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A Numerical Simulation Model for the Vortex Induced Vibration of Flexible Risers Using Dynamic Stiffness Matrices

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Abstract

In this study, a time-domain numerical method with dynamic stiffness matrices is proposed to obtain the characteristics of the vortex induced vibration (VIV) of a flexible riser. Due to the large ratio of length to diameter, the marine riser can be approximated as a cable model. Based on the finite volume method (FVM), the cable model is divided into a certain number of segments. The dynamic stiffness matrix of each segment reflecting the time varying strain energy is updated in each step-time. The hydrodynamic forces affecting on segments are calculated by the instantaneous vorticity conserved boundary condition (IVCBC) method, and the nonlinear governing equation is solved by the step-by-step time integration method. Validation of this numerical approach is carried out by comparing the static balance and the dynamic responses with quantities in classical experimental cases. The results show that: when the vibration of riser is high order mode vibration, the dynamic stiffness matrix cannot be ignored, and also indicates that the present method has strong applicability for VIV research.

Introduction

With the exploitation of offshore oil resources moving into deeper water, the ratio of length to diameter (L/D) of marine risers has increased rapidly[1], which leads the appearance of more complex vibration issues. Significant vibration with large amplitude induces a riser to experience fatigue damage and reduces its service life.

So far, many researchers adopted numerical methods to investigate the vortex-induced vibration of riser. [2] used the discrete vortex method and the stream function based on the boundary integral method to assess hydrodynamic forces. A consistent static stiffness matrix was obtained through a finite element technique to solve an Euler-Bernoulli beam equation and a truss bar equation. [3] implemented numerical simulations of risers with finite element methods that are tolerant of sparse meshes and high element aspect ratios. [4,5] performed time domain simulations using an unsteady overset-grid Navier-Stokes method. The fluid domain was discretized into nearly one million elements. At each time step, the drag and lift forces were computed by solving the Navier-Stokes equation. [6,7] obtained the vortex-induced force on a flexible riser in the cross flow (CF) direction using an inverse estimation method based on the state-space formulation of a finite element beam model and an inverse method based on the optimal control theory. [8] developed a fluid-structure-interaction model by solving the viscous Navier-Stokes equation, turbulence model, and structural dynamic response equation with the dynamic meshing technique. [9] presented a bound and positively preserving variation method for the turbulence transport equation of Spalart-Allmaras based delayed detached eddy simulation (DDES); a standing wave response and complex chaotic response of the riser VIV were confirmed. [10] carried out numerical studies on the fluid–structure interaction of an elastic cylinder in turbulent fluid. The Arbitrary Lagrange-Euler (ALE) Navier-Stokes equations with the large eddy simulation model were applied to model the turbulent flow and the Euler–Bernoulli beam dynamic equation was solved for the vibration of the elastic cylinder. [11] proposed a method to obtain the hydrodynamic forces of a flexible riser undergoing VIV using the measured strain. The tensioned riser was approximated as an Euler–Bernoulli beam and an inverse method was adopted for the calculation of the hydrodynamic forces in the CF and inline (IL) directions. [12] investigated the IL and CF VIVs of a long cylindrical tensioned beam using 3-D direct numerical simulation. They found that the flow supplies energy to the structure mainly at the local lock-in frequency

Above all, the stiffness matrix is generally considered as an invariant. However, the stiffness matrix is calculated by the strain energy in the finite volume method (FVM), and the strain energy is obtained using the deformation of the riser. Meanwhile, the riser deformation is time-varying. Hence, the stiffness matrix should consider time varying characteristics. In this paper, a time-domain numerical method using dynamic stiffness matrices is proposed to obtain the characteristics of the VIV of a flexible riser.

Numerical method

The marine riser is approximated as a cable model, which is divided into a certain number of segments. The dynamic stiffness matrix and bending stiffness matrix are updated in calculations. The hydrodynamic forces are calculated by the instantaneous vorticity conserved boundary condition (IVCBC) method newly proposed by [13~15].

To derive the governing equation for riser vibration[15], the Hamilton theory is used.

$$\int_{\tau_1}^{\tau_2}\delta\left(T-U\right)dt+\int_{\tau_1}^{\tau_2}\delta W_{nc}dt=0 \tag{1}$$

where δ and W_{nc} are the variation within a specific time and the work done by non-conservative force, respectively, and U and T are strain energy and kinetic energy, respectively.

The equation based on the Lagrange theory can be written as:

$$\begin{gathered}\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{x}}\right)+\frac{\partial W_c}{\partial x}+\frac{\partial U}{\partial x}=\frac{\partial W_e}{\partial x}\\ \left\{F\right\}-\left\{R\right\}=0\end{gathered} \tag{2}$$

where W_c and W_e are damping work and external force work, respectively, and F and R are the internal forces and the external forces, respectively. It follows that the equation of vibration can be written as:

$$[M]\{\ddot{X}(t)\}+[C]\{\dot{X}(t)\}+[k]\{X(t)\}=\{R(t)\} \quad (3)$$

where M, C, and K are the mass matrix, the damping, and the stiffness matrix, respectively. The nonlinear governing equation of the riser's vibration is solved to obtain the accelerations, velocities, and displacements of the riser using the step-by-step time integration method in the [16]. The numerical procedure for the dynamic analysis is as follows:

(1) Initialize meshing and calculate the initial coordinates of all nodal points
(2) Calculate external loads at every time interval
(3) Determine the riser balance position;
(4) Formulate the total damping matrix and total mass matrix;
(5) Calculate the tangential stiffness matrix and formulate the total dynamic stiffness matrix;
(6) Calculate the effective increment load;
(7) Solve the stiffness equation to obtain the incremental displacement;
(8) Obtain the incremental velocity and acceleration;
(9) Obtain the displacement, velocity, and acceleration at each time interval.

Considering that the cable model is based on the FVM, the stiffness matrix at every time instant is the tangent stiffness matrix of the riser, which is dependent on the strain energy. As the flexible riser deforms, the strain is time-varying. So, the tangent stiffness matrix continues to change during the vibration. Hence, at each time interval, the dynamic stiffness matrix needs to be updated in the order (5).of the above numerical calculation procedure. According to the theory the numerical procedure for the dynamic stiffness matrix is as follows:

(1) Formulate the internal nodal force vectors of all cable elements;
(2) Formulate the tangent stiffness matrices of all cable elements;
(3) Calculate the unbalanced force of all nodes;
(4) Determine the initial positions of all nodes;
(5) Obtain the incremental nodal coordinates using the nonlinear equation of strain energy;
(6) Update the new coordinates of all nodes;
(7) Check convergence of each iteration;

If convergence of the iteration cannot be achieved, go to step (1); when all nodal points satisfy $\beta \leq \beta_d$,(precision value) terminate the program.

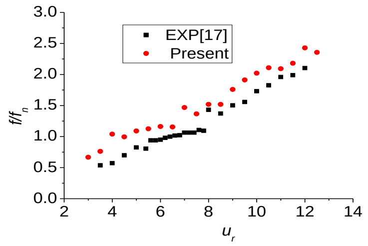
Verification of the numerical method

To further validate the accuracy and the reliability of the IVCBC method in simulating the VIV of a marine riser, a single cylinder with elastic supports and in 2-D uniform flow was investigated. The parameters in Table 1 come from the experiment by [17]. M^* is the mass ratio, ζM^* is the damping ratio, D is the cylinder diameter, and F_n is the fund a mental frequency. It can be seen that high Re falls in the range from 12500 to 66700.

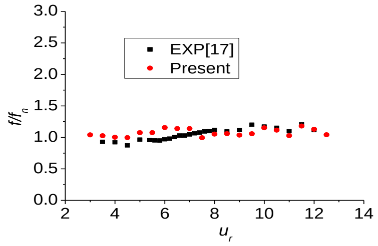
Table1.Parameters defining the single cylinder

M^*	ζM^*	D(m)	R_e	$F_n(H_z)$
10.63	0.053	0.0554	12500-66700	1.084

Fig.1 shows the vortex shedding frequencies and vibration frequencies computed by the IVCBC vortex method and those measured by [17] respectively. The calculated results are in good agreement with the experimental results. Therefore, the IVCBC vortex method is an effective approach to simulate the VIV of a cylindrical structure in 2-D flow.



(a) Vortex shedding frequency.



(b) Vibration frequency.

Fig.1.Non-dimensional shedding frequencies and vibration frequencies for cylinder in 2-D flow.

The influence of the discretized number (50, 80, 100) of cable elements of a marine riser on the 3-D numerical results was investigated. In order to analyze the discretized numbers of the different length risers, the length of the riser was elected to be 3000*m*. The static analysis of the flexible riser was performed under a uniform current speed of 0.3 *m/s*. Draft of platform is 144 *m*, under water length is 2300 *m*, the elastic modulus is 2.07×1011 P_a, the density of water is 1025 kg/m^3, the internal fluid density is 865 kg/m^3, the density of riser is 7850 kg/m^3, the kinetic viscosity is 1.3×106 m^2/s, the inner diameter is 0.3206 *m*, and the outer diameter is 0.3556 *m*.

The results of the equilibrium positions for the three discretized numbers are very close to each other, as shown in Fig.2. This indicates that numerical convergence has been reached. A discretized number of 80 suffice the discretization requirement in 3-D numerical calculations.

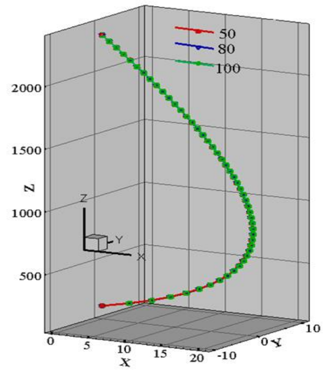


Fig.2.Equilibrium positions of a riser with different numbers of cable elements.

In many previous studies, the riser is modeled as a beam. To compare the cable model with the beam model, the classical VIV experiment carried out by [18] is taken as an example. The main parameters for the experiment are: the length of the riser is 13.12 *m*, of which 6.5 *m* is in water; the riser's outer diameter is 0.028 *m*; the top tension is 939 *N;* the mass ratio of the riser is 3.0; its submerged weight is 12.1 *N/m*; its bending stiffness is 29.9 $N.m^2$; the structural damping measured in free decay tests in air is 0.33%; and the flow speed is 0.85*m/s*. Fig.3 gives the typical instantaneous mean deflected shapes of the riser by[18], the beam model in the [19], and the present cable model. All of them are comparable.

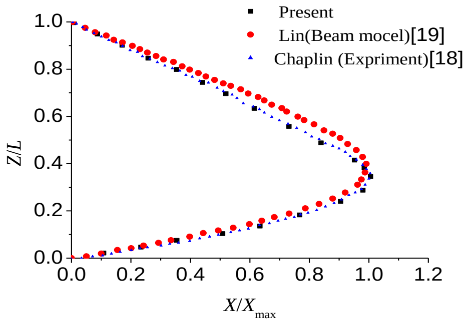
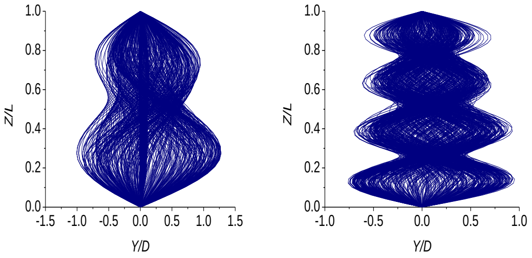


Fig.3.The static mean deflected shapes of the riser(Z is the position of the segments).

Numerical results

In order to test the computational accuracy of the present model, a classical experiment in the [20] was selected for comparison. Using by an ordinary personal computer (PC), it took 5 hours 35 minutes and 28 seconds to complete the calculation with the present numerical method. Length of riser is 9.63 *m*, under water length is 9.63 *m*, elastic modulus is $9.\times10^{10}$ P_a, density of water is 1000 kg/m^3, internal fluid density 1000 kg/m^3, density of riser is 8900 kg/m^3, Inner diameter is 0.02 *m*, Outer diameter is 0.0101 *m*. Top tension is 817 *N*, and the impact of the bottom is ignored.

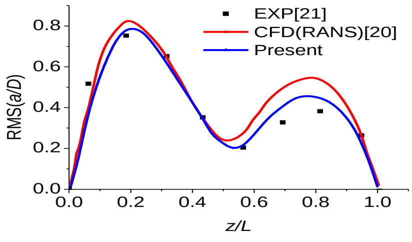
Fig.4.shows the vibration modals with respect to U=0.42 *m/s* and 0.84 *m/s*. It can be seen that the main vibration under U=0.42 *m/s* is the second order modal and the main vibration under U=0.84 *m/s* is the forth-order modal.



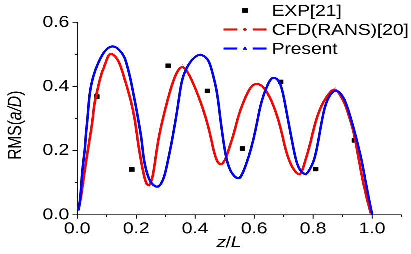
U=0.42*m/s* U=0.84*m/s*

Fig.4.Vibration modes with respect to U=0.42*m/s* and 0.84 *m/s*.(Y is vibration displacement in CF)

Fig.5 shows the root mean square (RMS) of riser VIV at CF. For comparison, the numerical results in the [21] calculated by the Renolds Average Navier-Stoks (RANS) method and the classical experimental results[20] are shown in Fig.5. The trend for riser RMS calculated by the present method is consistent with that of the classical experiment, and these results agree with the results computed by the RANS method. Fig.6 clearly shows the vibration pattern and vortex contours.

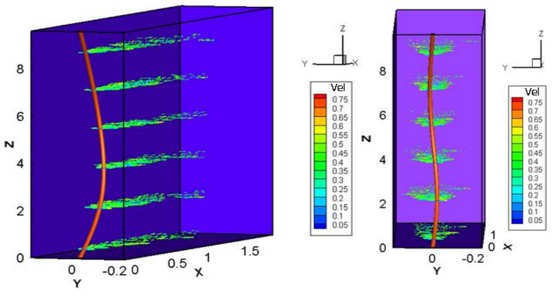


U=0.42*m/s*

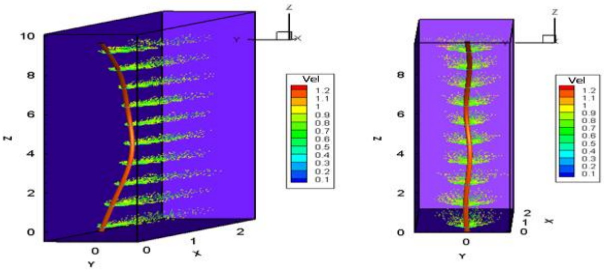


U=0.84*m/s*

Fig.5. Root mean square of vortex-induced vibration in the cross flow(a is amplitude of vibration)



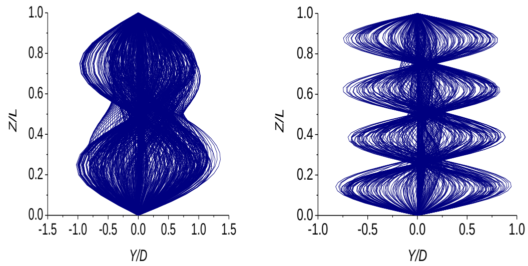
U=0.42*m/s*



U=0.84*m/s*

Fig.6.Vibration of validation model (t=100).

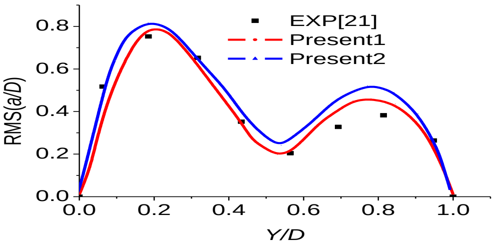
With constant coefficient stiffness, Fig.7 shows the vibration modals of the riser for U=0.42*m/s* and U=0.84 *m/s*. When compared with Fig.4, we can see that the vibration modals have changed slightly for U=0.42*m/s*; however, the vibration modals for U=0.84*m/s* have changed significantly.



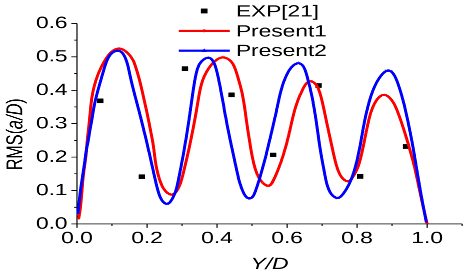
U=0.42*m/s* U=0.84*m/s*

Fig.7.Vibration modes with respect to U=0.42*m/s* and 0.84*m/s* with constant coefficient stiffness.

Fig.8 shows the cross flow VIV RMS comparison between the simulation results and the experimental data. It can be seen that the trend calculated by the dynamic stiffness matrix is the same as that calculated by the constant coefficient stiffness matrix when the velocity is 0.42*m/s*. This indicates that the deformation of the riser has a very small effect for the stiffness matrix. When the velocity is equal to 0.84*m/s*, the RMS trends between the constant coefficient stiffness matrix and dynamic stiffness matrix change significantly. This indicates that the deformation of the riser begins to affect the stiffness matrix. Thus, the influence of deformation of the riser cannot be ignored in high modes of vibration.



U=0.42*m/s*



U=0.84*m/s*

Fig.8.Root mean square of vortex-induced vibration in the cross flow (Present1 (dynamic stiffness matrix), Present2 (constant coefficient stiffness matrix)).

Conclusions

A new numerical method with high efficiency in time domain introducing the dynamic stiffness matrix and the bending stiffness to study the VIV of a flexible riser is proposed. By analyzing and comparing the calculation results of dynamic stiffness matrix and constant coefficient stiffness matrix, the results adopting the dynamic stiffness matrix agree very well with the experimental results. The result shows that when the vibration of riser is high order mode vibration, the dynamic stiffness matrix cannot be ignored. All of the above indicates that the present method has strong applicability for VIV research.

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