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The appearance of subharmonic modes in bluff body wakes

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Abstract

This paper presents a series of observations of the onset of subharmonic modes in vortex wakes. Two main scenarios are investigated: the transition from two-dimensional to three-dimensional flow; and the appearance of subharmonic modes in flow-induced vibrations. Observations across flows past static bodies, bodies moving with a prescribed trajectory, and bodies freely responding to the flow forces are presented.

In all cases, it is shown that the spatiotemporal symmetry of the classic Kármán vortex street must first be broken to allow a subharmonic mode to occur. The symmetry breaking can be introduced by modifying the geometry or some external forcing, or it can come from a prior spontaneous symmetry breaking bifurcation. However, the broken symmetry is not sufficient for the appearance of a subharmonic mode, and the strength of the asymmetry needs to reach a finite threshold before such modes arise. It is hypothesized that this finite threshold is related to the point at which interaction between vortices on the same side of the wake are stronger than interactions between vortices on opposite sides.

Introduction

In many fluid mechanics problems, it is important to understand the state of the flow as a function of a control parameter, or a set of control parameters. For example, the flow past a circular cylinder is governed by a single parameter, the Reynolds number $Re = UD/\nu$, where U is the freestream speed, D is the cylinder diameter, and ν is the kinematic viscosity of the fluid. As Re is increased, the flow moves from a creeping flow (reflection symmetric about the wake centreline and up and down stream) to a steady flow with standing vortices (reflection symmetric about the wake centreline), to periodic vortex shedding (spatiotemporally symmetric), to three-dimensional vortex shedding (which may or may not be spatio-temporally symmetric). In some sense, with every transition from one state to the next, there is a loss of order. More formally, it can be stated that every transition is a *symmetry breaking bifurcation*, so that the symmetry group is reduced with each bifurcation.

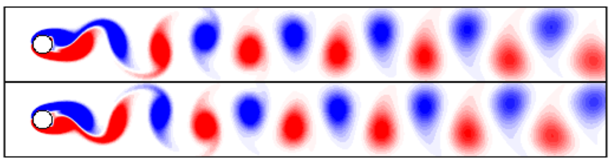

Figure 1: The cylinder wake visualised using vorticity contours at Reynolds number $Re = 100$. The second image is exactly half a period in time after the first. Reflecting the velocity field of the second image about the centreline (i.e, combining evolution in time by half a period with a spatial reflection) will produce the first image, showing that the flow is spatiotemporally symmetric.

Here, a symmetry refers to any operation on the flow solution that produces the same flow as that to which it was applied. For $47 < Re < 190$, the cylinder wake as shown in figure 1 displays periodic vortex shedding in the Kármán vortex street, and this flow has two important symmetries:

- the spatiotemporal symmetry of evolution forward in time by half a period and then relection about the wake centreline
- the translation of any distance in the spanwise dimension.

In [1], it was shown that for this symmetry group, there are only three generic symmetry breaking bifurcations that can lead to a three-dimensional flow (or equivalently, a loss of the translation symmetry in the spanwise direction):

- break the translation symmetry along the span, and maintain the spatiotemporal symmetry
- break the translation symmetry along the span and break the spatiotemporal symmetry, but maintain periodicity
- break the translation symmetry along the span, and break the spatiotemporal symmetry and break the periodicity.

Note that a subharmonic mode or a period-doubling bifurcation (where the flow maintains periodicity, but at a period twice the original) is not a generic feature of this bifurcation scenario. However, there are numerous flows similar to that of the circular cylinder wake that do undergo period-doubling bifurcations, and the feature that allows this is the breaking of the spatiotemporal symmetry of the vortex shedding.

This paper presents a number of examples highlighting this phenomenon in two main categories, which are the transition to three-dimensional flow, and examples from flow-induced vibrations. In the first category, the period-doubling bifurcation also results in breaking the translational symmetry, whereas in the second category it does not. While the details of the flow physics differ in each example, the common point is that the period-doubling only arises once the spatiotemporal symmetry is broken, and by a finite amount, i.e, an infinitesimal breaking of symmetry or bias is not enough to induce the subharmonic mode.

Two-dimensional to three-dimensional transitions

A prime example of how a subharmonic mode only appears once the spatiotemporal symmetry of the basic flow is broken is given by the transversely oscillating cylinder in a free stream studied in [7]. In this flow, the symmetry of the transverse forcing is the same as that of the Kármán vortex street. If the frequency of oscillation is at, or near, the vortex shedding frequency from an unperturbed body, then the vortex shedding is locked to the oscillation and for moderate amplitudes the symmetry of the flow is unchanged, with a single vortex shed each half cycle in a 2S mode (two single vortices per cycle - the same as shown for the fixed cylinder in figure 1). However for high amplitudes, a spontaneous symmetry breaking occurs, and the flow transitions to a P+S mode, with a single vortex in one half cycle, and a pair of oppositely-signed vortices in the other. The flow remains periodic and synchronized to the cylinder oscillation frequency, but loses its spatiotemporal symmetry. An example of this flow is shown in figure 2(a).

Using Floquet stability analysis, [7] showed that for low amplitudes (before the symmetry breaking) the three-dimensional modes are effectively the same as those for the static cylinder. However for high amplitudes after the symmetry breaking, the stability analysis recovers two subharmonic modes, either of which could produce a period-doubling bifurcation from the two-dimensional P+S wake. This example highlights that the necessary condition is that the basic flow is not spatiotemporally symmetric, as the boundary conditions and applied forcing are all spatiotemporally symmetric and it is only after the occurence of a spontaneous symmetry breaking that the subharmonic modes arise.

Of course, this symmetry can also be broken by modifying the geometry or the boundary conditions of the flow. [12] presented a study of the bifurcations to three-dimensional flow for a square cross section as a function of the angle of incidence. For non-zero angles (where the relection symmetry about the wake centreline is broken) the square presents a subharmonic mode dubbed mode C, which could be the first mode to bifurcate from the two-dimensional base flow with increasing Re, when the angle of incidence was in the range $12^\circ < \alpha < 25^\circ$. An example of the subharmonic mode C is shown in figure 2(b). Further study on the same problem in [2] very carefully showed an important point - that a finite “amount” of symmetry breaking was required to produce a subharmonic mode. An infinitesimal disturbance of the spatiotemporally symmetric wake was not enough to produce the subharmonic mode, which was only seen once the angle of incidence of the square reached $\alpha = 5.8^\circ$.

A related case for the emergence of the subharmonic mode C is that of a cylinder with a small control cylinder or wire placed in its near wake, but off the wake centreline to induce some asymmetry. The study of [14] presented experimental evidence of this subharmonic mode, showing that the repetition over two periods was caused by the streamwise vortex structures forming on each of the wake vortices being out of phase. This out of phase relationship was shown to be driven by the vortex induction of one positive-signed wake vortex on the next. Here, then, is a physical explanation of the finding from [2] that there needs to be a certain amount of asymmetry - the flow needs to be such that the vortex induction is strongest between two vortices of the same sign shed one period apart, rather than between two vortices of opposite sign shed one half period apart. The flow visualization of [14] is reproduced in figure 2(c).

This is also supported by the stability results for the flow past a torus. In that case, as the torus aspect ratio increases, the local curvature in the azimuthal or “spanwise” direction decreases, and the body approaches a straight circular cylinder. The symmetry of the flow past a local circular cross section is broken due to the curvature, which forces the flow to accelerate through the centre of the torus. Therefore, the asymmetry is stronger for lower aspect ratios. The study from [13] shows that the subharmonic mode C becomes unstable at lower Re as the aspect ratio is lowered, i.e, for a given Re, the growth rate is higher for higher asymmetry.

Similarly, [10] showed that in the wake of an ellipse at an angle of incidence to the freestream, the subharmonic mode C was not present for an aspect ratio of 1.1 at any angle, but it appeared when the aspect ratio was increased to 1.5, apparently increasing the level of asymmetry for a given incidence angle. The same paper also showed that there was an upper bound on the incidence angle for which the subharmonic mode C could become unstable. For an aspect ratio of 2.5, the domain of instability for the subharmonic was around $5^\circ < I < 12^\circ$. An example of this flow is shown in figure 2(d).

Other examples of a subharmonic mode arising after breaking the reflection symmetry are rotating cylinders [9], bodies with trip wires [15, 11] or some other small finite perturbation [4], and cylinders in staggered arrangements [3]. All of these examples also show that the asymmetry needs to reach a finite threshold before a subharmonic mode can become unstable.

Flow-induced vibrations

The previous section has shown that three-dimensional subharmonic modes only bifurcate from the two-dimensional flow once the spatiotemporal symmetry of the wake has been broken and the asymmetry has reached a threshold. However the spanwise translation symmetry does not have to be broken to allow a subharmonic mode to occur, i.e, a subharmonic mode can arise that is still two-dimensional in a two-dimensional flow, and a nominally subharmonic mode can arise in a flow that is already turbulent and three-dimensional. Here, we present a number of examples taken from flow-induced vibration phenomena over a range of geometries and Reynolds numbers.

The first example is that of an elliptical cross section that is elastically mounted, constrained to oscillate in the transverse direction relative to the incoming flow, and mounted such that it has an angle of incidence relative to the free stream direction.

A previous study [5] showed that this configuration, even at the low $Re = 200$ and a moderate aspect ratio of the elliptical cross section of 1.5, could produce a synchronized, period doubled mode. In this mode, the body oscillates with a base frequency similar to the natural structural frequency but at half the vortex shedding frequency of the stationary body, and there are two vortex shedding cycles per body oscillation cycle. This mode can be achieved for angles of attack between 30° and 50°. Similar to the three-dimensional mode transitions, not only does the reflection symmetry need to be broken, but a certain amount of asymmetry is required to capture the subharmonic mode.

This behaviour is not limited to laminar flows. Two studies [8, 16] investigated the vibration of a square cross section mounted with an angle of incidence to the incoming flow but free to oscillate transverse to the flow direction. These studies discovered a “higher branch” mode of vibration response, where the vortex shedding again repeated twice over each cycle of oscillation or in a subharmonic mode (interestingly, the amplitude of vibration was also larger than that seen in the harmonic modes). This higher branch subharmonic mode was identified for angles of incidence between 5° and 25°, again confirming the idea that a finite amount of asymmetry is required for the development of the subharmonic mode.

Other examples of subharmonic modes arising once the spatiotemporal symmetry has been broken are also found where the geometry and forcing remain symmetric, but the symmetry is broken spontaneously. A study of the flow-induced vibration of a diamond cross section (a square oriented at 45° to the incoming flow) [6] showed a rich variety of modes generated by the complicated vortex shedding from the sharp corners, which included non-zero-mean lift modes, and a subharmonic mode

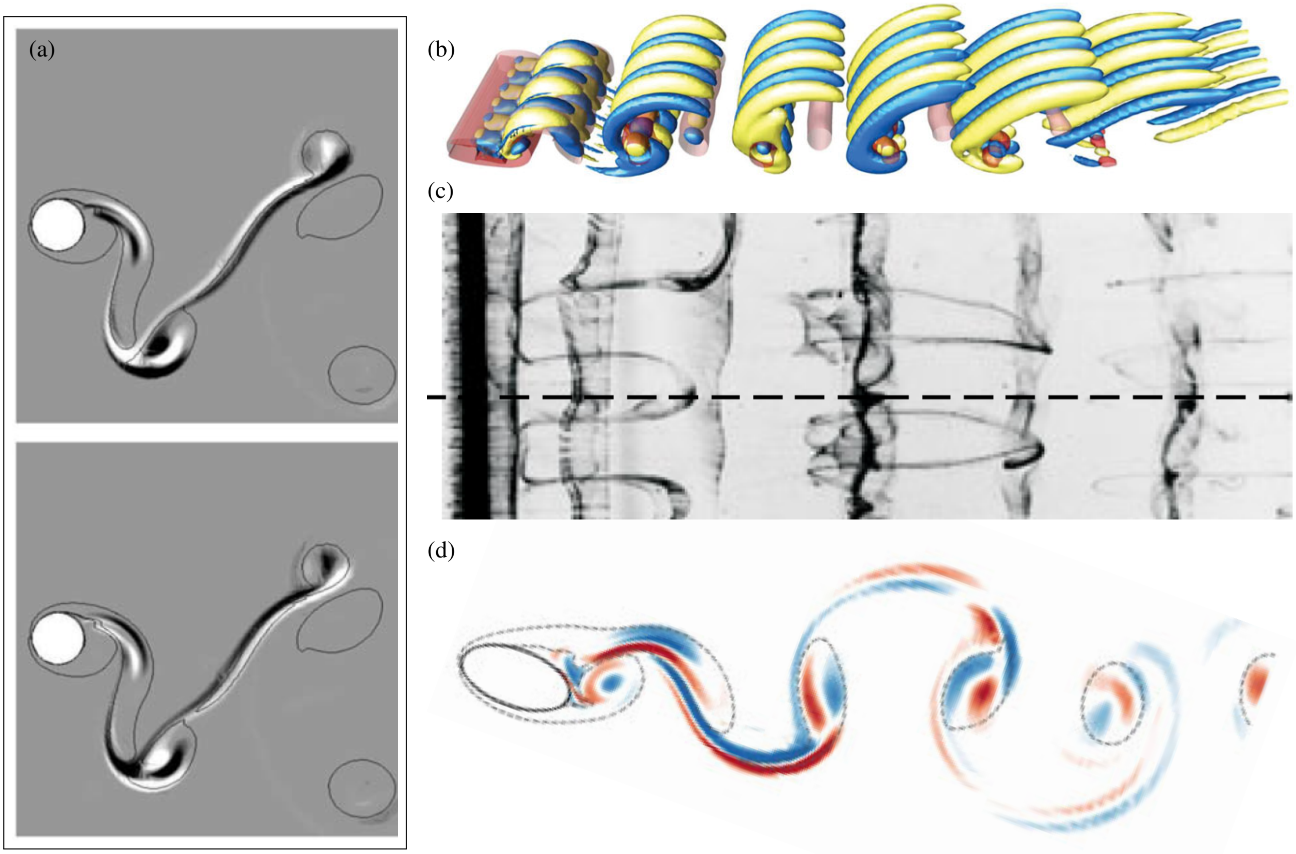

Figure 2: A series of images displaying subharmonic modes in the transition to three-dimensional flow.
(a) The transverse oscillating cylinder at high amplitude, where the amplitude of oscillation is $A/D = 0.75$ and the frequency $fD/U = 0.2$ and Reynolds number $Re = 260$. Contour lines represent the base two-dimensional flow vorticity at $\omega D/U = \pm 1$, and the greyscale contours represent the perturbation field spanwise vorticity calculated during Floquet stability analysis. The base flow is asymmetric leading to the development of two subharmonic modes, one of which (mode SS) is shown here. The two images are one period apart, and show the perturbation field is identical except for a change in sign, indicating it repeats over two periods.
(b) The subharmonic mode C in the wake of a square cylinder at an angle of incidence of 22.5°, and Reynolds number $Re = 183$. The blue/yellow isosurfaces represent positive/negative streamwise vorticity, the red surfaces the spanwise vorticity of the Kármán vortices, calculated via Direct Numerical Simulation. The subharmonic nature is shown by the alternating blue and yellow streamwise structures from one negative Kármán vortex to the next. Reproduced from [12].
(c) Flow visualisation of out-of-phase streamwise vortical structures in the wake of a cylinder with a smaller control cylinder in the wake, showing the interaction between wake vortices of the same sign one period apart. The dye clearly shows streamwise structures that are out of phase on every other positive Kármán vortex. Flow is from left to right, with the cylinder in black on the left hand side of the image. Reproduced from [14].
(d) The subharmonic mode C in the wake of an elliptical cylinder at an angle of incidence of 20°, and a Reynolds number $Re = 360$. The subharmonic nature of the mode can be inferred from the change in sign of the colour contours of spanwise perturbation vorticity in each subsequent braid region between vortices at the bottom of the image. Reproduced from [10].

apparently bifurcating from these non-zero-mean states.

Conclusions

This paper has presented a series of examples of wake flows that bifurcate to a subharmonic state, or a state that repeats over two cycles of oscillation. In particular two scenarios have been studied - the onset of three-dimensional flow from a two-dimensional periodic state, and the flow-induced vibration of a bluff cross section. In both scenarios, it has been shown that submarmonic modes can arise once the spatiotemporal symmetry of the basic state is broken, either by the geometry and/or boundary conditions, or by a spontaneous symmetry breaking. Importantly, in both scenarios it has been shown that the symmetry not only needs to be broken, but the level of asymmetry required to induce the subharmonic mode is finite. It has been hypothesized that this is due to vortex dynamics, and the fact that the wake needs to be such that vortices on the same side of the wake interact more strongly than vortices on opposite sides of the wake.

Acknowledgements

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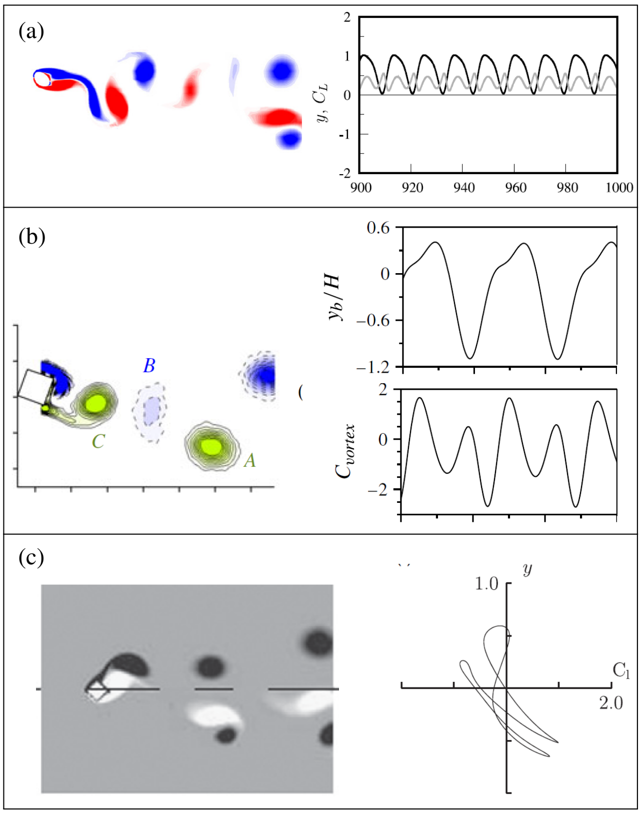

Figure 3: A series of images displaying subharmonic modes in flow-induced vibration problems.
(a) An ellipse of aspect ratio 1.5 elastically mounted and constrained to oscillate in the transverse direction. Here, the angle of incidence is 40°, Reynolds number based on frontal width to the flow is $Re = 200$, and the reduced velocity $U^* = U/(f_N D) = 11$ where f_N is the natural frequency. The flow is highlighted using contours of vorticity. This result is from a two-dimensional DNS simulation [5]. The right-hand side shows time histories of displacement (black) and lift (grey), showing the lift oscillates twice for every cycle of motion.
(b) A square cross section elastically mounted and constrained to oscillate in the transverse direction. Here, the angle of incidence is 20°, Reynolds number based on side length is $Re = 6052$, $U^* = 8$. The flow is highlighted using contours of vorticity from phase-averaged PIV. This result is from water channel experiments [16]. The right-hand side shows plots of displacement (top) and vortex lift (bottom), again showing the lift oscillates twice for every cycle of motion.
(c) A diamond cross section (a square at 45° to the flow) elastically mounted and constrained to oscillate in the transverse direction. Reynolds number based on frontal width is $Re = 200$, $U^* = 7$. Even though the geometry is symmetric, a spontaneous symmetry breaking leads to a non-zero-mean lift subharmonic mode. The right-hand side shows a Lissajou plot of displacement y against the lift coefficient C_L, clearly showing two loops representative of the flow repeating over two oscillation cycles.

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