8] as an analogy to the solutions of some heat flow problems in solids, leads to evaluation of lower moments of the distribution function without the evaluation of the function itself. But this method cannot be naturally and simply generalized and extended to include, in particular, first, second, etc. collisions between molecules (the price paid for omission of the basic microscopic level of description). On the other hand such generalizations and extensions can be made if the explicit free-molecular solutions are used as the zeroth approximations in certain iterative schemes of solution of the Cauchy problem for the non-linear Boltzmann equation. Various iterative schemes were investigated from a theoretical point of view in [9], [10], [11] and [12] for some classes and spaces of continuous or Lebesgue measurable functions. A practical formal application of one of the iterative schemes*** (more precisely, that of [11]) to some of the abovementioned problems, assuming a cut-off of intermolecular interaction, has been given in [16] where main considerations were restricted to first collisions and final computations were performed in the case of pseudo-Maxwellian molecules. It is in order to point out here the existence of some other results for smoothing out of weak initial discontinuities [17], [18], [19] obtained by using more sophisticated methods of evaluation than that of [16]. (For a review of other problems in which free-molecular solutions are involved see, for example, Schaaf [20] and Kogan's monograph [6], Ch. VI.)

The contents of this paper are as follows. The problems to be solved are formulated in Section II following the Introduction. In Section III solutions of the problems are obtained analytically and numerically. Section IV contains a comparison of the results of Section III with some related free-molecular or classical gasdynamic solutions (by means of appropriate graphs) as well as a further discussion of the results with some conclusions. (A full discussion of the results obtained, in particular a detailed comparison with gasdynamic solutions [21], [22], [23], is given in [16]. Such a discussion, connected with consideration of various cases, should not be given here, but not simply because of the restricted size of the presentation. Clearly, to have a complete and precise picture of similar, but more comprehensive, problems it certainly is much more profitable if the (limiting) free-molecular solutions are compared not only with the (limiting) continuum solutions but also with some, at least, intermediate solutions; for example, with the first-collision results evaluated in [16].)

---

* For a precise mathematical definition and range of validity of the free-molecular limit, see Grad [3], Sect. 7 and 12. (Also see Truesdell [4], especially §22, Lebowitz & Frisch [5] and Kogan [6], §2.11.) An extension of the definition to the case of a flow of a rarefied gas mixture is simple (cf. Kogan [6], §1.4).

** Note an error in [1], which has been pointed out by Truesdell in [4], §22, as well as a number of typographic errors and omission of the drawings to which reference is made, in [2].

*** One should exercise some care in choosing an appropriate scheme. A somewhat striking example of the incorrect use of iterative procedures by early contributors is given by the so-called Knudsen iteration, especially in the case of certain one-dimensional geometries. [The fact was pointed out and partly explained by Willis (e.g. in [13] and [14]). For a short explanation from a functional analysis view-point the reader is advised to consult Cercignani [15], p.136 and p.204.]

II. Formulation of the problems

Two initial-value problems are to be considered under the assumption of the absence of external forces, for a certain system of the Boltzmann equations, in the case of two distinct* neutral, collisionless gases initially separated by an impenetrable boundary and each in absolute equilibrium, with initial (not necessarily weak) discontinuities between the number densities, temperatures and (only in the first problem) mean velocities. Thus the initial distribution function of the i^{th} gas, f_i^0, is as follows:

$$f_i^0 \equiv f_i^0(\underset{\sim}{x},\underset{\sim}{\xi}_i) \equiv \phi_i \zeta(X_i^0).$$

Here

$$\phi_i \equiv \frac{n_i^0}{(\pi^{\frac{1}{2}} h_i^0)^3} \exp\left[-\left(\frac{\underset{\sim}{\xi}_i - \underset{\sim}{u}_i^0}{h_i^0}\right)^2\right],$$

$$\overline{X}_i^0 \cup X_j^0 = X, \quad \overline{X}_i^0 \cap X_j^0 = \emptyset, \quad i,j \in \{1,2\} ,$$

where n_i^0, $\underset{\sim}{u}_i^0 \equiv (u_{i1}^0, u_{i2}^0, u_{i3}^0)$ are the initial mean number density and mean velocity of the i^{th} gas, $\xi_i \equiv (\xi_{i1}, \xi_{i2}, \xi_{i3})$ is its molecular velocity and $h_i^0 \equiv (2kT_i^0/m_i)^{\frac{1}{2}}$ is the so-called most probable (initial) random speed; T_i^0 is the initial i^{th} gas temperature, m_i is the constant mass of a molecule of the i^{th} gas. [Clearly, the superscript "0" distinguishes initial parameters and functions, while the subscript 'i' refers to the i^{th} gas.] $\zeta(X_i^0)$ is the characteristic function of the region X_i^0, i.e. $\zeta(X_i^0) \equiv 1$ if $\underset{\sim}{x} \in X_i^0$, $\zeta(X_i^0) \equiv 0$ if $\underset{\sim}{x} \notin X_i^0$. $\overline{X}_i^0$ denotes the closure of X_i^0; $\emptyset$ denotes the empty set.

The considerations are restricted to some initial configurations which are generalizations of those of [2], namely

(i) $X_1^0 = \{\underset{\sim}{x}: |x_3| < a = \text{const} > 0\}$, $\qquad u_{i1}^0 = u_{i3}^0 = 0$ for $i = 1, 2$;

(ii) $X_1^0 = \{\underset{\sim}{x}: r < a = \text{const} > 0; \ \underset{\sim}{r} \equiv (x_1, x_2, x_3), \ r \equiv |\underset{\sim}{r}|\}$, $\underset{\sim}{u}_i^0 = \underset{\sim}{0}$ for $i = 1, 2$.

In what follows the resulting expansions are called (i) infinite layer and (ii) spherical cloud, respectively.

After the boundary is instantaneously removed at $t = 0$, diffusion of the gases and smoothing out of the initial discontinuities begins. Our main aim is not so much the evaluation of the distribution functions for $t > 0$ (this is trivial in both problems) but (a) to find analytically and numerically the mean ordinary density, mass-average velocity, diffusion velocities of particular gas constituents, components of the mass-average stress tensor (and thus by implication the mass-average temperature), etc. for the mixture as a whole, and (b) to investigate their behaviour.

III. Solutions of the problems

Under the free-molecular conditions and in the absence of external forces any system of Boltzmann equations which describes the behaviour of a gas mixture [6], [10], [24], [25] reduces to a system of independent equations, each containing only one distribution function as an unknown function so that the type of Cauchy problem of Section II has the following form:

$$\mathcal{D}_i f_i = 0, \quad t > 0, \qquad f_i(0,\underset{\sim}{x},\underset{\sim}{\xi}_i) = f_i^0 \quad (i = 1, 2),$$

where the linear free-streaming operator $\mathcal{D}_i$ is reduced to the directional derivative

$$\mathcal{D}_i \equiv \frac{\partial}{\partial t} + \underset{\sim}{\xi}_i \cdot \frac{\partial}{\partial \underset{\sim}{x}} .$$

Solution to the Cauchy problem are known to be**

$$f_i(t,\underset{\sim}{x},\underset{\sim}{\xi}_i) = f_i^0(\underset{\sim}{x} - \underset{\sim}{\xi}_i t,\underset{\sim}{\xi}_i) \equiv \phi_i^0 \zeta(X_i),$$

* By distinct gases we mean hereafter gases having distinct masses of their molecules [although strictly speaking this would be too general if a consistency in matching the boundary conditions prior to $t = 0$ is required (cf. [6], p.69, and [15], p.54)]. In a general "collisional" case distinct molecules may of course have many other important distinct properties, e.g. collision cross-sections, degrees of freedom etc.

** Actually the class of the solutions is restricted both from the physical and mathematical point of view. From the physical point of view, the functions should approach zero sufficiently rapidly for $\xi \to \infty$ to guarantee the existence of moments (at least as high as the kinetic energy moment). From the mathematical point of view, to achieve the a.e. existence of $\mathcal{D}_i f_i$, f_i^0 should be continuous a.e. In the problems to be considered the f_i^0's are either absolute Maxwellian or identically vanishing in some regions, thus all necessary requirements are satisfied.

where for our initial configurations, and $t > 0$,

(i) $X_1 = \{\underset{\sim}{x}: |x_3 - \xi_{13}t| < a\}$, $\quad X_2 = \{\underset{\sim}{x}: |x_3 - \xi_{23}t| > a\}$,

(ii) $X_1 = \{\underset{\sim}{x}: |\underset{\sim}{x} - \underset{\sim}{\xi}_1 t| < a\}$, $\quad X_2 = \{\underset{\sim}{x}: |\underset{\sim}{x} - \underset{\sim}{\xi}_2 t| > a\}$.

For these problems various mean quantities of interest have been evaluated and the results are given below. Definitions of the quantities (in the case of general gas mixtures) are found in [6], [24], [25].

(i) Infinite Layer

The X_i may be rewritten for this problem as follows:

$$X_1 = \{\underset{\sim}{x},\underset{\sim}{\xi}_1: z_- < \xi_{13} < z_+\}, \qquad X_2 = \{\underset{\sim}{x},\underset{\sim}{\xi}_2: \xi_{23} < z_-,\ z_+ < \xi_{23}\},$$

where

$$z_- \equiv (x_3 - a)/t, \qquad z_+ \equiv (x_3 + a)/t.$$

Also, an inertial frame of reference is introduced, so that $\underset{\sim}{u}_2^0 = \underset{\sim}{0}$ and thus $u_{12}^0 = U = \text{const.}$

(a) Mean densities

The number and mass densities of the i^{th} gas component in the mixture are given respectively by

$$n_i(t,\underset{\sim}{x}) = (-1)^{i-1} n_i^0 Ef_i + \delta_{i2} n_2^0 \text{ and } \rho_i(t,\underset{\sim}{x}) \equiv m_i n_i,$$

where δ_{ij} is the Krönecker delta tensor, and

$$Ef_i(t,\underset{\sim}{x}) \equiv \tfrac{1}{2}[\mathrm{erf}(z_+/h_i^0) - \mathrm{erf}(z_-/h_i^0)].$$

The number and mass densities for the mixture are respectively

$$n(t,\underset{\sim}{x}) = \sum_i n_i, \qquad \rho(t,\underset{\sim}{x}) = \sum_i \rho_i.$$

(b) Average velocities

Components of the mean velocity of the i^{th} gas are found to be

$$u_{i1} = 0, \quad u_{i2} = (1 - \delta_{i2})U, \quad u_{i3} = (-1)^{i-1} n_i^0 h_i^0 \mu_{i,1}/n_i,$$

where

$$\mu_{i,1} \equiv \tfrac{1}{2}\pi^{-\frac{1}{2}}\{\exp[-(z_-/h_i^0)^2] - \exp[-(z_+/h_i^0)^2]\}.$$

Thus the components of the mass-average and diffusion velocities are respectively,

$$\bar{u}_1 = 0, \quad \bar{u}_2 = \rho_1 U/\rho, \quad \bar{u}_3 = \sum_i (-1)^{i-1} \rho_i^0 h_i^0 \mu_{i,1}/\rho,$$

and

$$\overline{V}_{i1} = 0, \quad \overline{V}_{i2} = U(\rho_2/\rho - \delta_{i2}), \quad \overline{V}_{i3} = (-1)^{i-1}(1 - \rho_i/\rho)\sum_j (n_j^0 h_j^0 \mu_{j,1}/n_j).$$

(Obviously $\rho_i^0 \equiv m_i n_i^0$.)

(c) Stress tensor and temperature

The expression for the components of the stress tensor, as given in say [25], may easily be simplified to

$$p_{jk}(t,\underset{\sim}{x}) = \sum_i m_i \int \xi_{ij}\xi_{ik} f_i d\underset{\sim}{\xi}_i - \rho \bar{u}_j \bar{u}_k$$

(integration is over the whole Ξ_i-space), and evaluation gives

$$p_{11} = \tfrac{1}{2}\sum_i (h_i^0)^2 \rho_i, \quad p_{22} = p_{11} + \rho_1\rho_2 U^2/\rho,$$

$$p_{33} = p_{11} + \sum_i (-1)^{i-1} \rho_i^0 h_i^0 \mu_{i,2} - [\sum_i (-1)^{i-1} \rho_i^0 h_i^0 \mu_{i,1}]^2/\rho,$$

$$p_{12} = p_{21} = 0, \qquad p_{13} = p_{31} = 0,$$

$$p_{23} = p_{32} = U\sum_i \rho_i^0 h_i^0 \mu_{i,1}(1 - \rho_i/\rho).$$

where

$$\mu_{i,2} \equiv \tfrac{1}{2}\pi^{-\frac{1}{2}}\{z_-\exp[-(z_-/h_i^0)^2] - z_+\exp[-(z_+/h_i^0)^2]\} .$$

The temperature is given by the following tensor contraction:

$$T = \sum_j p_{jj}/(3kn) .$$

Recalling that $(h_i^0)^2 \equiv 2kT_i^0/m_i$, we obtain

$$T = \sum_i[n_i + \tfrac{2}{3}(-1)^{i-1}n_i^0\mu_{i,2}/h_i^0]T_i^0/n + \tfrac{2}{3}(n_2/n)(\rho_1/\rho)(U/h_2^0)^2T_2^0$$

$$- \tfrac{2}{3}[\sum_i(-1)^{i-1}n_i^0T_i^0\mu_{i,1}/h_i^0]^2/[n\sum_i n_iT_i^0/(h_i^0)^2].$$

(ii) Spherical Cloud

The positive quantities r, ξ_i are defined as $r \equiv |\underset{\sim}{x}|$, $\xi_i \equiv |\underset{\sim}{\xi}_i|$; the angle θ_i is defined to be the angle between the $\underset{\sim}{x}$ and $\underset{\sim}{\xi}_i$ vectors, i.e. $\cos\theta_i = (\underset{\sim}{x}\cdot\underset{\sim}{\xi}_i)/(r\xi_i)$; $\theta_i\in[0,\pi]$. To complete our description of the Ξ_i-space, the co-ordinate $\psi_i \in [0,2\pi)$, is introduced so that ξ_i, θ_i, ψ_i form spherical polar co-ordinates. It can be readily shown that sets X_1 and X_2 may be rewritten as follows (with $t > 0$ as previously):

$$X_1 = \{\underset{\sim}{x},\xi_1\text{: } 0\leqslant\xi_1\leqslant R_{12},\ 0\leqslant\theta_1\leqslant\pi,\ 0\leqslant r<a;\quad R_{11}\leqslant\xi_1\leqslant R_{12}, 0\leqslant\theta_1\leqslant\sin^{-1}\left(\tfrac{a}{r}\right),\ a<r<\infty\},$$

$$X_2 = \{\underset{\sim}{x},\xi_2\text{: } R_{22}\leqslant\xi_2<\infty,\ 0\leqslant\theta_2\leqslant\pi,\ 0\leqslant r<a;\quad (0<\xi_2\leqslant\infty,\ 0\leqslant\theta_2\leqslant\pi) \ominus (R_{21}<\xi_2<R_{22},\ 0<\theta_2<\sin^{-1}\left(\tfrac{a}{r}\right)),\ a<r<\infty\},$$

where '$\ominus$' denotes set subtraction, and

$$R_{ik}(t,r,\theta_i) \equiv [r\cos\theta_i + (-1)^k(a^2 - r^2\sin^2\theta_i)^{\frac{1}{2}}]/t.$$

The following notation will be used in this section:

$$r_+ \equiv (a + r)/t, \qquad r_- \equiv (a - r)/t,$$

$$\mathrm{Erf}_{i,k}(t,\underset{\sim}{x}) \equiv \tfrac{1}{2}[\mathrm{erf}(r_+/h_i^0) + (-1)^k\mathrm{erf}(r_-/h_i^0)], \qquad k = 1,2 ,$$

$$v_i(t,\underset{\sim}{x}) \equiv [2r^2 - 2a^2 + 3(h_i^0)^2t^2]/(4rt),$$

$$\nu_{i,m}(t,\underset{\sim}{x}) \equiv \tfrac{1}{2}\pi^{-\frac{1}{2}}\{\exp[-(r_-/h_i^0)^2] - (-1)^m\exp[-(r_+/h_i^0)^2]\}, \qquad m = 1,2 ,$$

$$\nu_{i,m}(t,\underset{\sim}{x}) \equiv \tfrac{1}{2}\pi^{-\frac{1}{2}}\{r_-\exp[-(r_-/h_i^0)^2] + (-1)^m r_+\exp[-(r_+/h_i^0)^2]\}, \qquad m = 3,4 .$$

(a) Mean Densities

The number and mass densities of the i^{th} gas constituent are respectively,

$$n_i(t,\underset{\sim}{x}) = (-1)^{i-1}n_i^0[\mathrm{Erf}_{i,2} + h_i^0\nu_{i,2}t/r] + \delta_{i2}n_2^0 , \text{ and } \rho_i(t,\underset{\sim}{x}) = m_in_i.$$

The number and mass densities of the mixture are given by the same formulae as in the problem of the infinite layer.

(b) Average velocities

Clearly, the macroscopic flow of the gas constituents and the whole mixture will be in a radial direction only. We denote the radial component of the mean velocity of the i^{th} gas by u_i. Since, by [25], p.453, eq.(7.2-2), $u_i(t,\underset{\sim}{x}) \equiv (1/n_i)\int\xi_if_id\underset{\sim}{\xi}_i$, we find on integration that,

$$u_i = (n_i^0/n_i)\{(-1)^i[v_i\mathrm{Erf}_{i,k} - (-1)^kh_i^0(\tfrac{3}{2}t\nu_{i,k+2}/r + 2\nu_{i,k})] + \delta_{ik}2\pi^{-\frac{1}{2}}h_k^0\}, \qquad k = 1,2 ,$$

where for $k = 1$, $r \leqslant a$; for $k = 2$, $r \geqslant a$.

The mass-average and diffusion velocities will also have only radial components, and these are given respectively by

$$\bar{u} = \sum_i\rho_iu_i/\rho \text{ and } \bar{V}_i = u_i - \sum_j\rho_ju_j/\rho.$$

(c) Temperature*

Using the formulae from (i) for the stress tensor and for the temperature, it can be seen that for the mixture of gases in the case of the spherical cloud

$$T(t,\underset{\sim}{x}) = (\sum_i m_i \int \xi_i^2 f_i d\underset{\sim}{\xi}_i - \rho\bar{u}^2)/(3kn).$$

Evaluating this,

$$T = (n_2^0/n)T_2^0 + \sum_i (-1)^i\{- \mathrm{Erf}_{i,2} + \frac{2}{3}[2(h_i^0)^2 t\nu_{i,2} + a\nu_{i,3}]/(rh_i^0)\}(n_i^0/n)T_i^0$$
$$- \frac{2}{3}[\sum_i n_i T_i^0 u_i/(h_i^0)^2]^2/[n\sum_i n_i T_i^0/(h_i^0)^2].$$

IV. Discussion and conclusions

The diagrams, following at the end of this section, have been presented mainly to display the most typical or striking features of the formulae involved, rather than to give a complete and comprehensive description of the many situations possible (which is given elsewhere, [16], for the reasons specified at the end of Section I of this paper). Diagrams for the case of the spherical cloud were not included here due to the restrictions imposed on the size of this paper. However, even comparing the formulae obtained for each configuration, it can be seen that basic trends will apply equally to both problems. In particular, the number and mass density diagrams for both problems resemble one another very closely. (This has also been shown graphically, for the case $\rho_2^0 = 0$, in [8], where the author, using a different theoretical approach, obtains the same expressions as ours when such a simplification is considered.)

On examination of the diagrams**, the most interesting point to be made is that any highly non-monotonic behaviour observed in the diagrams is mainly associated with differences in the initial most probable random velocities, h_i^0, for example see Fig. 2. In most cases, the effect of a difference in other initial parameters (assuming $h_1^0 = h_2^0$) is continuously and monotonically in space and time smoothed out (e.g. see Fig. 1). Exceptions to this are where the unequal initial parameters are not directly related to the quantities described. For example, if the initial densities are different, they produce an effect on the mass-average velocity that is not monotonic (see Fig. 3), and similarly in Fig. 6, where there is the addition of a term depending on the initial velocity U, to the expression for T. However, these exceptions are predictable, and certainly do not represent disturbances of such highly non-monotonic type as those due to differences in the h_i^0. The behaviour of the gases as shown in Fig. 2 is explainable in terms of a statistical approach. The h_i^0 represents statistically the standard deviation of the initial Maxwellian distribution function f_i^0, from the inital mean velocity u_i^0 of the i^{th} gas. Hence, if $h_1^0 > h_2^0$ (say), whilst $n_1^0 = n_2^0$ and $u_{13}^0 = u_{23}^0 = 0$, then $n_1^0(\xi_{13}) > n_2^0(\tilde{\xi}_{23})$, where $n_i^0(\xi_{i3})$ denotes the initial number density of particles of the i^{th} gas, having velocity component $\xi_{i3} > h_i^0$. As the membrane is removed, more particles of the first gas will move during an initial, sufficiently short, time interval ($\tau = 0.1$ say) in the direction of increasing x_3 (or r in the case of the spherical cloud) than those of the second gas will move in the opposite direction (Fig. 2). It seems that the phenomena in Fig. 2, and consequently in Figs. 4 & 5, could not be predicted by a continuum theory, since it needs a kinetic theory approach to explain it. In Fig. 4, we see that the initial motion is directed towards increasing x_3, i.e. there is an initial basic surge of the first gas in this direction (in agreement with our foregoing discussion), since the second gas is not able to match the (initially more quickly moving) first gas. The presence of the lower maximum point in Fig. 4 is related to the interaction between a small number of fast particles of the first gas, and a great number of slow particles of the second gas moving in the opposite direction. However, the point is that with $h_1^0 > h_2^0$, the first gas appears to dominate initially the motion of the second gas even in the region $x_3 > a$ (or $r > a$) for x_3 (or r) close to a.

Although the diagrams discussed here cannot be directly compared with the graphical results of a one-dimensional discontinuity, [17]-[19], as well as Keller's [1] expansion into a half-space vacuum, the analytical results can be compared by introducing into our formulae a new variable $x_3' \equiv x_3 + a$, and then letting $a \to \infty$. [The limiting results are related to the (initially separating) plane $x_3' = 0$.] Appropriate graphical results obtained for this case can also be compared with the available gas-dynamical solutions. The comparison exhibits various features previously pointed out in [17]-[19] as well as those mentioned in this section (e.g. one can see the importance of the ratio of the initial most probable velocities). In particular, a great similarity between the free-molecular solutions and the appropriate Navier-Stokes solutions as well as "the best smooth mean curve" behaviour in respect of the Euler inviscid

* The components of the pressure tensor for the spherical cloud are found to be too lengthy to be included in this paper.

** In view of the symmetry of the infinite layer problem with respect to the plane $x_3 = 0$, only the diagrams for $x_3 \geq 0$ are presented.

solutions have been observed. (For an explanation of this fact see Bienkowski [19], Sect. 2.A4; see also [16].)

It should be mentioned that, in the case of the infinite layer, non-zero shear terms are present in the stress tensor, even though there is no interaction between the gas molecules themselves (cf. the corresponding result of [1], in which it was found that the stress tensor was diagonal for the case of the half-space expansion into vacuum). Also it is readily shown, both analytically and numerically, that as $t \to \infty$, all mean quantities for the mixture as a whole tend to those initially associated with the second gas, as would be expected. Finally, it may be observed that the solutions obtained above should remain correct even for the case of higher densities, if the mixture is considered over a sufficiently short time of evolution $\tau < \tau_\alpha$ where τ_α is the average time between collisions, i.e. it is possible then to treat the mixture as a Knudsen-like gas. Theoretically, the time interval of evolution can be enlarged by using the approach proposed by Grad at the end of Sect. 19 of [3].

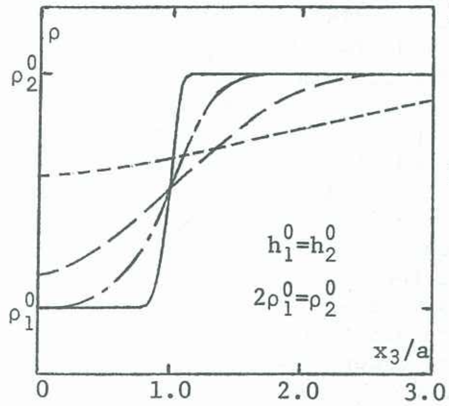

FIG. 1.
MASS DENSITY OF MIXTURE

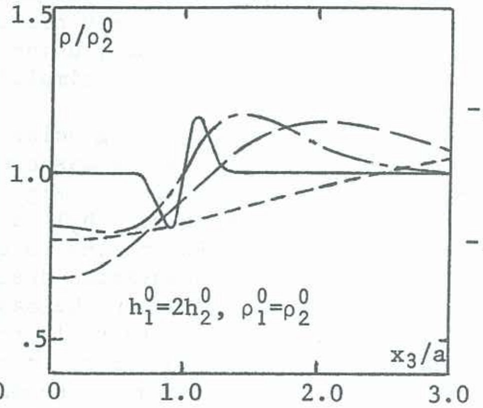

FIG. 2.
MASS DENSITY OF MIXTURE

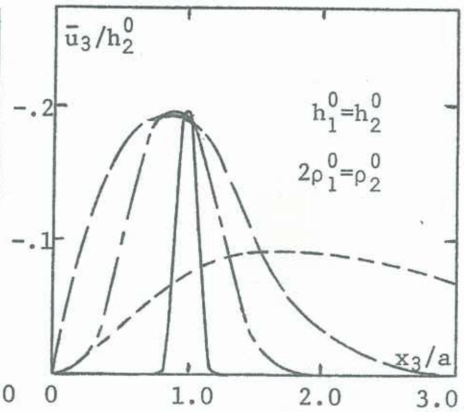

FIG. 3.
MASS-AVERAGE VELOCITY

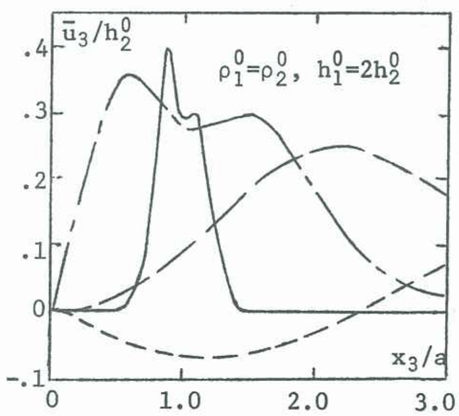

FIG. 4.
MASS-AVERAGE VELOCITY

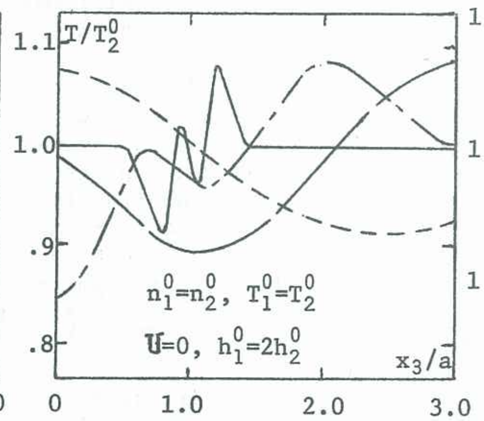

FIG. 5.
TEMPERATURE OF MIXTURE

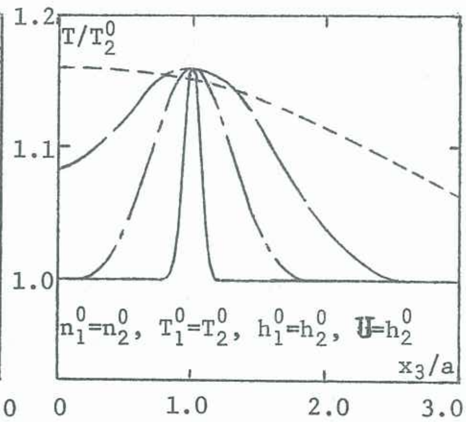

FIG. 6.
TEMPERATURE OF MIXTURE

Legend: $\tau = h_2^0 t/a$; ——— $\tau = 0.1$; — — $\tau = 1.0$; — - — $\tau = 0.5$; - - - - - $\tau = 2.5$

REFERENCES

[1] KELLER, J.B., On the solution of the Boltzmann equation for rarefied gases. Comm. Pure Appl. Math. 1, 275-285 (1948).

[2] KORNOWSKI, E.T., On some unsteady free molecular solutions to the Boltzmann equation. Gen. Elec. Rept. R59SD463, November (1959).

[3] GRAD, H., Principles of the kinetic theory of gases, in: Handbuch der Physik. Vol. XII, Ed. by S. Flügge. Berlin-Göttingen-Heidelberg: Springer 1958, pp. 205-294.

[4] IKENBERRY, E. & C. TRUESDELL, On the pressures and the flux of energy in a gas according to Maxwell's kinetic theory, I. J. Rational Mech. Anal. 5, 1-54 (1956).

[5] LEBOWITZ, J.L. & H.L. FRISCH, Model of nonequilibrium ensemble: Knudsen gas. Phys. Rev. 107, 917-923 (1957).

[6] KOGAN, M.N., Rarefied Gas Dynamics. New York: Plenum Press 1969.

[7] STALKER, R.J., Shock tube developments at the Australian National University. Austral. Physicist 7 (7), 99-102 (1970).

[8] MOLMUD, P., Expansion of a rarefied gas cloud into a vacuum. Phys. Fluids 3, 362-366 (1960).

[9] MORGENSTERN, D., Analytical studies related to the Maxwell-Boltzmann equation. J. Rational Mech. Anal. 4, 533-555 (1955).

[10] GLIKSON, A., On the existence of general solutions of the initial-value problem for the nonlinear Boltzmann equation with a cut-off. Arch. Rational Mech. Anal. 45, 35-46 (1972).

[11] GLIKSON, A., On solution of the nonlinear Boltzmann equation with a cut-off in an unbounded domain. Arch. Rational Mech. Anal. 47, 389-394 (1972).

[12] GLIKSON, A., Applications of constructive analysis to the theory of Boltzmann equations for gas mixtures. [The main contents of this paper have been presented at the Newcastle meeting of the Australian Mathematical Society, May 1974. (To be submitted to Arch. Rational Mech. Analysis.)]

[13] WILLIS, D.R., A study of some nearly-free molecular flow problems. Ph.D. Thesis, Princeton University (1958).

[14] WILLIS, D.R., On the flow of gases under nearly free molecular conditions. AFOSR TN 58-1093, AD207 594. Rept. no. 442 (1958).

[15] CERCIGNANI, C., Mathematical Methods in Kinetic Theory. New York: Plenum Press 1969.

[16] GLIKSON, A., Some problems of mutual diffusion of two initially uniform and separated gases. (To be submitted.)

[17] SONE, Y., & M. SHIBATA, Kinetic theory analysis of diffusion of discontinuity plane of flow velocity. AIAA Journ. 3 (9), 1763-1764 (1965).

[18] CERCIGNANI, C., & R. TAMBI, Diffusion of a velocity discontinuity according to kinetic theory. Meccanica 2, 25-33 (1967).

[19] BIENKOWSKI, G., Propagation of an initial density discontinuity, in: Rarefied Gas Dynamics, Vol. I, Ed. by J.M. de Leew. New York: Academic Press 1965, pp. 71-95.

[20] SCHAAF, S.A., Mechanics of rarefied gases, in: Handbuch der Physik, Vol. VIII/2, Ed. by S. Flügge. Berlin-Göttingen-Heidelberg: Springer 1963, pp. 591-624.

[21] KELLER, J.B., Spherical, cylindrical and one-dimensional gas flows. Quart. Appl. Math. 14, 171-184 (1956).

[22] ROGERS, M.H., The isothermal expansion of a gas cloud into a non-uniform atmosphere. Arch. Rational Mech. Anal. 1, 22-34 (1957).

[23] FREEMAN, N.C. & S. KUMAR, On the solution of Navier-Stokes equations for a spherically symmetric expanding flow. J. Fluid Mech. 56, 523-532 (1972).

[24] CHAPMAN, S., & T.G. COWLING, The Mathematical Theory of Non-Uniform Gases. New York: Cambridge University Press 1970, 3rd edition.

[25] HIRSHFELDER, J.O., C.F. CURTISS & R.B. BIRD, Molecular Theory of Gases and Liquids. New York: Wiley & Sons 1954.