21st Australasian Fluid Mechanics Conference
Adelaide, Australia
10-13 December 2018

A framework for understanding the flow around groups of cylinders

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Abstract

The interaction between fluid flow and an array of cylinders in a tandem arrangement with $Re = UD/\nu = 200$ (where U is the free stream velocity, D is the diameter of the cylinder and ν is the kinematic viscosity) is studied numerically in this paper. All the cylinders are rigidly mounted and equal in size and the pitch (p = distance between centre of cylinders) is consistent between any two consecutive cylinders. The sharp interface immersed boundary method is used to simulate the fluid and cylindrical structures in two dimensions. In the first step of this study, the two-cylinder system with varying pitch, $1.1 \leq p \leq 10$ is studied. There are four distinct regimes as a function of p, delineated by the maximum lift coefficient and mean drag coefficient data. In the second step of this study, one representative pitch, $p = 6.0$, is selected and the fluid-structure interaction problem is extended systematically by adding further cylinders in the wake. The contours of streamwise mean flow velocity for two to five cylinders show that there is a region in which the flow is convectively unstable. This region starts after interaction of fluid with the second cylinder and its length is dependent on the number of cylinders. The length of the convective region increases when the third cylinder is added to the system but it follows by a decrease when adding further cylinders in the wake.

Introduction

The flow past a single cylinder has been a canonical topic for many years due to its practical importance in many engineering applications such as bridges, heat exchangers, chimneys and marine platforms. Fluid flow over a cylinder with $Re > 45$ can cause vortices to be shed at a distinct frequency alternatively from each side of the structure, and the resulting pattern is named the Von Kármán vortex street. This phenomenon is the source of oscillating forces in the cross-flow and streamwise directions. One of the parameters that describes the properties of this oscillating flow and vortex formation phenomena is the Strouhal number $St = fL/U$ (Where f, L and D are vortex shedding frequency, characteristic length that is taken as the cylinder diameter D and U is free stream velocity) which is the function of the single parameter, Re [1].

However there is a wide range of flow applications where multiple cylindrical structures are in close proximity. Research for such systems is less developed and the study of this phenomenon in multi-structure systems has became more popular recently. When the cylinders are placed in the wake of each other, identifying flow properties becomes more complicated. Hence, fundamental investigation of how the flow changes in multi-cylinder systems are not fully and well studied [2].

The simplest first step to extend studies from one cylinder towards arrays is the study of a two-cylinder system. It is worthwhile to note that p and Re are the key parameters in determination of the wake flows [3].

When cylinders are in a tandem arrangement, the line connecting the centre of cylinders is parallel with the free stream, the downstream cylinder is exposed to the flow seperated from the upstream cylinder, i.e, mutual interference happens. This mutual interference means that the presence of the upstream cylinder changes the incoming flow characteristics to its wake [4] and the next cylinder interferes with the wake characteristics and vortices shed from the upstream cylinder [2].

Zdravkovich was a pioneer in the study of two-cylinder systems and conducted a comprehensive experimental study to find the effect of p in different configurations [5]. According to the high Re experimental results of this study, four distinct regimes as a function of p for the flow in the gap between two cylinders was recorded when cylinders are in tandem arrangement. The flow behavior in the gap between two cylinders varies for each regime. In the first regime, when p is very small, the seperated flow from the first cylinder reattaches to the next cylinder and two cylinders behave as single body. In the second regime, there is alternate reattachment in the gap. A quasi-steady reattachment can be seen in the gap between two cylinders in the third regime. In the fourth regime, a clear vortex shedding can be seen in the gap.

Ohya [6] presented experiments with Reynolds number 80 to 2.3×10^5. The focus was on drag coefficient measurements in a system with two cylinders in tandem arrangement and varying p. He found a critical spacing of approximately 3.6 in which the drag coefficient dramatically increased.

Xu & Zhou [7] conducted experiments with $Re = 8004.2 \times 10^4$ and $p = 1 - 15$ and classified the flow to four different regimes for constant Reynolds number. The proposed threshold pitches to move to another flow regime in this study are different with the reported data from [5] but the main features of the regimes are similar. The experimental results also showed that, once the vortex formation starts in the gap between two cylinders, both vortex streets shed in the same frequency and they correspond to each other.

Liu et al. [8] investigated the laminar flow ($Re = 200$) over two identical and in-line cylinders spacing ratios of $1.1 < p < 7$, numerically. They classified flow into two and three regimes based on the hydrodynamic jumps in the drag and lift forces on the cylinders in terms of p. It was stated that the hydrodynamic jumps are induced by changes in the flow pattern.

Igarashi & Suzuki [9] expanded the study to a three-cylinder system in tandem arrangement with $1 \leq p \leq 4$ at $Re = 2.2 \times 10^4$. Similar regimes as the two-cylinder system were observed for three-cylinder system.

Harichandan & Roy [10] investigated three cylinders in tandem arrangement with pitch $= 2$ and 5 at $Re = 200$ numerically. The presented results indicated that although the separated shear layer from the upstream cylinder reattaches to immediate downstream cylinder in pitch $= 2$, a Kármán vortex street is observed between the cylinders when the pitch is increased to 5. Comparison of results with single cylinder data demonstrated that along with the increase in pitch, the measured parameters of the upstream cylinder get closer to the single cylinder system; while the effect of flow impingement on the downstream cylinder was confirmed because of its difference with the single cylinder system.

Multi-cylinder systems are important not only from fluid dynamics perspective, but also in thermodynamics. Aiba & Yamazaki [11] also studied a three-cylinder system with $1 \leq p \leq 5$ but from thermodynamic and heat transfer perspectives. It was found that the strength of vortex shedding from the first cylinder has the highest influence in heat transfer when $1.3 \leq p \leq 3.8$.

Igarashi [12] investigated the flow pattern of four in-line cylinders with $1 \leq p \leq 2.65$ and $8.72 \times 10^3 \leq Re \leq 3.92 \times 10^4$. The experimental results showed that the flow pattern in each gap can differ by changing the p and Re.

Hetz, Dhaubhadel & Telionis [13] investigated a five-cylinder system in different configurations such as adding a splitter plate behind the last cylinder and using stiff cardboard for taping the top and bottom of the array. Different patterns of vortex formation were recorded as a result of this experiment.

This previous research has identified flow patterns or aerodynamic forces on structures. However, a missing point in the studies of multi-cylinder systems is to investigate the effect of adding extra cylinders on the system properties. This is the focus of this study.

The problem setup is shown in Figure 1:

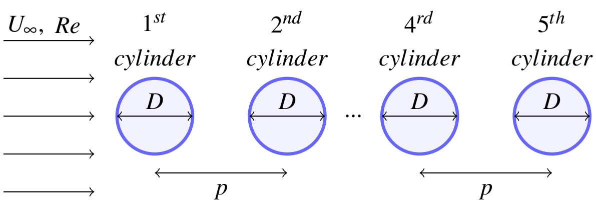

Figure 1: Schematic description of the present study.

Methodology

The sharp interface immersed boundary method is utilized to simulate the interaction of fluid and multiple cylinders numerically in two dimensions. In this method, the cylinders are modeled by a Lagrangian set of finite immersed elements in an underlying Cartesian grid. Details and validation of the method can be found in [14] and its implementation are provided in [15], and only a brief overview is provided below.

The incompressible Navier-Stokes equations govern the motion of the fluid and are shown in equation 1:

$$\frac{\partial \mathbf{u}}{\partial \tau} = -(\mathbf{u}\cdot\nabla)\mathbf{u} - \nabla P + \frac{1}{Re}\nabla^2\mathbf{u} + \mathbf{A}_b, \qquad \nabla\cdot\mathbf{u} = 0, \tag{1}$$

where $\tau = tU/D$, $\mathbf{u}$, P and $\mathbf{A}_b$ are the non-dimensionalised time by the advective time scale, the non-dimensionalised velocity field by the free-stream velocity U, the non-dimensionalised pressure field by ρU^2, and a generic acceleration term that models the presence of an immersed boundary, respectively.

A two-way time-splitting scheme in employed to integrate the equations forward in time. Initially, the advection and diffusion terms (the first and third terms in equation (1) respectively) and the acceleration term are integrated to an intermediate time between the start and end of the time step, over a first "sub-step".

In the second stage, the pressure term is integrated from this intermediate time to the end of the step, over a second sub-step. By enforcing continuity at the end of this step, a Poisson equation for the pressure correction (the change in pressure since the previous timestep) is formed. Once this is solved, the second substep equation can be solved to obtain the velocity field at the end of the timestep.

For the velocity field, a Dirichlet condition is applied upstream ($u = 1, v = 0$), a Neumann zero-stress conditions at the lateral boundaries ($\partial\mathbf{u}/\partial y = 0$), and a Neumann condition ($\partial\mathbf{u}/\partial x = 0$) at the outlet. For the pressure, a Neumann condition was applied at the upstream and lateral boundaries ($\partial\mathbf{u}/\partial\mathbf{n} = 0$), and a Dirichlet condition ($p = 0$) at the outlet. At the cylinder surface, a no-slip condition was applied or the velocity field, and a Neumann condition $\partial p/\partial\mathbf{n}$ applied to the pressure.

Results

The base investigation starts with simulating the interaction of two rigidly-mounted cylinders and fluid with $Re = 200$ and varying p. The results of aerodynamic forces including maximum lift coefficient and mean drag coefficient as a function of p are presented in figure 2:

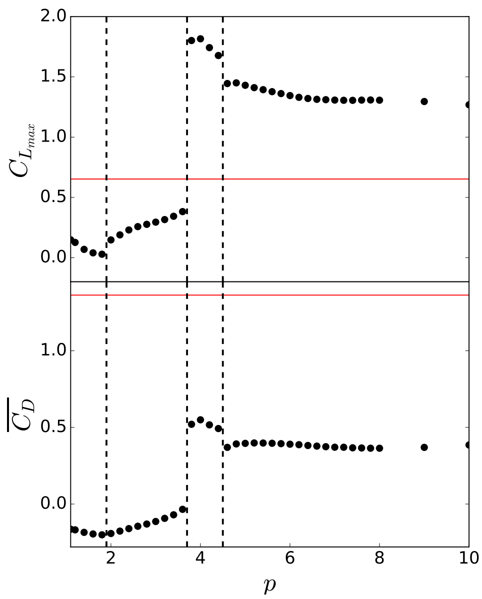

Figure 2: The maximum lift coefficient, $C_{L_{max}}$ (top), and mean drag coefficient, $\overline{C_D}$ (bottom), on the rear cylinder of two-cylinder system as a function of p. The red line is representative of the corresponding force for a single-cylinder system. The dashed lines are used to separate the regimes.

There are four distinct regimes as a function of p. These regimes match with regimes recorded in the high Re and experimental studies [5]. By moving from one regime to another, there is a significant change in value or trend of both forces simultaneously. The critical pitch, at which the vortex shedding starts in the gap (third regime) is $p_{cr} = 3.8$. At this p_{cr} the third regime starts and a significant change in aerodynamic forces is evident. This result accords with the critical pitch in previous studies [6].

The red lines in figure 2 represents the corresponding parameter for an isolated cylinder. It can be seen that the parameters converge to a value which is different from the values in single cylinder system. The drag coefficient on the second cylinder is considerably less than this parameter in single-cylinder system. Lee & Basu delineated the upstream and downstream cylinders as turbulence generators for drag reduction or “wake stabilizers” in the interference in the two cylinder system [16].

Importantly, when $p < p_{cr} = 3.8$ there is no vortex formation in the gap, while this phenomenon can be seen when $p \geq p_{cr} = 3.8$.

In the next step, one representative pitch from the last regime with vortex formation in the gap is selected to extend the study and find the effect of adding additional cylinders to the system.

The streamwise velocity contours of the mean flow for two to five cylinders are presented in figure 3.

In this figure, the region with blue color shows the negative velocity which is actually a recirculation region. The recirculation region behind the fist cylinder is identical in all the one- to five-cylinder systems. This result concludes that the properties for the first cylinder converges to single cylinder properties when the second cylinder is far enough and presence of additional cylinders in the wake, does not affect the single cylinder and the wake behind it.

On the other side, the properties including recirculation length are dependent on the number of cylinders. The long recirculation region formed in the mean flow represents the region of flow which is apparently convectively unstable. In convectively unstable flow, the effect of perturbation in a system moves only downstream [17]. This can be the reason why adding extra cylinders downstream does not affect the cylinders in convectively unstable region. Further evidence for a convective instability comes from the frequency selection of the flow. Convectively unstable flows act as amplitifiers - they respond at the same frequency that is supplied, and they do not generate a specific characteristic frequency of their own [17]. Particularly, in this study the imposed frequency to the convectively unstable region behind the second cylinder is the vortex shedding frequency in the first gap. Hence, the frequency in the first gap and the convectively unstable region locks to the vortex shedding frequency of the first gap.

This can explain why [7] measured similar vortex shedding frequencies in the wake of both cylinders in a two-cylinder system.

As it can be seen in figure 3, the L_C is dependent on the number of cylinders. The L_C as a function of number of cylinders in the system is shown in figure 4.

The L_C has the minimum value of 6.84 in two-cylinder system when $p = 6.0$. By adding the third cylinder system, this length dramatically increases and reaches $L_C = 7.90$. By adding the the next cylinders, the length decreases and becomes $L_C = 7.13$ in the five-cylinder system. Although the L_C is changing in different number of cylinders at $p = 6.0$, as its value is higher than p, hence it covers the entire second gap and some part of third cylinder’s wake.

These results indicate that in these cylinder arrays, it is likely that the flow will be synchronized (responding at the same, single frequency) at least over a distance of 15D from the front cylinder. Further work is required to establish whether this frequency locking continues if other bodies are added in this region, for instance by placing another cylinder, or cylinders, between the current 2nd and 3rd cylinders.

Conclusion

The numerical results for the interaction of two cylinders with varying p and $Re = 200$ indicates four distinct regimes. The maximum lift and mean drag coefficients experience a sudden change in value or have different trend by moving from one regime to another. The onset of vortex shedding in the gap is cocurrent with $p_{cr} = 3.8$ in this system where there is a jump in aerodynamic forces.

The study is extended by adding more cylinders in the wake of two cylinder system at a specific pitch, $p = 6.0$. The streamwise velocity contours of mean flow for the two- to five-cylinder systems indicates a convectively unstable flow behavior behind the second cylinder. The length of this region is variable and is the function of number of cylinders. At $p = 6.0$ the length is long enough to cover a small region behind the third cylinder. By adding the third cylinder to the system, the length of this region increases while adding the fourth cylinder results in decrease of the length of this region. Although adding more cylinders to the wake changes the wake and the convective length behind the second and third cylinders, it does not affect the recirculation behind the first cylinder and it can be an indication for lack of feedback in this part of the system.

Acknowledgement

This work has been financially supported by the Australian Research Council Discovery Projects scheme via grant DP150103177 and Swinburne University Postgraduate Research Award. The computational work has been supported by the National Computational Infrastructure Merit Allocation Scheme via grant IZ4, and the Swinburne Centre for Astrophysics and Supercomputing.

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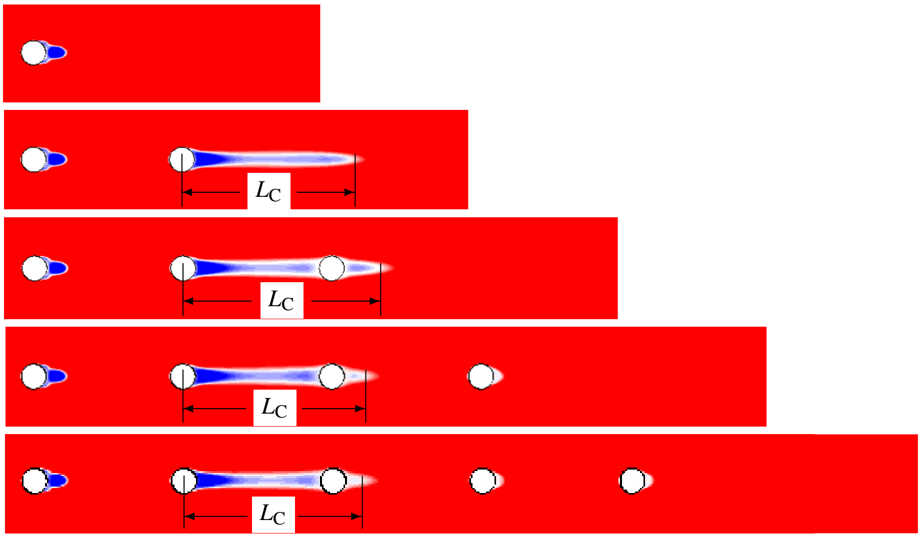

Figure 3: The contours of stream wise velocity in mean flow with one to five number of cylinders (top to bottom). The negative and positive velocities are shown by blue and red colors, respectively. The range for the velocity is from -0.2 to 1.2. The convective length, L_C, is shown in the figures for each system.

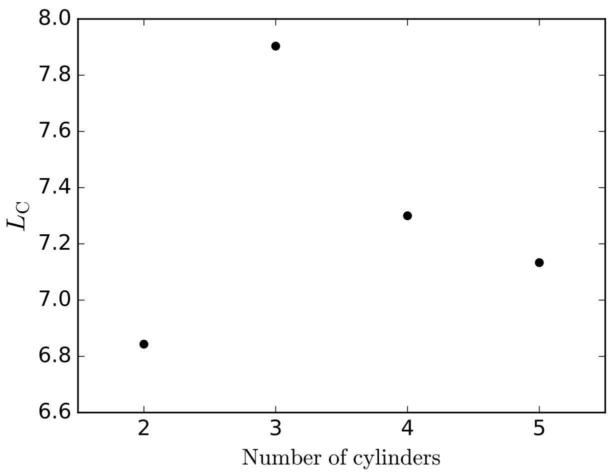

Figure 4: The L_C as a function of number of cylinders in the system at $p = 6.0$. This length has been measured from the center of second cylinder for all the systems.

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