Flow Past a Diamond Cylinder at Moderate Reynolds Numbers

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Abstract

Three-dimensional (3D) wake instabilities for flow past a diamond cylinder are investigated using direct numerical simulation. The neutral instability curve for mode A is mapped out, while that for mode B does not exist. Nevertheless, mode B flow structures are still captured in the fully developed 3D flows. The variations of the Strouhal number and drag coefficient with the Reynolds number are also presented.

Introduction

Steady incoming flow past a smooth and nominally two-dimensional (2D) bluff body is a classical problem in fluid mechanics. It is governed by a single dimensionless parameter, the Reynolds number Re (≡ UD/ν), which is defined based on the incoming flow velocity (U), the length scale of the cylinder perpendicular to the incoming flow (D), and the kinematic viscosity of the fluid (ν). For a circular cylinder the length scale is the diameter of the cylinder, while for a square cylinder aligned with sides perpendicular and parallel to the incoming flow (simply referred to as a square cylinder) the length scale is the side length of the cylinder. However, for a square cylinder aligned with all four sides 45 degrees to the incoming flow (referred to as a diamond cylinder), the length scale is \( \sqrt{2} \) times the side length of the cylinder. To distinguish the length scales for a square and a diamond cylinder, the length scale for a diamond cylinder is denoted as \( h \) in this study.

The wake instabilities for flow past a circular and a square cylinder have been studied extensively in the literature by using physical experiments and direct numerical simulations (DNS) (e.g. Williamson, 1996; Jiang et al., 2016, 2018), where two three-dimensional (3D) wake flow structures have been discovered with increasing Re.

(i) The first 3D wake flow structure is a relatively large-scale mode A flow structure, with a spanwise wavelength of approximately 4D for a circular cylinder and approximately 5D for a square cylinder, and an out-of-phase sequence between the neighbouring streamwise vortices.

(ii) The second 3D wake flow structure is a finer-scale mode B flow structure, with a spanwise wavelength of approximately 0.8D for a circular cylinder and approximately 1.1D for a square cylinder, and an in-phase sequence between the neighbouring streamwise vortices.

In addition to the physical experiments and DNS, Floquet stability analysis has also been adopted to identify the 3D wake instability modes based on time-periodic 2D base flows. Both mode A and mode B and their neutral instability curves have been reported for the cases of a circular and a square cylinder (e.g. Barkley and Henderson, 1996; Park and Yang, 2016).

In contrast, for flow past a diamond cylinder only a mode A wake instability has been reported by Sheard et al. (2009) through Floquet stability analysis, while both mode A and mode B are observed through physical experiments (Tong et al., 2008) and DNS (Sheard et al., 2009). This study will complement previous studies on flow past a diamond cylinder by investigating the neutral instability curves of mode A and mode B and the 3D flow characteristics.

Numerical Model

Numerical Method

In the present study, the flow around a diamond cylinder is solved with DNS. The governing equations are the continuity and incompressible Navier-Stokes equations:

\[
\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_i} (\rho u_i) = 0 \tag{1}
\]

\[
\frac{\partial u_i}{\partial t} + \frac{\partial}{\partial x_j} (\rho u_i u_j) = -\frac{1}{\rho} \frac{\partial p}{\partial x_i} + \nu \left( \frac{\partial^2 u_i}{\partial x_j \partial x_j} + \frac{\partial^2 u_j}{\partial x_i \partial x_j} \right) \tag{2}
\]

where \((x_1, x_2, x_3) = (x, y, z)\) are the Cartesian coordinates, \(u_i\) is the velocity component in the direction of \(x_i\), \(t\) is the time, and \(p\) is the pressure. The numerical simulations are carried out with an open-source code OpenFOAM (www.openfoam.org). The finite volume method and the PISO (Pressure Implicit with Splitting of Operators) algorithm (Issa, 1986) are adopted for solving the equations. The convection, diffusion and time derivative terms are discretized, respectively, using a fourth-order cubic scheme, a second-order linear scheme, and a blended scheme consisting of the second-order Crank-Nicolson scheme and a first-order Euler implicit scheme, respectively. The same numerical formulation has been used in Jiang et al. (2016, 2018) for the simulations of flow past a circular and a square cylinder.

Computational Domain and Boundary Conditions

The 2D and 3D simulations adopt a rectangular and a hexahedral computational domain, respectively. As sketched in Fig. 1(a), the centre of the diamond cylinder is located at \((x, y) = (0, 0)\). The computational domain size is \(-40 \leq x/L \leq 40\) in the streamwise direction and \(-40 \leq y/h \leq 40\) in the transverse direction. The blockage ratio in the transverse direction is 1.25%.

The boundary conditions are specified as follows. At the inlet boundary, a uniform flow velocity \(U\) is specified in the \(x\)-direction. At the outlet, the Neumann boundary condition (i.e., zero normal gradient) is applied for the velocity, and the pressure is specified as a reference value of zero. Symmetry boundary conditions are applied at the top and bottom boundaries, while periodic boundary conditions are employed at the two lateral boundaries perpendicular to the cylinder axis. For the lateral boundaries, Jiang et al. (2017a) showed that periodic boundary conditions are more suitable than symmetry boundary conditions in simulating fully developed
Table 1 lists the hydrodynamic forces on the cylinder at $Re = 255$ calculated with different meshes. The drag coefficient ($C_D$) and the Strouhal number ($St$) are defined as:

$$ C_D = \frac{F_D}{\frac{1}{2} \rho U^2 h / 2} \quad (3) $$
$$ St = \frac{f_\lambda h}{U} \quad (4) $$

where $F_D$ is the drag force on the cylinder, and $f_\lambda$ is the frequency of the fluctuating lift force. The time-averaged drag coefficient is denoted with an overbar. As shown in Table 1, the hydrodynamic forces calculated by the two variation cases are within 1% of those calculated with the reference mesh. Hence the reference mesh is used in the present study.

<table>
<thead>
<tr>
<th>Case</th>
<th>$St$</th>
<th>$C_D$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reference</td>
<td>0.1967</td>
<td>1.8970</td>
</tr>
<tr>
<td>Refined mesh</td>
<td>+ 0.76%</td>
<td>+ 0.81%</td>
</tr>
<tr>
<td>Doubled domain size</td>
<td>- 0.49%</td>
<td>- 0.92%</td>
</tr>
</tbody>
</table>

Table 1. Mesh independence check of the hydrodynamic forces on the cylinder at $Re = 255$. The results other than the reference case are shown by the relative differences with respect to those of the reference case.

The 3D mesh is constructed by replicating the 2D mesh along the $z$-axis with a spanwise cell size of 0.1$h$. The secondary wake instability of flow past a diamond cylinder is a mode A wake instability (Sheard et al., 2009). As will be shown in the next section, the critical $Re$ for the onset of mode A and the associated critical spanwise wavelength of mode A calculated in the present study are $(Re_{cr}, (\lambda h)_{cr}) = (120.7, 4.00)$. By using a spanwise cell size of 0.1$h$, 40 spanwise layers are used to capture one spanwise period of mode A structure. Such a spanwise mesh resolution is the same as those used in Jiang et al. (2016, 2018) for flow past a circular and a square cylinder. The spanwise domain length is 12$h$, namely three times the $(\lambda h)_{cr}$ value, which is also the same as those used in Jiang et al. (2016, 2018) for flow past a circular and a square cylinder.

**Numerical Results**

**Onset of Wake Instability**

For flow past a circular or a square cylinder, the neutral instability curves of the 3D wake instability modes have been predicted by both Floquet stability analysis (e.g. Barkley and Henderson, 1996; Park and Yang, 2016) and DNS (Jiang et al., 2017b, 2018). Floquet stability analysis is performed based on a perfectly time-periodic 2D base flow (Barkley and Henderson, 1996). However, it is noted that the development of the secondary vortex street in the far wake of a bluff body would result in flow irregularity in time (Kumar and Mittal, 2012). For the 2D base flow in the wake of a circular or a square cylinder, the irregular secondary vortex street develops at more than 30D downstream of the cylinder for the wake transition regimes of $Re \leq 300$, while the near-wake flow is time-periodic, such that Floquet stability analysis can be performed by choosing a computational domain excluding the far-wake irregularities.

However, our 2D simulations show that for flow past a diamond cylinder the irregular secondary vortex street develops closer to the cylinder, for example at $x/h = 14$ for $Re = 200$ and at $x/h = 7.5$ for $Re = 300$. The irregular development of the secondary vortex street would also result in flow irregularity to a certain distance upstream of the onset of the secondary vortex street (which may be very close to the cylinder), and consequently Floquet stability analysis could not be performed precisely. For example, Floquet stability
analysis was conducted by Sheard et al. (2009) for Re up to approximately 140, since Sheard et al. (2009) found that for Re ≥ 140 the 2D base flow became aperiodic. For Re ≤ 140, Sheard et al. (2009) identified a mode A wake instability at \( (Re_c, (\lambda/h)_c) = (116, 4.0) \).

To investigate the wake instabilities for Re ≥ 140, DNS has been adopted in the present study. The spanwise mesh resolution for one spanwise period of mode A is 20 layers of mesh. Fig. 2 shows the neutral instability curve of mode A for flow past a diamond cylinder calculated with DNS, determined through running simulations at small increments of Re around the neutral curve. The critical point at the left tip of the neutral curve is \( (Re_c, (\lambda/h)_c) = (120.7, 4.00) \).

![Neutral instability curve of mode A for flow past a diamond cylinder.](image)

A mesh convergence check of the Re_c value is carried out with three variations of the mesh (with a fixed \( \lambda/h \) of 4.00):

(i) A mesh refined in the x- and y-directions.

(ii) A mesh with a doubled computational domain size. In particular, the blockage ratio in the transverse direction reduces from 1.25% to 0.625%.

(iii) A mesh refined in the z-direction with doubled number of layers in the z-direction.

Table 2 shows the Re_c values predicted with different meshes, which are all within 1% of that predicted with the reference mesh. It is also noticed that the Re_c value reported in Sheard et al. (2009) is 3.89% smaller than the present result. Since a reduction in the blockage ratio from 1.25% (the reference case) to 0.0625% would result in a 0.75% increase in the Re_c value (see Table 2), it is speculated that the 3.89% decrease in the Re_c value by Sheard et al. (2009) is largely attributed to a relatively large blockage ratio of 3.54%.

<table>
<thead>
<tr>
<th>Case</th>
<th>Re_c (Relative difference)</th>
<th>Blockage ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reference</td>
<td>120.7</td>
<td>1.25%</td>
</tr>
<tr>
<td>Refined in the x-y plane</td>
<td>120.6 (− 0.08%)</td>
<td>1.25%</td>
</tr>
<tr>
<td>Doubled domain size</td>
<td>121.6 (+ 0.75%)</td>
<td>0.625%</td>
</tr>
<tr>
<td>Refined in the z-direction</td>
<td>120.7 (+ 0%)</td>
<td>1.25%</td>
</tr>
<tr>
<td>Sheard et al. (2009)</td>
<td>116 (− 3.89%)</td>
<td>3.54%</td>
</tr>
</tbody>
</table>

Table 2. Mesh independence check of the Re_c value.

In addition to the mode A wake instability, a mode B wake instability with a higher Re_c value and a smaller \( (\lambda/h)_c \) value than those of mode A has been discovered for flow past a circular and a square cylinder (e.g. Barkley and Henderson, 1996; Park and Yang, 2016). The critical point for the onset of mode B from a 2D base flow is \( Re_c = 259 \) for a circular cylinder (Barkley and Henderson, 1996) and \( Re_c = 201.4 \) for a square cylinder (Jiang et al., 2018). However, for flow past a diamond cylinder, the mode B neutral instability curve is not discovered in the present study in the area below the lower branch of the neutral curve for Re ≤ 280.

Three-dimensional Flows

Fig. 3 shows the Strouhal number and time-averaged drag coefficient for flow past a diamond cylinder for Re = 60 – 280. The 2D hydrodynamic forces are calculated based on the time-histories of a complete vortex shedding cycle of the fully developed flow, since the fully developed 2D flows are time-periodic. However, the fully developed 3D flows are irregular in time. For each 3D case, the simulation is run for approximately 500 non-dimensional time units (defined as \( t^* = Ut/D \)) to obtain the fully developed flow. After that, approximately another 500 non-dimensional time units of the fully developed flow are used to calculate the fully developed hydrodynamic forces on the cylinder. For 3D flows, \( f_s \) in equation (4) is determined as the peak frequency derived from the fast Fourier transform (FFT) of the time-history of \( C_L \).

As shown in Fig. 3, beyond the onset of the mode A instability at \( Re_c = 120.7 \), the 3D results are slightly smaller than their 2D counterparts due to the flow three-dimensionality. Some typical 3D flow structures are shown in Fig. 4. At Re = 125, ordered mode A structures are observed at early stages of the simulation (Fig. 4a). However, the ordered mode A is an unstable state. With the evolution in time, vortex dislocations would appear in the entire wake region (Fig. 4b). This feature is similar to the case of flow past a circular and a square cylinder (Williamson, 1996; Jiang et al., 2018). For Re ≥ 200, mode B structures are observed in the wake (Fig. 4c,d), albeit the inexistence of the neutral instability curve of mode B for Re ≤ 280.

![Variations of the hydrodynamic forces with Re for flow past a diamond cylinder.](image)
Fig. 4: Instantaneous vorticity fields in the near wake of a diamond cylinder for (a) $Re = 125$ and $t^* = 400$ (with ordered mode A structures), (b) $Re = 125$ and $t^* = 900$ (with disordered mode A having vortex dislocations), (c) $Re = 200$ and $t^* = 1000$ (with mode B structures), and (d) $Re = 280$ and $t^* = 900$ (with mode B structures). The translucent iso-surfaces represent spanwise vortices with $\omega_z = \pm 1.0$, while the opaque iso-surfaces represent streamwise vortices with $\omega_x = \pm 0.5$ for graphs (a) and (b) and $\omega_x = \pm 1.0$ for graphs (c) and (d). Dark grey and light yellow denote positive and negative vorticity values, respectively. The flow is from left to right past the cylinder on the left.

Conclusions

This paper presents a DNS study of the 3D wake instabilities and flow characteristics in the wake of a diamond cylinder. Due to the aperiodicity of the 2D base flow, the neutral instability curve of mode A is predicted with DNS rather than Floquet stability analysis. The critical point for the onset of mode A is $(Re_{cr}, (\lambda/h)_{cr}) = (120.7, 4.00)$. It is also found that the mode B neutral instability curve does not exist for $Re \leq 280$. Nevertheless, mode B flow structures are still captured in the fully developed 3D flows for $Re \geq 200$.

Acknowledgments

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References