Ultimate thermal turbulence and asymptotic ultimate turbulence induced by wall-roughness

Xiaojue Zhu, Ruben A. Verschoof, Dennis Bakhuis, Varghese Mathai, Sander G. Huisman, Richard J. A. M. Stevens, Roberto Verzicco, Chao Sun, Detlef Lohse

Physics of Fluids Group and Max Planck Center for Complex Fluid Dynamics, MESA+ Institute and J. M. Burgers Centre for Fluid Dynamics, University of Twente, P.O. Box 217, 7500AE Enschede, The Netherlands

Abstract

Turbulence is omnipresent in Nature and technology, governing the transport of heat, mass, and momentum on multiple scales. One of the paradigmatic turbulent flows is Rayleigh-Bénard convection, i.e., a flow heated from below and cooled from above. Here, the possible transition to the so-called ultimate regime, wherein both the bulk and the boundary layers are turbulent, has been an outstanding issue, since the seminal work by Kraichnan [Phys. Fluids 5, 1374 (1962)]. Yet, when this transition takes place and how the local flow induces it is not fully understood. By performing two-dimensional simulations of Rayleigh-Bénard turbulence covering six decades in Rayleigh number $Ra$ up to $10^{14}$ for Prandtl number $Pr = 1$, for the first time in numerical simulations we find the transition to the ultimate regime, namely at $Ra_* = 10^{13}$. We reveal how the emission of thermal plumes enhances the global heat transport, leading to a steeper increase of the Nusselt number than the classical Malkus scaling $Nu \sim Ra^{1/3}$ [Proc. R. Soc. London A 225, 196 (1954)]. Beyond the transition, the mean velocity profiles are logarithmic throughout, indicating turbulent boundary layers. In contrast, the temperature profiles are only locally logarithmic, namely within the regions where plumes are emitted, and where the local Nusselt number has an effective scaling $Nu \sim Ra^{0.38}$, corresponding to the effective scaling in the ultimate regime.

For real-world applications of wall-bounded turbulence, the underlying surfaces are virtually always rough; yet characterizing and understanding the effects of wall roughness for turbulence remains an elusive challenge. By combining extensive experiments and numerical simulations, here, taking as 2nd example the paradigmatic Taylor-Couette system (the closed flow between two independently rotating coaxial cylinders), we uncover the mechanism that causes the considerable enhancement of the overall transport properties by wall roughness. If only one of the walls is rough, we reveal that the bulk velocity is slaved to the rough side, due to the much stronger coupling to that wall by the detaching flow structures. If both walls are rough, the viscosity dependence is thoroughly eliminated and we thus achieve what we call asymptotic ultimate turbulence, i.e. the upper limit of transport, in which the scalings laws can be extrapolated to arbitrarily large Reynolds numbers.

This Proceeding contribution summarizes and reproduces the main results of our recent references [56, 57].

Introduction

Rayleigh-Bénard (RB) flow, in which the fluid is heated from below and cooled from above, is a paradigmatic representation of thermal convection, with many features that are of interest in natural and engineering applications. Examples range from astrophysical and geophysical flows to industrial thermal flows [3, 26, 7]. When the temperature difference between the two plates (expressed in dimensionless form as the Rayleigh number $Ra$) is large enough, the system is expected to undergo a transition from the so-called “classical regime” of turbulence, where the boundary layers (BLs) are of laminar type[47, 55, 54, 8], to the so-called “ultimate regime”, where the BLs are of turbulent type, as first predicted by Kraichnan [24] and later by others [43, 11, 12, 13, 14]. In the classical regime, the Nusselt number $Nu$ (dimensionless heat transfer) is known to effectively scale as $Ra^\beta$, with the effective scaling exponent $\beta \leq 1/3$ [11, 12, 45, 27, 37]. Beyond the transition to the ultimate regime, the heat transport is expected to increase substantially, reflected in an effective scaling exponent $\beta > 1/3$ [24, 3, 13].

Hitherto, the evidence for the transition to the ultimate regime has only come from experimental measurements of $Nu$. In fact, the community is debating at what $Ra$ the transition starts and even whether there is a transition at all. For example, in ref. [30] it was observed that $\beta$ first increases above $1/3$ around $Ra \approx 10^{14}$ and then decreases back to $1/3$ again for $Ra \approx 10^{15}$. Subsequently, Urban et al. [48] also reported $\beta \approx 1/3$ for $Ra = [10^{12}, 10^{15}]$. However, Chavanne et al. [5, 6] found that the effective scaling exponent $\beta$ increases to 0.38 for $Ra > 2 \times 10^{11}$. In the experiments mentioned above, low temperature Helium was used as the working fluid and Prandtl number $Pr$ changes with increasing $Ra$. In contrast, using high pressure SF$_6$ which has roughly pressure independent $Pr$ instead of Helium, a more conclusive realization of ultimate regime was achieved by He et al. [18, 17], who observed a similar exponent 0.38, but this exponent was found only to start at a much higher $Ra \approx 10^{14}$ (the transition starts at $Ra \approx 10^{13}$). This observation is compatible with the theoretical prediction [11, 12] for the onset the ultimate regime. It is also consistent with the theoretical prediction of Refs. [24, 13], according to which logarithmic temperature and velocity BLs are necessary to obtain an effective scaling exponent $\beta \approx 0.38$ for that $Ra$. The apparent discrepancies among various high $Ra$ RB experiments have been attributed to many factors. The change of $Pr$, the non-Boussinesq effect, the use of constant temperature or constant heat flux condition, the finite conductivity of the plates, and the sidewall effect can all play different roles [3, 44].

Direct numerical simulations of 2D RB

Direct numerical simulations (DNS), which do not have these unavoidable artefacts as occurring in experiments, can ideally help to understand the transition to the ultimate regime, with the strict accordance to the intended theoretical RB formulations. Unfortunately, high $Ra$ simulations in three dimensions (3D) are prohibitively expensive [41, 46]. The highest Rayleigh number achieved in 3D RB simulations is $2 \times 10^{13}$ [44], which is one order of magnitude short of the expected transitional $Ra$. Two-dimensional (2D) RB simulations, though different from 3D ones in terms of integral quantities for small $Pr$ [39, 51], still capture the many essential features of 3D RB [51]. Consequently, in recent years, 2D DNS has been widely used to test theories, not only for normal RB [20, 53], but also for RB in porous media [19]. Although also expensive at high $Ra$, now we have the chance to push forward to $Ra = 10^{14}$ using 2D simulations. Another advantage of DNS as compared to experiment is that velocity and temperature profiles can be easily measured,
to check whether they are logarithmic in the ultimate regime, as expected from theory. Specifically, for the temperature, only a few local experimental measurements were available in the near-sidewall regions of RB cells, which showed logarithmic profiles [1, 2]. Even worse, for velocity, there is almost no evidence for the existence of a logarithmic BL, due to the experimental challenges. For instance, in cylindrical cells with aspect ratio $\Gamma = O(1)$, the mean velocity profile cannot be easily quantified because of the absence of a stable mean roll structure [24]. In situations where stable rolls do exist (e.g. narrow rectangular cells), the highest $Ra$ available are still far below the critical $Ra$ at which logarithmic velocity BLs can manifest themselves [47, 8].

As numerical simulations provide us with every detail of the flow field which might be unavailable in experiments, they also enable us to reveal the links between the global heat transport and the local flow structures. A few attempts (both 2D and 3D) have been made in the classical regime, in which logarithmic temperature BLs were detected, by selectively sampling the regions where the plumes are ejected to the bulk [1, 49]. However, it is still unclear how these local logarithmic BLs contribute to the attainment of the global heat transport enhancement during the transition to the ultimate regime.

In ref. [56] we have observed a transition to the ultimate regime in 2D, namely at $Ra^* = 10^{13}$, similar as in the 3D RB experiments of Ref. [18]. The DNS of [56] have provided evidence that the mean velocity profiles follow the log-law of the wall, in analogy to other paradigmatic turbulent flows, e.g. pipe, channel, and boundary flows [35, 28, 42]. In Fig. 1, we show $Nu(Ra)$ compensated with $Ra^{0.33}$, for the range $Ra = [10^8, 10^{14}]$. Up to $Ra = 10^{11}$ (blue symbol), the effective scaling is essentially the same ($\beta \approx 0.29$) as has been already observed [23, 51, 50] in the classical regime where the BLs are laminar [55, 54]. This trend continues up to the transitional Rayleigh number $Ra^* = 10^{13}$ (green symbol). Beyond this, we witness the start of the transition to the ultimate regime, with a notably larger effective scaling exponent $\beta \approx 0.35$, as evident from the plateau in the compensated plot.

We next focus on the mean velocity field at the transitional $Ra$. Remarkably, even after 500 dimensionless time units, the flow domain still shows a stable mean roll structure, i.e. the rolls are pinned with clearly demarcated plume ejecting and impacting regions. The mean temperature and velocity fields display horizontal symmetry, which enables us to average them over a single LSR instead of the whole domain (as the velocity averaged horizontally for the whole domain will be zero). Figure 2 shows the temporally and spatially averaged velocity profiles, performed on one single LSR. We plot the profiles in dimensionless wall units, in terms of $u^+ = u/\nu$ and $y^+ = y|\tau/\nu|$. Here $u_\tau$ is the friction velocity $u_\tau = \sqrt{\langle \theta \rangle_{x,z=0}}$ [36]. Similar to channel, pipe, and boundary layer flows, we can identify two distinct layers: a viscous sub-layer where $u^+ = y^+$, followed by a logarithmic region, where the velocity profile follows $u^+ = 1/\kappa \ln y^+ + B_\kappa$ [36]. The inverse slope gives $\kappa = 0.4$, which is remarkably close to the Kármán constant in various 3D canonical wall-bounded turbulent flows [28, 42]. However, the parameter $B_\kappa$ varies with $Ra$. With increasing $Ra$, the logarithmic range grows in spatial extent, until at $Ra^* = 10^{13}$, it spans one decade in $y^+$.

We next explain how the global heat transport scaling can still undergo a transition to the ultimate regime, though only the local temperature profile is logarithmic, not the globally averaged one. We recall that by definition on the plate surface, $Nu = (-\partial_\theta A)$. We compute the local $Nu_{i}$ on the plate surface from ejecting ($Nu_{e}$) and impacting ($Nu_{i}$) regions separately. These are shown in Fig. 3, compensated by $Ra^{1/3}$. Up to $Ra^*$, both $Nu_{e}$ and $Nu_{i}$ follow a similar trend, with their respective local scaling exponents $\beta_{e}$ and $\beta_{i} < 1/3$. However, beyond $Ra^*$, $Nu_{e}$ and $Nu_{i}$ diverge. The ejecting regions show an increased heat transport, with $\beta_{e} = 0.38$, which is precisely the ultimate scal-
The global dimensionless torques, \( \Nu \sim Ta^\alpha \), for the four cases, with increasing \( Ta \) and fixed outer cylinder, are shown in Fig. 4a. Combining EXPs and DNSs, the range of Taylor number studied here extends over five decades. Similarly to what was shown in various recent studies \[18, 52, 32, 33, 4, 16\], for the SS case, an effective scaling of \( \Nu \sim T_a^{0.38 \pm 0.02} \) is observed in the DNS, corresponding to the ultimate regime with logarithmic corrections \[24, 13\]. A very similar scaling exponent \( \Nu \sim T_a^{0.39 \pm 0.01} \) is found in EXPs, demonstrating the excellent agreement between DNS and EXPs.

Dramatic enhancements of the torques are clearly observed with the introduction of wall roughness, resulting in the transition of \( \Nu \) from \( O(10^2) \) to \( O(10^3) \). Specifically, when only a single cylinder is rough, the logarithmic corrections reduce and the scaling exponents marginally increase, implying that the scaling is dominated by the single smooth wall. For the RR case, the best power law fits give \( \Nu \sim T_a^{0.50 \pm 0.02} \), both for the numerical and experimental data, suggesting that the logarithmic corrections are thoroughly canceled. This state with the scaling exponent 1/2 corresponds to the asymptotic ultimate turbulence predicted by Kraichnan \[24\]. The compensated plots of insets of \( \Nu/Ta^2 \) show the robustness and the quality of the scalings.

When expressing the relation between the global transport properties and the driving force in terms of the Reynolds number dependence of the friction factor \( c_f \), we obtain Fig. 4b. For the SS case, the fitting parameters \( a \) and \( b \) yield a von Kármán constant \( \kappa = 0.44 \pm 0.01 \), which is slightly larger than the standard value of 0.41 due to the curvature effect \[21, 34, 15\]. This agrees very well with the previous measurements on TC with smooth walls \[25\]. For the RR case, in both DNS and EXP, for large enough driving the friction factor \( c_f \) is found to be independent of \( Re_i \), but dependent on roughness height, namely \( c_f \sim 0.21 \) in the DNS and \( c_f \sim 0.23 \) in the EXP for roughness height \( h = 0.075d \), thus showing good agreement also for the rough cases. The results here are consistent with the asymptotic ultimate regime scaling 1/2 for \( \Nu \) and indicate that the Prandtl-von Kármán log-law of the wall \[38, 36\] with wall roughness can be independent of \( Re \), \[31, 38, 36, 22, 10\], which has been verified recently for Taylor-Couette flow \[58\]. For the RS and SR cases, one boundary is rough and the other is smooth such that the friction law lies in between RR and SS lines.

We further show the RR case with ribs of different heights, ranging from 1.5% to 10% of the gap width \( d \) in Fig. 4c, displaying its similarity with the Nikuradse \[31\] and Moody \[29\] diagrams for pipe flow. It can be seen that once \( h \geq 0.05d \) and \( Re_i \geq 8.1 \times 10^4 (Ta \geq 10^4) \), the asymptotic ultimate regime can always be achieved in both DNS and EXP.

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**References**


[40] See Supplemental Material at [URL will be inserted by publisher] for numerical details.


