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Entrainment in Pulsing Plumes

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Abstract

Turbulent axisymmetric lazy plumes formed by the issue of saline fluid downwards into a less dense fresh water ambient from a round pipe are examined experimentally. The source flow rate Q_0, momentum M_0 and buoyancy F_0 control the development of the plume. We study the case where flow rate $Q_0(t)$, momentum $M_0(t)$ and buoyancy $F_0(t)$ are sinusoidal functions of time. We produce this pulsing flow using a programmable ISMATEC gear pump. The maximum frequency f of the pulsing plume is of $O(1/\tau)$, where τ is the eddy turn over time scale at the source. Strouhal number $St = fD/u_0$ for the pulsing frequency is within the range of 0.0123 to 1.18 at the source, and the amplitude of Q_0 ranges from 0 to 80%. Measurements of bulk dilution and entrainment are made using the experimental approach of Hunt and Kaye [6]. Average local entrainment is obtained via the integral relationship for $Q(z)$ and $M(z)$ from the MTT model [10] for continuous sources. The relationship between the entrainment coefficient α and the Strouhal number St in pulsing plumes is examined and no significant increase of α was observed compared with unforced plumes for the range of source conditions considered.

Introduction

Plumes are free shear flows driven by buoyancy. They are often formed in industry and nature, such as where smoke is discharged from chimneys and in volcanic eruptions. In turbulent plumes, ambient fluid is entrained into the core flow. The entrainment coefficient α was first introduced by Taylor [15], and has been widely used to parameterize the entrianment process. Morton and Turner introduced it into their MTT model [10]. The entrainment assumption assumes the entrainment velocity is proportional to the average axial velocity of the plume. The value of the entrainment coefficient α for asymmetric turbulent plumes with constant source conditions is reported as being in the range 0.11 ± 0.034 [14].

The value of the entrainment coefficient α has only been reported for plumes which have constant source conditions rather than time dependent source conditions (i.e. flow rate $Q_0(t)$, momentum $M_0(t)$ and buoyancy $F_0(t)$). Several unsteady plume models have been established. Scase [11, 12] investigated numerically and experimentally turbulent plume models where the source was changed from one constant condition to another constant condition. A constant entrainment coefficient α was used in their model. Comparison between their simulations and experiments show good agreement. Plumes with time-varying sources were investigated in a "filling box" [9] problem. Killworth and Stewart [8] conducted the plume experiments with time dependent buoyancy flux including sinusoidal, square wave and sawtooth functions, whose forcing frequency was close to the time scale of the "filling box", which is the time taken by a source to completely fill a room in the absence of ventilation. Results show that the behaviour of forced plumes is similar to unforced plumes. Bolster and Caulfield [2] performed laboratory experiments of the "filling box" problem, where the plume buoyancy flux was forced with a square wave function at a frequency close to the filling box time scale. The higher the forcing frequency was, the larger the average flow rate through the box. In both [2] and [8] the forcing frequency was low compared with the time scale of turbulence in the plume. Measurements at higher forcing frequency have not been reported.

In contrast to plumes, the entrainment in pulsing jets has been widely investigated. Forcing (either by introducing a sinusoidal source or perturbing the source at the exit) can enhance the entrainment rate of jets, mainly at the potential core region where $z/D < 10$ [4, 5, 13]. The natural puffing frequency of unforced jets is $St = 0.30$ [5]. When forcing the jet close to this frequency, the entrainment rate can be increased. Crow and Champagne [5] found that the entrained volume flow of pulsing jets was increased by 32% ($St = 0.3$) compared with unforced case, and [13] shows it was increased by 25% ($St = 0.25$). The entrainment coefficient α has a experimentally determined value of 0.054 [3] for unforced jets. For fully pulsing jets, whose flow rate returns to zero between pulses, the entrainment coefficient α was shown to increase to 0.07-0.09 [3], with the larger value being at smaller z/D. In these conditions, the entrained flow rate, turbulent intensity and radial expansion of the pulsing jets increased.

It is unclear what influence forcing frequency and amplitude have on plumes at different z/D. We carried out experiments on a lazy pulsing flow, whose flow rate $Q_0(t)$, momentum $M_0(t)$ and buoyancy $F_0(t)$ are sinusoidal functions of time. We aim to find the relationship between the entrainment coefficient α and the dimensionless forcing frequency St of the pulsing plume. The frequency f is of $O(1/\tau)$ where τ is the eddy turn over time scale at the source.

Experimental set-up

Our experiment set-up was based on the experimental approach of Hunt and Kaye [6] and is shown in figure 1.

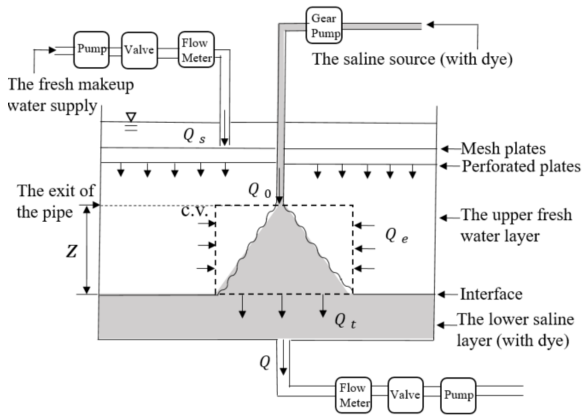

Figure 1: Schematic of the experiment set-up

Plumes were generated by discharging the saline source from a round pipe via the ISMATEC MCP-Z Standard gear pump. The pipe was situated at the upper middle of a 1m cubic tank filled with ambient fresh water. The gear pump is progamable using a Moxa C168 card and a LabVIEW driver. The plume formed a saline intrusion at the bottom of the tank. Dye was added to the saline source to enable the Basler camera pilot piA640-210gc to visualize the plumes behavior and track the height of the bottom saline layer. The upper fresh layer and the dyed bottom saline layer were separated by a sharp interface. Fluid was continually drained from the bottom of the tank and the drainage flow rate Q could be adjusted by an outlet valve and read by the MESLCD5 flow meter (accuracy $\pm 1.5\%$). The interface height is determined by the source flow rate Q_0, drainage flow rate Q and entrained flow rate Q_e. Reducing the drainage flow rate Q results in a higher interface or a smaller Z, where Z denotes the actual distance from the exit of the pipe to the interface. Thus, by adjusting the outlet valve, different interface heights can be obtained. The camera was utilised to measure the height of the horizontal interface and a time series of interface positions were tracked to ensure a steady state for each experiment was reached. According to the conservation of mass $c_0 Q_0 = cQ$, the concentration of salt at the bottom of the tank c was measured as a second indicator of the magnitude of the drainage flow rates Q to ensure steady state was achieved. The error in the reading of the interface height was ± 2mm. However, the uncertainties in our measurement was taken to be the thickness of the interface and was estimated to be $\pm 2.5\%$ to $\pm 2.9\%$. The interface thickness increased with increasing Z and the uncertainty at $Z = 400$mm was ± 10mm. Therefore, the interface height was measured at the middle of the thickness while analysing pictures taken by the camera. In order to maintain the free surface at a forced height with a continuous source flow, a fresh water make up flow was introduced into the top of the tank (below the free water surface), fresh water with the flow rate $Q_s = Q$, from four mesh plates above perforated plates suspended horizontally to ensure the make up water was supplied uniformly. Therefore, as shown in figure 1, by drawing a control volume over the plume region from the exit of the pipe to the interface, the entrainment volume flux Q_e can be deduced by $Q_e = Q_t - Q_o$, where Q_t denotes the volume flux of the plumes at the interface. One value of the drainage flow rate Q corresponds to one specific height of the interface. Once the steady state was reached, the height of the interface remained stationary, Q equals Q_t and can be read by the flow meter. The magnitude of the make up flow velocity was up to 4.1% of the source velocity u_0.

MTT model

We adopt the power law solution for a point source pure plumes discharged in a uniform environment in the MTT model [10], with a self-similar Gaussian velocity profile. For an axisymmetric plume, actual fluxes of buoyancy $\hat{F}(t)$, momentum $\hat{M}(t)$ and volume $\hat{Q}(t)$ are defined as

$$\hat{F}(t) = \frac{1}{2}\pi u_m b^2 g\prime, \quad \hat{M}(t) = \frac{1}{2}\pi u_m^2 b^2, \quad \hat{Q}(t) = \pi u_m b^2, \quad (1)$$

where b denotes the radius of the plume, r the radial coordinate, z the vertical coordinate, $u(r,z,t)$ the vertical velocity, $u_m(z,t)$ the vertical average center line velocity, $g\prime = g(\rho_0 - \rho_a)/\rho_a$ the reduced gravity, ρ_0 and ρ_a is the density of the saline source and ambient fresh water respectively. Source conditions are represented by $(F_0(t), M_0(t), Q_0(t))$, $F_0(t) = 2\hat{F}_0(t)/\pi$, $M_0(t) = 2\hat{M}_0(t)/\pi$, $Q_0(t) = \hat{Q}_0(t)/\pi$ are proportional to actual fluxes of buoyancy $\hat{F}(t)$, momentum $\hat{M}(t)$ and volume $\hat{Q}(t)$. In the experiments, the saline source came from a finite area pipe, so the virtual origin correction method for lazy plumes was used [6]. Following the MTT model [10]

$$Z + Z_v = C^{-3/5} Q^{3/5} {F_0}^{-1/5}, \quad (2)$$

where

$$Z_v = \frac{5\Gamma^{-1/5}(1-\delta)Q_0}{6\alpha M_0^{1/2}}, \quad (3)$$

$$C = \frac{6\alpha}{5}\left(\frac{9\alpha}{5}\right)^{1/3} \quad (4)$$

and

$$\delta = \frac{3}{5}\sum_{n=1}^{\infty}\left(\frac{(1-\frac{1}{\Gamma})^n}{5^{n-1}n!(10n-3)}\prod_{j=1}^{n}(1+5(j-1))\right), \quad (5)$$

where Z denotes the actual distance from the pipe to the interface, Z_v is the distance from the pipe to the virtual origin with the summation δ deduced by Hunt and Kaye [6], α the entrainment coefficient and $\Gamma = 5Q_0^2 F_0/4\alpha M_0^{5/2}$ the source condition. Jets are represented by $\Gamma = 0$ and are dependent on momentum only; $\Gamma = 1$ indicates pure plumes; $0 < \Gamma < 1$ indicates a forced plume, whose initial momentum flux M_0 is greater than that of an equivalent pure plume of the same initial buoyancy flux F_0; $\Gamma > 1$ denotes a lazy plume, whose initial momentum flux M_0 is less than that of an equivalent pure plume of the same initial buoyancy flux F_0. The pure plume characteristic would be reached sooner compared with experiments using forced plumes [7]. All source conditions Γ chosen in our experiments are for lazy plumes (see table 1), and the jet length L_m represents the distance after which the source buoyancy flux plays a significant role in the plume dynamics. The pure plume characteristic would be achieved at $z/L_Q \sim 1$, where L_Q denotes the source length scale. As L_Q was 2 to $3D$ and Z was 10 to $40D$, the flows behaved as pure plumes over most of their height.

Results

Unforced plume experiments

We performed experiments for unforced lazy plumes with multiple source conditions Γ. The accuracy of the gear pump was determined by measuring flow rates with a measuring cylinder and a stop watch, and we determined the pump flow rate Q_0 was accurate to $\pm 3\%$. For unforced plumes, $F_0(t) = F_0$, $M_0(t) = M_0$, $Q_0(t) = Q_0$. Table 1 shows that at the exit of the pipe, Reynolds number Re ranged 140-520 and a parabolic velocity profile was assumed. Plumes became turbulent a diameter downstream of the exit of the pipe. During one experiment, multiple drainage flow rates Q and different heights of the interface could be measured by adjusting the outlet valve. The actual distance between the interface and the pipe Z was determined. The virtual correction Z_v was determined by equation (3) based on different source conditions. The concentration of salt c at the outlet of the tank was measured and was used as a second check of the magnitude of the drainage flow rate Q. We determine the entrainment coefficient α following equation (2) and (4). For each source condition, a series of data is plotted as shown in figure 2.

A linear regression fit was obtained for each series of data and the constant of proportion is $C^{-3/5}/D$. We obtain α by solving equation (4). Adding a linear fit through the case $\Gamma = 22.90$ gives $\alpha = 0.0940$ and is within the range of entrainment coefficient α for unforced plumes. In the insert plot in figure 2, our unforced plumes experimental data with error bars are shown together with results from Hunt and Kaye [6]. Ideally, after applying the virtual origin correction Z_v in equation (2), the intercept of a straight line is expected to be at the origin of the

	Unforced plumes			Pulsing plumes			
$\Gamma = 5Q_0^2 F_0/4\alpha M_0^{5/2}$	1.95	5.54	22.90	5.93	6.95	22.97	142.71
$Re = \rho_0 u_0 D/\mu$	520	300	160	310	310	170	140
$St = fD/u_0$	0	0	0	0.16	0.082	0.30	1.0
$St_z = fD_z/u_z$	0	0	0	$\sim$0.64	$\sim$0.26	$\sim$0.87	$\sim$0.73
$L_m/D = 2^{3/2}\alpha^{-1/2}M_0^{3/4}/F_0^{1/2}D$	0.89	0.51	0.27	0.49	0.50	0.28	0.11
$L_Q/D = 5Q_0/6\alpha M_0^{1/2}D$	2.6	2.6	2.8	2.5	2.8	2.8	2.8

Table 1: Source conditions of unforced and pulsing plumes experiments. The jet length L_m and the source length scale L_Q are scaled on pipe diameter D, μ represents the dynamic viscosity of the saline source, St_z the Strouhal number at the maximum Z achieved by each experiments, D_z the diameter of the plume at Z, u_z the average center line velocity at Z.

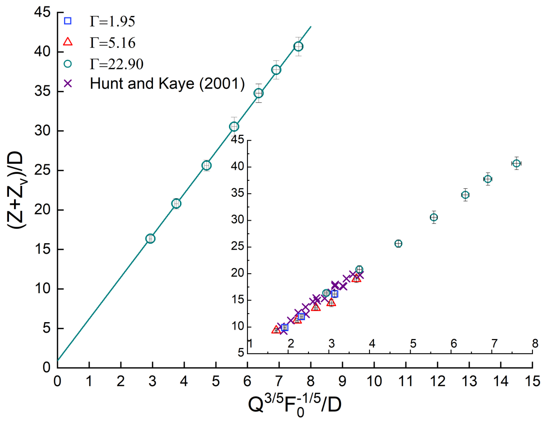

Figure 2: The interface and drainage flow rate measurements with error bars for unforced plumes. The solid straight line represents linear fit to $\Gamma = 22.90$ is $(Z+Z_v)/D = 5.29Q^{3/5}F_0^{-1/5}/D + 0.91$. The subplot shows all experimental results with error bars.

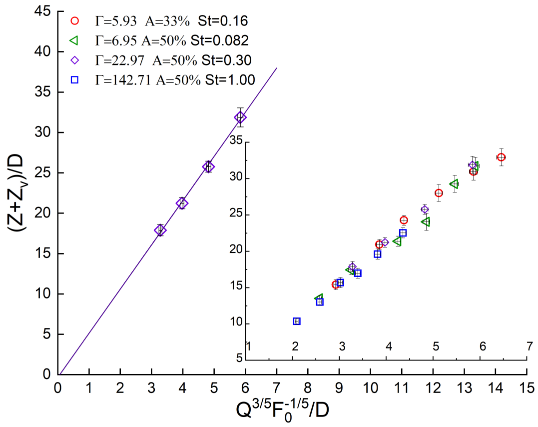

Figure 3: The interface and drainage flow rate measurements with error bars for pulsing plumes. The solid straight line represents linear fit to $\Gamma = 22.97$ is $(Z+Z_v)/D = 5.48Q^{3/5}F_0^{-1/5}/D - 0.34$. The subplot shows all experimental results with error bars.

coordinates. In our experiments, for the straight fit shown in figure 2, the intercept shows that the location of the virtual origin is shifted by $0.91D$ from the origin of the coordinates, which indicates that the virtual origin correction method [6] gives a reasonable correction for unforced plumes.

Pulsing plume experiments

The accuracy of the gear pump for sinusoidal flow rates was determined again by measuring flow rates over multiple cycles with a measuring cylinder and a stop watch, and we determined the pump flow rate $Q_0(t)$ was accurate to $\pm 3\%$. For pulsing plumes, source conditions $(F_0(t), M_0(t), Q_0(t))$ are sinusoidal functions of time. When estimating Γ and F_0 in equation (2), source conditions are considered to be average fluxes (F_0, M_0, Q_0) over a cycle. Multiple experiments with amplitude $A = 50\%$ and different Strouhal number $St = fD/u_0$ (f denotes the forcing frequency, u_0 the average velocity at the exit of the pipe) were carried out and results are shown in figure 3. The average entrainment coefficient α is deduced the same way as for unforced plumes.

The data in figure 3 is quite linear. We add a linear fit for the case that has similar average source condition $\Gamma = 22.97$, $A = 50\%$, $St = 0.30$ with unforced plumes displayed in figure 2. For this case, $\alpha = 0.0899$, which is 4.36% lower than for the unforced plume. With $\Gamma = 22.97$, the intercept shown in figure 3 is within $0.34D$ of the origin, which means the virtual correction method gives a reasonable correction for pulsing plumes under the same average source condition. By fitting a straight line through each experiment with different Γ, α can be deduced and is shown in figure 4.

Figure 4 shows experimental results of α for unforced and pulsing plumes with multiple source conditions. Solid markers indicate α for unforced plumes and hollow ones for pulsing plumes. The α for unforced plumes is at $St = 0$. The results match well with published experiment outcomes for unforced plumes [14]. Compared with unforced plumes, α for pulsing plumes with the same amplitude $A = 50\%$ are lower than the smallest α for unforced ones. It has been shown that forcing jets at $St = 0.30$ can increase entrainment [4, 5]. However, α for $St = 0.30$ in our case $(\Gamma = 22.97, A = 50\%)$ is the lowest in figure 4.

Pulsing plume frequency and amplitude study

Our results show that no significant change can be seen in α under three different St when $A = 50\%$. In order to find if there is any specific Strouhal number St and amplitude A which significantly affects α, a new experimental method to test multiple source conditions was introduced. First, an unforced plume test was conducted, then St or A was modified. According to equation (2), one specific Q and F_0 corresponds to one Z. Thus, after switching into pulsing plumes by adjusting A or St, if the height of the interface remains the same (i.e. same Z), then α deduced from that corresponding source condition for pulsing plumes would be similar with unforced plumes. If the height of the interface changes, then α deduced from that correspond-

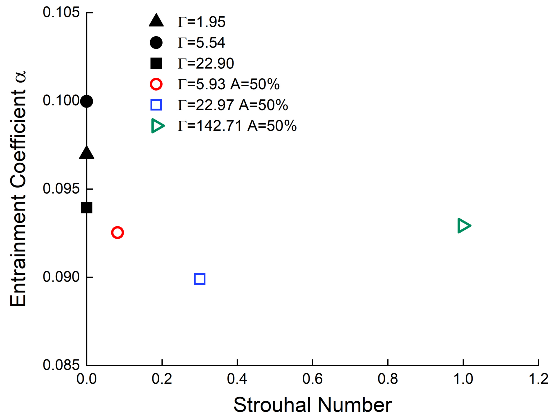

Figure 4: The entrainment coefficient α for unforced and pulsing plumes with $A = 50\%$. Solid markers stand for α for unforced plumes and hollow markers for pulsing plumes.

ing source condition for pulsing plumes would be different from unforced plumes. Three series of tests were carried out. When fixing A at 40% and 50%, nine tests with St from 0.0123 to 1.07 and three tests with St from 0.0164 to 0.164 were performed respectively. No more than 5% increase of the height of the interface was observed. Another three tests with $St = 1.18$ and $A = 50\%$, 66%, 80% were carried out. The height of the interface increased by 10 mm. Analysis of α suggests that only a 2% difference in α occurred. Over the entire range, the results suggest a maximum change of α is 4.36%.

Discussion

The data in figure 3 is well represented by a linear fit over all distances from the pipe. This suggests the entrainment coefficient α is constant over the length of the plume. This also suggests that the local entrainment coefficient α is proportional to the local plume velocity even in a pulsing plume where the local velocity is varying in time. The results suggest that the entrainment assumption and the average plume characteristic predicted by the MTT model [10] are applicable for pulsing plumes forced at time scales close to eddy turn over times at the source.

Bharadwaj and Das [1] showed that plumes have a natural puffing frequency. The stability of plumes is determined by the density ratio, Froude number and Reynolds number at the source. At lower density ratio (e.g. Boussinesq plumes), puffing is confined to lower Froude number and higher Reynolds number. All source conditions of our pulsing experiments are outside the neutral curve. This suggests that the natural puffing frequencies are not contributing to behaviors observed in our results.

Conclusion

Turbulent axisymmetric lazy plumes and pulsing plumes with sinusoidal source conditions $(F_0(t), M_0(t), Q_0(t))$ were obtained experimentally in a laboratory rig. The bulk entainment coefficient α was determined. In these flows, pure plume conditions were achieved within $z/D = 2 \sim 3$ downstream. The entrainment coefficient α for unforced plumes ranges between 0.0940-0.100 and matches the range reported in the literature [14]. The entrainment coefficient α of pulsing plumes ranges from 0.0899 to 0.0929. Experimental results with different source conditions shown in figure 3 are well represented by a linear fit, which suggests that the entrainment assumption and MTT model [10] are applicable for pulsing plumes forced at time scales close to eddy turn over at the source. Forcing jets at $St = 0.30$ can increase entrained flow rates [5], our experiments show that forcing plume at $St = 0.30$ does not result in higher entrained flow rates, but in fact may reduce the entrainment. With source flow rate amplitude up to $A = 80\%$ and $St = 1.18$, no more than 4.36% change of α is obtained in both tests and experiments with pulsing plumes.

Acknowledgement

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