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Model-Based Estimation of Vortex Shedding in an Unsteady Cylinder Wake

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Abstract

This paper considers the estimation of vortex shedding in cylinder wakes. Two methods are proposed to estimate time-resolved flow fields using a single velocity sensor based on a harmonic model. This model is developed to captures the periodic dynamics of the vortex shedding, and a model-based estimator (Kalman Filter) is used to estimate the flow given the measurement. The methods are applied to an unsteady cylinder flow at a Reynolds number of 1036. The estimated fields are compared with the flow fields measured experimentally using a two-dimensional time-resolved PIV. Physical interpretation of the harmonics is suggested based on the estimation results.

Introduction

Vortex shedding, a key feature in cylinder flows, has a strong periodic nature and appears in cylinder wakes at a Reynolds number higher than 49 [8]. Time-resolved velocity fields of vortex shedding are helpful in identifying the evolution of spatial structures in the flow. They also help to reveal the dynamics of the flow and therefore have potential for flow control applications. Time-resolved particle image velocimetry (TRPIV) is a commonly used technique for velocity field measurement, but it is costly. This motivates the estimation problem: we want to estimate the entire flow fields using only limited measurements.

In previous studies, a commonly used method in flow estimation is Linear stochastic estimation (LSE), introduced by Adrian [1]. This method assumes a joint probability distribution for the velocity fluctuation at each point in the field. Based on the statistics obtained from offline measurements, one can estimate the most statistically likely value at any chosen points in the flow, given measurements at other points [3]. However, LSE only reveals the statistical connection between flow quantities, and does not necessarily reflect the dynamics of the fluid system.

Model-based estimation, an alternative to LSE, typically consists of two parts: i) forming a Reduced Order Model (ROM) for capturing the dynamics of the system and ii) forming a dynamic estimator (e.g. Kalman Filter) to estimate the flow field. This method has been shown to outperform LSE when measurements are contaminated by noise [6]. Model-based estimation has been applied to flow estimation problems in previous studies, including cavity flow [6] and cylinder flow [2].

This paper develops model-based estimation methods to estimate vortex shedding in an cylinder flow using only a single sensor measurement in the wake. The model used exploits the periodicity of the flow.

Experimental Setup and PIV Data Processing

The experiment is conducted in a water tunnel facility located in the Wet Laboratory at the University of Melbourne shown in figure 1. The free-stream velocity in the experiment is $U_f = 41.5mm/s$. The tunnel has an open top, from which the cylinder is suspended in water. The test cylinder is a sealed aluminum pipe with outer diameter $D = 25mm$ and a thickness of $3mm$. This gives a Reynolds number of 1036. The total length of the cylinder is $495mm$, and hence it has an aspect ratio of 19.8.

The velocity fields are obtained using 6000 TR-PIV snapshots with a sampling frequency of 50Hz and a spatial resolution of 1920×1080 pixels. This frame rate ensures that at least 50 snapshots are avaliable for one vortex shedding period. A 60-watt continuous green laser source is placed on the bottom of the tunnel, providing a laser sheet with 3 mm thickness perpendicular to the cylinder axis. The tracer particles used are silver-coated hollow glass spheres (10 μm in diameter) from DANTEC DYNAMICS. The field of interest (shown in Figure 1) covers the vortex formation region and a downstream region containing roughly two vortices simultaneously. The total 6000 snapshots obtained are divided into two equally sized sets, referred as the modelling set and the reference set, for system modelling and validation purposes respectively.

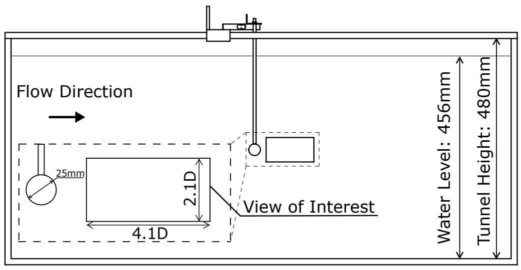

Figure 1: A schematic showing the experimental setup and the view of interest (PIV) in the test section of the water tunnel

Harmonic-Based System Modelling

Harmonic Decomposition

For the flows dominated by periodic structures, such as vortex shedding, one is often more interested in the periodic features than the aperiodic features. To focus more on such periodic features, a decomposition technique, introduced in [4], is used to decompose the unsteady velocity field into two parts. One is a time-independent mean flow, and the other is the summation of harmonic perturbations. The decomposition of the velocity gives:

$$u = \bar{u} + \sum_{n=1}^{N} [\alpha_{2n-1} \cos(n\omega t) + \alpha_{2n} \sin(n\omega t)], \quad (1)$$

where ω is the known fundamental harmonic (i.e. vortex shedding frequency). The coefficients α_{2n-1} and α_{2n} contain the real and imaginary parts of the nth harmonic for N harmonics in total. To further extend this equation to the entire domain, one can represent the evolution of the velocity fluctuation of the entire field in terms of a finite number of harmonics and the corresponding modeshapes Φ, given as:

$$U = \bar{U} + \tilde{U}, \quad (2)$$
$$\tilde{U} = \Phi X, \quad (3)$$

where U represents a 2-D velocity field as a single column vector. The term $\bar{U}$ corresponds to the mean velocity field, and $\tilde{U}$

corresponds to the fluctuations about the mean. The term Φ is a row vector of modeshapes, with each ϕ_i containing the α coefficients. The term X contains all harmonics. These terms are given as:

$$
\begin{aligned}
U &= \begin{bmatrix} u_1 & \dots & u_I & v_1 & \dots & v_I \end{bmatrix}^T, \\
\bar{U} &= \begin{bmatrix} \bar{u}_1 & \dots & \bar{u}_I & \bar{v}_1 & \dots & \bar{v}_I \end{bmatrix}^T, \\
\Phi &= \begin{bmatrix} \phi_1 & \dots & \phi_{2N} \end{bmatrix} = \begin{bmatrix} \alpha_{1,1} & \dots & \alpha_{1,(2N)} \\ \vdots & \ddots & \vdots \\ \alpha_{2I,1} & \dots & \alpha_{2I,(2N)} \end{bmatrix}, \\
X &= \begin{bmatrix} \cos(\omega t) & \sin(\omega t) & \dots & \cos(N\omega t) & \sin(N\omega t) \end{bmatrix}^T,
\end{aligned}
$$

where I is the number of grid points. Each harmonic has two modeshapes, corresponding to $cos(n\omega t)$ and $sin(n\omega t)$ respectively. Harmonic decomposition reduces the degrees of freedom to the number of harmonics chosen if the time-invariant modeshapes Φ are known. In experiments, these modeshapes can be estimated from the measurement of the time-evolution of the entire velocity field.

Formation of Harmonic Modeshapes

Knowing the fundamental harmonic, one can estimate all the harmonic modeshapes Φ using a least-square-error method. This method gives,

$$\hat{\Phi} = \tilde{U}X^T(XX^T)^{-1}. \tag{4}$$

Estimating modeshapes could be problematic if the velocity field measurement is contaminated by noise. Therefore, in this case, we estimate the modeshapes based on the velocity fluctuation of the phase-averaged flow instead of the direct measurement of the flow fields. By doing so, we can filter out much of the energy of any aperiodic structures.

The current method of phase averaging is based on Proper Orthogonal Decomposition (POD). POD captures the most energetic structures in the flow, which makes it suitable for identifying the vortex shedding phase of such flow ([7]). The method has two steps: i) the phases of velocity fields are obtained by computing the phases of the time coefficient of the first two POD modes (the most energetic modes). ii) the phase-averaged flow is then obtained by averaging the velocity fields based on these phases.

Once the harmonic modeshapes are obtained, the corresponding time coefficient $\hat{X}$ can be obtained by the least-square-error method, given as $\hat{X} = (\hat{\Phi}^T\hat{\Phi})^{-1}\hat{\Phi}^T\tilde{U}$, where $\tilde{U}$ is the direct measurement of the velocity fluctuation. Now we can reconstruct the velocity field using the harmonic modeshapes $\hat{\Phi}$ estimated and the corresponding time coefficients $\hat{X}$. In the current case, we only consider the first three harmonics because clear peaks can only be observed at these harmonics in the power spectral density of the entire velocity field measured in PIV, shown in Figure 2. Figure 3a shows the instantaneous vorticity field obtained directly from the PIV measurement. Figure 3b shows the reconstructed field from the fundamental harmonic alone. The streamwise positions of the vortex cores are correctly captured, but the transverse positions and the tilting angles of the vortices are not correctly captured. The vorticity field reconstructed using the first three harmonics, as shown in Figure 3c, better capture the streamwise positions, as well as the transverse positions and the tilting angles of the vortices. This indicates that one should consider the first three harmonics if one intends to estimate the cylinder wake with correct features of vortex shedding.

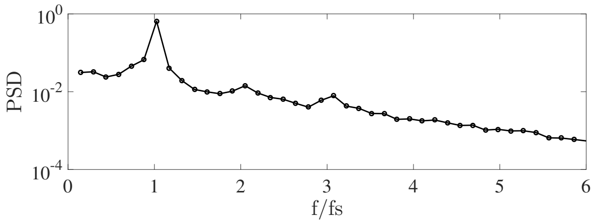

Figure 2: Power spectral density of the entire velocity field is obtained from the PIV measurement. The frequencies are normalized by the vortex shedding frequency f_s.

Therefore, the estimation of the entire flow fields can be done by estimating the time coefficients of the harmonic modeshapes (the first three harmonic in the current case). The following sections demonstrate methods to estimate the time coefficients and the corresponding flow fields.

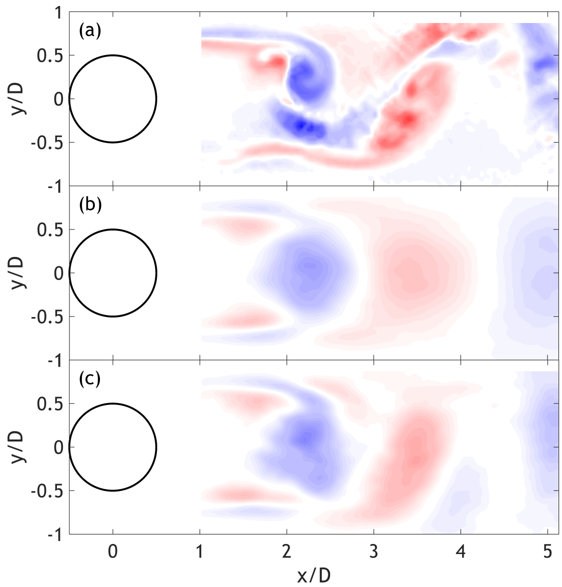

Figure 3: The instantaneous vorticity field obtained from PIV (a) is compared with reconstructed fields using the fundamental harmonic alone (b) and using the first three harmonics (c).

Direct Estimation of Cylinder Flow

Estimation Method

One way to estimate the flow field is to build a linear model based on the harmonic decomposition, and then directly estimate the time coefficients of the harmonic modeshapes using the measurement from a single velocity sensor in the wake. The linear model is a marginally stable oscillator with multiple harmonic frequencies. This model has the form:

$$
\begin{aligned}
\dot{x} &= Ax, \\
\tilde{U} &= \Phi x,
\end{aligned} \tag{5}
$$

where the modeshape matrix Φ is regarded as an output matrix in the model. The term x represents the states of the model, containing the time coefficients of the harmonic modeshapes. The state matrix A is given as:

$$A = \begin{bmatrix} A_{\omega_1} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & A_{\omega_N} \end{bmatrix}, \qquad A_{\omega_n} = \begin{bmatrix} 0 & -n\omega \\ n\omega & 0 \end{bmatrix}, \tag{6}$$

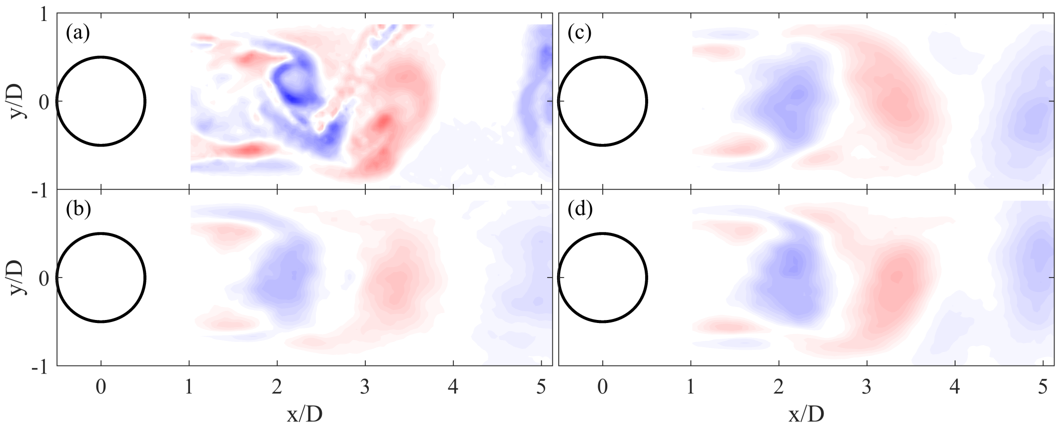

Figure 4: The instantaneous Vorticity field obtained from PIV (a) is compared with the fields estimated from LSE (b), DE (c) and IE(d).

where ω_n indicates the nth harmonic, and N is the number of harmonics considered in the model.

Once the state space model of the cylinder wake is found, one can design the estimator, a Kalman Filter. This estimator assumes a zero-mean Gaussian disturbance $w(t)$ for the linear model and a zero-mean Gaussian noise $n(t)$ for the single sensor measurement. The system takes the form:

$$\begin{aligned} \dot{x}(t) &= Ax(t)+w(t), \\ u_m(t) &= C_m x(t)+n(t), \end{aligned} \tag{7}$$

where $u_m(t)$ is the single sensor measurement, and C_m is the corresponding output matrix. This output matrix is a certain row of the modeshape matrix Φ, corresponding to the location of the sensor. The Kalman Filter, acting as an observer, uses the knowledge of the output measurement $u_m(t)$ to estimate the full state $\hat{x}(t)$ and then the output $\tilde{U}(t)$.

$$\begin{aligned} \dot{\hat{x}}(t) &= A\hat{x}(t)+L(\hat{u}_m(t)-u_m(t)), \\ \hat{\tilde{U}}(t) &= \hat{\Phi}\hat{x}(t), \end{aligned} \tag{8}$$

where the gain L is obtained by solving a Riccati equation, which ensures that the error $e(t)=\hat{x}(t)-x(t)$ converges to zero.

Estimation Results

In the current case, vortex shedding frequency is 0.35Hz obtained from the power spectral density of the velocity. This gives a Strouhal Number of 0.21, which is consistent with the previous literature [5]. The input is chosen as the transverse velocity fluctuation at $x_0 = 2.8D, y_0 = 0D$, where the perturbation energy is maximized. The transverse velocity is chosen as the input signal because the magnitude of its fluctuation is greater than the streamwise velocity. This provides a higher signal-to-noise ratio.

Figure 4 compares the vorticity fields obtained from the PIV reference set with the estimated fields. The LSE field (Figure 4b) only correctly estimates the streamwise positions of the large vortices, similar to the reconstructed field using only the fundamental harmonic (Figure 3b). This suggests LSE correctly estimates the features related to the fundamental harmonic. As for the field estimated using the current Direct Estimation (DE, Figure 4c), the streamwise positions of the vortices are estimated correctly, but the vortices are in the wrong phase of the vortex shedding comparing to the PIV field (i.e. the large vortices are tilted in the wrong angle as shown in Figure 4a and 4c).

The above observation can be explained by using harmonic decomposition. We decompose the PIV velocity field and the estimated velocity fields from LSE and DE into different harmonics. The time coefficients of the fundamental harmonic obtained from the PIV field (Figure 5a,b,c solid lines) are compared with the coefficients estimated from LSE (Figure 5a dashed line) and DE (Figure 5b dashed line). This explains the correct estimation of the streamwise positions by both methods. However, a clear mismatch of the time coefficients at the second harmonic can be observed for both LSE (Figure 5d) and DE (Figure 5e), which explains the discrepancies observed in Figure 4a and 4c, regarding the transverse positions and the tilting angles of the vortices.

Both methods fail to estimate the time coefficients of the higher harmonics. This can be explained by looking at the power spectral density (Figure 2). The peaks at the second and the third harmonics are much lower than the peak at the fundamental harmonic, which means that it is difficult to obtain information about the higher harmonics directly from the measurement. To solve this problem, an indirect method is proposed in the following section.

Indirect Estimation of Cylinder Flow

Estimation Method

Indirect Estimation (IE) provides a different way to estimate the time coefficients of the higher harmonics. IE uses the same method as DE to estimate the time coefficients of the fundamental harmonic. Then instead of obtaining information about higher harmonics directly from the measurement, IE estimates the time coefficients of the higher harmonics from the estimated fundamental harmonic. This can be achieved by using the Trigonometric Addition Formulas iteratively, given as:

$$\begin{aligned} \begin{bmatrix} x_{2n-1} \\ x_{2n} \end{bmatrix} &= \begin{bmatrix} \cos(n\omega t) \\ \sin(n\omega t) \end{bmatrix}, \\ &= \begin{bmatrix} \cos((n-1)\omega t)\cos(\omega t) - \sin((n-1)\omega t)\sin(\omega t) \\ \sin((n-1)\omega t)\cos(\omega t) + \cos((n-1)\omega t)\sin(\omega t) \end{bmatrix}, \\ &= \begin{bmatrix} x_{2n-3}x_1 - x_{2n-2}x_2 \\ x_{2n-2}x_1 + x_{2n-3}x_2 \end{bmatrix}, \end{aligned} \tag{9}$$

where x_{2n-1} and x_{2n} are the states containing the coefficients of the nth harmonic. Based on (9), any harmonic can be expressed as a nonlinear function of the fundamental harmonic alone. Therefore, the estimate of the velocity fluctuation by IE

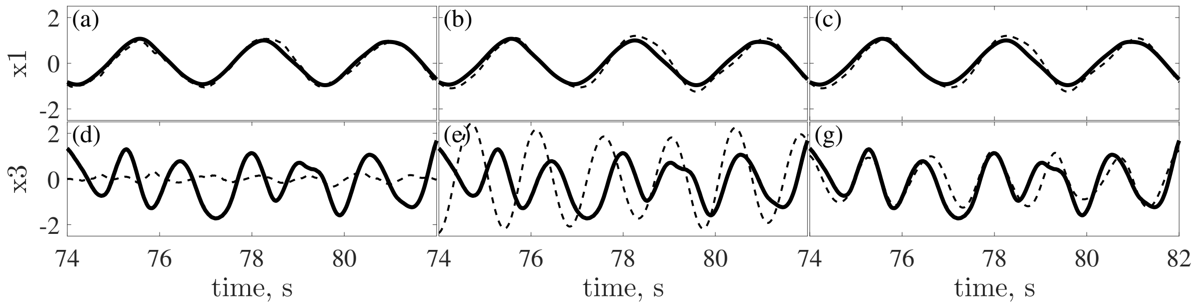

Figure 5: Time Coefficients of the Fundamental Harmonic (dashed lines) from LSE (a), DI (b) and IE (c) and the second harmonic (dasged lines) from LSE (d), DE (e) and IE (f) Compared with PIV (solid lines)

takes the form:

$$\hat{\hat{U}} = \Phi_{12}\begin{bmatrix}x_1\\x_2\end{bmatrix} + \Phi_{3N}\mathcal{N}(\begin{bmatrix}x_1\\x_2\end{bmatrix}) \qquad (10)$$

where x_1 and x_2 represent the states (time coefficients) of the fundamental harmonic. These coefficients are estimated following the same procudures as DE. The term $\mathcal{N}(\begin{bmatrix}x_1\\x_2\end{bmatrix})$ contains the higher harmonics, where $\mathcal{N}$ is the nonlinear operator described in (9). The matrix Φ_{12} contains the first two columns of Φ, and Φ_{3N} contains the remaining columns of Φ.

Estimation Results

The time coefficients of the first three harmonics are estimated using IE, given the same measurement signal as used in LSE and DE. Figure 5f shows good agreement between the amplitudes and phases of the time coefficients of the second harmonic from IE and PIV. The fact that IE is able to estimate the time coefficients of the higher harmonics indicates that the higher harmonics (up to third harmonic in the current case) are non-linearly dependent on the fundamental harmonic because the time coefficients of the higher harmonics are non-linearly estimated using the fundamental harmonic alone. This may suggest that the flow features related to the higher harmonics can be regarded as the "slave" features of the fundamental harmonic.

Based on the estimated time coefficients, the flow field is reconstructed using (10). The estimates from IE are an improvement over DE as we see in Figure 4d. The streamwise and transverse positions and the tilting angle of the large vortices are correctly estimated. To quantitatively demonstrate the performance of all three methods, Figure 6 shows the power spectral density of the estimation error from LSE, DE and IE. The error from DE and IE has lower energy than that from LSE at the fundamental frequency. At the second and the third harmonics, the error from IE has the lowest energy. High energy of error from DE can be observed at the second harmonic, suggesting the flow features related to the second harmonic are over-estimated by DE.

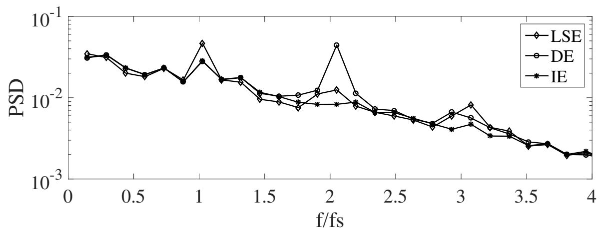

Figure 6: Power spectral density of the estimation error from LSE, DE and IE

Conclusions

The estimation in a cylinder wake at $Re = 1036$ has been considered. We first build a harmonic model based on Harmonic Decomposition. The reconstructed vorticity field based on the first three harmonics shows good agreement with the PIV measurement. Then estimation from Linear Stochastic Estimation, Direct Estimation and Indirect Estimation of the cylinder wake is compared to the PIV measurement. Indirect Estimation outperforms the other two methods by better estimating the flow features related to higher harmonics. Two conclusions are drawn based on the estimation results. i) The estimation of vortex shedding in cylinder flows can be improved if both the fundamental and the higher harmonics are considered. ii) The fact that the time coefficients of the higher harmonics can be correctly estimated based on the fundamental harmonic alone indicates that, for the flow under consideration, the flow features related to the higher harmonics may be slave to the dynamics of the fundamental harmonic.

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