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Direct Numerical Simulation of Two-Dimensional Stratified Gravity Current Flow with Varying Stratification and Aspect Ratio

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Abstract

Direct numerical simulations (DNSs) of two-dimensional stratified gravity current are carried out using OpenFOAM. Three different aspect ratios, h_0/l_0 (where h_0 is the height of the dense fluid and l_0 is the length of the dense fluid) are simulated with stratification strength, S=0.2, 0.5, and 0.8. Reynolds number is kept constant at Re=4000. For the equivalent h_0/l_0 ratio, the stratification strength reduces the front velocity, u_f and results in a shorter front location, x_f-l_0. Stratification strength has a strong effect on the size and shape of the gravity current. Gravity current with aspect ratio, h_0/l_0 of 0.25 propagating in the stratified ambient of S=0.2 and 0.5 experienced two acceleration phases. Qualitiative analysis of contours ranging from subcritical to supercritical flow in comparison to the flow structures was presented in this study. Subcritical flow was smooth and minimal vortices formed in the head of the gravity current was obtained in all the cases with strong stratification strength, S=0.8. Supercritical flow occurs in weak stratification with h_0/l_0=1 and 4. Strong vortices formed in the head of the gravity current followed by Kelvin-Helmholtz (K-H) billows at the back of the head. The propagation of the internal waves was faster than the gravity current flow in the subcritical regime. Case h_0/l_0=4 with S=0.5 had the Froude number closest to the critical regime (Fr=0.324 $\approx 1/\pi$) and can be deemed to be in the critical regime.

Introduction

Gravity current or buoyancy current, is a type of flow that occurs due to the density difference of two fluids. Gravity current is observed in many naturally occurring phenomena such as sandstorms, powder-snow avalanches, pyroclastic flow and haboob. A comprehensive review of gravity current flow in geophysical flows and laboratory experiments can be referred in Simpson[1].

Gravity current can be categorized into two types which are compositional gravity current (can be Boussinesq, where the density differences between two fluids is less than 5% or non-Boussinesq gravity current) and particle-driven gravity current (where the density differences is caused by the presence of sediments in the dense fluid.). The gravity current flow has been widely researched with a variety of lock configurations such as lock shape[8] (elliptical release[12, 9] and circular release[11]), lock depth ratio, and initial aspect ratio. The flow regime of the gravity current flow is determined by the Froude number (Fr) which is a dimensionless parameter defined as the ratio of the inertial forces relative to the gravitational forces.

The purpose of this study is to investigate the propagation of the gravity currents into a stratified ambient. Experimental and numerical study have been conducted by Maxworthy et.al [3] where the relationship between Froude number (Fr) and the density ratio (R) were studied. In subcritical regime ($Fr<1/\pi$), the gravity current flow propagates slowly, whereas in the supercritical regime ($Fr>1/\pi$), the gravity current flow becomes turbulent. The main difference between the subcritical flow regime and supercritical flow regime is the turbulence structures behind the head of the gravity current. A similar study has been conducted by Ungarish, et.al [5] using shallow-water approximation and the validation of the results have an excellent agreement with the experimental results presented by Maxworthy et.al [3]. It is worth noting that the propagation speed of the gravity current is reduced by the stratification ambient compared to the homogeneous ambient (where the density of the ambient fluid is constant throughout the domain.). Birman et al. [2] examined the theory developed by Ungarish (2006) [6] (where the Benjamin's (1968) [4] theory is generalized to a linearly stratified ambient) on the dependency of the front velocity on the stratification strength (S) and was found to work well for the weak stratification ($S\leq0.5$). Ungarish et al. (2008) [7] analyzed the energy-balances of a axisymmetric gravity current in the homogenous ambient and linearly stratified ambient with a two-dimensional geometry.

The purpose of the study is to investigate the effects of the initial aspect ratio and the strength of stratification on the dynamics of the gravity current flow. Two-dimensional stratified gravity current has been simulated and the front location and front velocity of the stratifed gravity current and qualitative analysis of the contours will be used to compare the flow structures of the three different flow regimes (subcritical, critical, and supercritical).

Computational Setup

The two-dimensional stratified gravity currents were simulated using OpenFOAM with the solver *twoLiquidMixingFoam*. The governing equations employed in the study take the form

$$\nabla \cdot \mathbf{u} = 0 \tag{1}$$

$$\rho \frac{Du}{Dt} = \rho \mathbf{e^g} - \nabla p + \frac{1}{Re}\nabla^2 \mathbf{u} \tag{2}$$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = \frac{1}{ScRe}\nabla^2 \rho \tag{3}$$

where the ρ, $\mathbf{u}$=[u,v], and p are the density of the fluid, velocity vector, and pressure respectively. $\mathbf{e^g}$ is the unit vector for the direction of gravity. The dimensionless density (ρ) is defined as

$$\rho = \frac{\rho^* - \rho_0^*}{\rho_c^* - \rho_0^*} \tag{4}$$

where ρ^*, ρ_0^*, and ρ_c^* are the density of the local, ambient and dense fluid respectively. The variables with asterisk are dimensional. The value of ρ is bounded between 0 and 1.

Sc=ν^*/κ^* is the Schmidt number (where ν^* is the kinematic viscosity and κ^* is the molecular diffusivity) and Reynolds number, Re=U^*L^*/ν^* is based on the characteristic length. Saline (salt water) has Sc=700 but it is found that when Sc is in the order of 1, there is a weak scaling with the dynamics of the gravity current [10]. The velocity scale, U^*, the characteristic length L^*, and characteristic time T^* are defined as

$$U^* = \sqrt{\frac{g^*(\rho_c^* - \rho_0^*)}{\rho_0^*} L^*}, \quad L^* = (V_0^*)^{\frac{1}{2}}, \quad T^* = \frac{L^*}{U^*} \tag{5}$$

where g^* is the gravitational acceleration, $V_0^* = l_0^* \times h_0^*$ is the initial volume per spanwise unit of the dense fluids (ρ_c) and T^* is the time scale.

The magnitude of stratification (S) can be determined by calculating the reduced density differences of the bottom fluid with the ambient fluid (ε_b) and the lock fluid with the ambient fluid (ε).

$$S = \frac{\varepsilon_b}{\varepsilon}, \quad g\prime = \varepsilon g \tag{6}$$

where $g\prime$ is the reduced gravity and g is the gravitational acceleration. The reduced density differences of the bottom fluid (ρ_b) with the ambient fluid and the lock fluid with the ambient fluid (ρ_0) is defined as

$$\varepsilon_b = \frac{\rho_b - \rho_0}{\rho_0}, \quad \varepsilon = \frac{\rho_c - \rho_0}{\rho_0} \tag{7}$$

where ρ_b is the density at the bottom, ρ_0 is the density at the top, and ρ_c is the density of the dense fluid. The equation takes the form of

$$\rho_c = \rho_0(1+\varepsilon), \quad \rho_a(y) = \rho_0\left[1 + \varepsilon S\left(1 - \frac{y}{L_y}\right)\right] \tag{8}$$

where ρ_a is the density with respect to the height, y and L_y is the height of domain in wall-normal direction.

The second-order *backward* implicit time scheme was used for integration of time and divergence *vanLeer* scheme used to discretise the convection terms in the transport equation (3). The *twoLiquidMixingFoam* solver has been validated with a test case from Dai [10] who used pseudospectral code to conduct the simulation. The front velocity and energy budgets results from OpenFOAM have a good agreement with the case with a slope angle, θ of 9° and depth ratio, $\phi = h_0/L_y$ of 0.16 where L_y is the wall-normal length of the domain. The stratified ambient is added to the domain by using the *funkySetFields* utility. A test case (case 19, subcritical) from Maxworthy et al. [3] is used to validate with Fr=0.2556 is close to the Fr of 0.26 reported by [3]. The configuration of the domain is consistent with a test case where S=0.2 and h_c=0.5 from Birman et al. [2]. The contour and the propagation speed from OpenFOAM have a good agreement with [2].

Figure 1 shows the sketch of the computational domain with the streamwise length L_x=20 and the wall-normal length L_y=4. No-slip boundary condition is employed for top and bottom (streamwise) of the walls, however, slip or symmetry boundary condition is applied at the vertical side walls ($\Delta v/\Delta x$=0, u=0). The lock with a density of ρ_c inside a rectangular channel containing a linearly stratified ambient with an aspect ratio of h_0/l_0 (where h_0 is the height of the lock and l_0 is the length of the lock).

Three aspect ratio of the lock, h_0/l_0=0.25, 1, and 4 with S=0.2, 0.5, and 0.8 are simulated in this study. Re=4000 is used for all simulations. The computational grid domain is equally spaced and has a streamwise length of $\Delta x \approx 0.0078$ and wall-normal length of $\Delta y \approx 0.0156$. This grid resolution is sufficient to resolve all of the turbulent length scales and is smaller than the requirement of $\Delta x \approx (ReSc)^{-1/2}$ [10].

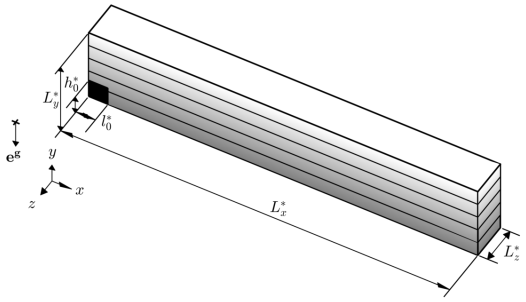

Figure 1: Sketch of the initial condition of the simulation. The black represents the dense fluid (ρ_c) with h_0 and l_0 are the height and length of the dense fluid. The density of the fluid at the top, ρ_0 (white) is increased linearly to ρ_b (grey) at the bottom of the domain where L_x, and L_y are the streamwise length and wall-normal length of the domain. L_z is the unit depth of the domain required for the two-dimensional simulations.

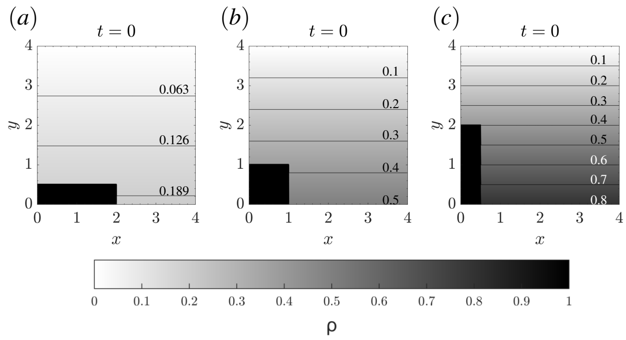

Figure 2: Contour of the gravity current flow propagating in the stratified domain where h_0/l_0 of (*a*) 0.25 with S=0.2, (*b*) 1 with S=0.5, and (*c*) 4 with S=0.8. The dense fluid is represented in black, where ρ=1. The density of the fluid at the top, ρ=0 (white) increases linearly to ρ=(*a*) 0.2, (*b*) 0.5, and (*c*) 0.8 at the bottom (light-grey to grey) respectively. Black contour lines indicate the interval of the stratification where the numbers of contour line increases as the stratification strength increases.

Results

Figure 3 shows the front location plot for the cases with S=0.2, 0.5, and 0.8 with different aspect ratio, h_0/l_0. The wall-normal averaged streamwise gradient of density of the gravity current is used to determine the front of the gravity current. The front location increases at a slower rate with increasing stratification strength and decreasing aspect ratio. At t=20, the front location for the case with h_0/l_0=4 with S=0.8 is about 36% smaller than the same case with S=0.2. A larger drop 47% in the front location is found for case h_0/l_0=0.25, clearly indicating that the strength of the stratification greatly affects the propagation of the gravity current.

The front velocity of the gravity current is determined by differentiating the front location of the gravity current with time (where $u_f = d(x_f - l_0)/dt$). Figure 4 shows the front velocity against time plot for the cases with S=0.2, 0.5, and 0.8 with different aspect ratio, h_0/l_0. For the case h_0/l_0=0.25 with S=0.2 in Figure 4(a) and S=0.5 in Figure 4(b), the gravity current experiences first acceleration phase, followed by short deceleration then undergoes a second acceleration phase, and lastly decelerates. The second acceleration is due to the interaction between the primary and secondary vortices which pushes dense fluid to the front of the gravity current [13]. The maximum front velocity of the gravity current decreases as the stratification increases.

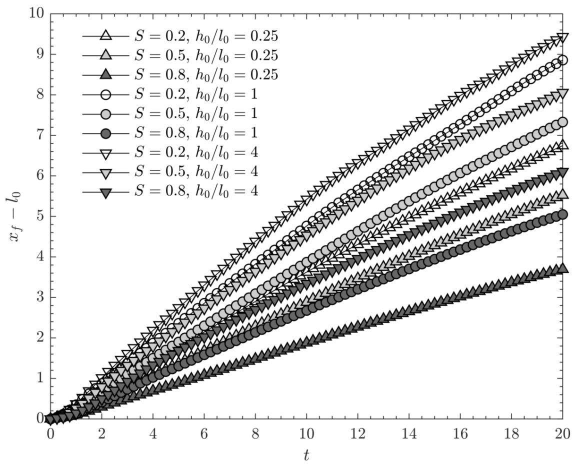

Figure 3: Plot of the front location against time for gravity current with h_0/l_0 of 0.25, 1, and 4 with S=0.2, 0.5, and 0.8. White marker represents stratification of 0.2, light-grey marker represents stratification of 0.5 and dark-grey marker represents stratification of 0.8.

This phenomenon is obtained in the study conducted by Ungarish et al.[5] who used shallow-water approximation to study the propagation of gravity current in the stratified ambient.

Figure 5 shows the contour of the gravity current flow with h_0/l_0 of 0.25 and S=0.8. Figure 5(a) shows the slumping of the gravity current due to gravity at t=1.5. Two small vortices are formed at the top and bottom of the head of the gravity current. The propagation of the dense fluid initiates the movement of the internal waves above the head at y=0.5. At t=5 (Figure 5(b)), the head of the gravity current is located at $x_f \approx 3$. At the same time, the position of the internal waves lies between $4<x<5$ at y=0.5 and $5<x<5.5$ at y=1. As the head of the gravity current reaches $x_f \approx 4$ at t=10 (Figure 5(c)), the propagation of the internal waves lies between y=0.5 reached $6<x<6.5$ and $7<x<7.5$ at y=1. Figure 5(d) shows the location of the front of the gravity current at $x_f \approx 5$ at t=15. The propagation of the internal waves on the other hand, is located at $x \approx 8$ at y=0.5 and $8<x<9$ at y=1. The motion of the waves can be seen in the line contour.

The Froude number, Fr is a dimensionless parameter defined as the ratio of inertial forces relative to the gravitational forces and is calculated by

$$Fr = \frac{u_{f\,\mathrm{mean}}}{NL_y}, \quad N = \sqrt{\frac{g(\rho_b - \rho_0)}{\rho_0 L_y}} \tag{9}$$

where $u_{f_{mean}}$ is the mean of the front velocity and L_y is the height of the domain, g is the gravitational acceleration, ρ_b is the density at the bottom and ρ_0 is the density at the top of the domain

The quasi-steady front velocity of gravity current is employed to calculate Froude number, Fr where it is indicated with a horizontal black line in Figure 4(a) to 4(c) respectively. Table 1 shows the flow regime of the gravity current. Maxworthy et al. [3] reported that the flow is in the subcritical regime when $Fr<1/\pi$ and supercritical when $Fr>1/\pi$. In this study, gravity current cases in weak stratification ambient, S=0.2 with the aspect ratio, h_0/l_0=1 and 4 is in the supercritical regime. Subcritical flow is obtained in all the cases with strong stratification strength, S=0.8. Critical flow is obtained when $Fr=1/\pi$ but it is difficult to predict a simulation case that can produce the exact value of $Fr=1/\pi$. Case h_0/l_0=4 with S=0.5 has the Froude number closest to the crtical regime (Fr=0.324 $\approx 1/\pi$)

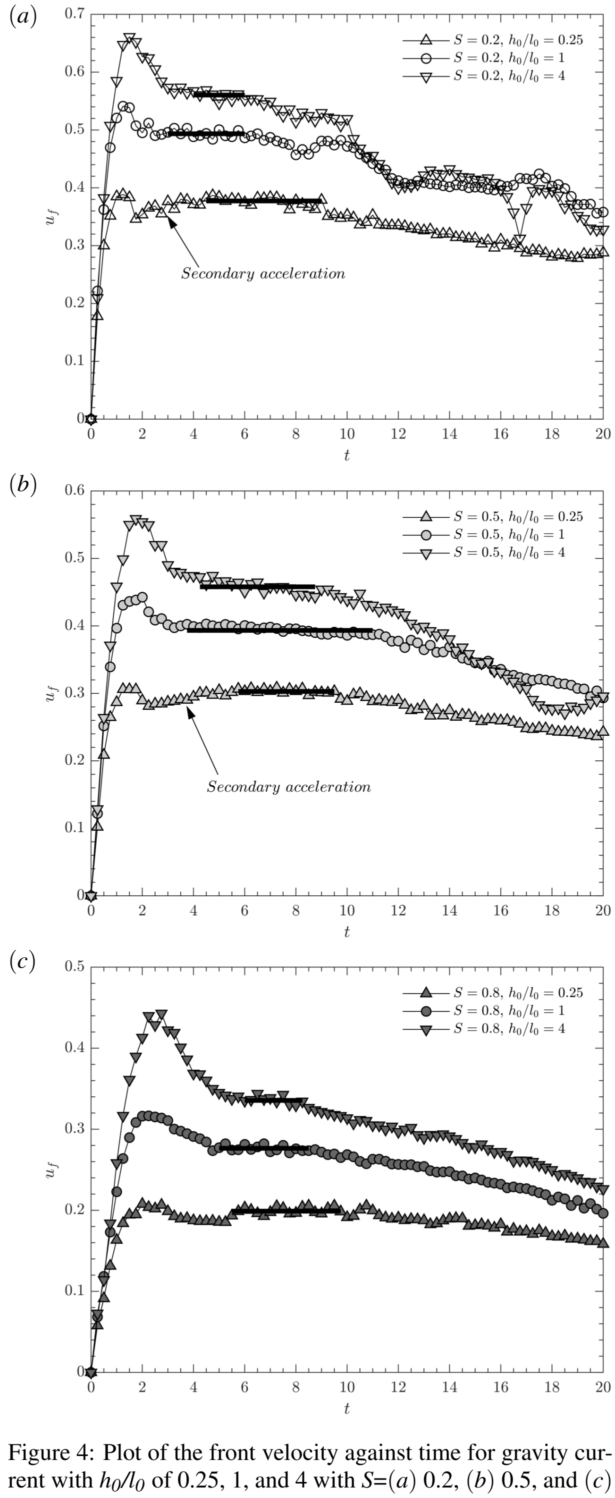

Figure 4: Plot of the front velocity against time for gravity current with h_0/l_0 of 0.25, 1, and 4 with S=(a) 0.2, (b) 0.5, and (c) 0.8. The white marker represents stratification of 0.2, light-grey marker represents stratification of 0.5 and dark-grey marker represents stratification of 0.8. The black horizontal line indicates the mean of the front velocity, $u_{f_{mean}}$.

and can be deemed to be in the critical regime. In subcritical regime, the gravity current contains minimal (or no) vortices in the gravity current. The gravity current is smooth and the internal waves which form in the flow propagate faster than the head of the gravity current as shown in Figure 5 (the arrows indicate the position of the waves). When the gravity current is in the supercritical regime, the gravity current becomes turbulent. Strong vortices presented in the head of the gravity current and

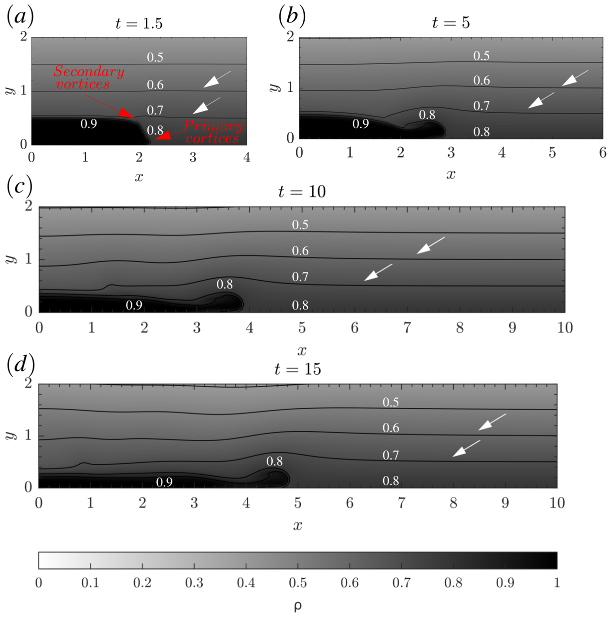

Figure 5: Contour of the gravity current propagating in the stratified domain with h_0/l_0=0.25 and S=0.8 at t=(a) 1.5, (b) 5, (c) 10, and (d) 15. The white arrows show the propagation of the internal waves.

Kelvin-Helmholtz (K-H) billows are formed behind the head of the gravity current. Based on the simulation results, subcritical flow occurs in all the cases with strong stratification strength and supercritical flow occurs in weak stratification for the large h_0/l_0 ratios simulated in the study.

AR \ S	0.2	0.5	0.8
0.25	subcritical (Fr=0.2667)	subcritical (Fr=0.2142)	subcritical (Fr=0.1406)
1	supercritical (Fr=0.3489)	subcritical (Fr=0.2782)	subcritical (Fr=0.1955)
4	supercritical (Fr=0.3963)	critical (Fr=0.3238)	subcritical (Fr=0.2373)

Table 1: Results of the flow regime with varying aspect ratio and stratification

Conclusions

Direct numerical simulations (DNSs) of a two-dimensional stratified gravity current is simulated with varying aspect ratios and the stratification strengths using OpenFOAM. The front location and front velocity plot shows that as the stratification strength increases, the propagation speed of the gravity current decreases. The size and shape of the head became smaller as the stratification strength increases and Kelvin-Helmholtz (K-H) billow behind the head of the gravity current is reduced or is completely suppressed. For the aspect ratio, h_0/l_0=0.25 with S=0.2 and 0.5, the gravity current flow experiences two acceleration phases. The occurrence of the second acceleration is due to the interaction between the secondary vortices behind the head of the gravity current and the primary vortices inside the head. Subcrtical and supercritical flow is presented in the simulation. Subcritical flow is obtained in all the cases with strong stratification, S=0.8 and supercritical flow is obtained in the cases h_0/l_0=1 and 4 with weak stratification, S=0.2. In the subcritical regime, the gravity current contained minimal (or no) vortices in the head. The formation of internal waves which propagate faster than the gravity current occur when the flow is in the subcritical regime. When the gravity current is in the supercritical flow, strong vortices are formed in the head of the gravity current followed by Kelvin-Helmholtz (K-H) billows behind the gravity current.

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