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# MAXIMUM SCOUR DEPTH AT A BRIDGE PIER FOR SEDIMENT CARRYING FLOWS

by
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SUMMARY

The velocities of flow are increased locally near a bridge pier due to the combined effect of constriction and curving of stream-lines. This increases the capacity of the flow to carry sediment initially. As a result, scour occurs and it continues until, the capacity is so reduced due to enlarged flow section that it can carry no more than that brought from upstream reaches. For a safe design of the foundation of a bridge pier one should know the dimension of the deepest scour hole.

In this paper a model based on mean velocity is proposed to estimate the maximum depth of scour. The experiments conducted with circular piers and rectangular piers with triangular noses have shown good agreement with the solution obtained based on the model.

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## 1.- Introduction

At a bridge pier the flow deviates from its original path and takes a curved path around the pier. As a consequence of this, greater velocities and bed shear stresses occur near the pier than in the flow upstream. If the bed is movable, a further consequence is that the sediment carrying capacity of the flow is locally increased near a pier. Depending on the relative magnitudes of the bed shear stresses $\tau_u$ of the flow upstream and $\tau_p$ at the pier and the critical shear stress $\tau_c$ for the bed material, the following three types of flow are possible:

1. $\tau_u < \tau_p \leqslant \tau_c$ 2. $\tau_p > \tau_c \geqslant \tau_u$ 3. $\tau_p > \tau_u > \tau_c$

In the first type of flow there will not be any movement of the sediment and hence does not pose a problem in the design of the foundation of the piers.

In the second type of flow, the flow in the channel is free from sediment, but the flow starts scouring the bed material at the pier. With time,the scour hole gets enlarged and as a result the bed shear stress gets reduced. The scouring process continues until the scour hole attains such dimensions that the local shear stress on the surface of the scour hole becomes equal to the critical shear stress. Though the time taken to attain this equilibrium is infinite a practical equilibrium profile is obtained in a finite time. This type of equilibrium has been referred as static equilibrium (1).

On the third type, the flow carries sediment all along the channel. Scouring process at the pier, in this case, continues until the capacity to carry sediment at the pier is no more greater than that which is brought from the upstream reaches. When this happens then the dynamic equilibrium is said to have been achieved. A study of the maximum scour depth occuring at a bridge pier under such dynamic equilibrium conditions is presented in this paper. Although some of the earlier investigators have studied (2, 3, 4, 5) the case of dynamic equilibrium analytically, it is felt that this study is important because more appropriate bed load and friction equations are used in this study than in the earlier ones.

## 2. Solutions

The solution presented in the following pages is obtained for zero angle of attack of the bridge piers and non-cohesive bed material. The assumptions made are that at section yy (passing through the end points of the cut-waters) in Fig. 1,

1. the depth of flow above the bed level is the same as the depth of flow D at the far upstream section xx,
2. the slope of the scour hole is equal to the tangent of the angle of repose, Θ, of the bed material,
3. the flow is normal to the section yy,
4. the flow is in the forward direction.

### 2.1 Equation of Continuity

Let 2b be the width of the pier measured normal to the direction of flow, 2Kb be the centre to centre spacing of two adjacent piers and $V_x$ be the mean velocity of flow at Section xx. Then the mean velocity $V_y$ at Section yy can be obtained from the equation of continuity as,

$$V_y = V_x/\left(\frac{K-1}{K} + \frac{S_e^2}{2bDK \tan \Theta}\right) \qquad ...(1)$$

where $S_e$ is the equilibrium depth of scour at Section yy. The average depth $D_y$ at Section yy is given by,

$$D_y = D\,\frac{(P+M)}{P} \qquad ...(2)$$

where

$$P = \frac{K-1}{K}$$

$$\text{and } M = \frac{S_e^2}{2bDK \tan \Theta}$$

The Froude number in terms of $V_y$ is given by

## 3. Experiments

The experiments were conducted in a 4ft wide 18 in. deep and 50ft long masonry flume. A pump with a design discharge of 5 cfs delivered water into a stilling reservoir. From this reservoir water was allowed to flow to a reservoir of 8ft x 8ft cross section through a pipe where in the water was further stilled before it was allowed to pass on to the masonry flume through a smooth transition.

Sand was retained between two masonry aprons, 9 in. high and 32ft apart, to serve as the movable bed. The sand used was fairly uniform with a mean particle size of 0.55 mm. Sand was supplied at the upstream end through manual means and collected between the downstream apron and a 9 in. high sharp crested weir located at the end of the flume.

The piers were located 20ft downstream of the upstream apron. The piers were made of seasoned teak wood. Each run was made with two similar piers placed symmetrically about the centre line of the flume with zero angle of attack. Copper tubes 6 in. intervals on either side of the centre line, along the width of the flume. The piers were provided with the threaded rods projecting from the centre to fit the copper tubes. Small lengths of rubber tubes were provided at the bottom end of the copper tubes and were closed with pinchclips to prevent water from leaking. Experiments were done with circular piers .8 in. diameter and rectangular piers of 0.8 in. wide and having triangular cut and ease waters. Triangular cut-waters have either $90^o$ noses or $60^o$ noses.

At the beginning of each run the entire flume was filled with water by allowing such a low discharge over the initially levelled bed that no particle moved anywhere over the entire bed. After filling the flume upto the crest of the weir, the required discharge was allowed to pass. Discharge measurements were made over a calibrated right angled V-notch situated at the end of a channel running at right angles to and below the downstream end of the 50ft long flume.

Equilibrium scour conditions were obtained within a few hours after the commencement of each run. When equilibrium was attained, the scour hole dimensions and the bed pattern was statistically stable over a period of time. Water and bed levels were measured with a point gauge which could measure upto .001 ft. The flow was stopped when a ripple or dune just reached the scour hole so that the depth of scour measured would be the maximum scour depth. At the end of the run the average bed level was determined at a far upstream section. The difference between this level and the water level at that section was taken as the depth of flow.

Experiments were done for values of K equal to 7.5, 15 and 30. For each value of K and each shape of the cut-water, six runs were made.

For the experiments conducted the value of J was 0.0065 and the value of tan $\Theta$ was 0.77.

## 4. Results

Experimental verification of the solution is done by checking whether the measured values of $S_{max}$ are proportional to the calculated values of $[\Delta]$ $(2bDK \tan \Theta)^{\frac{1}{2}}$ as indicated by equation 19 or not. Figure 3 which shows a plot of these two quantities one against another, shows that for each shape of the cut-water a straight line can be fitted reasonably well. Figure 4 also shows that the value of $A_1$ which is given by the slope of the line is different for different shapes of the cut-water, in accordance with the meaning attached to it. Hence it can be concluded that the experiments support the solution obtained in this paper. The values of $A_1$ are listed below.

| Shape of the cut-water | Value of $A_1$ |
|---|---|
| Semicircular | 1.0 |
| $90^o$ | 0.9 |
| $60^o$ | 0.6 |

The list of values of $A_1$ is not complete. But it is felt that the value of $A_1$ for any particular geometry of the cut-water can easily be determined in the laboratory when needed. So in the opinion of the author, equation 19 can be used for design purposes. It is, however, again emphasized that the solution is not valid for angles of attack other than zero.

Experiments also showed that on equal width basis the $S_{max}$ for given conditions of flow and sediment was more for circular piers than that for rectangular piers with a triangular $90^o$ nose which in turn was greater than that for rectangular piers with triangular $60^o$ nose.

An interesting result can be obtained from equation 19 for the case when $K \rightarrow \infty$ .

$$F_y = \frac{V_y}{(gD_y)^{1/2}} = \left(\frac{F_x^{2/3}\, P^{1/3}}{P + M}\right)^{3/2} \qquad ...(3)$$

where $F_x$ is the Froude number at Section xx.

### 2.2 Bed Load Equation

Chien (7) showed that Meyer-Peter and Miiler bed load formula can be written as,

$$\phi = (4/\psi - 0.188)^{3/2} \qquad ...(4)$$

where $\phi$ and $\psi$ are defined as

$$\phi = \frac{q_s}{\{g(S_s - 1)\}^{1/2}\, d^{3/2} \lambda} \qquad ...(5)$$

$$\frac{1}{\psi} = \frac{1}{(S_s - 1)} \left(\frac{D}{d}\right) . S \qquad ...(6)$$

where

$$\lambda = \left\{\frac{2}{3} + \frac{36\nu^2}{gd^3\,(S_s - 1)}\right\}^{1/2} - \left\{\frac{36\nu^2}{gd^3\,(S_s - 1)}\right\}$$

$\nu$ = Kinematic viscosity,
$S_s$ = specific gravity of the sediment,
$S$ = slope of the energy line of the flow,
$q_s$ = volume rate of sediment transported as bed load per unit width of the flow,
$d$ = particle size.

Equation 4 fitted the data analysed by Chien (7) quite well for values of $\phi$ upto 10. In the present solution equation 4 is used to estimate the bed load in preference to the usual Du Boys' type of bed load equations, for its dimensional homogenity and in preference to Einstein's bed load equation for its simplicity and equal goodness of fit for the data.

### 2.3 Equation of Resistance

The slope of the energy line S in equation 6 can be expressed in terms of velocity of flow and depth of flow using the equation of resistance. For sediment carrying channels the value of f depends on the bed form which in turn depends on flow characteristics and the particle size of the sediment. So to evaluate S from Mannings equation it is not enough if the velocity, depth and particle size are known, the bed form should also be known beforehand. So it is felt that it would be better to choose an equation from the available literature which connects f with the hydraulic characteristics and the particle size, over a wide range of bed forms from plane to antidunes. One such equation is that proposed Vanoni & Brooks (8). This equation shows very good agreement with experimental data and hence is chosen. It can be transcribed as (8)

$$f = \frac{17\nu^2}{F^{1/2}\, gd^3} \qquad ...(7)$$

where F is the Froude number and f is the friction factor

also

$$f = \frac{8gDS}{V^2} \qquad ...(8)$$

$$\therefore S = \frac{V^2}{8gd} . f = \frac{V^2}{8gd} . \frac{17\nu^2}{F^{1/2}\, gd^3} = \frac{1.125\nu^2}{gd^3} . F^{3/2} \qquad ...(9)$$

By substituting for S from equation 9 in equation 6 the following relationship is obtained.

$$\frac{1}{\psi} = [JF^{3/2} - 0.188]^{3/2} \qquad ...(11)$$

where

$$J = \frac{8.5}{S_s - 1} \cdot \frac{\nu^2}{gd^3}$$

## 2.4 Equation of Continuity for Bed Load

The total amount of bed load under transport at Section xx and Section yy should be the same under equilibrium conditions. The equation of continuity of sediment under such conditions can be written as,

$$\phi_x \cdot g^{\frac{1}{2}} (S_s - 1)^{\frac{1}{2}} \cdot d^{3/2} \cdot \lambda \cdot 2Kb$$

$$= \phi_y \cdot g^{\frac{1}{2}} (S_s - 1)^{\frac{1}{2}} \lambda \; 2 \; (K - 1)b \qquad \ldots(12)$$

where $\Phi_x$ and $\phi_y$ are values of $\phi$ at Section xx and Section yy respectively.

$$\therefore \phi_x = P \cdot \phi_y \qquad \ldots(13)$$

Substituting for $\phi_x$ and $\phi_y$ from equation 11 we get,

$$[J \, F_x^{\;3/2} - 0.188]^{3/2} = P \, [J \, F_y^{\;3/2} - 0.188]^{3/2} \qquad \ldots(14)$$

By substituting for $F_y$ from equation 3 in equation 14 and re-arranging terms, we get,

$$M = P^{1/3} \left[ \frac{J \, F_x^{\;3/2} \, (D/d)}{J \, F_x^{\;3/2} \, (D/d) + (P^{2/3} - 1) \; 0.188} \right]^{4/5} - P \qquad \ldots(15)$$

$$\therefore S_e = [\Delta] \; (2bDK \tan \Theta)^{\frac{1}{2}} \qquad \ldots(16)$$

where

$$[\Delta] = \left[ \left\{ \frac{P^{5/12} \, J \, F_x^{\;3/2} \, (D/d)}{J \, F_x^{\;3/2} \, (D/d) + (P^{2/3} - 1) \; 0.188} \right\}^{4/5} - P \right]$$

It may be noted that equation 16 is obtained on the basis of average velocity at Section yy without any regard for the velocity distribution. So a coefficient A has been introduced in equation 16, to represent the effect of velocity distribution. Equation 16 now becomes,

$$S_e = A \; [\Delta](2bD \; K \tan \Theta)^{\frac{1}{2}} \qquad \ldots(17)$$

As velocity distribution depends on the shape of the cut-water, the coefficient A can also be expected to depend on the shape of the cut-water.

It may also be noted that in the case of scour near an obstruction under dynamic equilibrium conditions, $S_e$ is less than the maximum scour depth $S_{max}$ that occurs at times due to the aperiodic oscillations of scour depth. In order to make the design safe, the foundations of bridge piers have to be designed for $S_{max}$ and not $S_e$. Now the problem is to estimate $S_{max}$ after estimating $S_e$ from equation 17.

Using the data of Narasimhulu (9) who has recently studied the time variation of the scour depth at bridge piers a plot of $S_{max}$ versus $S_e$ is made in Figure 2. It shows that

$$S_{max} = 1.075 \; S_e \qquad \ldots(18)$$

$$\therefore S_{max} = A_1 \; [\Delta] \; (2 \; bDk \tan \Theta)^{\frac{1}{2}} \qquad \ldots(19)$$

where

$$A_1 = 1.075 \; A$$

$$\lim_{K \to \infty} S_{max} = A_1 \left(\frac{4}{3} bDK \tan \Theta\right)^{1/2} \qquad \ldots(20)$$

Equation 20 implies that when the spacing between two piers is very large $S_{max}$ depends only on depth of flow and the geometry of the obstruction for flows carrying sediment in alluvial channels. This aspect requires further study.

## 5. Conclusions

The maximum scour depth caused by flow carrying sediment near a bridge pier situated in alluvial channels chan be estimated by using equation 19, when the angle of attack is zero. Further studies are necessary for the case when the spacing between twa adjacent piers becomes very large. From the point of view of scour depth, it is better to have rectangular piers with triangular noses than circular piers.

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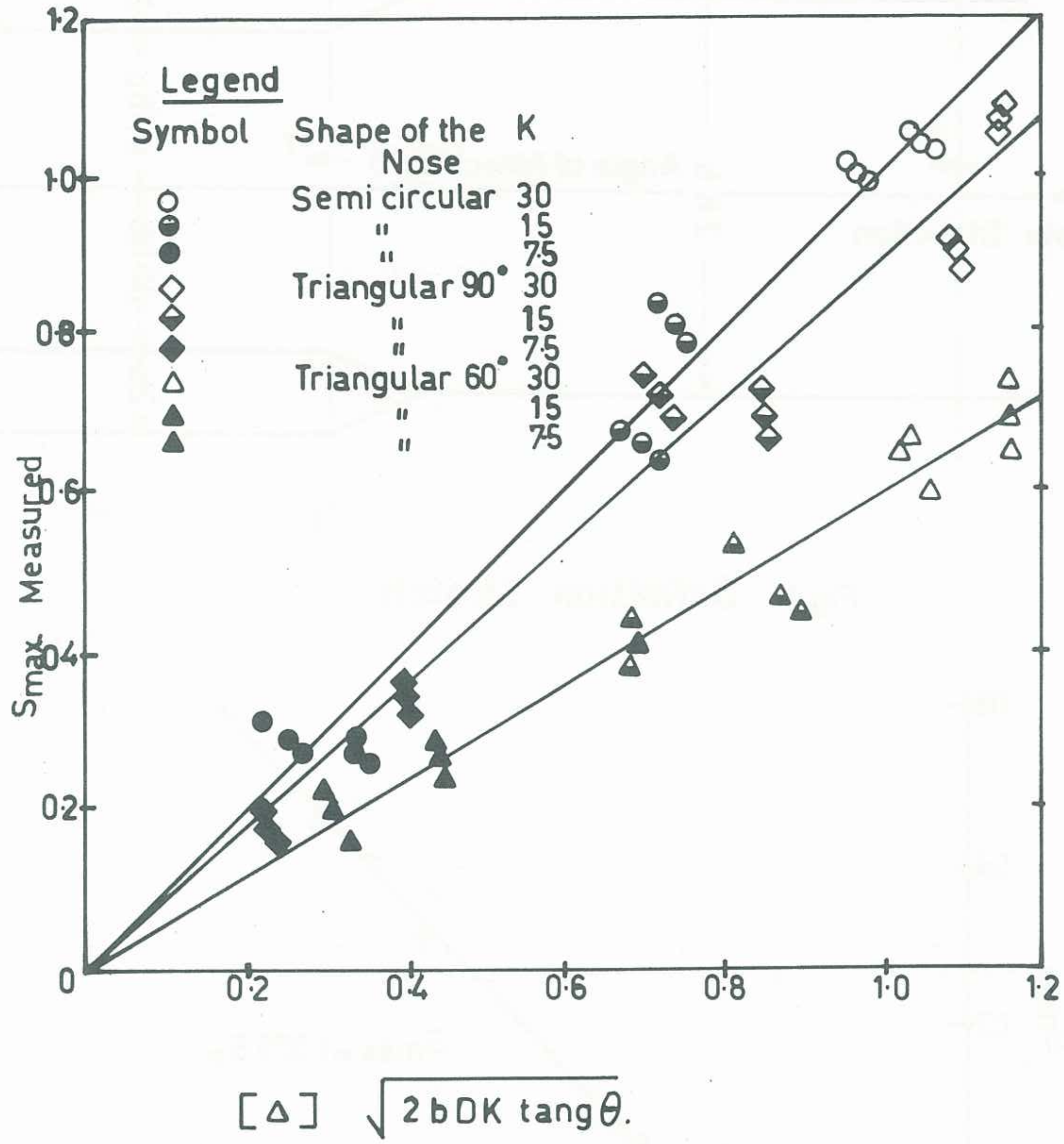


Fig. 3 **Results.**

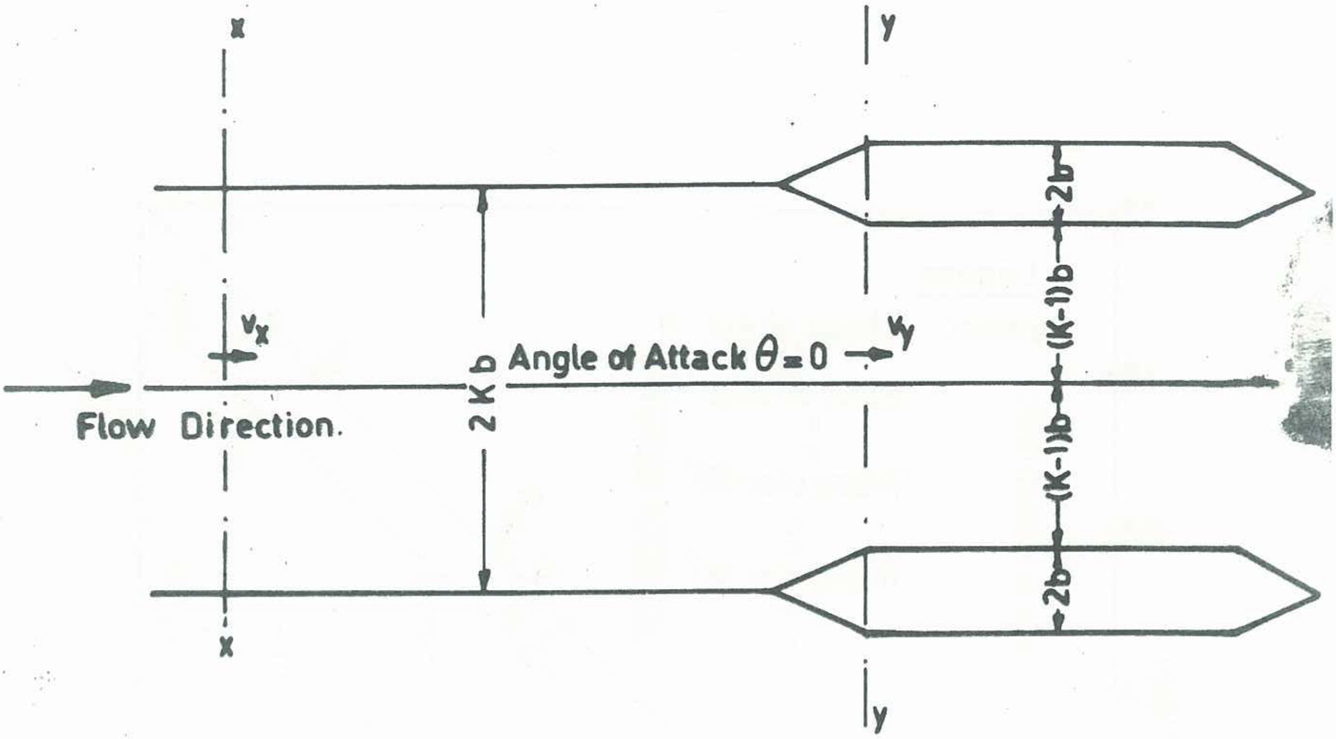


Fig.1 Definition Sketch

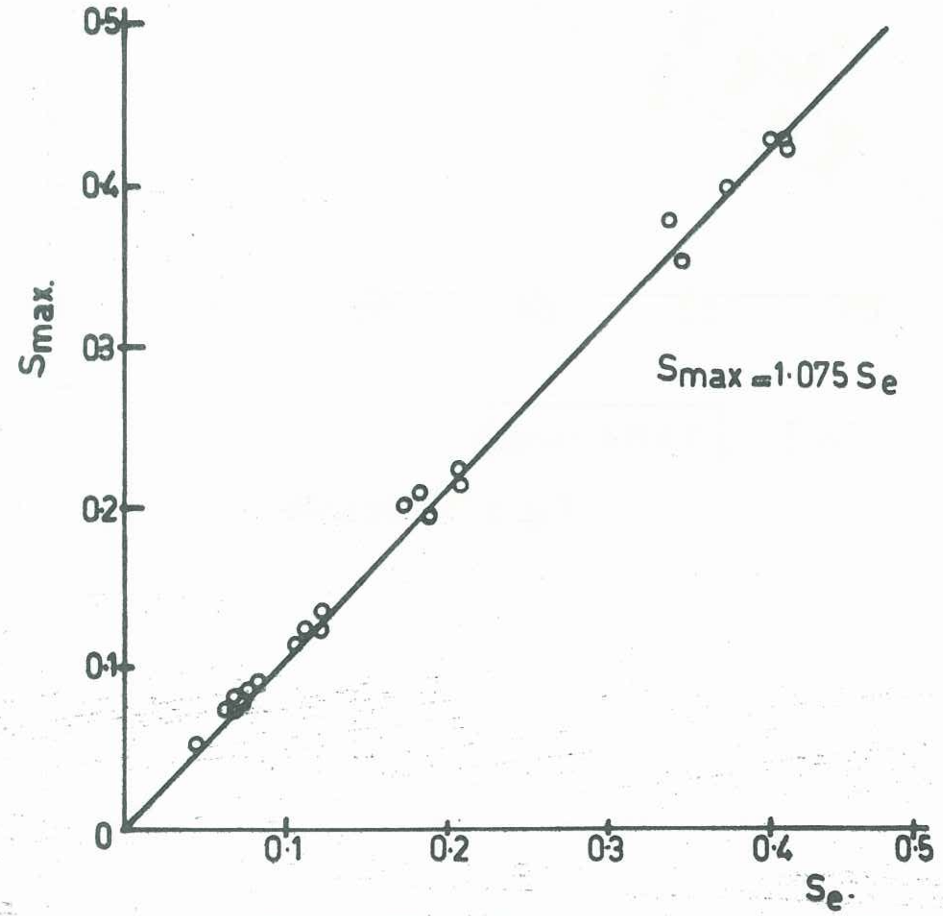


Fig. 2 Relationship between $S_{max}$ and $S_e$.