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Hydrodynamics around two identical circular cylinders in tandem arrangements at different flow attack angles

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Abstract

Numerical simulations are conducted to investigate the effects of the flow attacking angles on the hydrodynamic forces on two smooth circular cylinders in a tandem arrangement in a fluid flow. Simulations are conducted at different gap to diameter ratios and a constant Reynolds number (based on the velocity perpendicular to the cylinder) of 500. The three-dimensional incompressible Navier-Stokes equations are solved using Petrov-Galerkin Finite Element Method to simulate the flow. The aim of this study is to investigate the effects of the flow attacking angle on the wake of the circular cylinders for two gap ratios (G/D) of 1 and 3 and its influences on the vortex shedding patterns. G/D = 1 and 3 are chosen because the wake flow at these two gap ratios are in the reattachment and co-shedding regimes, respectively. Very weak three-dimensional (3D) flow appears in the wake of the downstream cylinder at G/D=1. Clear 3D vortices are shed from both upstream and downstream cylinders for all the simulated flow attacking angles and lowered the spanwise coherence behind the downstream cylinder due to the effects of vortex impingement and streamwise vortex structures in the approaching flow.

Themes: Flow around two circular cylinders; Tandem arrangement; direct numerical simulation; wake flow.

Introduction

Flow past multiple cylindrical structures has attracted increasing interest due to its importance in many engineering applications. Remarkable complex flow patterns for a simple geometric configuration of two tandem cylinders with a gap G between them in a fluid flow (as shown in Fig. 1) has been reported. The characteristics of the flow around the two circular cylinders depend upon the ratio between the gap G and the cylinder diameter D (G/D is defiend as the gap ratio hereafter), the flow approaching angle (α) and the Reynolds number (Re) which is defined as $Re = U_N D/\nu$, where U_N is the component of free-stream velocity U in the perpendicular direction of the cylinder and ν is the kinematic viscosity of the fluid. When two circular cylinders are arranged in tandem, the wake of the two cylinders depends significantly on the gap between the two cylinders. If the gap between the cylinders is very small, the two cylinders may behave as a single bluff body. If the gap between the cylinders is sufficiently large, they may behave as two independent bluff bodies, although the synchronization between the adjacent vortex streets may occur.

Previous studies on flow around two circular cylinders show that the classification of flow regimes depends upon the gap ratio between the cylinders [1, 5-8, 13]. The discontinuous jump of the base pressure of the downstream cylinder at gap ratio of about 2.5 were reported in [14]. Flow around two circular cylinders in a tandem arrangement in subcritical Reynolds numbers $3.3\times10^3 < Re < 12\times10^3$ at various gap ratios have been studied in the past by using smoke-wire technique [6]. It was reported that the change in the flow mode due to the change in the gap ratio has a significant effect on the Strouhal number. Alam et al. [1] carried out an experimental study of aerodynamics characteristics of two circular cylinders in a tandem arrangement at Re = 6.5×10^4 and reported a bistable flow at G/D = 3 (the critical gap), and the reattachment of shear layer on to the downstream cylinder occurred intermittently at $G/D < 2$. The experimental research of flow in the wake of two tandem cylinders based on hot-wire measurement and LIF flow visualization at Reynolds numbers between 800 and 4.2×10^4 and various gap ratios (G/D = 0 – 14) was conducted by Xu & Zhou [12] and it was reported that the critical gap ratio (G_{cr}/D) decreases with increasing Reynolds numbers (G_{cr}/D = 3.5 for Re = 1400, G_{cr}/D = 3 for Re = 8500 and G_{cr}/D = 2 for Re = 42000).

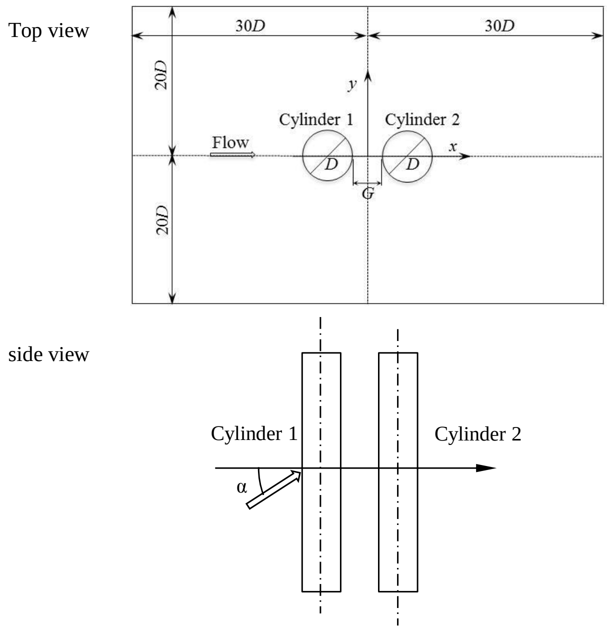

Fig.1 Sketch for flow past two cylinders in tandem arrangement

Extensive numerical studies on flow past two circular cylinders in tandem arrangements have also been reported in the literature. Meneghini et al. [7] simulated flow interference between two circular cylinders in tandem and side-by-side arrangements for Reynolds numbers of 100 and 200 using a two-dimensional fractional step finite element method and the computed interference regimes agree with the experimental results. By employing spectral finite element method to simulate 2D and 3D flow for various gap ratios from 0.5 to 7 at Reynolds numbers 160 – 320 [2], it was found that the 3D vortical structures have significant effects on the fluid loads and 2D simulations are not able to accurately determine the interference regime in the vicinity of critical spacing. Papaioannou et al. [9] studied flow around two tandem cylinders at gap ratios ranging from 0.1 to 4 and Reynolds numbers from 100 to 1000 using spectral/hp element method and revealed the 3D effects on the critical gap ratio at various Reynolds numbers. Carmo et al. [3] carried out linear stability analysis and 3D direct numerical simulations of flow past two circular cylinders in tandem arrangements at various gap ratios from 0.2 to 9 and identified the boundaries between three different vortex shedding regimes (SG: symmetric in the gap, AG: alternating in the gap and WG: wake in the gap). 3D simulations of flow past two identical circular cylinders at tandem, side-by-side and staggered arrangements at Reynolds number of 1000 and the gap ratios (*G*/*D*) from 0.5 to 3 were performed by Tong et al. [11]. For a tandem arrangement, small drag coefficients were reported due to complete or partial immerse of downstream cylinder in between two shear layer from the upstream cylinder.

Most of previous studies of flow past two tandem cylinders focused on cases where the flow direction is perpendicular to the cylinders. In many engineering applications, the flow may approach the cylinder at an oblique attack angle. It is expected that the inclination angle of the flow affects the vortex shedding flow patterns [4, 10]. The focus of this study is to investigate the effect of flow attack angle on the formation vortex shedding and the force coefficients for flow past two tandem cylinders by means of 3D direct numerical simulations at a low subcritical Reynolds number (*Re* = 500) in two gap ratios (*G*/*D* = 1 and 3), which are in the re-attachment and co-shedding regimes, respectively.

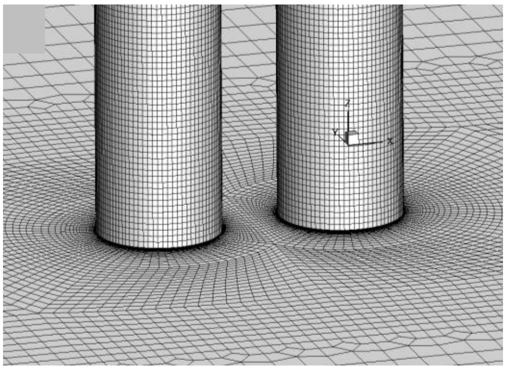

Fig. 2 Computational mesh used for simulation

Methodology

The three-dimensional incompressible Navier-Stokes (NS) equations are solved by the Petrov-Galerkin finite element method developed by Zhao et al. [15]. The velocity, time and pressure are non-dimensionalized by $u_i = u_i'/U_N$, $t = U_N t'/D$, $p = p'/(\rho U_N^2)$, respectively, where a prime stands for dimensional values, u_i is fluid velocity component in x_i-direction, $(x_1, x_2, x_3) = (x, y, z)$ are Cartesian co-ordinates, *t* is time and *p* is pressure. The non-dimensional NS equations are written as

$$\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} + \frac{\partial p}{\partial x_i} - \frac{1}{\mathrm{Re}} \frac{\partial^2 u_i}{\partial x^2} = 0, \qquad (1)$$

$$\frac{\partial u_i}{\partial x_i} = 0, \qquad (2)$$

The flow past circular cylinder(s) at oblique attacks for various Reynolds numbers have been successfully simulated using this numerical model [10, 16]. The non-dimensional rectangular computational domain is 60×40 in the flow direction and cross-flow direction, respectively, and the length of the cylinder is kept at a value of 9.6. Initially, the velocity and pressure in the whole domain are set to zero. The non-dimensional velocity at the inlet boundary is given by

$$(u_1, u_2, u_3) = (1, 0, \tan\alpha) \qquad (3)$$

We employed free slip boundary condition at the two side boundaries which are parallel to *x*-*z* plane where the velocity component and the pressure gradient perpendicular to the boundary are zero. When the flow approaches the cylinders at an inclined angle, fluid mass needs to be allowed to pass through the two end boundaries (the boundaries perpendicular to the cylinder axis). A periodic boundary condition is imposed to allow the flow to pass through the two end boundaries by setting velocity and pressure gradients in all three directions on one of the end boundary to be equal to their counterparts on the other boundary. The periodic boundary condition at the two end boundaries allow the fluid flowing through the boundary. On the cylinder surfaces, no-slip condition is applied.

According to previous studies, the flow for *G*/*D*=3 is in the co-shedding regime [9]. In numerical simulation with a symmetric initial condition and uniform flow, it was found that co-shedding vortex shedding does not occur until *t* exceeds 300. To trigger vortex shedding from the downstream cylinder and save computational time, the two cylinders are rotated with a tangential speed at the cylinder surface described by $u_T = \sin(2\pi t/10)$ at initial duration of *t*<10.

Results and discussions

The focus of this study is to investigate the effect of flow approach angles at various gap ratios at a low Reynolds number in the subcritical flow regime. The numerical simulation for each case was carried out for at least a non-dimensional time of *t*=500 for all cases to ensure a fully developed wake flow have been obtained. The flow attacking angles were 0º, 15º, 30º and 45º for gap ratios of 1 and 3. The mesh created to simulate the flow is shown in Fig. 2, where 96 elements were distributed along the circumference of each cylinder and 192 layers of hexahedral 8-node linear finite elements were assigned along the cylinder’s 9.6-long span. The minimum radial mesh size on the cylinder surface was 0.002. Simulations were carried out using 96 Central Processing Units (CPU) with a non-dimensional computational time step Δ*t* of 0.003. Pawsey Supercomputing Centre supercomputing facility in Western Australia was used to perform all the simulations for this study. The mesh density used in this study is same as that used in Thapa et al. [10], where a mesh dependency study was conducted for Re =500.

The time series of lift coefficient of upstream and downstream cylinders are shown in Fig. 3 for two gap ratios and four different flow attack angles. Drag and lift coefficients are defined as $C_D = F_D/(\rho D L U_N^2/2)$ and $C_L = F_L/(\rho D L U_N^2/2)$, where F_D and F_L are the drag and lift forces, respectively. At a small gap ratio of 1 as shown in Fig. 3 (a & b), the amplitude of lift coefficient of cylinder 1 (upstream cylinder) is smaller than that of cylinder 2 (downstream cylinder) for all flow attack angles. For *G*/*D*=1, vortex shedding occurs quickly because the cylinders rotate at the beginning of the numerical simulations. A sudden reduction of lift coefficient after about non-dimensional time of 110 corresponds to the instant when the wake flow becomes three-dimensional and vortex shedding from the upstream cylinder ceases. The amplitudes of lift coefficient for both cylinders at *α* = 45º are noticed to be small. At co-shedding regime as shown in Fig. 3 (c & d), the fluctuating amplitude of lift coefficients increases significantly compared to Fig. 3 (a & b) due to the existence of strong vortex shedding from the both cylinders.

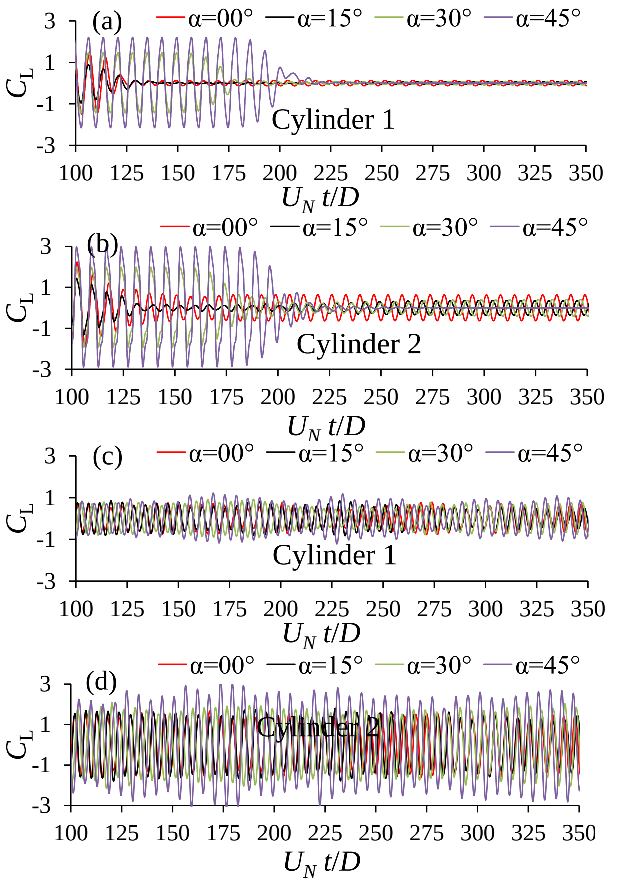

Fig. 3 Time histories of lift coefficient for Re = 500 at two gap ratios and four different flow attack angles; a & b correspond to G/D = 1 and c & d correspond to G/D = 3

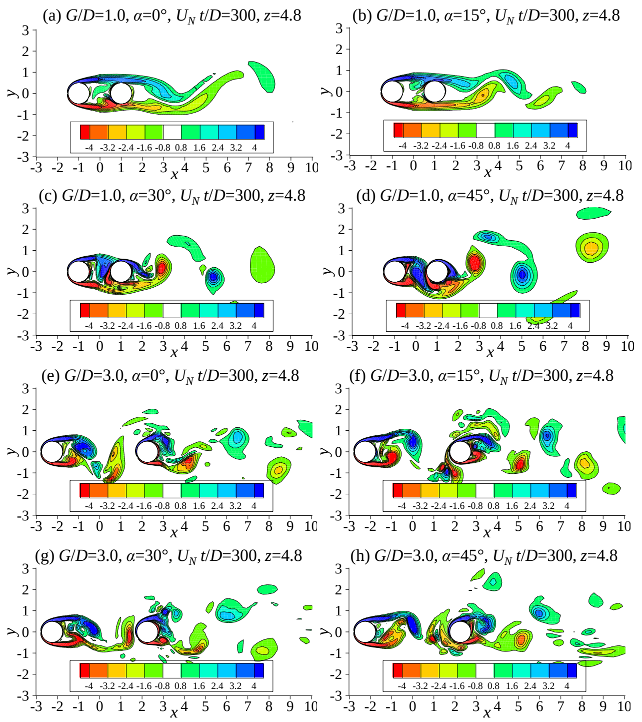

Fig. 4 Contour of axial vorticity at midsection of cylinders for G/D = 1.0 and 3.0 at Re=500

Fig. 4 shows the contours of the axial vorticity at the mid-section of the cylinders for both gap ratios and four different flow attacking angles. Regular vortex shedding as shown in Fig. 4 (a & b) appeared far away from cylinder 2 for α = 0º and 15º, leading to weak oscillation of lift. As the attacking angle increases, the regular vortex shedding beyond cylinder 2 becomes weak and the fluctuating lift is noticed to be further decreased. The reattachment of shear layers after separating from cylinder 1, started to occur at the front surface of cylinder 2 and the development of a vortex pair in the gap between the cylinders are noticed (Fig. 4 c & d). When the gap ratio between the cylinders is G/D = 3, vortex shedding started to occur from both cylinders, but the strength of vortices beyond cylinder 2 is found reduced compared to the vortices in re-attachment regime. It is observed that the vortices that are shed from both cylinders have the same frequency and are in in-phase as noticed from time series analysis.

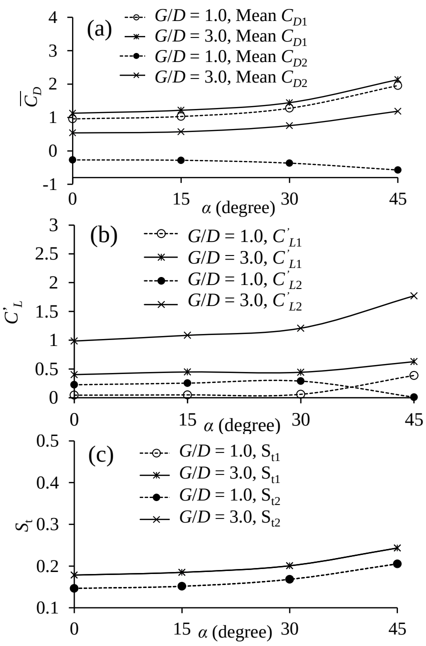

Fig. 5 Mean drag, RMS lift coefficients and Strouhal number at four different attack angles (α degree) experimental and numerical results

Fig. 5 shows the calculated mean drag coefficients, RMS lift coefficients and Strouhal numbers of both cylinders at four different flow attack angles. As gap ratio is increased from 1 to 3, a change of mean drag coefficient of cylinder 2 from negative to positive indicates the formation of vortex shedding from upstream cylinder. The very small values of RMS lift of cylinder 1 at G/D=1 as shown in Fig. 5 (b) indicate a lack of formation of vortex shedding up to α = 30º for small gap ratio, whereas vortices start to be shed when RMS lift coefficient increases from 0.00601 to 0.387 when α increases from 30º to 45º. But RMS lift of cylinder 2 at α = 45º decreases to almost zero due to a inclined wake structure shown in Fig. 6 (a), where the iso-surface eigenvalue of the velocity tensor λ_2=-0.25 are presented. This eigenvalue can be used to identify the vortices [17]. These inclined vortices in the wake of the cylinder can be clearly seen for α=45° but not for α=15° and 30°. The inclined vortex flow structure in the wake of the cylinder for G/D=1 and α=45° leads to a variation of the phase of lift coefficient as shown in Fig. 7, where the contours of sectional lift coefficient on t-z plane are presented. The phase variation of sectional lift coefficient leads to a very small lift coefficient on the whole cylinder. When gap ratio increases to 3, vortices are clearly shed from both cylinders with slightly increased RMS lift until α = 30º. At α = 45º, RMS lift from cylinder 2 increased abruptly, showing weak vortices from cylinder 2. Fig. 5 (c) shows the calculated Strouhal numbers at two gap ratios and four different flow attacking angles.

Strouhal numbers, defined as $S_t = fD/U\cos\alpha$, from both cylinders are the same for both gap ratios, but greater Strouhal number is noticed for a large gap ratio compared to that for a small gap ratio, indicated the clear effect of gap ratios on the wake flow. The slightly increased value of S_{t1} and S_{t2} at both gap ratios for all flow attack angles offers a further proof of attack angles' effect in the wake .

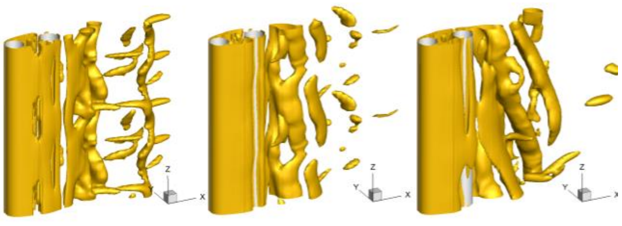

Fig. 6. Iso-surface of the eigenvalue of the vortex

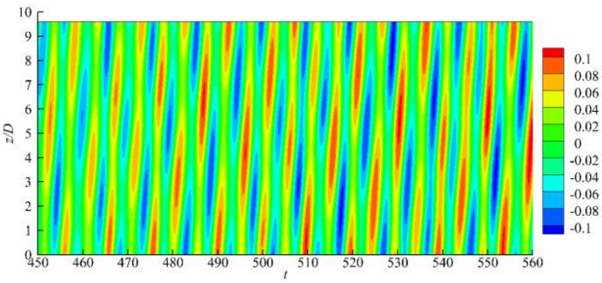
Fig. 7 Contours of the sectional lift coefficient of the downstream cylinder on the *t-z* plane

Conclusions

Flow past two cylinders in tandem has been simulated numerically at constant Reynolds number of 500. The effects of gap ratio and flow attack angle on mean drag coefficient and vortex shedding modes have been examined numerically at re-attachment and co-shedding regimes. The conclusions are summarized as follows:

The influence of the flow attack angle on the vortex flow structures has been clearly noticed. For a gap to diameter ratio of 1, vortices are not shed from the upstream cylinder at $\alpha = 0^o$ and 15^o. As the flow attack angle reaches 30^o and 45^o, vortices start to be shed from the upstream cylinder but they are very weak. In the re-attachment regime, the oscillation amplitude of the lift coefficient of the upstream cylinder is much smaller than that of the downstream cylinder for all flow attack angles. An inclined wake flow pattern is observed when G/D=1 and α=45°. Vortices are shed from both cylinders for G/D = 3 independent of flow attack angle. Both the mean drag and RMS lift coefficients were found to increase with increasing flow attack angle.

Acknowledgments

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