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Scaling for Transient Natural Convection Flow in an Enclosure with Isoflux Heating

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Abstract

The start-up and fully developed natural convection flow in a cavity subjected to isoflux heating on one sidewall, with all other walls adiabatic, is investigated via two-dimensional numerical simulation. The dominant features of the evolving flow are identified as: the development of a vertical natural convection boundary layer on the heated sidewall, heated fluid discharged as an intrusion layer layer beneath the upper boundary, and the formation of an approximately linear, stable, temperature stratification in the cavity core. Rayleigh number based scalings are suggested and tested for the time to full development, and for the intensity of the core stratification, validated against the numerical results, and are shown to perform well.

Introduction

Heat transfer by natural convection in an enclosure is widely encountered in engineering situations, such as energy conversion systems for thermal storage devices and solar collector-receivers, amongst others. Natural convection in a differentially side-heated cavity where heating and cooling is applied through opposing side-walls has been studied extensively, with both isothermal [1, 3, 8, 13, 14] and isoflux configurations [9, 11] examined. Those investigations provided a general description of the flow together with scalings of the transition behaviour, time to full development and overall flow structure. In the case of isoflux heating/cooling, Kimura and Bejan [9] obtained an exact solution for the stratification, temperature and velocity profiles in the cavity core away from the top and bottom boundaries.

The isothermal cooling case was considered by Lin and Armfield [10], where a fixed temperature, cooler than the initial fluid temperature, was applied to a single wall, for both rectangular and cylindrical configurations. The fluid is initially quiescent and isothermal. After the onset of cooling a descending natural convection boundary layer develops on the cooled wall, discharging cool fluid at its downstream end. That fluid forms a cold intrusion which, first, flows along the base of the cavity to the far wall and then fills the domain with cooled fluid from the bottom up. At full development the fluid is again quiescent, but at the same temperature as that of the isothermal cool wall. Scalings for the transient and fully developed flow were obtained.

Natural convection flow in cavities with vertical walls heated and/or cooled is largely mediated by the natural convection boundary layers that form adjacent to the heated/cooled walls. Those natural convection boundary layers are similar to the natural convection boundary layer that form adjacent to a vertical semi-infinite heated plate, a canonical natural convection flow that has received considerable attention over many years. A pertinent example here is the investigation by Armfield, Patterson and Lin [7] in which the flow adjacent to an isoflux heated semi-infinite plate immersed in an ambient fluid with a stable linear background temperature gradient was considered. Scaling relations for the start up, transition and fully developed flow were obtained and validated against numerical simulation results.

In the present study, we investigate the flow features from initiation until full development for a cavity with isoflux heating applied to one wall. The general structure of the flow is described and scalings are suggested and validated for the time to full development and the stratification intensity, in terms of the Rayleigh number in the range $10^6 \sim 10^9$, all for Prandtl number $Pr = 7.0$.

Governing equations and numerical method

The two-dimensional incompressible fluid development is described by the Navier–Stokes and temperature transport equations, with the Oberbeck–Boussinesq approximation for buoyancy, recast in non-dimensional form using the characteristic length H, characteristic velocity $U = \frac{\kappa}{H} Ra^{\frac{2}{5}}$, characteristic temperature, $\boldsymbol{T} = \Gamma_w H$, and characteristic time, $\boldsymbol{t} = \frac{H^2}{\kappa Ra^{\frac{2}{5}}}$ [7]:

$$u_t + uu_x + vu_y = -p_x + \frac{Pr}{Ra^{\frac{2}{5}}}(u_{xx} + u_{yy}), \quad (1)$$

$$v_t + uv_x + vv_y = -p_y + \frac{Pr}{Ra^{\frac{2}{5}}}(v_{xx} + v_{yy}) + PrRa^{\frac{1}{5}}T, \quad (2)$$

$$u_x + v_y = 0, \quad (3)$$

$$T_t + uT_x + vT_y = \frac{1}{Ra^{\frac{2}{5}}}(T_{xx} + T_{yy}). \quad (4)$$

where u and v are the horizontal and vertical components of velocity, T the temperature, and p the pressure. The Rayleigh number and Prandtl number are defined as $Ra = (g\beta\Gamma_w H^4)/(\nu\kappa)$ and $Pr = \nu/\kappa$. H is the height/width of the square cavity, Γ_w the negative of the temperature gradient at the left-hand wall, ν and κ are the kinematic viscosity and thermal diffusivity, g the gravity and β the coefficient of thermal expansion.

The governing equations are discretised on a non-staggered mesh using the finite volume method, with standard second-order central difference schemes used for the viscous, pressure gradient and divergence terms. The second-order Adams-Bashforth and Crank-Nicholson schemes are used for the time integration of the advection terms and the diffusive terms respectively. The Jacobi method is used to solve the momentum equations, while the pressure correction equation, which enforces continuity, is solved using a biconjugate-gradient stabilised method with Stone's strongly implicit procedure (SIP) used as a pre-conditioner [2, 4, 5, 6, 12, 15].

Geometry and boundary conditions

Under consideration is the unsteady natural convection flow in a two-dimensional square cavity of width and height 1.0, as shown in figure 1. The cavity, with rigid non-slip boundaries, contains a Newtonian fluid initially at rest and at temperature

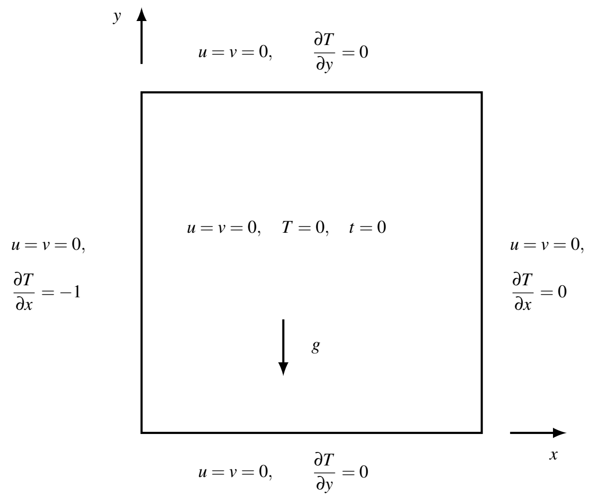

Figure 1: The cavity notation and boundary conditions.

$T = 0$. The top, right, and bottom boundaries are adiabatic. At time $t = 0$, the west wall is instantaneously heated with non-dimensional temperature gradient $\frac{\partial T}{\partial x} = -1$.

The domain is discretised with a non-uniform rectangular grid with Δx, $\Delta y = 5 \times 10^{-4}$ at the vertical/horizontal walls, expanding at a rate of 5% away from the walls. This gives a grid of 400×400 nodes in the x and y directions respectively. Variable time-stepping is used with the Courant number, $C = \frac{u\Delta t}{\Delta x}$, limited to be between 0.1 and 0.2. An extensive mesh and time-step dependency analysis has been carried out to ensure that the solution is accurate.

Results

Temperature contours for the developing and fully developed flow are shown in figure 2 for $Ra = 1 \times 10^9$ and $Pr = 7.0$. Immediately following the onset of heating a rising natural convection boundary layer is formed adjacent to the left, heated wall, seen in figure 2(a). The natural convection boundary layer discharges heated, buoyant fluid at its downstream end, forming a gravity intrusion immediately beneath the upper boundary that travels across the cavity. In figure 2(b) the gravity intrusion can be seen to have just reached the right hand side wall. The buoyant fluid continues to be discharged from the boundary layer and fills the cavity, from the top down, with heated fluid. In figure 2(c), it has filled approximately half the cavity. Ultimately the heated fluid will fill the entire cavity, as shown in figure 2(d), and the flow is then fully developed. At full development the temperature continues to increases everywhere at the same, linear rate to match the continued heating applied at the left wall, while maintaining the same stratification and overall structure shown in figure 2(d).

Velocity vectors for the $Ra = 1 \times 10^9$ flow, at full development, are shown in figure 3(a). The narrow rising natural convection boundary layer is seen adjacent to the left wall, discharging fluid that travels across the cavity immediately below the upper boundary, with part of the flow turning through 180° at the right hand wall and travelling back towards the left hand wall, forming a two-layer flow structure similar to that observed in [11] for the isoflux heated and cooled cavity. The discharged fluid ultimately forms a weak downwards flow in the central part of the cavity, away from the rising boundary layer on the left hand side. Over most of its height the rising natural convection boundary layer is parallel, with a small y-dependent region at the top and bottom where fluid is discharged and entrained.

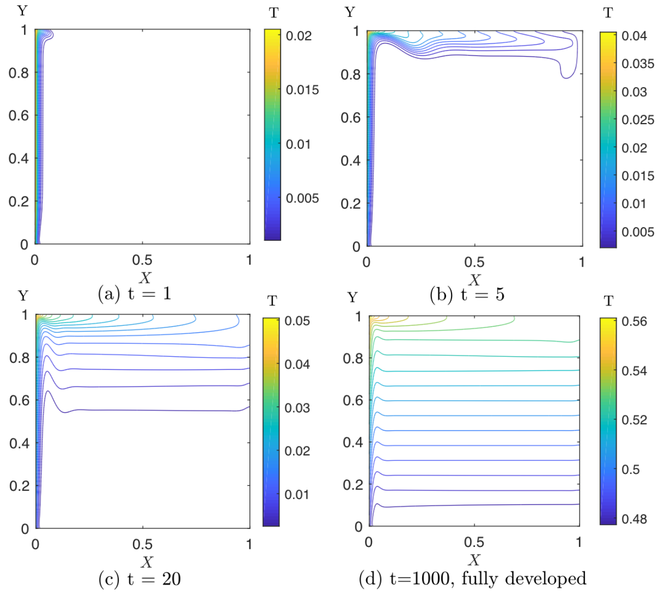

Figure 2: Temperature contours from start up to full development for $Ra = 1 \times 10^9$

This structure is also seen in figure 3(b) where vertical velocity profiles at a range of y locations are plotted. The velocity profiles are seen to be y-dependent below approximately $y \cong 0.15$, and y-independent above that location for the profiles shown.

For comparison the velocity vectors and vertical velocity profiles for $Ra = 1 \times 10^6$ are shown in figure 4. The basic flow structure is similar to that for $Ra = 1 \times 10^9$, but with a thicker boundary layer and with the vertical extent of the y-independent, central region of the boundary layer reduced, with the location of the transition point in the lower part of the boundary layer now at $y \cong 0.4$. This structure, whereby the lower part of the boundary layer is y-dependent and the upper part y-independent, is identical to that observed in [7] for natural convection flow on a semi-infinite vertical plate with stratified ambient.

One of the main features of this flow is the development of the approximately linear temperature stratification in the cavity core. The overall stratification may be parameterised by integrating the vertical temperature gradient over the domain, to give an average stratification,

$$\bar{\Gamma} = \iint \frac{\partial T}{\partial y}\, dx\, dy. \tag{5}$$

The average stratification is plotted against time for a range of Ra in figure 5. The average stratification reduces with increasing Ra, while the time to full development increases with Ra.

Two scalings are suggested for the average stratification at full development. In [7] it was found that, for the case with an imposed background stratification, a height existed at which the fully developed boundary layer transitioned from a y-dependent to a y-independent structure, similar to the behaviour noted above for the cavity flow. It was shown in [7] that the transition height is dependent on the strength of the background stratification via an identified scaling relation. For the cavity flow, considered here, a scaling for the transition location y_{trans} in terms of Ra may be obtained from the numerical results, and the scaling in [7] then inverted, to give a relation for the background stratification, $\bar{\Gamma} \sim Ra^{-\frac{1}{10}}$. An alternative approach is

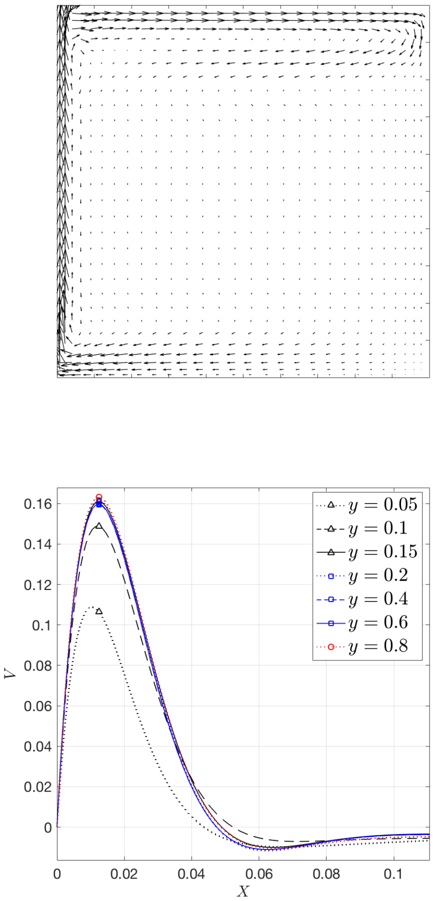

Figure 3: Velocity vector and vertical velocity profiles at full development for $Ra = 1 \times 10^9$, (a) Velocity vector field (top), (b) Vertical velocity profile at a range of y locations (bottom)

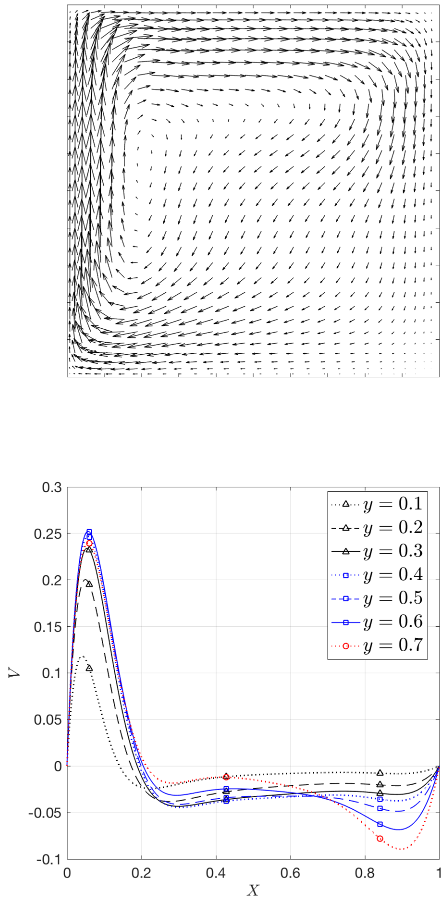

Figure 4: Velocity vector and vertical velocity profiles at full development for $Ra = 1 \times 10^6$, (a) Velocity vector field (top), (b) Vertical velocity profile at a range of y locations (bottom)

to use the scaling given in [9] for the stratification in the core region of the differentially isoflux heated/cooled cavity. Applied to the configuration considered here their result suggests a scaling $\bar{\Gamma} \sim Ra^{-\frac{1}{9}}$. A possible scaling for time to full development is the diffusive time scale, obtained from equation (4) as $t_{full} \sim Ra^{\frac{2}{5}}$.

The stratification time series shown in figure 5 are plotted in scaled form in figure 6. In figure 6(a) the stratification is scaled with $Ra^{-\frac{1}{10}}$, and in 6(b) with $Ra^{-\frac{1}{9}}$. In both cases the time is scaled with the diffusive scale $Ra^{\frac{2}{5}}$. In both cases the diffusive time scale brings the results for all Rayleigh numbers close to a single curve, showing that the overall time to full development is well approximated by this scaling. Both stratification scalings are also working well, bringing the values at full development close to a single value. However some variation in the performance of the two scalings is observed. The scaling based on the y transition location, $Ra^{-\frac{1}{10}}$, performs best for the lowest two Rayleigh numbers, while the scaling based on the core stratification in the differentially heated cavity, $Ra^{-\frac{1}{9}}$, performs best for the larger Rayleigh numbers. As can be seen in figures 3 and 4, the y-independent boundary layer region spans a much greater proportion of the cavity height for the higher Rayleigh number, while for the lower Rayleigh number the y-dependent part of the boundary layer spans the greater proportion of the cavity height. The second scaling is only valid for the y-independent, core region of the cavity, and so it would be expected to perform better for the higher Rayleigh numbers.

Conclusions

Numerical simulations of a cavity with a single isoflux heated wall, with all other walls adiabatic, have demonstrated the basic structure of the start-up and fully developed flow, showing that a rising boundary layer forms on the heated wall, discharging heated fluid into the cavity, and entraining fluid from the cavity. At full development the flow consists of the rising natural convection boundary layer, interacting with a stable stratification formed in the core of the cavity. For $Ra \gtrsim 10^7$ the majority of the vertical extent of the boundary layer flow is y-independent, while the discharge flow is largely horizontal, finally forming an overall falling flow in the cavity core. At full development the flow structure and the gradient temperature structure are maintained, with the temperature increasing linearly in line with the continuing heat flux applied at the heated wall.

Two possible scalings were suggested for the overall temperature stratification of the flow at full development, one based on the relation between the vertical transition location from y dependence to y independence in the boundary layer and the background stratification for the semi-infinite isoflux heated plate, and the other based on the exact solution for the stratification in the cavity core for a differentially isoflux heated/cooled cavity. Both approaches perform well, with the former being best for

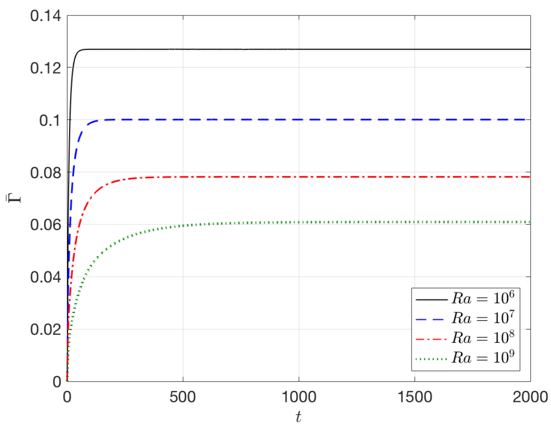

Figure 5: Unscaled average stratification plotted against time

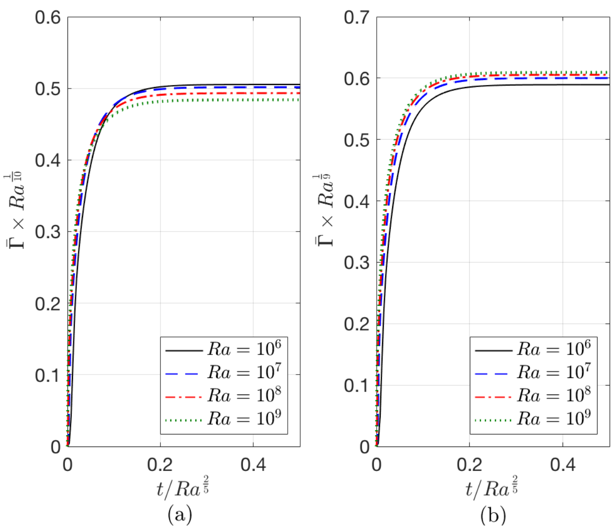

Figure 6: Scaled average stratification plotted against $\frac{\tau}{Ra^{\frac{2}{5}}}$: (a) scaled with $Ra^{-\frac{1}{10}}$;(b) scaled with $Ra^{-\frac{1}{9}}$

the lower Rayleigh numbers, and the latter best for the higher Rayleigh numbers. A diffusive time scaling was suggested to parameterise the time to full development, and shown to perform well.

Acknowledgement

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