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Numerical Analysis of Jamming Conditions in a Fluid-Driven Granular Flow

Wenwen Chen, Dao Ho-Minh and Danielle S. Tan
Department of Mechanical Engineering
National University of Singapore, Singapore 117575

Abstract

Jamming is a phenomenon commonly seen in granular flows, whereby the solid particles are unable to pass through a (typically) narrow opening due to the particles interlocking and forming a solid block or plug. Intuitively, we expect that jamming might occur when the particles are relatively large compared to the opening, are angular in shape and/or have rough surfaces; however it is still possible for fine, smooth circular particles to form a jam due to other reasons such as cohesive attraction. Thus it is reasonable to conclude that the factors significant to jamming vary for different particle lengthscales. Here, we investigate the case of jamming when relatively large solid particles are being driven by a fluid medium towards an opening, such as might occur when debris or ice are carried down a river in which there are obstructions or constrictions, or through the legs of an offshore structure. This opens up the field of potentially-significant factors to include local concentration of the solid particles, density of the particles relative to the fluid, surface tension between the particles and the fluid, as well as the change in surface friction due to the presence of the fluid. We use the discrete element method (DEM) to numerically simulate a 2D flow of solid particles being driven by a flow of constant velocity towards a stationary obstacle, represented by 4 solid clusters arranged in a square. From our simulations, we show that the particle size relative to the opening size is strongly significant. In addition, we demonstrate that inhomogeneity in terms of size distribution of particles may be accounted for by using an 'equivalent' mean particle size in our Weibull-based probability function to predict jamming occurrence.

Introduction

The jamming of flowing particles is a ubiquitous phenomenon seen in granular flow, whereby the particles form a strong arch capable of stopping flow when a stream of granular particles channel through a small opening. It is common in both natural and industrial scenarios, such as river ice jams and plugging of granular pipe flow, which may cause environmental hazards and economic losses[1-2]. Therefore, understanding and controlling the jamming is of significant technological importance. Although theoretical, laboratory experimental and computer simulation studies have showed that the nature of jamming is analogous in granular systems[3-5], similar to a liquid-solid phase transition, the conditions under which jamming occurs are still not properly understood. Intuitively, one might assume that jams would occur when the particle's size is relatively large compared to the opening, has sharp corners and angular sides, or its surface is rough. However, an arch or bridge could also be formed by fine, smooth circular particles which are much smaller than the opening. It may be that an arch is mainly stabilized by external mechanical stresses instead of internal strength between particles [6] and can be released by continuous force imparted by other particles acting on it. In addition, relevant studies [5] suggested that the jamming of fluid-driven flow differs from that of gravity-driven flows due to the hydrodynamic effect of fluid. Jams from fluid-driven flows started from dilute flow which has to be dense in flows under gravity. Therefore, the effect related to surface tension between the particles and the fluid, fluid properties, local concentration of the solid particles, as well as the change in surface friction due to the presence of the fluid should also be taken into consideration when using a fluid as the driving medium for granular flow. *Hopkins et al.* [7] proposed that other factors such as the size distribution of the solid particles and the global solid area fraction also affect jamming as found in the ice passage around bridge piers. The mixed ice floes seemed to jam easier than uniform-diameter small floes because the largest ice floe in the mixture was caught between piers or bank stopping the passage of ice. Hence, it is still a challenging problem to quantitatively describe the conditions that lead to jamming. The purpose of this work is to examine the effects of particle size, as well as inhomogeneity in terms of particle size distribution, affecting the jamming of relatively large solid particles suspended in flowing fluid toward an opening. The eventual goal is to build a model or equation relating these key parameters to predict the probability of jamming occurrence. Furthermore, knowledge of jamming mechanism can be useful in design of obstructions or constrictions to meet the requirements of jamming prevention, such as bridge or legs of an offshore structure.

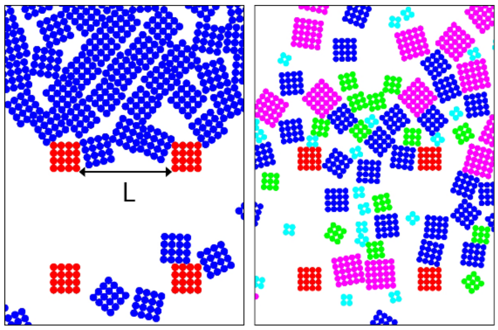

Figure 1. Diagram illustrating the two different flows of particles: (left) homogenous particle flow and (right) inhomogeneous particle flow. The red clusters represent a 4-legged obstacle with gap spacing *L*.

We use the discrete element method (DEM) [8] to study the jamming events that appear in the 2-dimensional (2D) flow of solid particles being driven by a fluid medium towards a stationary obstacle, represented by 4 solid clusters arranged in a square (figure 1). The DEM is a popular numerical simulation method widely used to study the dynamic behaviour of particles [9], including particle jams. It provides detailed analysis and its results are comparable to that of experiments. In order to investigate the effect of concentration and size distribution of particles, two particle conditions will be studied: both uniform-sized particle cluster (referred to as homogenous particle flow) and mixed-size particle cluster (referred to as inhomogeneous particle flow), depicted in figure 1. The influence of both flow and particle conditions on jamming can be measured by the statistical probability of jamming events observed in numerical simulations.

The remainder of this paper is organised as follows. First, the details of DEM simulation scheme are described. Then the simulation details and results, as well as the resulting jamming predictive model are reported. Finally, the main conclusions of this work are summarized.

Methodology

In this work, a soft-sphere DEM [10] with bonded-particles [11] was developed to model the large solid particle cluster that represented by a packing of uniform-sized spherical particles that are connected together by parallel bonds to form a desired shape with different sizes as shown in figure 1 and figure 2. The interaction between the particles is given by

$$m_i \frac{dv_i}{dt} = \sum_{j \neq i} F_i^c + F_i^f + F_i^g \tag{1}$$

where v and m are the mass and velocity of the i^{th} particles, F_i^c is the force exerted on the i^{th} particle by j^{th} particle, and F_i^f and F_i^g are the force acting on the i^{th} particle due to the fluid and gravity, receptively. The force F_i^c can be further decomposed into two components of the normal force F_n and the tangential force F_t between a pair of contacting particles i and j.

$$F_n = -k_n \delta_n^{1/2} n_{ij} - \gamma_n \delta_n^{1/4} \dot{\delta}_n \tag{2}$$

$$F_t = \min\left\{ -k_n \delta_n^{1/2} \delta_t - \gamma_t \delta_n^{1/4} \dot{\delta}_t, \mu^c F_n \right\} \tag{3}$$

where k_n and k_t are the normal and tangential stiffness coefficients, γ_n and γ_t are the corresponding viscoelastic damping coefficients, and μ^c is the inter-particle friction coefficient. All these coefficients here are evaluated by Hertz's theory using the material properties. δ_n and δ_t are the overlaps in the normal and tangential directions and the dots overhead represents its rate of change with the time. n_{ij} is the unit vector in the normal direction for the particle pair.

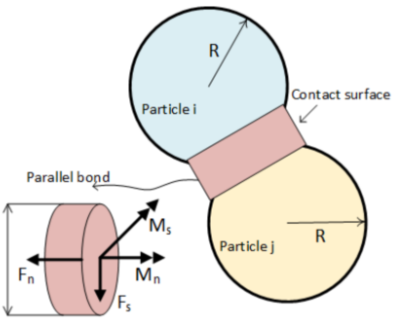

Figure 2: Bonding model.

The inter-particle forces on a particle due to parallel bonds is

$$F_i = F_n n_i + F_s t_i \tag{4}$$

$$M_i = M_n n_i + M_s t_i \tag{5}$$

where F_i and M_i denote the total force and torque associated with parallel bonds on the particle i. F_n, F_s, M_n and M_s are the normal and sheer components of force and torques with respect to the contact plane, respectively, as shown in figure 2. The maximum normal and shear stress in the bonding model can be calculated by

$$\sigma_{\max} = -\frac{F_n}{A} + \frac{|M_s|}{I} \bar{R} \tag{6}$$

$$\tau_{\max} = \frac{|F_s|}{A} + \frac{|M_n|}{J} \bar{R} \tag{7}$$

where A and $\bar{R}$ are the area and radius of the bonding disk with $A = \pi \bar{R}^2$, J and I are the polar inertia moment and inertia moment of the bonding disk cross-section with $J = 1/2\,\pi \bar{R}^4$ and $I = 1/4\,\pi \bar{R}^4$, respectively.

In addition, hydrodynamic forces are also considered, which includes the drag force F_d and added mass force F_{am}:

$$F_d = \tfrac{1}{8} \pi \rho_f C_D D^2 \,|U_f - u_p|\,(U_f - u_p) \tag{8}$$

$$F_{am} = \tfrac{1}{2} \rho_f V_p \left(\frac{dU_f}{dt} - \frac{dv_i}{dt}\right) \tag{9}$$

where ρ_f, C_D, C_D, u_p, U_f, V_p are density of fluid, drag force coefficient, diameter of particle, velocity of the i^{th} particle, fluid velocity and volume of particles, respectively.

Simulation Details and Results

The DEM simulation of 2D flow was performed using open-source software LIGGGHTS[12]. The simulation domain is a rectangle of dimension 0.9 (x) × 3.5 (y) m where the stationary obstacle is located at the bottom centre of y direction. The top and bottom boundaries are fixed and the two sides are periodic. The obstacle is represented by 4 solid square clusters arranged in a square of spacing L as red in figure 1. The mobile clusters are all square-shaped, with 4 possible sizes D: 0.016, 0.024, 0.032 and 0.040m, respectively formed by 4, 9, 16 and 25 uniform spherical particles of diameter 0.008m. In figure 1, the homogeneous case is composed of 16-particle clusters, while the inhomogeneous case has all 4 cluster sizes. The material properties, such as friction, damping and stiffness coefficients, as well as fluid density and viscosity were chosen to match the physical experiments. Generally, spacing-to-size ratio L/D range from 2 to 8, relative approach velocity V from 0.02 to 0.06m/s, global area concentration C from 40% to 80% and mixing area fraction from 10% to 70% were investigated. The probability of jamming P for certain simulation condition (L/D, C, V) was determined by

$$P\left(\frac{L}{D}, C, V\right) = \frac{number\ of\ runs\ wherein\ a\ jam\ occurred}{total\ number\ of\ simulation\ runs\ conducted} \tag{10}$$

For each condition, at least 10 independent DEM simulation runs using identical material properties and physical conditions were conducted with a slightly different initial cluster arrangement. Additional runs were conducted if needed to eliminate statistical errors due to the complexity of lengthscales in inhomogeneous particle flow. Once P is determined from simulations, the data sets (P, L/D, C, V) are fitted to equation (11) using L/D, C and V as variables. We first proposed an initial expression of P as a function of L/D, C and V for homogenous particle flow. After that, the effect of non-homogeneity in particle cluster in terms of size was investigated and the model adjusted to incorporate the size inhomogeneity.

Homogenous Particle Flow

From intuition and published literature[5], we know that the probability function should taper off to zero for large L/D ratios, and plateau at one for small L/D ratios (similar for other variables) for homogenous particle flow. Hence our probability expression should look similar to a Weibull function (which exhibits such behaviour):

$$P\left(\frac{L}{D}, C, V\right) = e^{\left(-\alpha \frac{L}{D}^{\beta}\right)} \tag{11}$$

where α and β are the scale and slope parameters respectively. These are empirically-determined functions of C and V. The spacing-to-size ratio plays a dominant role as the jamming probability is exponentially related to L/D as seen in figure 3 for two concentrations C=0.4 and 0.8 of 16-particle clusters at the lowest (0.02m/s) measured velocities. It is expected that the probability of jamming is strongly determined by this ratio, as it increases with decreasing size ratio for all measured conditions. It is close to unity when L/D is smaller than 2, and zero for L/D is larger than 10 at high concentration — this agrees with those of similar studies available in literature [5]. The same trend is observed at low concentration with only slightly differences at upper and lower L/D limits.

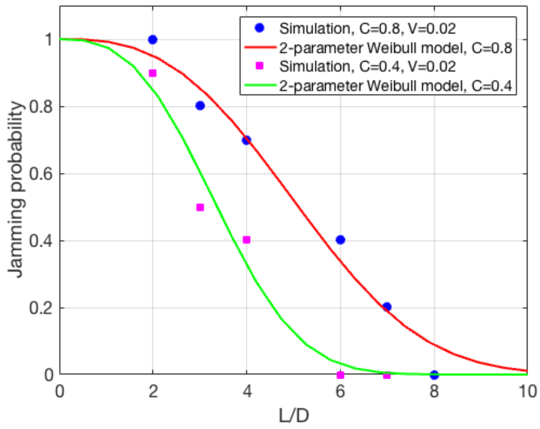

Figure 3. Weibull-based jamming probability fit (lines), as a function of spacing-to-size ratio L/D at low velocity V=0.02m/s for high concentration (C = 0.8) and low concentration (C = 0.4), superimposed on simulation data (markers).

Figure 4 shows the effect of global concentration on jamming probability, for which all size ratios exhibit the same trend: the probability of jamming increases with the concentration. Intuitively, it is easier to form a jam at high concentration. Comparing the different size ratios though, it is clear that the dependence on global concentration is greater at higher L/D.

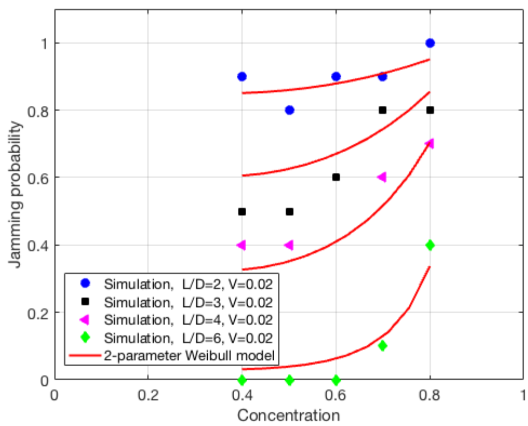

Figure 4. Weibull-based jamming probability fit (lines), as a function of global particle concentration C at low velocity V=0.02m/s with spacing ratio L/D = 2, 3, 4 and 6, superimposed on the simulation data (markers).

We also investigate the effect of velocity on the jams as shown in figure 5. The velocity has a strong influence on the jamming probability. It is found that the probability decreases with increasing relative velocity. This relationship also varies according to the spacing-to-size ratio and the decrease is even faster at high L/D. When the velocity is high (V=0.06m/s), no jam can be formed until the ratio L/D = 2. Since velocity V affects the magnitude of jamming probability more than concentration C as in figure 4-5, it is reasonable to conclude that both concentration and velocity act as the scale parameter of the probability function, while velocity is the slope parameter of the probability function. Using the simulation data, we fitted a quadratic function of C and V to α, and a quadratic function in V only to β. (Due to confidentiality issues, we are unable to provide the fit coefficients here.) This 2-parameter Weibull function is seen to fit the simulation data well as illustrated in figures 3-5 and can be expected to accurately predict the probability of jamming for homogenous cases.

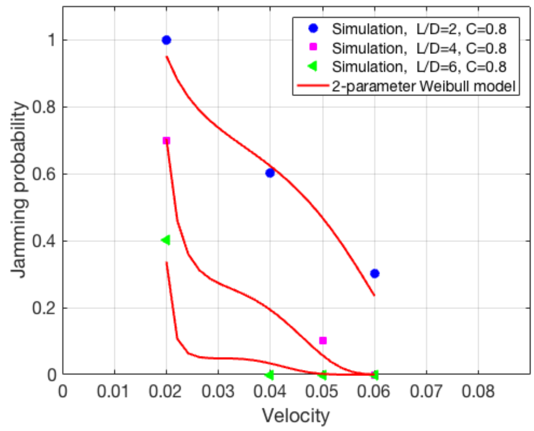

Figure 5. Weibull-based jamming probability fit (lines), as a function of relative velocity V at high concentration C = 0.8 with spacing ratio L/D = 2, 4 and 6, superimposed on simulation data (markers).

Inhomogeneous Particle Flow

In reality, an inhomogeneous particle flow is more likely than a homogeneous one and thus we need to have a model describing the occurrence of jamming formed by mixed size particles. To investigate the effect of size distribution, 5 sets of 4-component mixtures, defined in terms of mixing area fractions of components as listed in table 1, were studied. While not all-encompassing, these may give a good representation of the inhomogeneous particle condition in nature to some extent.

Set ID	Area Fraction			
	4-particles	9-particles	16-particles	25-particles
1	25%	25%	25%	25%
2	10%	10%	70%	10%
3	70%	10%	10%	10%
4	10%	70%	10%	10%
5	10%	10%	10%	70%

Table 1. Simulation sets defined by mixing area fractions of the 4 sizes of square cluster particles.

We simulated these 5 sets for global concentration C=0.4, relative velocity V=0.03m/s, and gap space L=0.064m. Jamming occurred for all 5 sets of mixing ratios investigated, although the cases with larger mean particle size seem to jam easier.

As mentioned previously, the spacing-to-size ratio L/D is the dominant factor that controls the jamming and its relationship satisfies the modified 2-parameter Weibull function quite well. In analogy to L/D for a homogenous flow, a natural question arises as to how the size distribution of particles affects this spacing-to-size ratio. In other words, we aim to determine an expression relating the various sizes in the mixture, to an 'equivalent' uniform size D in Weibull model yielding the same jamming probability. In this way, our previously proposed Weibull-based function can be easily modified for inhomogeneous conditions by replacing D with D_{avg}. Here, we define a mean particle size:

$$D_{avg} = \sum_i \lambda \phi_i D_i \quad (12)$$

where λ is a scaling factor, and ϕ_i and D_i are the area fraction and particle size of component i of the mixture, respectively. The fitting results superimposed on the dataset are shown in figure 6. We found that λ = 0.794 gives the best fit; the difference between all D_{avg} and the corresponding D (calculated by inverting our Weibull-based function for the same jamming probability) is smaller than the diameter of the individual spheres making up the clusters.

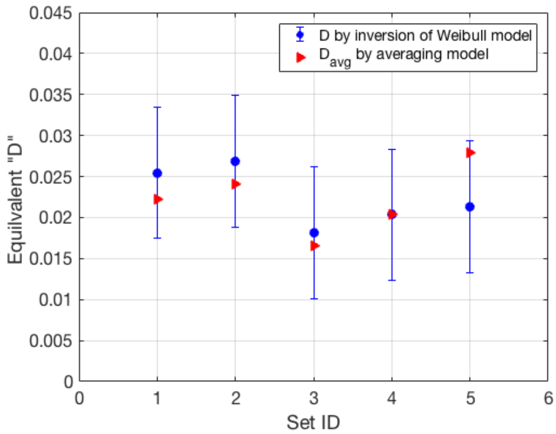

Figure 6. The particle-size-averaging model fit, plotted by equivalent "D" vs. Set ID. The error bars shown are one particle size (0.008m).

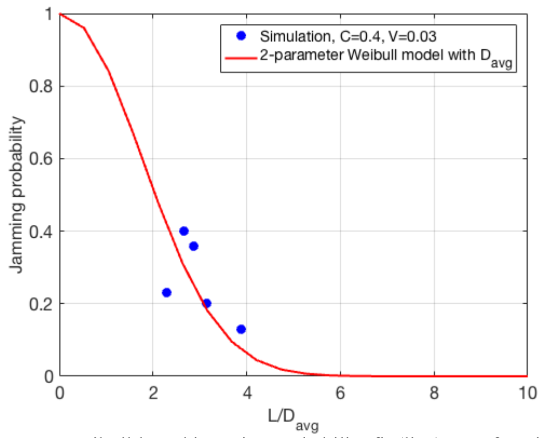

Figure 7. Weibull-based jamming probability fit (line), as a function of spacing-to-mean-size ratio L/D_{avg} at velocity V=0.03m/s with concentration C=0.4, superimposed on simulation data (dots).

In figure 7, we plotted the modified jamming probability function against spacing-to-mean-size ratio L/D_{avg}, with the simulation data as comparison. We observe that the modified function fits the data fairly well. In principle, a homogenous case could be treated as one-component mixture; however, it must be noted that D_{avg} does not reduce to D for one-component mixtures due to the introduced scaling factor λ. As such, we are unable to use the same probability function for both homogeneous and inhomogeneous cluster size flow cases.

Conclusions

In this paper, we used a DEM model to investigate the jams formed by both uniform-sized cluster particles and mixed-size cluster particles driven by a fluid medium towards an opening, such as might occur when debris or ice are carried down or through the legs of an offshore structure. The relationships of jamming probability with the spacing-to-size ratio L/D, concentration C, velocity V and size distribution were investigated, and the results used to develop a 2-parameter Weibull-based function. As the actual conditions of full-scale ice-infested waters are complex, of which inhomogeneity in terms of size distribution of particles is important, we examine an approach to account for the size inhomogeneity by defining an 'equivalent' mean size D_{avg} to replace the uniform size D in the Weibull-based probability function. The predictions of our Weibull-based function are shown to be in good agreement with the numerical results for both homogeneous particle flow and inhomogeneous particle flow. In the future, to further improve the jamming probability function, the effect of cluster shape can be taken into account as that also affects the net drag force of fluid.

Acknowledgments

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