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SOME NUMERICAL STUDIES OF THE BIOMECHANICS OF BLOOD CIRCULATION

by

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S U M M A R Y

Study of the circulation is complicated by factors such as the visco-elastic properties of the vessel walls, the non-Newtonian behaviour of the blood and the deformable nature of the erythrocytes and other particles.

The effects and importance of these factors can be fruitfully studied by the use of mathematical models even though these necessarily incorporate simplifying assumptions.

Two selected aspects involving (a) micro-circulation - particulate flow, and (b) macro-circulation - pulsatile flow, are treated by numerical techniques. The simplifying assumptions necessary and the implications thereof are discussed.

(a) Particulate Flow Model. A simplified two-dimensional model of idealised particulate flow with (i) individual particles, (ii) groups of particles and (iii) a continuous array of particles in a tube is analysed to provide a detailed mapping of flow patterns around the particles and the effect of particle aggregation on pressure gradients.

(b) Pulsatile Flow Model. The model employed is one-dimensional, although a distensible vessel wall is incorporated. The effects of some system parameters on the character of the flow wave as computed from pressure data, are investigated.

Some of the results of these model analyses are described and their relevance in the overall blood circulation system is discussed.

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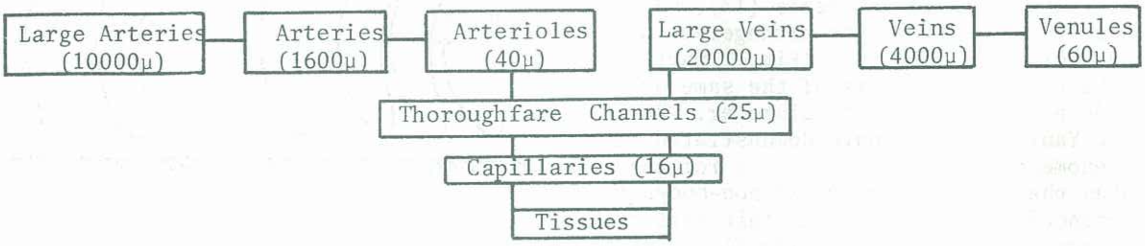
INTRODUCTION

The study of the biomechanics of the circulatory system has grown rapidly in the past decade. Qualitative studies such as the high speed colour motion photography of Bloch (1) vividly portray the complexities of flows in the microcirculation and such studies have stimulated the development of idealised models which not only allow quantitative information to be evaluated but can provide detailed flow characteristics in regions of the circulation which are virtually inaccessible to present day experimental techniques.

Studies of the circulation must eventually involve a combination of a macroscopic systems approach and a microscopic mechanics approach. The two areas of study are not unrelated as the overall macro-study relies on integration of the detailed fundamental analyses to provide bulk characteristics of the system.

The Circulation.

The circulation channels consist of two hierarchies - those leading from the heart and those carrying blood to the heart. The channel size reduces as distance from the heart increases and the two hierarchies meet in the thoroughfare channels (small arterioles and venules) and merge in the capillaries and the perivascular beds of tissue as outlined by Bugliarello (2). The average diameters of the various channels have been summarised by Whitmore (3) as -



Reynolds Number (RN) based on the channel diameter varies from 0.02 in the smallest capillaries to 3×10^5 in the largest veins.

The principal pressure gradients in the mammalian system occur in the arterioles where about 60% of the total pressure losses occur whilst about 25% of the pressure loss occurs in the capillaries and about 10% in the veins (3).

Although capillaries and the smaller arterioles account for most of the energy losses it is important to note that at any instant under normal circumstances only a fraction (1/5 to 1/3) of the capillaries are functioning.

The microcirculation is that part of the circulatory system where channel sizes are generally less than 100μ and in which, (i) most pressure losses occur and (ii) exchange of heat and mass between the circulation system and the surrounding beds of tissue take place.

Material Properties.

The material properties of the fluid - blood - and the channel wall have been the subject of many studies (4,5). Suffice it to say here that blood is a multi-phase fluid with the largest suspended phase being the red-blood cells erythrocytes. The dimensions of the erythrocyte are of the order of 2μ (thickness) and 10μ (diameter) and therefore such particles are particularly significant in studying flows in the microcirculation channels of diameters 5μ - 100μ. The smaller particulate constituents of blood attain importance in considering the mechanisms used for attaining effective mass (and heat) transfers.

No rigorous constitutive equations for blood or its components have been reported although the rheological properties of the erythrocyte and the conveying fluid are fundamental pre-requisites to any detailed understanding of the microcirculation.

The mechanical properties of the vessel walls particularly in the larger channels of the circulation introduce further complications into any analytical study. The arteries and arterioles and to some extent the veins alter their size under the action of muscle cells which line their walls (2), - the capillaries do not alter their diameter in this way but control flow by restrictions (sphincters) at the entrance to the channels. The visco-elastic properties of the larger vessel walls have been studied by many researchers, including Anliker, Histand and Ogden (6), Dick, Kendrick, Matson and Ridesut (7) and Gow and Taylor (8).

A further factor which must be included in any realistic study is the pulsatile characteristic of blood flow. Generally these characteristics have been neglected in studies of the microcirculation where the multi-phase aspects of the fluid need to be considered, however, in the larger vessels where the fluid may be assumed single-phased many studies involving pulsatile flow have been reported (9,10,11).

Idealised Models.

In this paper two studies involving idealised models are outlined - (a) a study in the microcirculation in which a two-phase fluid is considered in a very simplified model aimed at studying patterns of erythrocyte aggregation that have been observed experimentally and (b) a study of pulsatile flow behaviour in the larger arterial vessels in which the visco-elastic wall properties are incorporated in a one-dimensional model of an arterial segment.

In each study numerical techniques are applied to an idealised problem of biomechanics. The possible extensions of the analysis to more refined and realistic problems are obvious but, inevitably, restrictions will be imposed by present day computational capabilities. A further problem is imposed by the difficulty in obtaining adequate physical data to prove the analytical models.

Particulate Flow Model.

A number of observers of capillary flow, Merrill and Wells (12) and Benis (13), have described a typical erythrocyte arrangement as a 'stacked coin configuration' (Fig.1) particularly where the capillary size is of the same order of magnitude as the erythrocyte diameter. Bugliarello and Yanizeski (14) have demonstrated this same phenomenon experimentally in a rectangular Heli-Shaw channel in which rigid non-buoyant particles tended to attain and maintain particular cluster configurations for steady flow conditions. Prothers and Burton (15) 1962, suggested their now well known 'bolus flow' analogy, in which the red blood cells were modelled by large scale air bubbles within a cylindrical tube with the plasmatic gaps modelled by water, as the earliest study of the fluid dynamics of cluster flow involving axial gaps between the fluid.

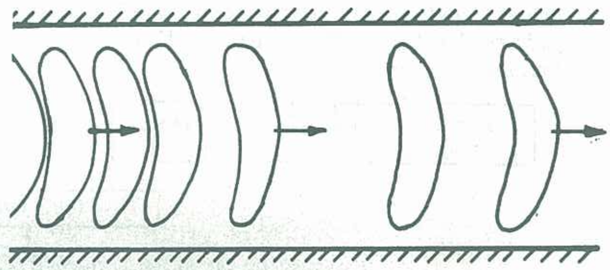


Figure 1. Stacked coin configuration.

Brandt and Bugliarello (16) 1965, considered an idealised two-dimensional model in which the red-cell particles were modelled by rigid rectnagular blocks occupying the full cross-section of the channel and the steady creeping flow equations (for zero Reynolds number) were solved numerically for the flow in the gaps between the particles with rigid boundary conditions. Bugliarello and Hsaio (17) extended this model to the corresponding axisymmetric case. The results were used to suggest that for a given hematocrit of 40% the pressure drop would be considerably reduced if the erythrocytes are grouped tightly into clusters of 4 or 5 cells rather than being spaced evenly and, further, increasing the number of cells in each cluster, beyond 5, does not gain any significant extra pressure reduction. These analyses assumed that the 'erythrocytes' spanned the complete cross-section and friction losses between the particle and the wall or within the narrow bands of flow between the particle and the wall or between particles were not taken into account. Inertia effects were ignored which is reasonable for the creeping flows of capillaries.

More recently Sutera and Hochmuth (18) have developed a sophisticated large scale experimental model involving buoyant discoidal and plane convex discs flowing at low RN in cylincrical tubes. Wang and Skalak (19) have obtained analytical solutions for the movement of arrays of rigid spherical and spheroidal particles (Chen and Skalak) (20) and, also, for the case of viscous liquid drops (Hyman and Skalak) (21).

These various solutions illustrate that the ratio of the lateral dimension of the particle to the tube diameter is a more important parameter than the cell spacing or orientation in affecting pressure drop and Sutera and Hochmuth (18) indicated that clusters of cells showed a lower energy loss than isolated cells separated by distances greater than the tube diameter.

Numerical Model.

With these various models and hypotheses in mind a series of numerical studies of an idealised two-dimensional, two-phased flow of single (and arrays of) buoyant particles in a tube have been undertaken. The particle shape for the earliest models was rectangular although circular, spherical and discoidal particles are currently being considered. Fig.2 shows a typical set of boundary conditions for the problem.

Fortunately some simplifications appear to be permissible in modelling cpaillary flow as a numerical problem - thus (i) the plasma may be considered as a Newtonian fluid (12), (ii) the flexibility of the 'red blood cell' can be neglected (18), (iii) the capillary walls may be assumed rigid and straight (12),(18),(22) and finally (iv) particulate constituents smaller than erythrocytes may be neglected in any fluid dynamic analysis. Laminar inertia terms have been retained in the solutions so that the results would be applicable for Reynolds numbers of the order found in flows in the larger arterioles.

The numerical studies have, in the first instance, assumed that the particles are moving with constant velocity. Under these circumstances zero net drag is experienced by the particles and if the particles are neutrally buoyant and moving with the fluid with a velocity V_p when the fluid has a mean velocity V_m then a steady-flow condition is appropriate for analysis in which the particle is held stationary and the walls are moved with a velocity (V_B) equal and opposite to V_p. The appropriate value of V_B that results in zero net drag on the particles must be determined for each configuration of particle, channel and RN. The complete flow field is determined by solving the Navier-Stokes equations, for the appropriate boundary conditions, using a numerical technique which has been developed from the squaring method of Thom and Apelt (23). Complete details of the technique and the solutions obtained will be published separately. The Navier-Stokes equations for zero RN are linear so that consideration of any two solutions gives the zero-drag case, however, for higher RN the equations are non-linear and interpolation of a number of solutions for different V_B values is required to deduce the zero-drag case.

The field equations are solved to give stream-function, vorticity, velocity and pressure values throughout the flow field and some typical results are plotted as Figs.3,4 and 5. Fig.3 gives the stream-function pattern relative to the particle for an array at zero RN and shows the relative vortex formation for α = .33, β = .75, γ = .25 as defined between particles in the two-phase flow.

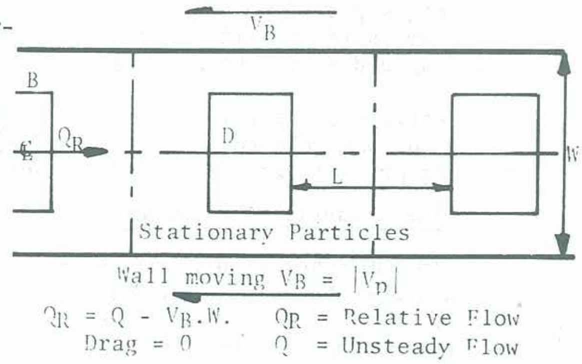


Fig.2 Typical conditions for array model.

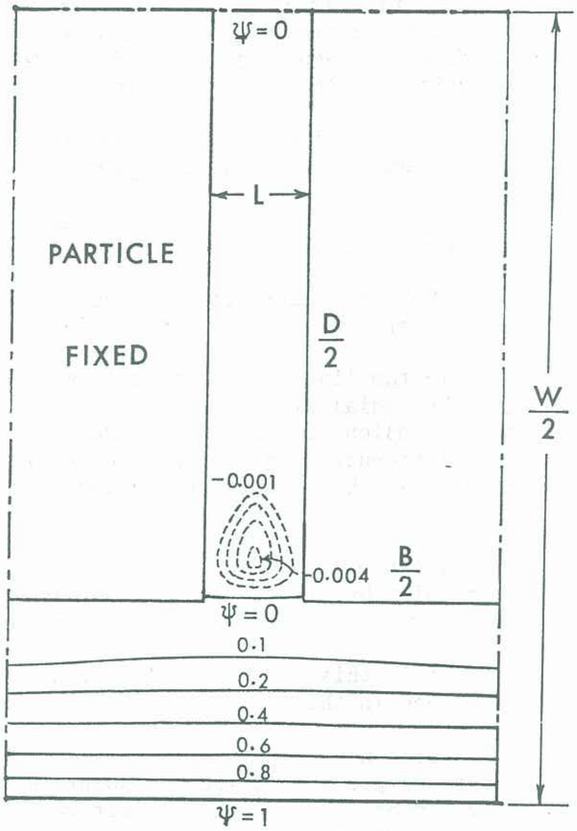


Fig.3 Streamlines relative to particle at RN=0.0 for arrays α=.33,β=.75,γ=.25.

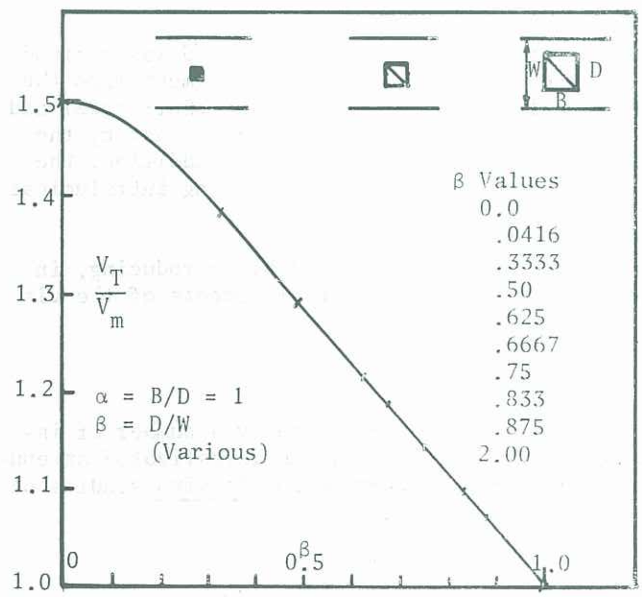


Fig.4 V_T/V_m for single particles α=1,β various.

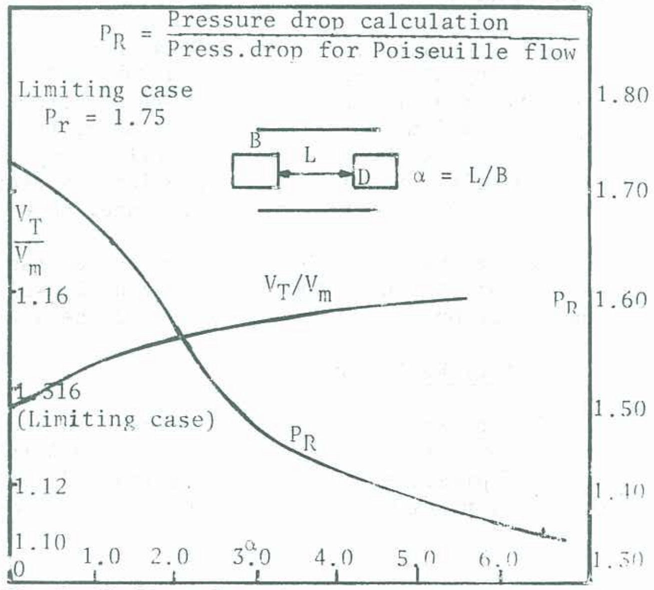


Fig.5 V_T/V_m and P_R for arrays with α=.33, β=.75, γ various.

Fig.4 plots the ratio of V_T/V_m for a single particle with $\alpha = 1.0$ and β varying from zero to 1.0. V_T is the terminal particle velocity - i.e. the constant velocity of the particle for zero net drag and V_m is the mean velocity of flow.

Fig.5 plots the variation of V_T/V_m for an array of particles with $\alpha = 0.33$, $\beta = 0.75$ and γ (i.e. the particle spacing) varying and also shows the ratio of pressure gradient to the corresponding pressure gradients for Poiseuille flow for the same variety of solutions.

The Pulsatile Flow Model.

In published studies of the pressure-flow relation in arteries, the objective, most commonly, has been to compute the flow from the pressure gradient. The complete system is, as Rudinger (5) says, far too complicated to be amenable to analysis and pressure-flow studies have generally been confined to short, straight, unbranched segments.

In the larger systemic arteries blood can be treated as an incompressible, Newtonian fluid (24) and the pressure-flow relationship described by a set of non-linear, second order, partial, differential equations, comprising the equations of motion in three directions, the continuity equation and an equation expressing the visco-elastic properties of the vessel wall.

The non-linearities are associated with the convective inertia forces, the viscous friction forces and the wall elastic properties.

The equations have defied exact solution and recourse has consequently been made to simplified models.

There are currently in vogue, two such simplified models of pulsatile arterial blood flow - the linear, axisymmetric model (25) and the non-linear, one-dimensional model (26).

In the linear, axisymmetric model, variations in flow properties in both the longitudinal and the radial directions are included. There are thus two physical dimensions in addition to the dimension of time. The equations are transformed from the time to the frequency domain, each frequency component is treated individually and, by virtue of the linearity of the equations, the results may be superimposed. The problem is thus reduced to a two-dimensional one.

The simplified non-linear model, on the other hand, can only be reduced to a two-dimensional problem by eliminating one physical dimension. Thus flow properties are assumed to be invariant in the radial direction and the problem treated is one-dimensional in the physical sense.

It is this second model, the non-linear, one-dimensional one, with which we will be concerned in this paper.

Being non-linear, solutions for this model can only be obtained by numerical methods. The equations are hyperbolic and solution is by the method of characteristics. Two non-linear terms in the model express the convective inertia forces of the motion and vessel taper due to pressure gradient and wall distensibility. These terms vanish in the case of an initially untapered vessel with non-elastic walls, but have been shown to be important in tapered and distensible vessels (27).

The non-linear one-dimensional model was first propounded by Lambert (28) and was refined somewhat by Streeter, Keitzer and Bohr (29) who computed the flow wave in the segment from the pressure data at the two ends. Dr. Streeter and his colleagues were aware that their model had some severe limitations. In particular, the vessel wall was treated as linearly elastic, the flow assumed to be turbulent and all energy losses were lumped in one term. In addition, the wave celerity expression used predicted a decrease in wave celerity for increasing intraluminal pressure, contradictory to experimental observation.

It has been our object to improve on the model used by Streeter et al by introducing, in particular, the non-linear, visco-elastic wall properties, and realistic statements of the viscous friction losses in the blood and the wave celerity function.

Artery Wall Properties.

The presence of a viscous element in the artery wall has been reported by a number of investigators, the most recent of whom are Anliker, Histand and Ogden (6) who investigated attenuation of pressure pulses in the aorta and Gow and Taylor (8) who carried out in vivo studies on the pressure-diameter relationship in arteries.

Dick, Kendrick, Matson and Rideout (7) and Gow and Taylor (8) from in vivo experiments on mongrel dogs, concluded that a small degree of visco-elastic non-linearity existed in the arterial wall and was more pronounced in the more peripheral arteries than in the central ones.

Flow Properties.

It is generally accepted that, in the larger arteries, blood may be assumed to have a constant viscosity. There are difficulties, however, in choosing an appropriate value of the viscosity. Anticoagulants added to blood are known to have a significant effect on viscosity (5). In addition there is the question of whether the flow is laminar or turbulent.

Energy losses due to viscous friction are known to be higher in pulsating flow than in steady flow (9,10,11).

Streeter et al (26,29) assumed viscous losses proportional to $V^n/2D$ and the values of the exponent and the constant of proportionality which yielded best agreement (on the basis of a least squares criterion) between their computed flow wave and the experimentally recorded wave, were determined. The exponent was invariably in the neighbourhood of 2, suggesting turbulent flow. The constant of proportionality varied over a considerable range depending on the value of the frequency parameter, $= R\sqrt{\omega\rho/\mu}$ where R = vessel radius, $\omega = 2\pi x$ frequency in cycles/second, ρ = fluid density and μ = absolute viscosity. For the femoral artery ($\alpha \simeq 3$) the constant was, typically, 0.5 for flow in the direction of vessel convergence and 0.76 for flow in the reverse direction.

Wiggert and Keitzer (9) called the constant of proportionality an "index of energy dissipation". The index is analogous for the case of pulsatile flow in tapered vessels, to the Darcy-Weisbach friction factor, f, for steady flow in uniform pipes. It takes on higher values than f because it incorporates, in addition to the steady flow friction losses, the energy losses due to the pulsatile flow component and the taper of the vessel cross-section. Experimenting with latex rubber tubing at α = 12-15, Wiggert and Keitzer found the index had a value of 0.05 for turbulent pulsating flow in uniform tube of unstressed diameter 0.595 cm. For tapered tubes (0.602 cm to 0.343 cm diameter over a length of 59.8 cm) the index varied with α from 0.1 to 0.2 for flow in the direction of convergence and was consistently 0.3 for flow in the reverse direction.

The higher values of the index found by Streeter et al for arteries indicate the presence of additional mechanisms of energy dissipation. The viscous element in the artery wall is obviously one such mechanism.

Visco-elastic Model.

We have attempted to separate the energy losses into those attributable to the vessel wall and the computations reported in this paper are based on a model incorporating a visco-elastic wall. Vessel diameter at each end of the segment was derived from the pressure data by shifting each harmonic of the pressure wave by a certain phase angle, resynthesising the wave and scaling the amplitude appropriately. Gow (30) suggested a constant phase shift of the harmonics of 0.10 to 0.15 radians. Accordingly a shift of 0.13 radians was used throughout these computations. This procedure results in a viscous, linearly elastic wall. Instantaneous diameter was assumed to vary linearly over the length of the segment between the corresponding values at the two ends. Wave celerity was expressed as a linear, increasing function of pressure. Laminar flow was assumed and the friction factor based on the instantaneous velocity at each step of the computations was derived from the Blasius expression f = 64/RN.

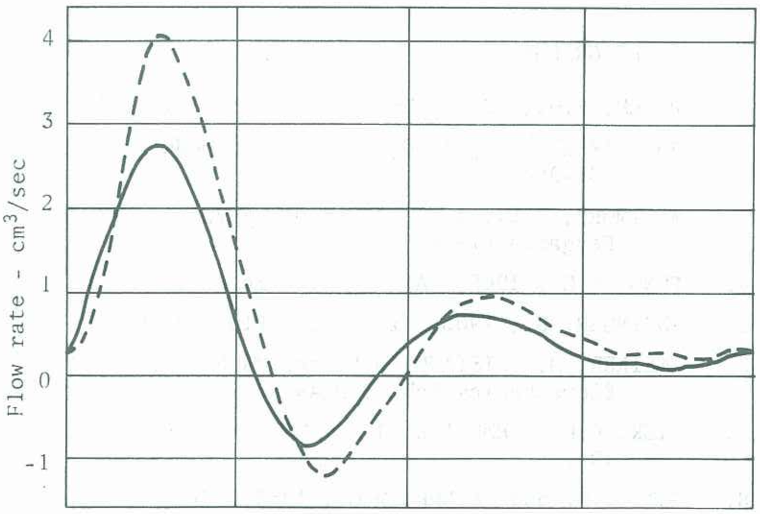


Figure 6. Broken line: Computed flow wave using visco-elastic model. Laminar flow, μ = 3 cP. Full line: Experimentally recorded flow wave.

The flow wave was computed in this manner from the pressure data published by Streeter et al (26) for the dog femoral artery. The result is shown in Fig. 6, with the experimentally recorded flow wave for comparison. The coefficient of absolute viscosity was taken in this computation as 3 cP.

The result should be compared with the flow wave plotted in Fig.7, computed by Streeter et al on the basis of turbulent flow in a purely elastic walled model, with an index of energy dissipation of 0.4 for flow in both the forward and the reverse directions.

Over the forward flow portions of the wave, Streeter et al's model yields closer agreement with the recorded wave, but over the reverse flow portion, the visco-elastic model yields the better result.

It should be remembered that the visco-elastic model assumed laminar flow and a friction factor appropriate to steady flow in an untapered vessel. To allow for the greater friction losses of pulsatile flow in a tapered vessel, artificially high values of the friction factor were introduced simply by using increased values of the coefficient of viscosity, μ. Wiggert and Keitzer found a 2.5- to 5-fold increase in index for pulsatile flow in a tapered vessle. Accordingly, a 2-2/3-fold increase in the friction factor (μ raised from 3 to 8 cP) was tried. The result is shown in Fig.8 and agreement in both forward and reverse portions of the wave is seen to be much improved.

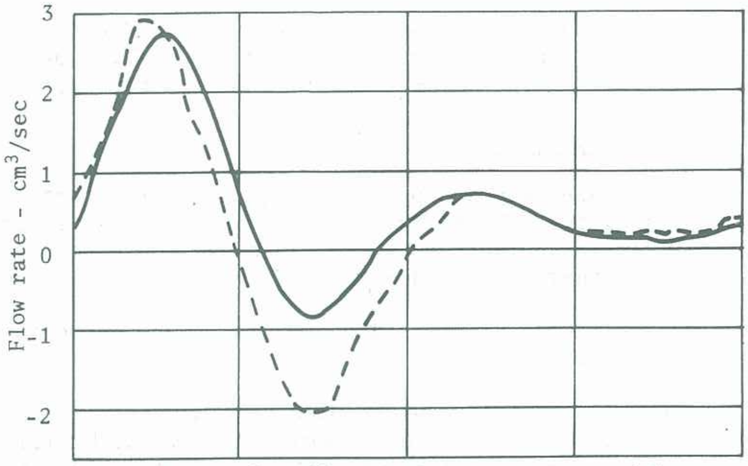


Figure 7. Broken line: Flow wave computed by Streeter et al. Turbulent flow, f = 0.4. Full line: Experimentally recorded flow wave.

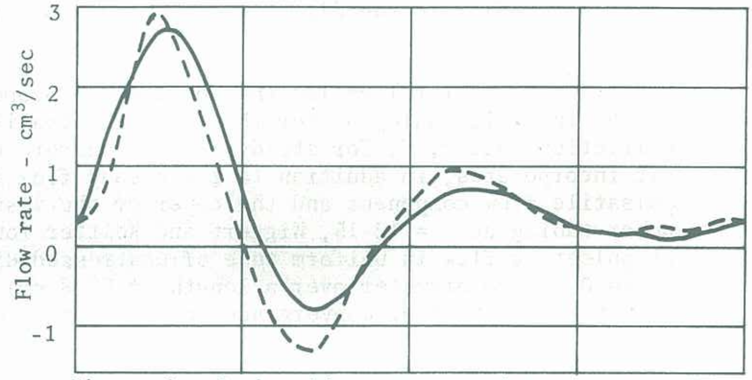


Figure 8. Broken line: Computed flow wave using visco-elastic model. Laminar flow, μ = 8 cP. Full line: Experimentally recorded flow wave.

CONCLUSIONS

The realisation of an accurate mathematical model of the complete circulatory system is still very remote. At this time the properties of the individual elements and their roles in the overall system are only imperfectly understood. The two models developed in this paper illustrate the advantages of applying mathematical techniques to the study of the parameters affecting such complex systems as the circulation.

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