21st Australasian Fluid Mechanics Conference
Adelaide, Australia
10-13 December 2018

Linear Stability of Hypersonic Flow with Moderate Cooling

F. Viera[1], S. O. Stephen[2] and V. Michael[3]

[1,2]School of Mathematics and Statistics
University of Sydney, Sydney 2006, Australia

[3]School of Mechanical, Aerospace and Civil Engineering
University of Manchester, Manchester M13 9PL, UK

Abstract

An asymptotic analysis is used to investigate the effects of surface cooling on the linear stability of a compressible boundary layer in hypersonic flow over a wedge. The derivation of the asymptotic structure relevant to the level of cooling applied to hypersonic vehicles is presented. Starting from previous results for supersonic flow, our analysis reveals that for hypersonic flow a reduction in surface temperature produces a region of large heat transfer and surface skin friction in the unperturbed boundary layer that leads to deformation of the classical triple-deck scales and a first new asymptotic structure emerges. Moreover, as the wall temperature is lowered further, to values of recent experiments of hypersonic flows, the upper region of the triple-deck structure collapses onto the main tier of the middle zone. The linear stability analysis of this second asymptotic stucture is presented.

Introduction

Efficient design of new high-speed vehicles is critically dependent on accurate knowledge of the behaviour of instabilities at hypersonic speeds. In particular, the ability to control the surface temperature of a hypersonic vehicle is a crucial aspect in controlling and delaying the transition from laminar to low-level free-stream turbulent flow. When this transition occurs it inevitably leads to a large increase in drag and surface temperature and consequent severe drop in overall performance.

It is well known [6] that the transition of hypersonic flow over smooth bodies to low-level free-stream turbulence occurs predominantly through either of two mechanisms. The so-called first Mack mode (lower-branch of neutral curve) is the high-speed counterpart of Tollmien–Schlichting waves, so is essentially a viscous travelling wave instability located close to the boundary. In this work we shall concentrate on Tollmien–Schlichting waves governed by a triple-deck structure. On the other hand, the second Mack mode (upper-branch of neutral curve) is inviscid in nature and occupies a greater proportion of the overall flow. The exact characteristics of any particular problem dictates whether it is the first or second Mack mode that possesses the greater growth rate and thus potentially dominates. First Mack modes are of particular interest since they may be excited by roughness effects.

The analysis of the stability of the Blasius boundary layer on a flat plate involving the triple-deck structure was introduced by Smith [10] for incompressible viscous flows using a formal matched-asymptotic-expansion approach for large Reynolds number. This work was later extended to compressible viscous flow past a wedge of small angle by Cowley and Hall [1] who also studied the effects of the attached shock. Their methods are the ones we follow in this article.

Recent studies on the effect of wall cooling in high-speed boundary layers [2, 7] have shown that localized cooling decreases the second-mode amplification and delays transition to turbulence. The effect of cooling on the linear stability of subsonic and supersonic boundary layers was considered by Seddougui *et al.* [9], who identified a new asymptotic regime for so-called "moderate cooling" where viscous modes were found to be destabilized. Moderate cooling introduces a sublayer in the basic flow close to the surface, that acts as a buffer layer, where large heat transfer and skin friction gradients occur. This process is required to reduce the high temperature in the boundary layer to the low temperature on the surface. This new sublayer, in turn, distorts the lower-branch Tollmien–Schlichting modes so as to increase their growth rates such that they become comparable with, or even exceed, those of inviscid modes. Moreover, it was shown in [9] that even at the so-called "moderate stage", cooling completely destabilizes otherwise stable viscous modes for any obliqueness of wave-angle, including the two-dimensional case. This is in contrast with the uncooled work of Smith [11]. Thus, first Mack modes may be important in the transition process in cooled high speed boundary layers and these are the focus of our current investigation.

The aim of the paper is to determine the structure of the boundary-layer flow for surface temperatures relevant to practical surface cooling in hypersonic flows. Our interest is in the theoretical solutions of this problem for instability modes which may promote transition to turbulence in hypersonic flows.

Basic Flow

The basic steady hypersonic flow whose stability we investigate consists of a compressible, viscous, perfect gas of constant velocity $\hat{U}$ over a wedge of small semi-angle θ. The wedge is symmetrically aligned with an oncoming flow with velocity magnitude $\hat{U}$. Shocks of semi-angle σ develop from the tip of the wedge and the acute angle between the shock and the wedge is $\phi = \sigma - \theta$. Introduce a Cartesian coordinate system $(\hat{x}, \hat{y}, \hat{z})$, where $\hat{x}$ is the distance along the upper surface of the wedge, $\hat{y}$ the distance normal to the wedge face and $\hat{z}$ is the spanwise coordinate and let $(\hat{u}, \hat{v}, \hat{w})$ denote the corresponding velocities and $\hat{t}$ the time. Quantities upstream of the shock are indicated by the subscript u and quantities in the so-called shock-layer between the shock and the wedge by the subscript s. Assume the fluid has an upstream Mach number M_u given by

$$M_u = \hat{U}/a_u; \qquad a_u^2 = \gamma(\hat{p}_u/\hat{\rho}_u) = (\gamma-1)\hat{h}_u,$$

where $\hat{p}_u$, $\hat{\rho}_u$ denote the pressure and density, respectively, $\gamma = c_p/c_v$ the ratio of specific heats, with c_p being the specific heat at constant temperature, a_u is the speed of sound in the upstream flow, $\hat{h}_u = c_p\hat{T}$ is the enthalpy and $\hat{T}$ is the temperature of the fluid.

Overall, the basic flow satisfies the compressible continuity, Navier-Stokes and heat-flux equations and they are non-dimensionalized using the flow quantities between the shock and the wedge surface. Let $(\hat{x}, \hat{y}, \hat{z}) = L(x, y, z)$ and $(\hat{u}, \hat{v}, \hat{w}) = \hat{U}_s(u, v, w)$, where the length scale L is the distance from the tip of the wedge to the location of interest and $\hat{U}_s$ is the magnitude of the fluid velocity in the region between the shock and the wedge defined in equation (1). Time is non-dimensionalized with respect to $L/\hat{U}_s$, pressure with respect to $\hat{\rho}_s\hat{U}_s^2$ and the other variables with respect to their values in the shock layer. The Reynolds number is defined by

$$Re = \hat{\rho}_s\hat{U}_s L/\hat{\mu}_s,$$

where $\hat{\mu}_s$ is a typical value of the viscosity. Away from the surface of the wedge viscous effects are neglected and the fluid velocities $(\hat{u},\hat{v},\hat{w})$, the pressure $\hat{p}$ and the density $\hat{\rho}$ satisfy the compressible Euler equations. This inviscid solution is well known (see for example [4], [1]). One important feature is that it is uniform between the wedge and the shock and given by

$$\hat{U}_s = \hat{U}\left(1+\varepsilon^2\tan^2\sigma\right)^{1/2}\cos\sigma, \qquad (1)$$

where $\sigma = \theta+\theta_s$ and $\varepsilon = \hat{\rho}_u/\hat{\rho}_s$, the ratio between the fluid density upstream of the shock and that in the shock layer. If ε is assumed to be small, then the density can be taken as constant in the shock layer. In this case the shock layer is thin and the viscosity μ may also be taken as constant with $\mu = \mu_s$.

Since the inviscid flow solution (1) specifies a non-zero slip velocity along the wedge, a boundary-layer region has to be introduced close to the surface in order to satisfy the viscous no-slip condition. Assuming $y = Re^{-1/2}\bar{y}$ and letting $Re \to \infty$ in the Navier-Stokes equations, we obtain at leading order the boundary-layer equations for compressible flow (e.g. [12]). We have $\rho T = \gamma M_s^2 P$ and the power-law form

$$\mu = CT^n, \ (n>0),$$

of the temperature-viscosity law, where C is a constant. Note that $n=1$ corresponds to Chapman's law, while $n = 1/2$ gives Sutherland's law. The boundary conditions on the boundary-layer equations are the no-slip condition $U = V = 0$ on $\bar{y} = 0$ and prescribed constant temperature $T = T_w$ at the wall and matching with the external inviscid flow, $U \to U_e,\ T \to T_e,\ \rho \to \rho_e$, as $\bar{y} \to \infty$, where the subscript e denotes external inviscid values obtained from the solution to the Euler equations. It is possible to obtain similarity solutions of the steady boundary-layer equations of the form

$$U = dF_B(\xi)/d\xi \quad \text{and} \quad T = H_B(\xi),$$

where ξ is the Dorodnitsyn-Howarth variable defined as $\bar{y} = \ x^{1/2}\int_0^{\xi} H_B(s)\,ds$ and F_B and H_B are Blasius type functions [12].

Effects of Surface Cooling

One of the effects of surface cooling is felt through the changes that occur in the mean-flow and mean-temperature profiles, U_0 and T_0, near the surface of the wedge, inducing large values of the heat-transfer and skin-friction coefficients. To see this consider the basic-flow momentum and heat-flux equations which are dominated by their viscous contributions near the surface. Introducing the scale

$$S_w = T_w/M^2, \qquad (2)$$

where M is the Mach number just behind the shock, the boundary-layer equations with $\bar{y} = S_w^{1+n}\tilde{y}$, give the following balances

$$\frac{\partial}{\partial\tilde{y}}\big(\tilde{T}_B^n\frac{\partial\tilde{U}_B}{\partial\tilde{y}}\big) = 0, \qquad \frac{\partial}{\partial\tilde{y}}\big(\tilde{T}_B^n\frac{\partial\tilde{T}_B}{\partial\tilde{y}}\big) = 0.$$

Integration then gives

$$\tilde{U}_B = \frac{c_2}{c_1}\Big\{\big[1+c_1(1+n)\tilde{y}\big]^{\frac{1}{1+n}}-1\Big\}, \quad \tilde{T}_B = \big[1+c_1(1+n)\tilde{y}\big]^{\frac{1}{1+n}},$$

where c_1, c_2, are $O(1)$ constants. Therefore, one of the effects of surface cooling is to introduce a buffer layer such that the skin friction and scaled heat transfer

$$\Big(\frac{\partial\tilde{U}_B}{\partial\tilde{y}}\Big)_{\tilde{y}=0} = \frac{c_2}{M^{1+n}S_w^n}, \qquad \Big(\frac{\partial\tilde{T}_B}{\partial\tilde{y}}\Big)_{\tilde{y}=0} = \frac{c_1M^2}{M^{1+n}S_w^n}, \qquad (3)$$

are substantially increased. These results are consistent with the work of Seddougui *et al.* [9] on surface cooling of subsonic and supersonic flows over a flat plate.

Triple-Deck Equations

The linear stability of the boundary-layer flow has been extensively studied (in the absence of the shock) using the Orr-Sommerfeld quasi-parallel approximation (Mack [5, 6]). However, the mathematical theory used to obtain the Orr-Sommerfeld equation is inconsistent in that the Reynolds number has been assumed to be simultaneously asymptotically large (in order that the basic flow is approximated by a parallel shear) and also of order unity so that a critical value of the Reynolds number can be calculated.

Smith [10] developed a self-consistent asymptotic framework for the description of the lower branch of the linear neutral stability curve for Tollmien–Schlichting waves of the viscous–inviscid type in an incompressible fluid. This high-Reynolds number asymptotic approach provides a mathematically consistent and rational method to incorporate linear growth (unsteadiness), non-parallelism and nonlinearity. The theory was later extended to compressible flows by Smith [11]. In this case the first modes of instability are three-dimensional waves directed outside of the local wave-Mach-cone directions, that is, the wave angle θ must satisfy $\theta > \tan^{-1}\sqrt{M_\infty^2-1}$, where M_∞ is the free-stream Mach number.

The asymptotic framework is based on the triple-deck structure developed some years earlier to explain the process of "self-induced separation". The triple-deck formulation assumes that, at large Reynolds numbers, the normal y-variation of the Tollmien-Schlichting disturbances comprises three main regions, each with its own characteristic scale: (a) the viscous sub-layer referred to as the **lower deck**, (b) the **main deck** comprising most of the boundary layer and (c) the **upper deck** consisting of potential flow just outside the boundary layer. It is assumed that the triple-deck structure lies in a weak interaction region defined by

$$\chi = Re^{-1/2}M^{2+n} \ll 1,$$

where χ is the hypersonic interaction parameter [12]. To study the effect of the shock on the instability waves without the effects of non-parallelism, the so-called Newtonian approximation $(\gamma-1) \ll 1$ must be made.

The asymptotic scalings may be obtained from [12] but extended to three-dimensional unsteady hypersonic motion. For moderate surface cooling they are an extension of [9] with consideration of the attached shock. Strange [13] studied the stability of Tollmien–Schlichting waves in the regime $Re \gg 1$, $M^2 \gg 1$, $T_w \ll 1$ without the effects of an attached shock. Since it is known for hypersonic flow that $T_w = O(M^2)$ for the adiabatic case, we shall consider the regime

$$S_w \ll 1,$$

where S_w is defined in equation (4). The streamwise and spanwise length scales and the time scale are given throughout the three decks by

$$
\begin{aligned}
\left(x-x_0, z-z_0\right) =& Re^{-\frac{3}{8}} S_w^{\frac{1+2n}{2}} \left(M^{\frac{12+3n}{4}} X, M^{\frac{8+3n}{4}} Z\right), \\
t =& Re^{-\frac{1}{4}} S_w^n M^{\frac{4+n}{2}} \tilde{t}.
\end{aligned}
$$

Introducing the constant $c_3 = (C/c_2)^{1/3}$ the triple-deck structure for the linear stability problem may now be specified as follows:

Lower deck. The overall three-dimensional behaviour of the Tollmien–Schlichting waves is governed by the lower-deck equations and boundary conditions. This is the region in which viscous effects are important and the nonlinearity of the problem appears in this layer. The scales for the leading-order terms are given by

$$
\begin{aligned}
&y = Re^{-\frac{5}{8}} S_w^{\frac{1+2n}{2}} M^{\frac{8+5n}{4}} c_3 Y && u = Re^{-\frac{1}{8}} S_w^{\frac{1}{2}} M^{\frac{4+n}{4}} c_2 c_3 U \\
&v = Re^{-\frac{3}{8}} S_w^{\frac{1}{2}} M^{\frac{3n}{4}} c_2 c_3^2 V && w = Re^{-\frac{1}{8}} S_w^{\frac{1}{2}} M^{\frac{n}{4}} c_2 c_3 W \\
&p = p_\infty + Re^{-\frac{1}{4}} M^{\frac{n}{2}} c_2^2 c_3^2 P && \rho = S_w^{-1} M^{-2} R.
\end{aligned}
$$

(Note that the factors of c_2 and c_3 are included in the expansions above and below to simplify the resulting equations to be solved.) These scales lead to the three-dimensional unsteady nonlinear boundary-layer equations (e.g. see [9]) subject to the no-slip boundary conditions $U = V = W = 0$ at the surface $Y = 0$ and the condition $U \sim Y + A(X, Z, \tilde{t}),\ W \to 0,\ Y \to \infty$ to match with the buffer layer detailed below. The function $A = A(X, Z, \tilde{t})$ is an unknown displacement independent of $\bar{y}$.

Middle deck. The middle deck covers the extent of the undisturbed boundary layer and consists of three sub-layers:

(a) Transition layer of thickness

$$
y = Re^{-\frac{1}{2}} M^{1+n} \bar{y}_\infty = Re^{-\frac{1}{2}} y_i,
$$

whose exact structure depends on the viscosity-temperature law. The transition layer adjusts the $O(M^2)$ temperature in the boundary layer to $O(1)$ in the free stream.

(b) Buffer layer of thickness $y = Re^{-\frac{1}{2}} M^{1+n} S_w^{1+n} \tilde{y}$ required to reduce the large temperatures in the boundary layer to the smaller value of the surface temperature.

(c) Main tier with leading order terms given by

$$
\begin{aligned}
y &= Re^{-\frac{1}{2}} M^{1+n} \bar{y} \\
u &= U_0(\bar{y}) + Re^{-\frac{1}{8}} S_w^{\frac{1+2n}{2}} M^{\frac{4+n}{4}} c_3 A \frac{dU_0}{d\bar{y}} \\
v &= -Re^{-\frac{1}{4}} M^{\frac{n-2}{2}} c_3 \frac{dA}{dX} U_0(\bar{y}) \\
w &= Re^{-\frac{1}{4}} M^{\frac{n-2}{2}} c_2^2 c_3^2 \frac{D}{R_0(\bar{y}) U_0(\bar{y})} \\
p &= p_\infty + Re^{-\frac{1}{4}} M^{\frac{n}{2}} c_2^2 c_3^2 P \\
\rho &= R_0(\bar{y}) + Re^{-\frac{1}{8}} M^{\frac{4+n}{4}} S_w^{\frac{1+2n}{2}} c_3 A \frac{dR_0}{d\bar{y}}.
\end{aligned}
$$

Upper deck. In this layer the basic flow quantities take their free-stream values. The scalings are given by

$$
\begin{aligned}
&y = Re^{-\frac{3}{8}} S_w^{\frac{1+2n}{2}} M^{\frac{8+3n}{4}} y_u && u = 1 + Re^{-\frac{1}{4}} M^{\frac{n-4}{2}} c_2^2 c_3^2 u^{(2)} \\
&v = Re^{-\frac{1}{4}} M^{\frac{n-2}{2}} c_2^2 c_3 v^{(2)} && w = Re^{-\frac{1}{4}} M^{\frac{n-2}{2}} c_2^2 c_3^2 w^{(2)} \\
&p = p_\infty + Re^{-\frac{1}{4}} M^{\frac{n-4}{2}} c_2^2 c_3^2 p^{(2)} && \rho = 1 + Re^{-\frac{1}{4}} M^{\frac{n}{2}} \rho^{(2)} \\
&T = 1 + (\gamma - 1) Re^{-\frac{1}{4}} M^{\frac{n}{2}} T^{(2)} && .
\end{aligned}
$$

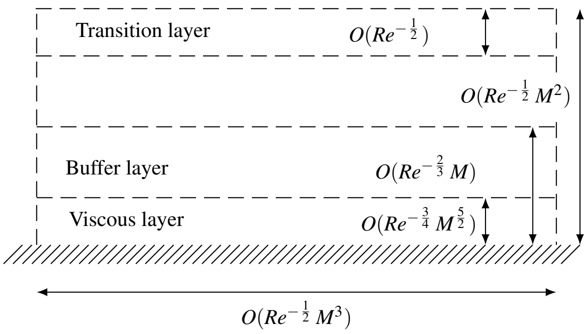

Figure 1: Schematic diagram showing the new cooled-structure of the main tier for $n = 1$.

Shock boundary conditions. To complete the definition of the problem we need to specify the boundary conditions at the shock which we suppose to be located at $\bar{y} = \bar{y}_s$. To ensure that the effects of the shock are included, the scalings are chosen so that it occurs in the upper deck of the triple-deck structure. Cowley and Hall [1] obtained general jump conditions at a shock for incident linearized inviscid waves that led to the boundary condition

$$
\bar{p} = 0 \quad \text{at} \quad \bar{y} = \bar{y}_s.
$$

The presence of the shock allows for an infinite discrete spectrum of unstable modes which are unstable over a relatively small distance and a high frequency range.

New Cooled Structure

The main effect of wall surface cooling occurs through the S_w factors in the scales given above, which result in the reduction in size of the thickness of all the layers, as might be expected physically. In particular the streamwise length and the maximum normal distance both decrease like $S_w^{\frac{1+2n}{2}}$, with similar deformations occurring in the lower-deck thickness. Consideration of the level of wall cooling applied in recent experiments investigating transition to turbulence in hypersonic flow reveals that further cooling needs to be applied to the system outlined above to be appropriate to the experimental surface temperatures. Thus, rather than solving the system just described attention will be focused on the case of further cooling. As the wall temperature is lowered to levels appropriate to hypersonic vehicles (where typically the ratio of the wall temperature to the adiabatic temperature is less than about 0.3) an additional asymptotic structure appears. In this case the upper-deck structure collapses onto the main tier of the middle deck and a new asymptotic structure emerges, as depicted in figure 1 (shown for $n = 1$). The collapse happens when the scales of the thickness of these these two layers coincide, that is,

$$
O\left(Re^{-\frac{3}{8}} S_w^{\frac{1+2n}{2}} M^{\frac{8+3n}{4}}\right) = O\left(Re^{-\frac{1}{2}} M^{1+n}\right),
$$

which gives

$$
S_w = Re^{-\frac{1}{4(1+2n)}} M^{-\frac{4-n}{2(1+2n)}} \tilde{S}_w, \tag{4}
$$

where $\tilde{S}_w$ is of order 1.

Considering S_w defined by equation (4), the expansions in the new main tier are now

$$
\begin{aligned}
&y = Re^{-\frac{1}{2}} M^{1+n} \bar{y} && u = U_0(\bar{y}) + Re^{-\frac{1}{4}} M^{\frac{n}{2}} \tilde{S}_w^{\frac{1+2n}{2}} c_3 A \bar{u} \\
&v = -Re^{-\frac{1}{4}} M^{\frac{n-2}{2}} c_3 \bar{v} && w = Re^{-\frac{1}{4}} M^{\frac{n-2}{2}} c_2^2 c_3^2 \bar{w} \\
&p = p_\infty + Re^{-\frac{1}{4}} M^{\frac{n}{2}} c_2^2 c_3^2 \bar{p} && \rho = R_0(\bar{y}) + Re^{-\frac{1}{4}} M^{\frac{n}{2}} \tilde{S}_w^{\frac{1+2n}{2}} c_3 A \bar{\rho}.
\end{aligned}
$$

These scalings in the main tier lead to the compressible pressure equation

$$\left(1-M_0^2\right)\bar{p}_{XX}+\bar{p}_{\bar{y}\bar{y}}+\bar{p}_{ZZ}=2M_{0\bar{y}}\bar{p}_{\bar{y}}/M_0, \tag{5}$$

where $M_0=U_0M\rho_0^{1/2}$. The boundary conditions on $\bar{p}$ are

$$\bar{p}\sim\ P+\frac{1+n}{2+n}\tau A_{XX}\ \bar{y}^{\frac{2+n}{1+n}}\quad \text{as}\quad \bar{y}\to 0, \tag{6}$$

and

$$\bar{p}'\sim\bar{y}_\infty-\bar{y}\ \text{ as }\ \bar{y}\to\bar{y}_\infty, \tag{7}$$

where $\bar{y}_\infty$ is the scaled boundary-layer edge. Condition (7) reflects the match with the oncoming undisturbed free-stream flow and (6) is needed for matching with the buffer tier. The $O(1)$ parameter τ is the cooling factor defined by

$$\tau=\frac{c_2^{1/3}\,2^{1/2}\,\tilde{T}_w^2}{c_1^{3/2}\,C^{1/3}},\quad \text{where}\quad T_w=Re^{-\frac{1}{4(1+2n)}}\tilde{T}_w. \tag{8}$$

Linear Stability Analysis for New Cooled Structure

We adopt the method of Smith [10] who implemented a weakly nonlinear analysis of an incompressible Blasius boundary layer to Tollmien-Schlichting waves by considering normal mode disturbances proportional to

$$E=\exp\left[i\left(\alpha X+\beta Z-\Omega\tilde{t}\right)\right],$$

where α and β are the streamwise and spanwise wavenumbers and Ω is the frequency. Assuming a solution of the form

$$\begin{aligned}(U,V,W,P,A,\bar{p})&=(Y,0,0,0,0)\\&\quad+\left[h(\tilde{U},\tilde{V},\tilde{W},\tilde{P},\tilde{A},\tilde{p})E+\text{c.c.}\right]+O(h^2),\end{aligned} \tag{9}$$

for $h\ll 1$ and substituting into the lower-deck equations, yields the relation

$$\frac{\tilde{P}}{\tilde{A}}=\frac{(i\alpha)^{-1/3}}{(1+\beta^2/\alpha^2)}\left[Ai'(\xi_0)\Big/\int_{\xi_0}^{\infty}Ai(s)\,ds\right],\quad \xi_0=-i^{1/3}\Omega/\alpha^{2/3}, \tag{10}$$

and $Ai(\xi)$ is the Airy function. Next, substituting the expression for the pressure into the main-tier pressure equation (5) we are left with solving the quasi-steady compressible Rayleigh equation,

$$\tilde{p}''-\frac{2M_0'}{M_0}\tilde{p}'-\left[\alpha^2(1-M_0^2)+\beta^2\right]\tilde{p}=0, \tag{11}$$

where $M_0=U_0M/\sqrt{T_0}$ and the prime denotes differentiation with respect to $\bar{y}$. The appropriate boundary conditions are

$$\tilde{p}'\sim-\alpha^2\ \tau\ \bar{y}^{\frac{1}{1+n}}\ \text{ as }\ \bar{y}\to 0, \tag{12}$$

and

$$\tilde{p}'\sim\bar{y}_\infty-\bar{y}\ \text{ as }\ \bar{y}\to\bar{y}_\infty. \tag{13}$$

Therefore, the linear stability of the new cooled system is given by the solution to equation (11) with the boundary conditions (12) and (13). The numerical solutions of this system is the focus of our current effort.

Conclusions

We have shown that a reduction in surface temperature leads to a region of large heat transfer and surface skin friction in the unperturbed boundary layer as indicated by the expressions (3), leading to deformation of the classical triple-deck scales. As the wall temperature is lowered further to levels appropriate to hypersonic flow, the upper region of the so-called triple-deck structure collapses onto the main tier of the middle zone and a new asymptotic structure emerges, as shown in figure 1. Strong interaction and non-parallelism have been neglected which places certain limits on the range of validity of the analysis [1]. Note, that it is expected that non-parallel flow effects will be insignificant.

The next stage of this study involves finding neutral and unstable solutions by solving the pressure equation (11) subject to the boundary conditions (12) and (13). Our methods have been guided by the numerical and asymptotic methods of [9] and [13].

Our intention is to extend this analysis to include passive porous walls [3] to determine their effect when significant levels of surface cooling are applied.

References

[1] Cowley, S.J. and Hall, P., On the instability of hypersonic flow past a wedge, J. Fluid Mech, **214**, 1990, 17- 42.

[2] Fedorov, A., Soudakov, V., Egorov, I., Sidorenko, A., Gromyko, Y., Bountin, D., Polovanov, D. and Maslov, A., High-speed boundary-layer stability on a cone with localized wall heating or cooling, AIAA J. 53(9), 2015, 25122524.

[3] Fedorov, A.V., Shiplyuk, A., Maslov, A.A., Burov, E. and Malmuth, N.D., Stabilization of a hypersonic boundary layer using an ultrasonically absorptive coating, J.Fluid Mech., **479**, 2003, 99–124.

[4] Hayes, W.D. and Probstein, R.F., Hypersonic Flow Theory, Academic Press, London, 1966.

[5] Mack, L.M., Boundary-layer stability theory. Document 900-277, Rev. A. Pasadena, Calif., JPL, 1969.

[6] Mack, L.M., Boundary-layer linear stability theory. Special course on stability and transition of laminar flow. *AGARD-709*, 1984.

[7] Poplavskaya, T. V., Bountin, D.A., Kirilovskiy, S.V. and Maslov A., Effect of local heating/cooling on the laminar-turbulent transition on a blunted cone, AIP Conference Proceedings 1770, 030056, 2016.

[8] Seddougui, S.O. and Bassom, A.P., Nonlinear instability of viscous modes in hypersonic flow past a wedge, J. Mech. appl. Math., **47**, 1994, 557-582.

[9] Seddougui, S.O., Bowles, R.I. and Smith, F.T., Surface-cooling effects on compressible boundary-layer instability, and on upstream influence, *Eur. J. Mech. B/Fluids*, **10**, 1991, 117–145.

[10] Smith, F.T., On the nonparallel flow stability of the Blasius boundary layer. Proc. R. Soc. Lond. A 366, 1979, 91-109.

[11] Smith, F.T., On the First-mode instability in subsonic, supersonic or hypersonic -boundary layers, J. Fluid Mech, **198**, 1989, 127-153.

[12] Stewartson, K., The theory of laminar boundary layers in compressible fluids, Oxford University Press, 1974.

[13] Strange, M., The effect of surface cooling on compressible boundary-layer instability, Ph.D. thesis, University of Hull, 1995.