Evolution of the Turbulent Far Wake of a Sphere

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Abstract

The classic turbulent axisymmetric wake derivation for the spreading of wake half-width, δ, and maximum mean velocity, $\bar{u}_{max}$ decay comes from arguments of a high local Reynolds number, Re, and thus negligible viscosity. If instead one assumes the local Reynolds number is small, then at some distance sufficiently far downstream the turbulent production term in the Reynolds shear stress equation will decay and a new similarity solution will arise: as shown by [2, 4]. This solution features the scaling of $\delta \sim (x/d)^{1/2}$ and $\bar{u}_{max} \sim (x/d)^{-1}$. In other words, the turbulent wake is scaling itself at rates that match the theoretical laminar wake, yet with a local Reynolds number high enough for the turbulent fluctuations to be non-negligible. Whilst the derivation of a low Reynolds number solution is a mathematical exercise, obtaining data to confirm or deny its existence has proved difficult. No experiment has been conducted at a combination of high enough initial Reynolds number and far enough downstream to capture this transition behaviour. Furthermore, only the DNS study of Gourlay [3] has been able to achieve this behaviour; leading some researchers to question whether this decay state would occur or if the wake instead would relaminarise [7].

This paper presents results for a towed a sphere through water at a Reynolds number, based on sphere diameter, of 13000. Our experiments have been able to capture the wake transitioning from the high local Reynolds number solution to the low local Reynolds number solution via high-speed time-resolved PIV. The value of local Reynolds number that exhibits itself in the extreme far wake during the low local Reynolds number solution suggests the wake is still turbulent, supporting the claim of [2, 4].

Introduction

Classical analysis of an axisymmetric turbulent wake [9] suggests that the wake will decay at a fixed rate. However, the appearance of two stable turbulent decay rates for the wake of an axisymmetric body was first highlighted by George [2]. After this initial derivation, a few *ad hoc* assumptions were rectified in a second paper by Johansson et al. [4], hereafter referred to as JGG. These papers show how the classic description of the turbulent axisymmetric wake is one that has a half-width, δ, growing as $(x/d)^{1/3}$ and with a maximum mean velocity decaying as, $\bar{u}_{max} \sim (x/d)^{-2/3}$. If instead one assumes the local Reynolds number is small, then at some distance sufficiently far downstream the turbulent production term becomes small and a new similarity solution will arise (specifically, $\overline{v^2}\partial_r(U - U_\infty)$ tends toward zero as $x^{-7/2}$ whereas the rest of the terms tend toward zero as $x^{-5/2}$; note that these powers are nearly equal, thus to notice the discrepancy in decay rates would take a large distance downstream). This solution features the scaling of $\delta \sim (x/d)^{1/2}$ and $\bar{u}_{max} \sim (x/d)^{-1}$. In other words, the turbulent wake is scaling itself at rates that match the theoretical laminar wake, yet with a local Reynolds number high enough for the turbulent fluctuations to be non-negligible. However, as shown by JGG, many turbulent axisymmetric wake experiments are conducted

at too low of an initial Reynolds number, Re_o, to achieve the inertial subrange required for the $x^{1/3}$ δ-growth-rate to appear. Those that do, are typically not continued far enough downstream capture the $x^{1/2}$ δ-growth-rate. JGG provides the estimate of $Re_{\delta^*} < 500$ for the viscous terms to become significant enough for the wake to begin the transition to the $x^{1/2}$. Here:

$$\delta^* = \lim_{r \to \infty} \frac{1}{\bar{u}_{max}} \int_0^r -Ur\,dr$$

is introduced as a wake-width measure. This argument is based on the turbulent wake data of Uberoi and Freymuth [11] who show that the $k^{-5/3}$ decay in a wake is very short for $Re_{\delta^*} \approx 400$ and completely disappears for $Re_{\delta^*} \le 100$. This value may appear low enough to be laminar, however the relaminarisation does not occur until Re_{δ^*} is order unity [9].

The only study that supports the transition of solution states of JGG is the DNS study of Gourlay et al. [3]. The DNS study was not able to resolve both the immediate and far wakes in the same computation, so instead an initially axisymmetric profile with spectrally specified incoherent turbulent fluctuations was employed to model the near wake. This was then allowed to evolve over time on an adaptive grid to a state equivalent of over 200,000 diameters (d) downstream. During this evolution the wake was found to attain the high local Reynolds number solution for the approximate range of 200 to 1600 d downstream, and the low local Reynolds number solution for the approximate range of 40,000 to 100,000 d downstream [3, 4].

The extreme distance downstream required for the recovery of this low Reynolds number solution has lead to authors questioning if the low Reynolds number solution is not the laminar solution. Redford et al. [7] in their extreme far wake DNS simulations found no existence of this low Reynolds number solution, despite a similar wake initiation scheme to Gourlay et al. [3]. Instead this DNS study found that the flow would relaminarise for their computational grid millions of diameters downstream. Redford et al. went on to argue that the low Reynolds number decay solution can occur either if the turbulence and viscous terms are the same order (i.e. the assumption of JGG) or if the solution is purely laminar. Furthermore, Redford et al. highlighted issues from the fixed streamwise size of the computational domain of Gourlay et al. [3], suggesting the solution was 'purely a numerical phenomenon'.

Typically, experiments would be implemented to provide insight on whether the second turbulent decay state of Gourlay et al. [3] or the relaminarisation state of Redford et al. [7] correctly describes the extreme far wake. However, despite large quantity of experimental data for axisymmetric wakes, no experiment has been conducted far enough downstream at a large enough initial Reynolds number to adequately capture the behaviour of the wake switching from the high to low-local-Reynolds-number/laminar decay.

Problem Setup

An experiments was conducted with a sphere, $d = 30.0$ mm, which was towed through the 60 m $\times$ 2 m $\times$ 2 m tow tank in the

Michell Hydrodynamics Laboratory at the University of Melbourne; the details on the facility and PIV set-up are provided by Lee [5]. This experiment used two vertically stacked high speed cameras, allowing for a 33.2 cm (tall) by 17.1 cm (wide) visualisation field. In this field, two velocity components were measured to a maximum distance of 340 d downstream. The sphere was towed past the stationary PIV setup with a velocity of 0.5 m/s, yielding an initial Reynolds number (based on d) of 13000. The effect of the free surface was mitigated by towing the sphere with a 17.3 d submergence depth. Thirty individual runs were performed with this configuration, such that statistical analysis could be performed on the turbulent wake data. Each run consisted of 5000 images from each camera. A settling time of 30 min was used between runs, to ensure that the turbulence from the previous run decayed to zero [5].

The hollow test sphere was held in tension along the tank centreline via two 0.8 mm nylon cords oriented in opposing directions, but both fixed coaxially in the spanwise direction. These cords were affixed to two NACA-0016 hydrofoils that were each located 870 mm away from the sphere centre, comprising a 1770 mm wide apparatus to hold the 30 mm sphere. The towing apparatus had previously found to achieve negligible influence in the PIV region for towed cylinder experiments [8].

In order to prevent the influence of the Kelvin-wake-waves caused by towing the hydrofoil struts through the free surface, a 1.26 m × 1.2 m floating beach for damping the waves was constructed. The beach was composed of foam and vinyl-coil-matting, which acted as the wave damping material. The beach attenuated the Kelvin wake generated by the towing struts, by 2 orders of magnitude, minimising towing-induced-wave influence, allowing for these far wake experiments to occur without the Kelvin-wake-waves disrupting the wake evolution.

The high-speed time-resolved PIV setup for this experiment captured the 2D, streamwise and transverse plane motions in the wake of the sphere. The measurement plane coincided with the midplane of the tow-tank. The water was seeded with 13 μm silver coated hollow glass spheres. The field was illuminated with a Photonics Industries DM50-527-DH, Nd:YLF laser that generated 527 nm light. Each of the two laser heads were set to generate 30 mJ/pulse, and were set externally to pulse at 250 Hz. The beam thickness was found to be 3.0 mm. The measurements were made using two high speed PCO Dimax HS4 cameras, each utilising 105 mm lenses. The measurements began at the trailing edge of the hydrofoils (approximately 5 d downstream from the trailing edge of the sphere). Measurements continued for 20 seconds which corresponds to a maximum measurement distance of 340 d downstream. The PIV images were processed on a 32 × 32 pixel^2 interrogation window with 50% overlap, resulting in 30,702 velocity vectors per image, after stitching the overlapping regions. The spatial resolution for the measurement was 1.4 mm, or 0.0467 d.

Due to the rapid decay of velocity for the axisymmetric wake, an increasing number of images were skipped in the PIV processing to allow for sufficient pixel movement during the PIV processing. A by-product of this was a reduction of the uncertainty in the PIV measurement. Thus for the results shown here the uncertainty in velocity for a standard PIV with subpixel interpolation given by [10], is ±0.63 mm/s through 110.6 $t\,u_\infty/d$ downstream. After this distance, additional spacing for consecutive images were employed to yield an uncertainty of ±0.28 mm/s.

The coordinate system for this experiment is the streamwise, spanwise, and transverse (i.e. vertical) directions, and u, v, and w the corresponding velocity components. The x origin for the experiment is considered to be the sphere trailing edge. The z origin for the experiment is considered to be the sphere centroid. For our coordinate system, our sphere towing velocity vector $\vec{u} = -u_\infty \hat{e}_x$.

The spatial velocity fields from PIV images are obtained (using an in-house code) at closely separated time instances. The mean of a flow variable was calculated at each spatial point, via averaging the value at each $t\,u_\infty/d$ for all images containing that spatial location. A similar field is created for the 30 individual realisations that we have. We then average the reconstructions for the 30 realisations to construct the mean. Note that with this technique, for most of the wake, each spatial location has approximately 2600 individual PIV realisations to generate an average. We denote mean quantities with an overbar and fluctuations about the mean with a prime symbol (e.g. $u = \bar{u} + u'$).

Results and Discussion

The reconstructed evolution of the mean streamwise velocity for the sphere wake is shown in figure 1. Note that the step change in the wake width at 110.6 $t\,u_\infty/d$ downstream corresponds to where the PIV uncertainty was lowered due to additional images being skipped in the processing, which allows for additional fringes of the wake to be included in the streamwise velocity isocontour as they are no longer under the uncertainty threshold.

In experiments where the sphere is fixed and air/water is sent past the body in a tunnel, the decay of velocity is taken to be at the geometric centreline. However, for towed far wake experiments (and DNS computations) the proclivity of the core of the wake to wander can affect the results [1]. As a result, the decay of local maximum velocity is instead shown for the sphere in figure 2. In the figure, it can be seen that the sphere takes on the high Reynolds number rate of velocity decay from 20 to 60 $t\,u_\infty/d$. After this, $\bar{u}_{max}/u_\infty$ takes an intermediate decay rate (i.e. between $-2/3$ and -1), before assuming the low Reynolds number decay from 80 to 150 $t\,u_\infty/d$. The decay rate then once again adjusts and takes on the low Reynolds number decay rate from 165 to 220 $t\,u_\infty/d$. It is these ranges of downstream distances that will be checked as the most promising candidates for the low Reynolds number decay solution.

JGG suggested that the easiest metric to test if the low Reynolds number decay rate has been achieved was to check how the value of the peak turbulence intensity, $u'_{rms,max}/\bar{u}_{max}$, evolved in the wake. Specifically, the value of $u'_{rms,max}/\bar{u}_{max}$ should initially be constant when the high Reynolds number solution is attained. This value would decay once the local Reynolds number dropped below a critical value, and would then restabilise once the low Reynolds number solution established itself. The evolution of $u'_{rms,max}/\bar{u}_{max}$ is shown in figure 3. The data shown in the figure suggest that the initial high Reynolds number decay should begin at approximately 32 $t\,u_\infty/d$ and terminate at approximately 43 $t\,u_\infty/d$. These results agree with the range discussed in the $\bar{u}_{max}/u_\infty$, however it is interesting to note the parameter does not change much over the range of 60 - 100 $t\,u_\infty/d$ encompassing the range the velocity decay suggested the wake was transitioning. The evolution of $u'_{rms,max}/\bar{u}_{max}$ further suggests that the low Reynolds number solution should establish itself at from 210 to 270 $t\,u_\infty/d$. Which does not agree with the velocity decay range previously outlined. This suggests that JGG's notion that the peak turbulence intensity would be the easiest metric perhaps is not correct, and that instead direct measurements of $\bar{u}_{max}/u_\infty$ and δ should be used to study the wake behaviour and state.
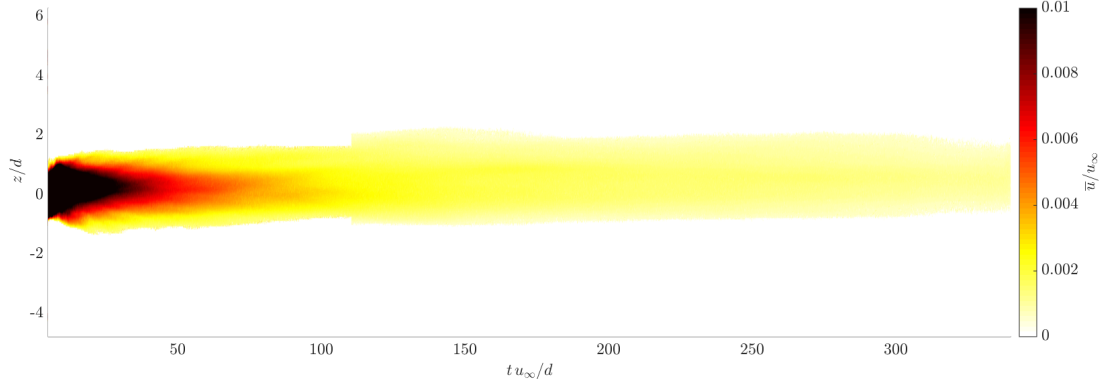
Figure 1: Normalised mean streamwise velocity ($\overline{u}/u_\infty$) evolution.
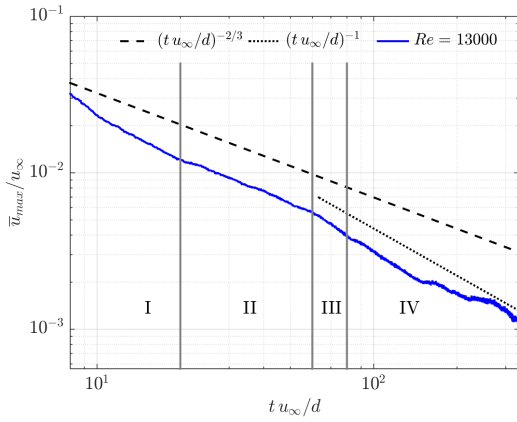

Figure 2: Maximum normalised mean streamwise velocity ($\overline{u}_{max}/u_\infty$) evolution. Note that region I corresponds to the near to far wake transition, II corresponds to the $-2/3$ decay, III corresponds to the decay rate transition, and IV corresponds to the -1 decay region.

The next variable to determine if both the high and low Reynolds number solutions has been attained is the wake half-width, δ, evolution. Wake half-width is defined as the location where the velocity decays to half that of its centreline value [6]. For our experiment we would be able to resolve two such wake edges, one for the upper half of the wake and another for the lower half of the wake. Thus we average these two edges, and introduce the notation $\delta_{0.5\,\mathbf{AVG}}$ to describe that wake edge. This evolution is shown in figure 4. Although a highly sensitive quantity, in the figure it can be seen that $\delta_{0.5\,\mathbf{AVG}}$ attains the high Reynolds number spreading rate from approximately 20 to 50 $t\,u_\infty/d$. After this the wake edge is seen to adjust before growing following the rate of the low Reynolds number solution from 75 to 145 $t\,u_\infty/d$. The wake edge evolution does not suggest a clear recovery of the low Reynolds number solution after 150 $t\,u_\infty/d$ as was suggested by the evolution of $\overline{u}_{max}/u_\infty$.

The results that have been outlined suggest that, by our velocity and wake width data, we have attained the high Reynolds number solution from 20 to 50 $t\,u_\infty/d$. The low Reynolds number solution has been found from 80 to 145 $t\,u_\infty/d$. However, as was previously highlighted by Redford [7], this rate of velocity decay and wake spreading is possible either if the turbulent and viscous terms are the same order (i.e. the assumption of JGG) or if the solution is purely laminar. Thus, in order to check if our solution had re-laminarised, we investigate both
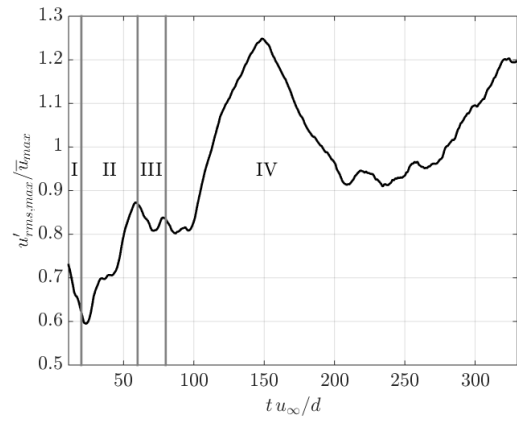

Figure 3: Streamwise peak turbulence intensity ($u'_{rms,max}/\overline{u}_{max}$) evolution.

our mean velocity local Reynolds number, Re, and rms velocity local Reynolds number, Re', as they evolve in the wake. However, the existing literature does not use the same local wake width definition in calculating local Reynolds number. Redford et al. [7] and JGG [4] in a towed sphere configuration use an integrated wake width, δ^*, whereas others [6, 11] use the previously outlined wake half-width metric $\delta_{0.5\,\mathbf{AVG}}$.

Figure 5 shows the Re and Re' evolutions for the outlined wake width definitions. In the figure it can be seen that the wake is still turbulent. Additionally, the local Reynolds number, $Re_{\delta_{0.5,\mathbf{AVG}}}$, is not of order unity, which would be required for relaminarisation we can conclude that our wake attains the low local Reynolds number decay whilst still remaining turbulent [9]. Using the estimate for the order of distance downstream for this transition to occur of Tennekes and Lumley [9]: $2x/c_d \sim (0.5u_\infty c_d/\nu)^3$, where c_d is the coefficient of drag for the sphere at the given Re_o. This yields a distance on the order of $4{\times}10^9\ d$ downstream for relaminarisation. Thus, we can conclude that our results meet the first condition of Redford [7], our flow is still turbulent and we are not measuring a laminar wake. We next turn our attention to the value of local Reynolds number at which the flow begins its second rate of decay. From the figure it can be seen that the Re_{δ^*} value is approximately 100 when the flow begins this rate of decay. At $t\,u_\infty/d = 60$ where the wake begins the transition to this solution, Re_{δ^*} has a value of approximately 130. This suggests the inertial subrange has to vanish before this state establishes itself in an experimental setting, rather than become very short as suggested by JGG.
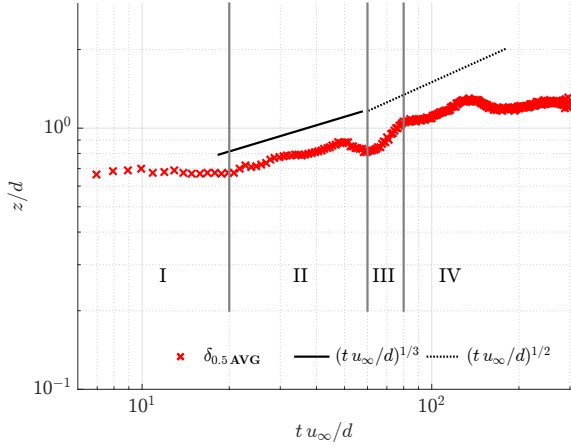
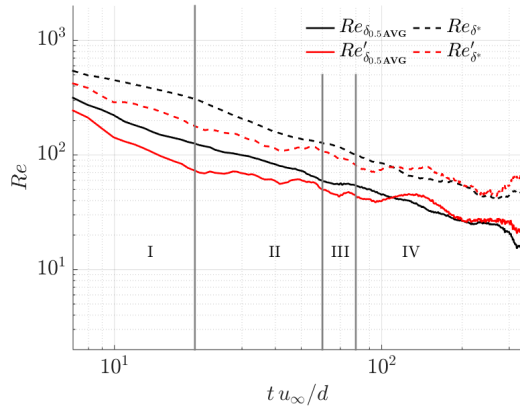Figure 4: Wake half-width ($\delta_{0.5\,\mathbf{AVG}}$) evolution.

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Figure 5: Local Reynolds number evolutions. Note that $Re_\alpha := (\overline{u}_{max}\alpha/\nu)$ and $Re' := (u'_{rms,max}\alpha/\nu)$, where α is a length scale.

Figure 6 shows the streamwise velocity traverses for downstream locations where the wake was exhibiting high and low local Reynolds number states in red and black, respectively. The intriguing result shown here is that the wake is able to maintain self-similarity despite covering the ranges of two different decay states; roughly collapsing on the constant-turbulent-viscosity solution given by the dashed line (i.e. $f(\xi) = \exp(-\xi^2 \ln 2)$ [6]).

Conclusions

We have experimentally demonstrated that it is possible to capture the axisymmetric transition from the high local Reynolds number to low local Reynolds number solution. The transition was captured well by the evolution of the maximum velocity and wake-edge, although it must be noted the wake-edge change is subtle but still matches the theory. The high local Reynolds number decay was found from 20 to 50 $t\,u_\infty/d$ and the low local Reynolds number decay was found from 80 to 145 $t\,u_\infty/d$. Examining the evolution of the local Reynolds number revealed that the wake is still turbulent when exhibiting this decay state, rather than relaminarising. However, the Reynolds number had to be sufficiently low for the inertial subrange to vanish.
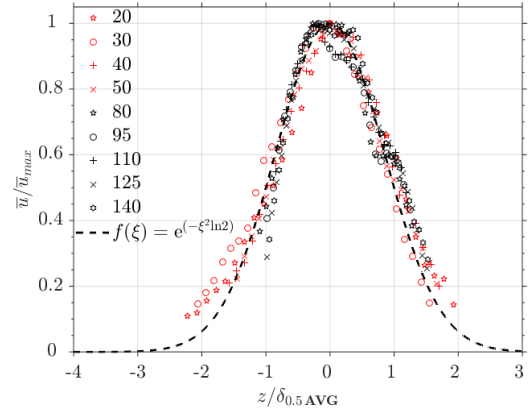

Figure 6: Mean streamwise velocity traverses at various distances downstream.

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