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PREDICTING THE PERFORMANCE OF INERTIAL SEPARATORS

by

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The methods currently available for the design of separation devices are briefly reviewed. The present work is an attempt to improve the prediction of performance by considering the general equation of motion for a single spherical particle surrounded by a moving fluid. The application of this method to a particular separating device requires a realistic description of the fluid motion within it. Fluid velocity fields can be devised by either experimental measurement or the solution of the equation of motion of the fluid. A general method for the solution of two dimensional and axisymmetric flows is described. Having established the velocity field, the particle equation can then be solved by a numerical integration routine which can allow the inclusion of terms for the fluid and particle accelerations.

Some solutions for a cyclone-type separator are presented to illustrate the method.

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Notation

D	characteristic dimension
F	external force
g	conversion constant
p	fluid static pressure
Q	volumetric flow rate of the fluid
t	time
v	velocity
V	characteristic velocity

Greek symbols

ρ	true density
μ	absolute viscosity
τ	dummy variable

Superscript

+	non-dimensional form

Overline

~	vector quantity

Subscripts

f	of the fluid
o	the initial value of
p	of the particle
s	of the separator

INTRODUCTION

The fundamental aim of any inertial separator is to cause the particles in a homogeneous particle/fluid mixture to move preferentially to some point where they can be collected.

In the past separators have been developed by trial and error to suit various types of service. The usual design procedure is to choose the type of separator found by past experience to suit the required service, then choose a size on the basis of semi-empirical relationships. This procedure gives reasonable results when the densities of fluid and particle are widely different, but is of little use when they are similar, and of course is of no value at all in the development of new designs.

The purpose of this paper is to consider alternative design procedures and to select the one which is considered to have the most promise in future design. The selected method - namely complete solution of the equations of fluid and particle motion for any chosen geometry - is considered in some detail to show what progress has been made and what difficulties still remain.

SOME ALTERNATIVE DESIGN PROCEDURES

A design procedure must not only be able to describe what is known to occur in a given situation, but also should be able to predict what will occur in different situations. It is in this light that each of the alternative methods will be judged. The following methods encompass the complete range of possible design procedures:

(1) Prototype Testing

Prototype testing is the full-scale testing of a given suspension in a separator of a new shape usually arrived at from a preconceived notion of the desired fluid-flow pattern. Although the method will correctly indicate the performance of the new system it is of little or no value in predicting the performance of other systems, and it is probably generous to describe it as a design method. We will consider it no further.

(2) Empirical Correlations

The correlation of the results from a number of prototype tests may lead to the development

of relationships between the variables of interest. The level of analysis is usually limited to a consideration of which variables should be used in the correlation, and predictions can be made with confidence only within the range of the prototype tests. Extrapolation of results from other systems, while often attempted, is not to be recommended. Nevertheless, as pointed out in the Introduction, this method is the main tool at present available to the designer. We will consider some of its limitations later in the paper.

(3) Model Testing and Scale-up

Model testing involves an approach similar to that used in prototype testing, except that a small-scale model of the new apparatus may be used to predict performance in the full-scale equipment. Proper modelling requires the various dimensionless groups which govern the motion to have the same values in model and full scale apparatus. The appropriate groups can be deduced from a simple analysis of the relevant equations of motion. This procedure can be followed even when the equations contain non-linear terms, because it is not necessary to solve the equations to obtain the groups. Many separation systems will require the scaling of more than one group to obtain dynamic similarity. In these cases the range of the scaling procedures will be restricted by the properties of available test materials. The limitations thus imposed on the designer will be considered later.

(4) Theoretical Prediction

This involves a rigorous theoretical analysis and the subsequent solution of the equations considered to describe the physical situation. The two limitations of such an approach are the ability to describe mathematically what is occurring and the ability to integrate the equations. Digital computers now permit the use of numerical techniques to overcome the latter problem (providing stable solutions can be obtained) and only the former remains.

As a result of this study we have reached the conclusion that this fourth method, although hardly yet considered in the research literature on inertial separators, holds considerable promise as a design tool. After a brief review of the equations to be solved we will proceed to an account of the progress made by ourselves and others on the solutions of the equations, and an assessment of the problems still remaining.

THE EQUATIONS TO BE SOLVED

A complete solution of the problem involves the prediction of the flow pattern of a homogeneous suspension of a solid in a fluid, together with the prediction of the relative movement of the solid and the fluid at any point in the flow field. Any completely rigorous solution procedure would require the solutions of the governing equations to be coupled. In practice, although the movement of the fluid very strongly affects the movement of the particles, the movement of the particles does not have nearly as great an effect on the movement of the fluid, and it may be possible to uncouple the solutions.

One way of describing momentum transfer in a two-phase suspension is to consider a volume element of the homogeneous suspension and deduce an equation analagous to the fluid motion equation, but incorporating the momentum changes of the particles and a modified shear stress tensor. This approach has recently been used by Crowe and Pratt[1] in an attempt to deduce the movement of particles in a swirling fluid suspension entering a sudden cylindrical expansion. In the absence of any appropriate theory these workers neglected the effect of the particles on the viscosity of the fluid.

The alternative approach of considering the separate equations of motion of the fluid and the particle reduces to the same result while providing a better understanding of their individual movements. Effects such as particle-particle interactions can be better studied in this form rather than in the form of the composite equation of Crowe and Pratt.

In the remainder of this paper, therefore, the procedure considered is the solution of the equation of motion of a single particle together with the solution of the equation of motion of the fluid phase.

THE EQUATION OF MOTION OF A SOLID PARTICLE

The following general form of the equation describing the rate of change of momentum of a particle moving in a variable velocity field is based on that suggested by Tchen[2]:

$$\frac{\pi}{6}D_p^3\rho_p\frac{dv_p}{dt} = 3\pi\mu D_p(v_f-v_p) - \frac{\pi}{6}D_p^3\nabla p + \frac{\pi}{12}D_p^3\rho_f\left(\frac{dv_f}{dt} - \frac{dv_p}{dt}\right)$$

$$+ \frac{3}{2}\sqrt{\pi\rho_f\mu}\cdot D_p^2\int_{t_o}^{t}\frac{\left(\frac{dv_f}{d\tau} - \frac{dv_p}{d\tau}\right)}{\sqrt{t-\tau}}d\tau + F_{pot} \qquad (1)$$

The unsteady form of the above equation should in general be retained even when the fluid motion is steady, because many of the forces acting are functions of a variable velocity field.

The first term on the right side is the steady drag resistance according to Stokes' Law. The second term is the force exerted on the particle by the pressure gradient within the fluid. The third term is the momentum transfer to the portion of the fluid which appears to move with the accelerating particle (often referred to as the "added mass" of the particle). The fourth term is the unsteady resistance that an accelerating particle experiences in a creeping motion, and is generally referred to as the Basset history term. The last term is the force due to any external potential field, which in this case is a gravitational potential field.*

$$F_{pot} = \frac{\pi}{6}D_p^3\rho_p\tilde{g} \qquad (2)$$

THE EQUATION OF MOTION OF THE FLUID

The general form of the equation that relates the rate of change of fluid momentum to the forces causing the movement is:-

$$\frac{\partial(\rho_f v_f)}{\partial t} = -\nabla\cdot\rho_f v_f v_f - \nabla p + \mu\nabla^2\cdot v_f + \rho_f\tilde{g} \qquad (3)$$

Since the fluid flows of interest will in general be steady, the left side of the equation usually reduces to zero.

The first term on the right side is the inertial force of the fluid and the second term the fluid pressure gradient. The third term is the viscous force due to the fluid viscosity; this may have to be modified to give an effective viscosity if the presence of very fine particles has appreciably affected the viscous properties of the fluid. The last term is the gravitational force on the fluid.

NON-DIMENSIONAL FORMS OF THE EQUATION

Equation (1) and (3) can be re-written in non-dimensional forms by expressing the variables in terms of characteristic properties of the system under examination. In the following treatment of the particle equation we have selected the particle diameter, D_p, as a characteristic dimension and the particle inlet velocity, V_{po}, as a characteristic velocity.

In the dimensionless form, equation (1) then becomes:-

$$\frac{dv_p^+}{dt^+} = 18\left[\frac{\mu}{D_pV_{po}\rho_p}\right](v_f^+-v_p^+) - \left[\frac{\rho_f}{\rho_p}\right]\nabla^+p^+ + \frac{1}{2}\left[\frac{\rho_f}{\rho_p}\right]\left(\frac{dv_f^+}{dt^+} - \frac{dv_p^+}{dt^+}\right)$$

$$+ 9\left[\frac{\mu}{D_pV_{po}\rho_p}\right]^{0.5}\int_{t_o^+}^{t^+}\frac{\left(\frac{dv_f^+}{d\tau} - \frac{dv_p^+}{d\tau}\right)}{\sqrt{t^+-\tau}}d\tau + \left[\frac{D_pg}{V_{po}^2}\right]\frac{\tilde{g}}{g} \qquad (4)$$

* Many authors prefer to write the individual contributions to the overall pressure gradient as one term involving acceleration of the fluid and another involving the steady pressure gradient within the fluid. In a stagnant fluid, the vertical component of the potential term then becomes $\frac{\pi}{6}D_p^3(\rho_p-\rho_f)\tilde{g}$

The dimensionless groups that appear in equation (4) are not unfamiliar. They are:

$$\text{Particle Reynolds Number, } Re_p = \frac{D_p V_{po} \rho_p}{\mu}$$

$$\text{Particle Froude Number, } Fr_p = \frac{V_{po}^2}{D_p g}$$

$$\text{and the Density Ratio, } DR_{pf} = \frac{\rho_p}{\rho_f}$$

In an analogous manner the separator diameter or width, D_s, and the velocity of the fluid as it enters the separator, V_{fo}, have been selected as characteristic of the fluid movement. In dimensionless form equation (3) becomes:-

$$-\nabla^+ \cdot v_f^+ v_f^+ - \nabla^+ p^+ + \left[\frac{\mu}{D_s V_{fo} \rho_f}\right] \nabla^{+2} \cdot v_f^+ + \left[\frac{D_s g}{V_{fo}^2}\right] \frac{\tilde{g}}{g} = 0 \qquad (5)$$

As might be expected, the dimensionless groups are:-

$$\text{Fluid Reynolds Number, } Re_f = \frac{D_c V_{fo} \rho_f}{\mu}$$

$$\text{Fluid Froude Number, } Fr_f = \frac{V_{fo}^2}{D_s g}$$

APPLICATION TO MODEL STUDIES AND SCALE-UP

In the general situation of a three dimensional motion of a two phase suspension it will be necessary for all five dimensionless groups to have identical values in model and full-scale apparatus. However, in some conditions partial scaling may suffice.

In the case of simple gravity settlers the gravity force does not affect the fluid flow and it could therefore be scaled using only the fluid Reynolds number. However particle paths would need to be scaled using both particle Reynolds and Froude numbers since inertial, viscous and gravity forces all affect the particle motion. The density ratio could be neglected on the grounds that in this type of equipment, accelerations are small and the pressure force and added mass terms of equation (4) therefore have a negligible effect.

In centrifugal separators such as the cyclone, particle accelerations are too high to be neglected, and the terms including them in equation (4) must be considered. These terms are the ones giving rise to the density ratio as a scaling group. Where this is virtually zero, as in gas cyclones, it may be omitted, but in liquid cyclones the density ratio should be included. As separation occurs in the r-θ plane, gravity forces have little influence on separation and fluid and particle Froude numbers may be neglected. Thus for liquid cyclones, scaling with Re_f, Re_p and the density ratio is required, whereas for gas cyclones Re_f and Re_p will suffice.

Most empirical correlations for particle separations within cyclones are based on the notion of balancing centrifugal and drag forces on the particles, and do not consider the fluid flow in the cyclone. This amounts to an attempt to scale with particle Reynolds number alone (the density ratio is usually not consciously included because of lack of appreciation of the importance of accelerative forces other than centrifugal forces).

Particle Reynolds scaling gives: $D_p \propto \frac{\mu}{\rho_p v_p}$

Since in the cyclone inlet $v_p = v_f$ and $V_{fo} \propto \frac{Q}{D_s^2}$ then $D_p \propto \frac{D_s^2 \mu}{\rho_p Q}$

If density ratio scaling is also applied this gives: $D_p \propto \frac{D_s^2 \mu}{\rho_f Q}$

When cyclone Reynolds is scaled also this finally becomes: $D_p \propto D_s$

Table 1 compares this rigorous scaling result with the semi-empirical correlations proposed by various workers[3-8]. The second column is the form given by the worker concerned, except that all correlations have been brought to a common basis by replacing inlet velocity of particle or fluid by Q/D_c^2. The third column transforms the result from the second column by combining it with density ratio where this has not already been done and the fourth column further incorporates fluid Reynolds number. The variation among these relationships is quite remarkable, and comparison of the transformed results in the fourth column with the correct result shows that in some cases appreciable scaling could lead to somewhat bizarre results (e.g. Stairmand's result appears to us to say that the particle size need not be scaled at all!).

Table 1. Relationships for Scaling Particle Size in Cyclones

Author	Suggested Scaling	Including sealing of the density ratio	Including, also, scaling of the fluid Reynolds No.
ter Linden (3) Kelsall (5)	$\left[\frac{D_s^3\mu}{Q(\rho_p-\rho_f)}\right]^{0.5}$	$\left[\frac{D_s^3\mu\rho_p}{Q\rho_f(\rho_p-\rho_f)}\right]^{0.5}$	$D_s\left[\frac{\rho_p}{\rho_p-\rho_f}\right]^{0.5}$
Stairmand (4)	$\left[\frac{D_s\mu}{Q\rho_p}\right]^{0.5}$	$\left[\frac{D_s\mu}{Q\rho_f}\right]^{0.5}$	1
Lilge (6)	$\frac{D_s^2\mu}{Q(\rho_p-\rho_f)}$	$\frac{D_s^2\mu\rho_p}{Q\rho_f(\rho_p-\rho_f)}$	$D_s\left[\frac{\rho_p}{\rho_p-\rho_f}\right]$
Rietema (7)	$\frac{D_s^2\mu}{Q(\rho_f(\rho_p-\rho_f))^{0.5}}$	$\frac{D_s^2\mu}{Q\rho_f}\left[\frac{\rho_p}{\rho_p-\rho_f}\right]^{0.5}$	$D_s\left[\frac{\rho_p}{\rho_p-\rho_f}\right]^{0.5}$
deGelder (8)	$\left[\frac{D_s^2\mu}{Q\rho_f(\rho_p-\rho_f)}\right]^{0.5}$	$\left[\frac{D_s^2\mu\rho_p}{Q\rho_f^2(\rho_p-\rho_f)}\right]^{0.5}$	$D_s^{0.5}\left[\frac{1}{\rho_p-\rho_f}\right]^{0.5}$
Present study	$\frac{D_s^2\mu}{Q\rho_p}$	$\frac{D_s^2\mu}{Q\rho_f}$	D_s

Only Stairmand[4] warns of the danger of scaling over a large range of variables. Provided this advice is adhered to the errors caused by deviations from the true scaling laws may not be too great, but at the same time the usefulness of the correlations is severely limited.

The rigorous scaling result for liquid cyclones, using Re_f, Re_p and density ratio is

$$D_p \propto D_s \propto \frac{1}{V_{fo}}$$

and for gas cyclones, where density ratio may be omitted, it is

$$D_p \propto \frac{D_s\rho_f}{\rho_p} \propto \frac{\rho_f}{\rho_p V_{fo}}$$

In both cases the velocity in the model has to increase by the same factor that the linear dimension decreases, which would usually lead to a prohibitively high pressure drop in the model. This restriction arises because Re_f is scaled in addition to Re_p. Many writers consider that in fully-turbulent situations it is unnecessary to scale Re_f so long as flow in the model is also turbulent. In our view it would be unwise to apply this philosophy to cyclone separators until it had been established that the rather complex flow pattern did have a general form over a wide range of turbulent flows.

INTEGRATION OF THE EQUATIONS OF MOTION

Neither the fluid equation (3) nor the particle-fluid equation (1) can yet be solved analy-

tically for general two or three-dimensional situations, and their coupled simultaneous solution is therefore out of the question.

However some simplified solutions are available. The particle equation has been solved analytically to give the unsteady motion of a particle in a stationary fluid, originally by Basset[9] and later by Brush, Ho and Yen[10] and Hjelmfelt and Mockros[11]. Solutions for some simple non-zero velocity fields have been obtained by considering only Stokesian drag. Thus Lapple and Shepherd[12] have obtained solutions for the case of two-dimensional motions with small slip velocity; Daniels[13], Davies[14] and Uematu et al[15] have obtained solutions for particle motion in ideal free-vortex flow with no radial component; and Kriebel[16] and Thompson and Strauss[17] have obtained solutions for ideal forced-vortex flow with no radial component.

In principle it is possible to obtain a numerical solution of the particle-fluid equation for any specified three-dimensional velocity field. Appendix I gives typical solutions obtained by one of the authors (RBB) in the fluid flow situation used by Thompson and Strauss.

The fluid-flow equation (3) can be solved numerically for most non-recirculating (boundary layer) flows (see, for example, Patankar and Spalding[18]), but stable solutions for recirculating flows, which are more usual in separators, are difficult to formulate, especially at high Reynolds number or with a high degree of swirl. Figure 1 gives a solution obtained by one of the authors (RBB) for fluid flow in a cyclone without swirl, using a modification of the method of Gosman et al (19). Attempts to obtain stable solutions with swirl have not yet become successful, but it is hoped to overcome the outstanding difficulties shortly. The relationship used to express the effective viscosity (turbulent momentum exchange coefficient) in the solution given in Figure 1 was the one suggested for cylindrical swirling flows by Deissler and Perlmutter[20]. This relationship is probably inadequate for turbulent cyclone flows since the character of the turbulence is rather sensitive to the flow geometry (21). There is an outstanding need for the formulation of more realistic turbulence models along the lines suggested by Launder and Spalding[22].

Provided equations (1) and (3) can be solved individually, their coupled solution can then proceed iteratively as follows:

(i) solve the fluid equation as described in the previous paragraph,

(ii) use the fluid velocity field so obtained as the first approximation for the integration of the fluid particle equation,

(iii) the momentum transfer between the fluid and the particle thus obtained can be used to correct the motion of the fluid,

(iv) the whole procedure is then repeated until changes between successive solutions are insignificant.

To perform step (iii) the net exchange of momentum between the particles and the fluid must be incorporated into equation (3).

Quite possibly, for dilute suspensions, the effect of particle movement on fluid flow will be so weak that little or no iteration will be required, but the only way to determine this will be to actually consider at least one iteration.

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APPENDIX 1: NUMERICAL SOLUTION OF THE PARTICLE-FLUID EQUATION

The method of Thompson and Strauss[17] represents the most elegant analytical solution available at the present time for an unsteady equation of particle motion. Figure 2 compares the results obtained by their method with those obtained by numerical integration of the particle fluid equation.

The examples considered are the movement of dust particles through air in a Stairmand-type cyclone and a similar motion of carbon particles in a water cyclone of the same geometry.

In both cases, the time for the particle to move from the inner wall of the inlet duct to the cylinder wall is calculated. In the shape under consideration this corresponds to the movement from0.6 to 1.0 of the dimensionless radial distance.

Table 2. Physical Properties

Quantity	Dimensions	Dust in air	Carbon in Water
D_s	m	0.610	0.152
Q	litre/s	568	1.015
V_{fo}	m/s	1.524	0.440
D_p	μm	5 to 160	40 to 1280
ρ_p	kg/m^3	2000	1500
ρ_f	kg/m^3	1.220	1000
μ	μPa.s	17.8	1000

Note that:

(i) For the air cyclone the Thompson solution agrees well with the numerical solution of the full equation for particle sizes up to 20 μm. Thereafter, as noted by Thompson, the simplifying assumption that he introduced is unreasonable, and negative collection times are obtained. The numerical solution, however, continues to give sensible solutions for the whole range of particle sizes.

(ii) For the water cyclone the Thompson solutions diverge from the numerical solutions for two reasons. The first is the same limitation mentioned above, while the second is a misunderstanding in the original formulation of the inertia term in the particle equation. Again the numerical solution is able to predict the separation times for all particles while the Thompson solution is useful only for particles smaller than 300 μm.

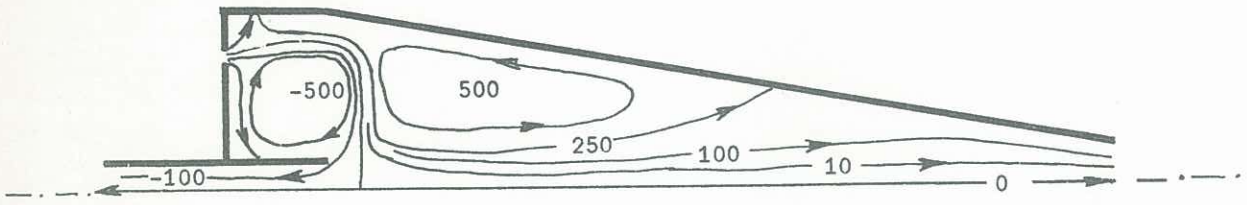

Fig. 1 Calculated fluid flow pattern for a cyclone separator without swirl.

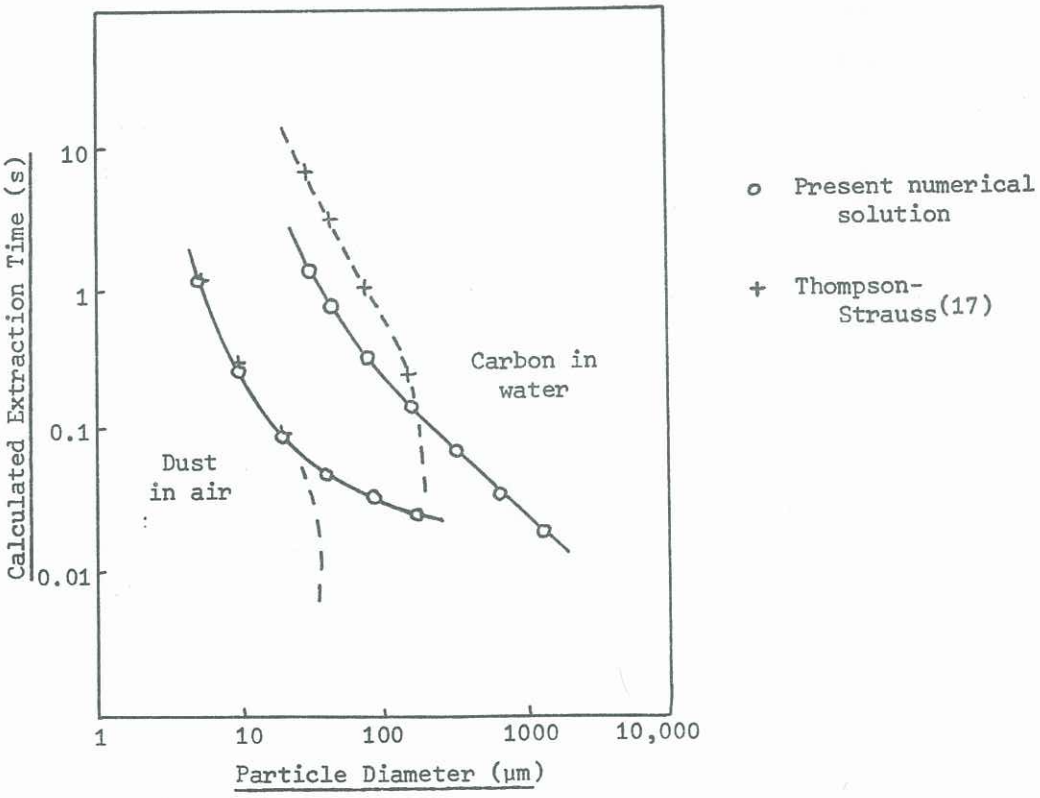

Fig. 2 Effect of particle size on calculated extraction time in a forced vortex flow.