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A Novel Method for Determination of Convective Velocity of Coherent Structures in High Speed Flows

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Abstract

A novel method to determine the average convection velocity of coherent structures in the shear layer of supersonic jets is presented in this paper. The method is based on the Proper Orthogonal Decomposition (POD) of a flow field and the standard advection equation. Substitution of a function satisfying the advection equation with its POD representation followed by the utilization of the fundamental properties of POD modes and coefficients yields an expression for the convection velocity in terms of the derivatives and the inner products of modes. The method is suitable for experimental data (like PIV) as it does not require the data set to be time resolved. For validation and practical applicability of the method, a sensitivity analysis has been performed using a synthetic data set followed by the application of the technique on the simulation data of a realistic research problem.

Introduction

Practical high speed jets (transonic and supersonic flow regime) are turbulent with coherent structures in the shear layer advecting at high subsonic or supersonic velocity. Understanding the shear layer instabilities in such flows is vital for applications like gas turbine engine exhaust noise, acoustics in impinging jets (VTOL aircraft) and surface finish in cold gas spray additive manufacturing processes. As discussed by [7, 5], the convection velocity of the large scale periodic structures in the shear layer is one of the important parameters governing the dynamics of these flows. In high speed flows, the convection velocity is of the order of the sonic speed (around $300\,\mathrm{m\,s^{-1}}$ at STP conditions) and hence, the corresponding time scales are very small i.e. of the order of milliseconds. The determination of the convection velocity is therefore, highly challenging for experimental data obtained using particle image velocimetry (PIV), which essentially consists of numerous ensembles of statistically independent data points i.e. no time resolution in the data is possible. Conventional methods like two point correlation, Taylor's approximation in space-time correlation [4] and phase domain analysis using wave number spectra [3] cannot be used as these require data to be resolved at the flow time scales or lower.

Experimental researchers have been exploring a few techniques for the computation of the convective velocity using PIV data. Krothapalli et al. [5] have estimated the values by tracing the vortex structures in the instantaneous velocity field followed by averaging of data obtained from each velocity field (snapshot). Weightman et al. [8] have used a phase averaging method for supersonic impinging jets having feedback loop formation (locked at a particular frequency). This technique involves measuring the resonating frequency using a microphone and hence, requires a lock-in frequency. A novel method is presented here which is more general and suitable for diverse range of flows.

Methodology

The method is based on the Proper Orthogonal Decomposition (POD) of a function. The theory and details of POD can be found in [2] and [6]. Let f be a general function of space and time which is approximated by POD as

$$f(\mathbf{x},t) \simeq \sum_{k=1}^{K} a_k(t)\phi_{\mathbf{k}}(\mathbf{x}). \tag{1}$$

If the function is purely advective, it will satisfy the advection equation -

$$\frac{\partial f}{\partial t} + \mathbf{U_c}\cdot\nabla f = 0 \; or \; \frac{\partial f}{\partial t} + U_c\frac{\partial f}{\partial x} = 0\,(for\,1-D). \tag{2}$$

Substituting f as approximated by equation 1 in equation 2 results in the following expression -

$$\sum_k^K \frac{\partial a_k}{\partial t}\phi_k + U_c\sum_k^K a_k\frac{\partial \phi_k}{\partial x} = 0\, or \sum_k^K a_{k,t}\phi_k + U_c\sum_k^K a_k\phi_{k,x} = 0. \tag{3}$$

As the POD modes ($\phi_{\mathbf{k}}$) are orthonormal and mode coefficients (a_k) are orthogonal, we have the following properties for the two (taking m and n as two general modes)-

$$(\phi_m,\phi_n) = \delta_{mn} \rightarrow Orthonormality, \tag{4}$$

$$< a_m a_n >= \delta_{mn}\lambda_m \rightarrow Orthogonality. \tag{5}$$

The symbol '$<>$' represents ensemble average (as the coefficients are only a function of time). The expression '$(,)$' represents the inner product.

Taking the projection of a general p^{th} mode on the advection equation gives

$$\sum_k^K a_{k,t}(\phi_k,\phi_p) + U_c\sum_k^K a_k(\phi_{k,x},\phi_p) = 0. \tag{6}$$

Using the orthonormal property of spatial modes, this yields

$$a_{p,t} + U_c\sum_k^K a_k(\phi_{k,x},\phi_p) = 0. \tag{7}$$

Multiplying the resultant equation by a general n^{th} mode followed by ensemble average results in

$$< a_{p,t}a_n > + U_c < a_n\sum_k^K a_k(\phi_{k,x},\phi_p) >= 0. \tag{8}$$

The summation is over space and ensemble averaging is over time. As the two are independent, the ensemble operator can be brought inside summation -

$$< a_{p,t}a_n > + U_c\sum_k^K < a_n a_k > (\phi_{k,x},\phi_p) = 0. \tag{9}$$

Using the orthogonal properties of coefficients yields

$$< a_{p,t}a_n > +U_c < a_n a_n > (\phi_{k,x}, \phi_p) = 0. \tag{10}$$

If $p = n$,

$$< a_{n,t}a_n > = \frac{1}{2}\frac{\partial < a_n^2 >}{\partial t} = \frac{1}{2}\frac{\partial \lambda_n}{\partial t} = 0. \tag{11}$$

This result when applied to equation 10 leads to the following test condition for advection -

$$(\phi_{n,x}\phi_n) = 0. \tag{12}$$

If $p \neq n$, we get the expression for the convection velocity -

$$U_c = \frac{- < a_{p,t}a_n >}{< a_n^2 > (\phi_{n,x}, \phi_p)}. \tag{13}$$

For a discrete data, the coefficients will be available as data values over time domain (discretized into N points). Hence, a_k's will be column vectors of size N which are orthogonal to each other. The inner product of a column vector with itself will give the eigen value of the mode ($(a_n, a_n) = \lambda_n$). The ensemble average for a discrete data can then be given as following -

$$< a_n a_n > = \frac{\sum_{i=1}^{N} a_{ni}^2 \Delta T}{T} = (a_n, a_n)\frac{\Delta T}{T}.$$

Equation 13 can now be re-written for a discrete data set to give the following expression for the convection velocity -

$$U_c = \frac{-(a_{p,t}a_n)}{\lambda_n(\phi_{n,x}, \phi_p)}. \tag{14}$$

The convection velocity can hence, be determined using the instantaneous temporal derivatives of the coefficients and the spatial derivatives of modes.

Application to Practical Data

For practical data (computational or experimental), the derivatives have to be evaluated using finite difference stencils. The inner products are also computed numerically using matrix multiplication of one vector with the transpose of the other. Depending on the spatial grid resolution (Δx) as well as the discretization schemes, there will be some numerical errors in the evaluation of expressions. The test condition will therefore, not give exactly zero but some small number. It is therefore, prudent to express the test condition for a discrete data in a modified non-dimensional form -

$$(\phi_{n,x}\phi_n)\Delta x = \varepsilon, \varepsilon \to 0. \tag{15}$$

The criteria for ε would be subjective and depend on the research application.

If a data set (values of a variable along a certain path at multiple instants of time) satisfies the test condition, the convection velocity of the variable along that path can be determined using equation 14. As mentioned previously, the data set need not be time resolved. However, for the computation of instantaneous derivatives, a pair of quick snapshots is required at each instant of time (figure1). As shown in the figure, the data is representative of a PIV data set acquired using two simultaneous acquisition systems. The data set from one system is shown in black and the data set from the other is shown in red. The two sets are offset by a small time interval 'Δt'. Such data sets are required for the determination of instantaneous temporal growth rates which is an important parameter for the analysis of flow dynamics and instabilities.

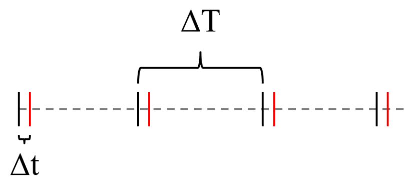

Figure 1: Experimental data set needed for the application of the proposed method

The derivatives required for the computation of the convection velocity can now be evaluated using a central difference stencil as follows -

$$\phi_{n,x} = \frac{\phi_n^{i+1} - \phi_n^{i-1}}{2\Delta x}, \tag{16}$$

$$a_{p,t} = \frac{a_p^{j+1} - a_p^{j-1}}{2\Delta t}. \tag{17}$$

The methodology can be used for a variety of research problems in fluids, heat transfer or combustion where a function (velocity, vorticity, temperature, pressure or any other flow variable) can be identified which is undergoing advection along a particular path. The convection velocity can then be computed using the following procedure -

1. The dominant modes are obtained using POD.
2. The test condition is applied on the first few dominant modes.
3. Using the two most dominant modes which successfully pass the test condition, the convection velocity can be estimated.

It is evident from equations 16 and 17 that the accuracy of the prediction is sensitive to the spatial grid resolution (Δx) as well as the time shift between the two arrangements (Δt). As the spatial resolution is fixed by the acquisition system (eg: the PIV system), the sensitivity analysis of the methodology has been done with respect to Δt.

Sensitivity Analysis

For the sensitivity analysis of the method, a synthetic data set has been generated using a standard analytical function satisfying the advection equation ($F(x - U_c t)$). To be closer to realistic experimental data, Gaussian noise having 10 % of the amplitude (taking a high noise level to check for the robustness of the technique) has been added to the function. The function used to generate a test data over a domain having a spatial resolution of Δx and temporal resolution of ΔT is following -

$$f = \sin(x - U_c t) + 0.1 * Noise. \tag{18}$$

Δx	50 µm
Spatial Domain	0.1 m (100 mm)
N_t *(No. of Ensembles)*	5000

Table 1: Fixed Parameters

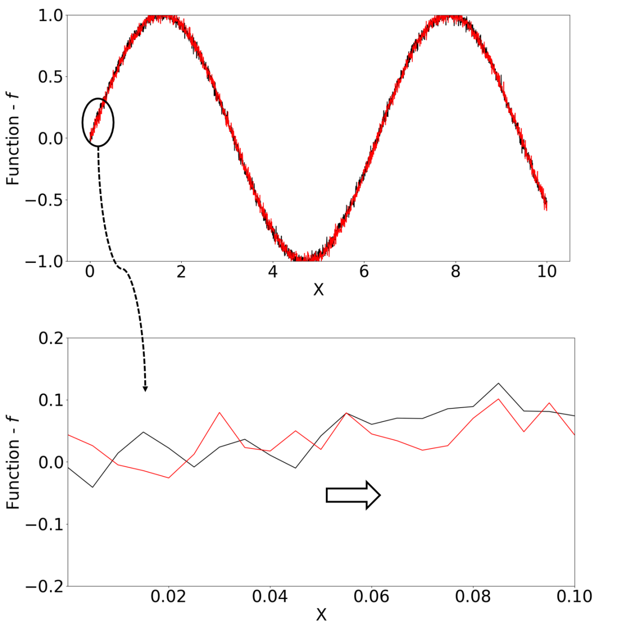

Figure 2: Noisy data generated by the test function along with a zoomed region showing the shift between the two data sets

Two parallel data sets have been generated which are offset by a small time step of Δt. A snapshot of the sample data is shown in figure 2. The same colors (black and red) are been used to distinguish between the two data sets. The spatial resolution, spatial domain and the number of ensembles for the test data set have been tabulated in table 1. These are representative of a typical experimental configuration.

The time difference (ΔT) between any two successive samples in a data set (determined by the camera acquisition rate), offset time step (Δt) and the convection velocity (U_c) are the variable parameters of the study. For jets investigated at low supersonic conditions (around Mach 1.5) at room temperature, the convective velocity of the coherent structures in the shear layer is expected to be about 200 m s^{-1} to 300 m s^{-1}. Centering around these conditions, the test matrix has been tabulated in table 2. The values of ΔT are kept of the order of a second which is typically the camera acquisition time.

ΔT (s)	0.1, 0.4, 0.5, 1.0
Δt (µs)	1, 10, 100
U_c (m s^{-1})	100, 300, 500

Table 2: Test matrix for the method

The prediction for the convection velocity using the proposed methodology is shown in figures 3, 4 and 5 which correspond to the actual values of 100 m s^{-1}, 300 m s^{-1} and 500 m s^{-1} (denoted by **$U_c$act**). As the noise is random and differs each time the same code is executed, the corresponding error bars (standard deviation) have also been shown in dotted lines for each case (computed for 100 iterations). Only two dominant modes are obtained (as expected) for all cases which easily satisfy the test condition. The convective velocity has therefore, been determined using the combination of modes 1 and 2. The prediction is very close to the actual value for all the cases which supports the proposed method. The variance is within 1% for all cases except for the low velocity (100 m s^{-1}) case with time shift of 1 µs where a variance of 5% is observed (figure 3). This is possibly due to the fact that for low advection velocity, the

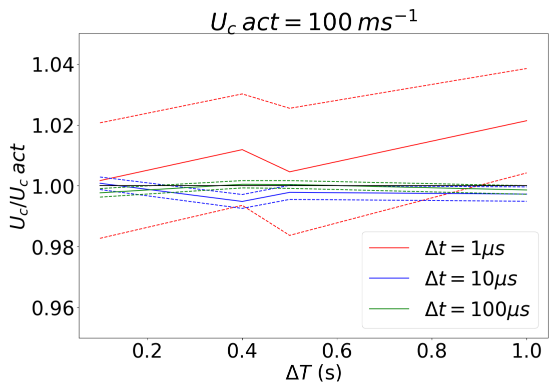

Figure 3: Prediction sensitivity for U_c = 100 m s^{-1}

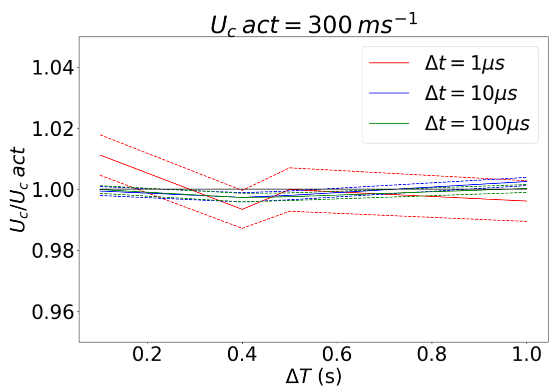

Figure 4: Prediction sensitivity for U_c = 300 m s^{-1}

two data sets are quite close to each other for a very small time shift. Hence, with high noise in the data, the time derivatives computed using the two data sets are significantly influenced by noise which affects the accuracy of the prediction. Hence, for processes with low advection speeds, it is important to ensure that the time shift is adequate and not too small.

Application to research problem

In addition to the sensitivity study on a synthetic data set, the methodology has also been tested on a simulated flow of an ideally expanded supersonic impinging jet at Mach 1.5. A 2-D axi-symmetric simulation has been carried out using OpenFOAM. The simulation domain and operating conditions have

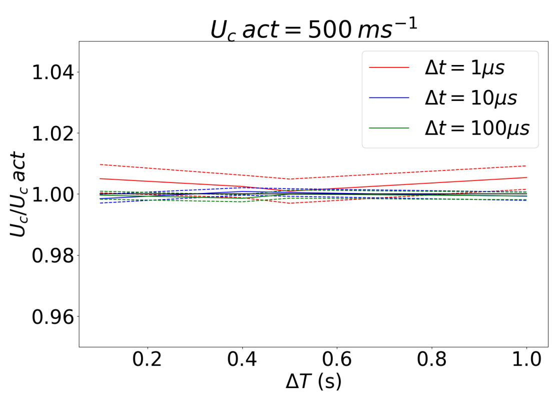

Figure 5: Prediction sensitivity for U_c = 500 m s^{-1}

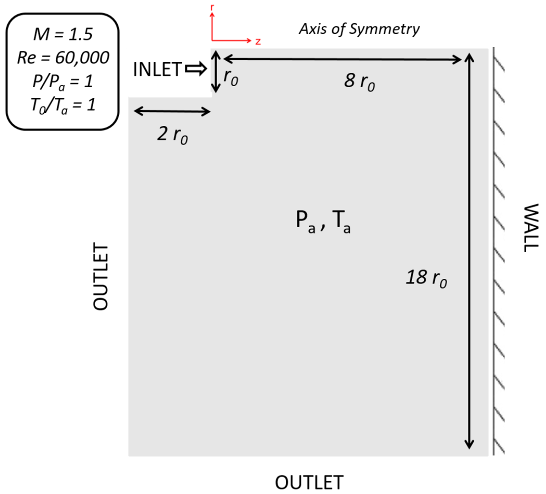

Figure 6: Simulation Domain and Conditions

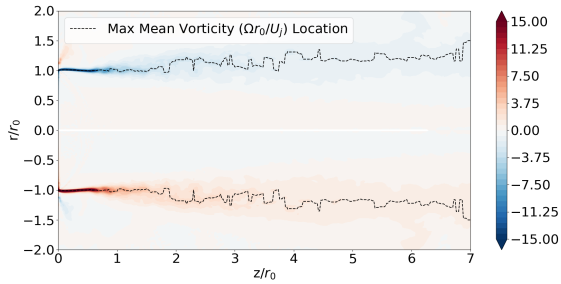

Figure 7: Mean Vorticity Contours with the location of maximum vorticity

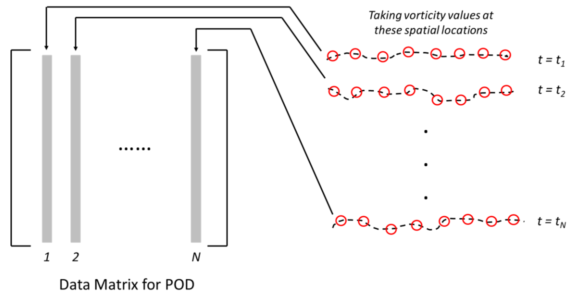

Figure 8: Formulation of advective function using maximum vorticity

been taken to be the same as those used by Bogey & Gojon [1] (figure 6). The simulation mimics one of the experiments by Krothapalli et al. [5] except for the fact that the Reynolds number for the simulation is an order of magnitude lower. A very fine spatial grid has been used which is identical to the setup of Bogey & Gojon [1].

The important aspect for the application of the proposed method is the identification of the advective function along with the path of advection. For impinging jets, the radial location of maximum vorticity at each streamwise location represents the path of advection in the shear layer and the corresponding value of vorticity represents the advective function (figure 7). The data set can now be created by populating the values of maximum vorticity at various instants of time (figure 8). As required by the method, two such data sets are created which are offset by a small time step.

POD of the resultant data matrices gives the dominant modes. The most dominant mode, i.e. mode 1, represents the mean value and hence, is neglected. The second and the third dominant modes are found to be quite close to the test condition requirement. The convective velocity is therefore, computed using the combination of modes 2 and 3. The value determined using this methodology for the simulation data is 0.48 U_j which is close to the estimation of 0.52 U_j by Krothapalli et al. [5] for experimental data having an order of magnitude higher Reynolds number. U_j refers to the jet velocity at the inlet.

Conclusions

The proposed methodology is useful for the determination of the convective velocity of coherent structures in the jet shear layer for diverse range of flows, especially when the data is not time resolved. The method requires an ensemble of two quick snapshots of flow field. Though the prediction is shown to have some dependence on the time difference between the two snapshots (Δt), the sensitivity to Δt is very low for high speed flows, even for data with high levels of noise. Therefore, having successfully demonstrated the applicability of the method on a synthetic experimental data as well as a simulated flow data, the method is validated and can be used in the analysis of experimental data.

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