Abstract

This talk and paper draw heavily on related published work by this author reviewing flow control and especially the use of resolvent analysis to characterize and design control approaches. In particular, a range of almost identical material was presented at the 12th International ERCOFTAC Symposium on Engineering Turbulence Modelling and Measurements [21].

The financial and environmental cost of turbulence is staggering: manage to quell turbulence in the thin boundary layers on the surface of a commercial airliner and you could almost halve the total aerodynamic drag on existing platforms, dramatically cutting fuel burn, emissions and cost of operation, and enabling new aircraft designs. Consequently, there have been large investments around the world in research into means to reduce and control turbulence [6]. However practical implementation, especially of active control schemes, has remained either impractical or not economically viable given relatively small net gains. A brief review of historical and current flow control efforts targeting skin friction reduction is given.

Systems-level tools to model scale interactions and control turbulence are given more detailed treatment. The resolvent analysis for turbulent flow proposed by McKeon and Sharma [23] provides a simple, but rigorous, approach by which to deconstruct the full turbulence field into a linear combination of modes which interact through the nonlinear term to provide self-sustaining turbulence. A review of the linear resolvent operator and the nonlinear feedback, or the inter-connection between different scales which provides the forcing required for self-sustaining turbulence, is provided, then the formulation is extended to include control through modifications to the wall boundary conditions. After considering the information given by the resolvent operator about the relative physical locations of input and corresponding response, framed in the context of flow control, the specific examples of experimental manipulation of both the linear response and the triadic scale interactions using dynamic roughness actuation, opposition control and compliant wall design for global drag reduction are considered. A brief statement on the outlook for modeling and control of turbulent flows using this approach is also presented, focusing on possibilities related to a more general shaping of flow properties, or “designer turbulence”, enabled by more detailed understanding of the mechanisms of turbulence sustenance.

Introduction to control of wall shear flows

Control of wall turbulence using passive or active means - as a sub-topic of the broader study of general flow control - has been a topic of scientific and especial practical interest for many years given the ubiquity of the phenomenon. Overviews of various approaches have been given by Gad-el-Hak [7], Kim [13, 14] and Kim and Bewley [15]. There have been various more recent efforts, as summarized by, e.g., McKeon, Jacobi and Duvvuri [22] as part of an AIAA Journal Special Issue on Flow Control. Henceforth in this contribution we focus specifically on the potential to use resolvent analysis for a system-level approach to flow control.

Closed Loop Resolvent Formulation

The resolvent formulation for wall turbulence has been described in detail in previous publications, e.g. [23, 24, 20]. The essential outline of the closed-loop resolvent analysis is shown in Figure 1, and the development here is for turbulent channel (plane Poiseuille) flow in a domain which is periodic in the streamwise and spanwise directions, x and z, respectively, with no-slip and no-penetration wall boundary conditions. Defining fluctuations relative to the spatio-temporal turbulent mean (which is assumed to be known from, e.g., experimental data, simulations or an eddy viscosity model), \( U(x, y, z, t) = \overline{U}(y) + u(x, y, z, t) \), and Fourier transforming the Navier-Stokes equations in \((x, z, t)\):

\[
\hat{u}(k_x, k_z, \omega; y) = \int \int \int_{-\infty}^{\infty} u(x, y, z, t) e^{-i(k_x x + k_z z - \omega t)} dx dz dt
\]

Rearranging, and formulating in velocity-vorticity form, \((v, \eta)\), we arrive at

\[
\left( \begin{array}{c}
v \\
\eta
\end{array} \right) = \left( \begin{array}{cc}
\mathcal{H}_v & 0 \\
\mathcal{H}_\eta & \mathcal{H}_\eta
\end{array} \right) \left( \begin{array}{c}
\hat{v} \\
\hat{\eta}
\end{array} \right)
\]

Here \( \hat{v} \) and \( \hat{\eta} \) represent a forcing of the linear terms at \( k = (k_x, k_z, \omega) \) arising from the nonlinear interactions between scales and the \( \mathcal{H} \) operators constituting the resolvent (transfer function) are related to the well-known Orr-Sommerfield and Squire operators as follows, where \( \kappa^2 = k_x^2 + k_z^2 \), \( \mathcal{D}^2 \) is the Laplacian and \( U^* \) is the local mean shear:

\[
\mathcal{H}_v = -i\omega + (\kappa^2 - \mathcal{D}^2)^{-1} \mathcal{L}_{OS}
\]

\[
\mathcal{H}_\eta = (-i\omega + \mathcal{L}_{SQ})^{-1}
\]

\[
\mathcal{H}_{\hat{v}} = -i\kappa \mathcal{H}_\eta U^* \mathcal{H}_v
\]

Input-output analysis of this form was described by Jovanovic and Bamieh [12] for laminar flow, and several other studies have considered the characteristics of the linear resolvent operator using an eddy viscosity rather than the explicit nonlinear term to provide a closure, e.g. Hwang and Cossu [10], or under a shaped stochastic forcing input, e.g. Zare, Jovanović and Georgiou [32].

Following the development of Rosenberg and McKeon [27], we further decompose the relationships above to reflect the potential independence of unforced Squire modes:

\[
\left( \begin{array}{c} v \\
\eta
\end{array} \right) = \left( \begin{array}{c} v \\
\eta_{os}
\end{array} \right) + \left( \begin{array}{c} 0 \\
\eta_{so}
\end{array} \right)
\]
The studies outlined above all pertain to changes to the boundary conditions on the linear operator, but note that Luhar [26], who investigated the effects of suboptimal control based on the shear stress measured on the wall, implementing wall transpiration proportional to the streamwise or spanwise wall shear stress.

Luhar, Sharma and McKeon [18] considered the interaction of simple compliant walls with turbulent channel flow. Consistent with earlier studies on compliant surfaces in the transitional flow regime, the dynamical wall boundary condition (for small deflections, constrained to be in the wall-normal direction only) was modeled using a complex admittance relating the wall-normal velocity and pressure at the wall,

$$Y = \frac{v_k(0)}{p_k(0)}$$

The analysis was subsequently expanded and developed as a potential design tool for compliant walls in Luhar, Sharma and McKeon [19].

The formulation can be modified to admit formal linear control, simply added here by implementing a wall boundary condition appropriate to the control law under consideration. These approaches have been advanced empirically and experimentally using dynamic roughness actuation at individual frequencies in a turbulent boundary layer (see, e.g., [24]) as well as more formally incorporating the true boundary condition change. Luhar, Sharma and McKeon [17] investigated opposition control (Choi, Moin and Kim, [2]) with a generalized (complex) amplitude, \(A_d\), by implementing a wall-normal velocity at the wall of the opposite sign to the signal detected at an off-wall detection plane at height, \(y_d\):

$$v_k(0) + A_d v_k(y_d) = 0$$

A similar approach was used by Nakashima, Fukugata and Luhar [26], who investigated the effects of suboptimal control based on the shear stress measured on the wall, implementing wall transpiration proportional to the streamwise or spanwise wall shear stress.

Conceptually, open loop excitation can be viewed as adding an additional, external component to \(f\) in the schematic of Figure 1. The formulation can be modified to admit formal linear control, simply added here by implementing a wall boundary condition appropriate to the control law under consideration. These approaches have been advanced empirically and experimentally using dynamic roughness actuation at individual frequencies in a turbulent boundary layer (see, e.g., [24]) as well as more formally incorporating the true boundary condition change. Luhar, Sharma and McKeon [17] investigated opposition control (Choi, Moin and Kim, [2]) with a generalized (complex) amplitude, \(A_d\), by implementing a wall-normal velocity at the wall of the opposite sign to the signal detected at an off-wall detection plane at height, \(y_d\):

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Opposition control in DNS

A dynamic roughness element using the framework of Luhar, in [22]. In ongoing work (Huynh and McKeon, [9]), we are
An extended summary of the results of these studies is given
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lar value (gain). Maximal gain reduction was obtained for dif-
corresponding to the near-wall cycle of turbulence (Figure 4),
Luhar, Sharma and McKeon [17]. For modes with a footprint in
The generalized opposition control described by equation 8 was
summarizes the linear and nonlinear responses observed for
Figure 3, from [3], shows the experimental configuration and
establishes downstream hot-wire and
particle image velocimetry measurements with respect to the
phase of the dynamic roughness. It was determined that the re-
sponse, which decayed in the downstream direction, was dom-
nated by a single streamwise wavelength for each frequency.
Figure 4, from [3], shows the experimental configuration and
summarizes the linear and nonlinear responses observed for a
two frequency input, i.e. forcing at $k_1 = (k_{11}, 0, 0)$ and
$k_2 = (k_{22}, 0, 0)$ which was shown to identify the sum and dif-
ference frequencies via the quadratic nonlinear interactions of
the excited synthetic modes.

An extended summary of the results of these studies is given in [22]. In ongoing work (Huynh and McKeon, [9]), we are
investigating the response of a compliant wall downstream of a
dynamic roughness element using the framework of Luhar,
Sharma and McKeon [18].

Opposition control in DNS
The generalized opposition control described by equation 8 was
implemented on a scale-by-scale basis in turbulent pipe flow by
Luhar, Sharma and McKeon [17]. For modes with a footprint in
$v$ at the detection plane, for example for the mode most closely
corresponding to the near-wall cycle of turbulence (Figure 4),
control modified both the response velocity field and the singu-
lar value (gain). Maximal gain reduction was obtained for dif-
dferent phases of control amplitude, $A_d$, for modes with different
wavespeeds, $c_k$. This behavior was captured by sweeps over
$k$ for the first response modes, i.e. a rank-1 approximation of
the resolvent. Using the FIK identity (Fukugata, Iwamoto and
Kasagi, [5]) to characterize the reduction in turbulent Reynolds
stress (a proxy for the true drag reduction which includes a con-
tribution from the laminar base flow) for traditional opposition
control, $A_d = 1$, it was shown that the variation of the drag re-
tection trends with detection plane location were captured by
the resolvent analysis. Further, an optimal drag reduction sig-
ificantly larger than for the traditional parameters could be ob-
served for control with $|A_d| = 1$ and $\Delta A_d \approx \pi/4$.

Note that these results were obtained employing the uncon-
trolled turbulent mean in the formulation of the resolvent. In
ongoing work, we have compared the predictions of the rank-
1 linear analysis detailed above with the output of DNS at
matched conditions, i.e. the full nonlinear solution (Toedtli,
Luhar and McKeon, [30]). Not only are the trends in drag re-
duction matched across detection plane and actuation phase, but

there is surprisingly good agreement between the conditions
for optimal drag reduction. The sweep through this parame-
ter space using the resolvent approach can be performed using
desktop computing power rather than high performance com-
puting capabilities, close to two orders of magnitude faster than
for the DNS for this one-dimensional base flow. Further, the
approach can be applied to Reynolds numbers beyond the ca-
pability of current DNS codes, however the gains decrease as
the complexity of the base flow increases, e.g. through a two-
dimensional base flow. Nonetheless, these results point to the
potential to use resolvent analysis as a design tool to evaluate
the likely efficacy of flow control techniques and the physics
underlying them before devoting large computational resources
at a particular control point.

Some Examples of Flow Control via Resolvent Analysis

Experiments using dynamic roughness actuation

Building on the results of Jacobi and McKeon [11], Duvvuri and
McKeon [3, 4] utilized a spatially-impulsive oscillating span-
wise strip - a dynamic roughness element - to provide external
actuation using open-loop thermal actuation at the wall.

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References

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isolated through scale synthesis in experimental wall tur-
Figure 2: Schematic of forcing (left) and response (right) mode shapes for an example two-dimensional operator with amplification due to the following properties (from Symon et al. [29]). Panels (a, b) show resonant forcing and response, respectively, where the operator is self-adjoint (normal). (c, d) show component-type non-normality associated with the lift-up mechanism. (e, f): non-self-adjoint operator. (g, h): Orr-type mechanism. (i, j): convective-type instability. (k, l): convective instability with streamwise overlap between forcing and response modes (absolute instability) between vertical dashed lines. Positive (negative) isocontours are denoted by solid (dotted) lines and blue (red) indicates streamwise (transverse) components, in the $x, y$ directions, respectively. Each mode is nonzero in one velocity component only.


Figure 3: Schematic of dynamic roughness experiments, from Duvvuri and McKeon [3]. A spanwise element is oscillated at one or two frequencies to excite linear and nonlinear responses in a turbulent boundary layer, which are reconstructed by phase-locking downstream measurements to the phase of the dynamic roughness actuation.

Figure 4: Isosurfaces of streamwise velocity (top) with cross-stream velocity vectors (bottom) for the first response mode with inner-scaled wavelengths and wavespeed $\lambda_1^+ = 1000, \lambda_2^+ = 100$ and $c^+ = 10$ in turbulent pipe flow at friction Reynolds number, $Re_\tau = 1100$. Left: uncontrolled flow. Right: under traditional opposition control, $A_d = 1$. Streamwise velocity isosurfaces are plotted at the same absolute level. The corresponding singular value for the controlled case is 0.6 times the uncontrolled value. Courtesy of M. Luhar.


